# The Nuclear Equation of State and the neutron skin thickness in nuclei

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Physics beyond the standard model and precision nucleon structure measurements with parity-violating electron scattering
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# INTRODUCTION

# The Nuclear Many-Body Problem:

- → **Nucleus:** from few to more than 200 strongly interacting and **self-bound fermions**.
- → **Underlying interaction** is **not perturbative** at the (low)energies of interest for the study of masses, radii, deformation, giant resonances,...
- → **Complex systems**: **spin, isospin, pairing, deformation,** ...
- → **Many-body** calculations based on **NN scattering data** in the vacuum are **not conclusive** yet:
  - → **different predictions** (interaction in the medium) are found **depending** on the **approach**
  - → EoS and (recently) **few groups** in the world are able to perform extensive calculations for **light and medium mass nuclei**
- → Based on effective interactions, **Nuclear Energy Density Functionals** are **successful** in the description of **masses, nuclear sizes, deformations, Giant Resonances,...**

**Nuclear energy density functionals $E[\rho]$ are commonly derived from an effective Hamiltonian solved at first order perturbation theory (Hartree-Fock)**

$$E = \langle \Psi | H | \Psi \rangle \approx \langle \Phi | H_{\text{eff}}(\rho) | \Phi \rangle \xrightarrow{\text{Exact EDF?}} E[\rho]$$

## Kohn-Sham iterative scheme (static)

→ Determine/copy/invent/derive... $E[\rho]$

→ Initial guess $\rho_0$

→ Calculate potential $V_{\text{eff}}$ from $\rho_0$ $(h = \delta E / \delta \rho)$

→ Solve single particle equation of motion $(h\phi_i = \epsilon_i \phi_i)$ $\phi_i$

→ Use $\phi_i$ for calculating new $\rho_1 = \sum_i^A |\phi_i|^2$

**Repeat until consistency between $\rho$ and $V_{\text{eff}}$**

## Runge-Gross Theorem (dynamic): exist also $E[\rho(t), t]$

$$\int dt \{ \langle \Phi(t) | i\partial_t | \Phi(t) \rangle - E[\rho(t), t] \} = 0$$

**Useful for the study of small perturbations of the gs $\rho$: GR**

# Nuclear Energy Density Functionals:

**Main types of successful EDFs for the description of masses, deformations, nuclear distributions, Giant Resonances, ...**

**Relativistic mean-field models,** based on Lagrangians where effective mesons carry the interaction:

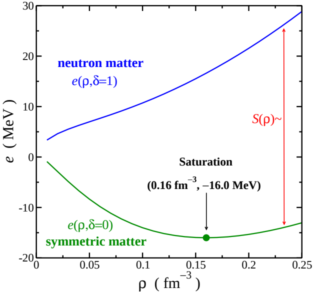
$$\begin{aligned}\mathcal{L}_{int} &= \bar{\Psi}\Gamma_\sigma\Psi\Phi_\sigma + \bar{\Psi}\Gamma_\delta\boldsymbol{\tau}\Psi\Phi_\delta - \bar{\Psi}\Gamma_\omega\gamma_\mu\Psi A^{(\omega)\mu} \\ &- \bar{\Psi}\Gamma_\rho\gamma_\mu\tau\Psi A^{(\rho)\mu} - e\bar{\Psi}\hat{Q}\gamma_\mu\Psi A^{(\gamma)\mu}\end{aligned}$$

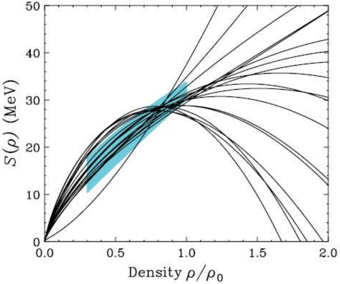
**Non-relativistic mean-field models,** based on Hamiltonians where effective interactions are proposed and tested:

$$V^{\mathrm{eff}}_{\mathrm{Nucl}} = V^{\mathrm{long-range}}_{\mathrm{attractive}} + V^{\mathrm{short-range}}_{\mathrm{repulsive}} + V_{\mathrm{SO}} + V_{\mathrm{pair}}$$

- $\rightarrow$ Fitted **parameters contain** (important) **correlations beyond the mean-field**
- $\rightarrow$ Nuclear energy functionals are **phenomenological** $\rightarrow$ **not directly connected to any NN** (or NNN) **interaction**

# The Nuclear Equation of State: Infinite System



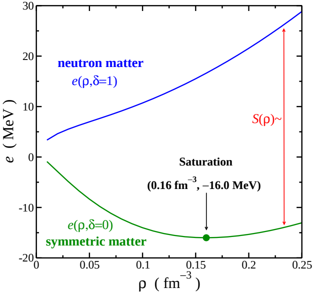



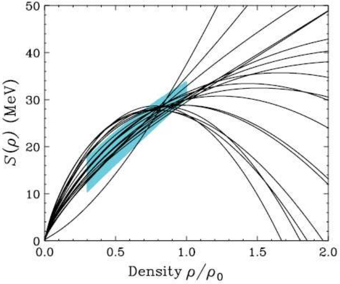

$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

→ Nuclear Matter

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

# The Nuclear Equation of State: Infinite System



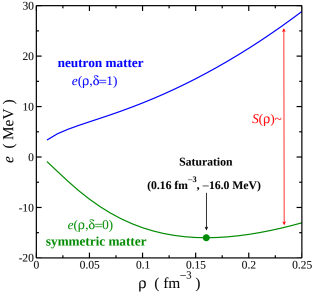

$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

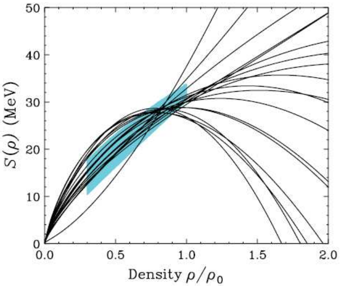
→ Nuclear Matter

→ Symmetric Matter

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

# The Nuclear Equation of State: Infinite System




$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

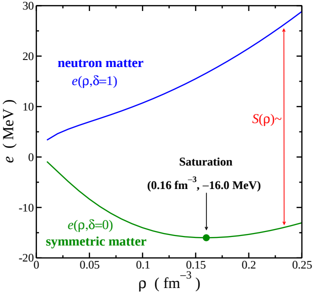
→ Nuclear Matter

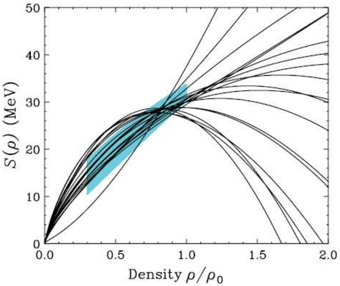
→ Symmetric Matter

→ Symmetry energy

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

# The Nuclear Equation of State: Infinite System



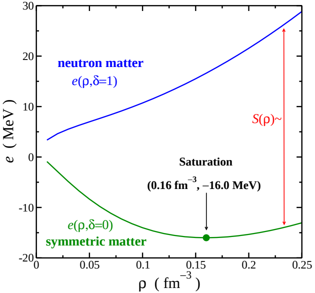



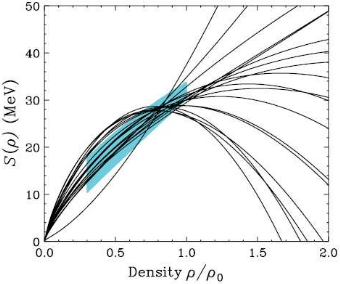

$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

$$= \frac{E}{A}(\rho,\beta=0) + \beta^2\left(J + Lx + \frac{1}{2}K_{sym}x^2 + \mathcal{O}(x^3)\right)$$

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}; \quad x = \frac{\rho - \rho_0}{3\rho_0}\right]$$

# The Nuclear Equation of State: Infinite System



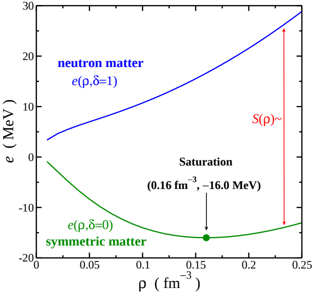



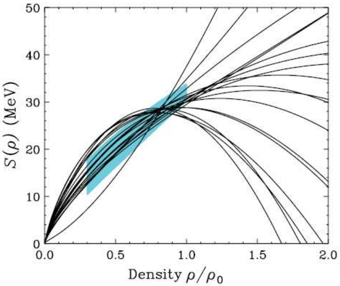

$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + \beta^2\left(J + L\,x + \frac{1}{2}K_{sym}\,x^2 + \mathcal{O}(x^3)\right)$$

$\rightarrow\ S(\rho_0) = J$ $\quad\rightarrow\ \frac{d}{d\rho}S(\rho)\Big|_{\rho_0} = \frac{L}{3\rho_0} = \frac{P_0}{\rho_0^2}$ $\quad\rightarrow\ \frac{d^2}{d\rho^2}S(\rho)\Big|_{\rho_0} = \frac{K_{sym}}{9\rho_0^2}$

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho};\quad x = \frac{\rho - \rho_0}{3\rho_0}\right]$$

# The Nuclear Equation of State: Infinite System




→ The uncertainties on $S(\rho)$ around saturation density (mainly due to $L$) **impact** on many nuclear physics and astrophysics **observables**.

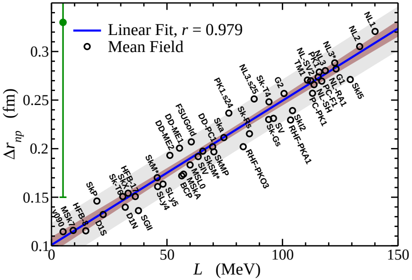
sym

$\rho_0^2$

# The symmetry energy and the neutron skin in $^{208}$Pb

$$\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$$

Simple macroscopic model: $\Delta r_{np} \sim \frac{1}{12}\frac{IR}{J} L \quad \left(L \propto p_0^{neut}\right)$



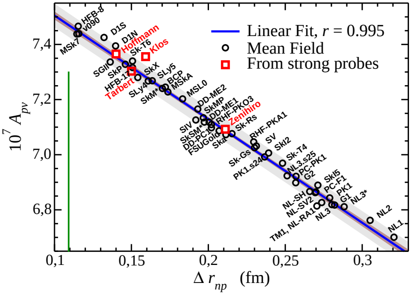

*Physical Review Letters* **106**, *252501 (2011)*

The faster the symmetry energy increases with density (L), the largest the size of the neutron skin in (heavy) nuclei.

**[Exp. from strongly interacting probes:** $\sim 0.15 - 0.22$ **fm (***Physical Review C* **86** *015803 (2012)***)].**

# The impact of the neutron skin on nuclear and astrophyiscs observables

# The neutron skin and the parity violating asymmetry in $^{208}$Pb




*Physical Review Letters* **106**, *252501 (2011)*

(Calculation at a fixed q equal to PREx)

- → Electrons interact by exchanging a $\gamma$ **(couples to** $p$**)** or a $Z_0$ **boson (couples to** $n$**)**
- → Ultra-relativistic electrons, **depending on their helicity** ($\pm$), will interact with the nucleus **seeing a slightly different potential**: **Coulomb** $\pm$ **Weak**
- → $A_{pv} \equiv \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega} \sim \frac{\text{Weak}}{\text{Coulomb}}$
- → Input for the calculation are the $\rho_p$ and $\rho_n$ **(main uncertainty)** and nucleon form factors for the e-m and the weak neutral current.
- → In PWBA for small momentum transfer:

$$A_{pv} \approx \frac{G_F q^2}{4\sqrt{2}\pi\alpha}\left(1 - \frac{q^2 \langle r_p^2 \rangle^{1/2}}{3F_p(q)} \Delta r_{np}\right)$$

The largest the size of the neutron distribution in nuclei ($\Delta r_{np}$), the smaller the parity violating asymmetry.

**[Exp. from ew probes:** $0.302 \pm 0.175$ **fm (***Physical Review C* **85**, *032501 (2012)***)].**

# Isovector Giant Resonances (some considerations)

→ In **isovector** giant resonances **neutrons and protons "oscillate" out of phase**

→ **Isovector** resonances will depend on oscillations of the density $\rho_{iv} \equiv \rho_n - \rho_p \Rightarrow S(\rho)$ will drive such "oscillations"

→ The **excitation energy ($E_x$)** within a **Harmonic Oscillator** approach is expected to depend on the symmetry energy:

$$\omega = \sqrt{\frac{1}{m}\frac{d^2U}{dx^2}} \propto \sqrt{k} \to E_x \sim \sqrt{\frac{\delta^2 e}{\delta\beta^2}} \propto \sqrt{\mathbf{S(\rho)}}$$

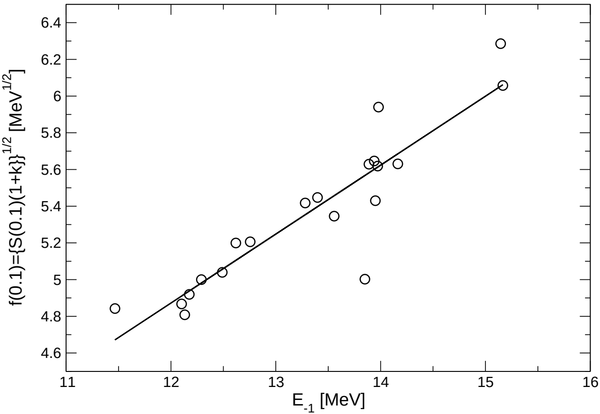
where $\beta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$

→ The **dipole polarizability ($\alpha \sim \int \frac{\sigma_{\gamma-abs}}{\text{Energy}^2} \sim$ IEWSR)** measures the tendency of the nuclear charge distribution to be distorted, that is, from a **macroscopic** point of view

$$\alpha \sim \frac{\text{electric dipole moment}}{\text{external electric field}}$$

# The neutron skin and the Giant Dipole Resonance in $^{208}$Pb

($E_x \approx f(0.1) \propto \sqrt{S(0.1\text{fm}^{-3})}$)

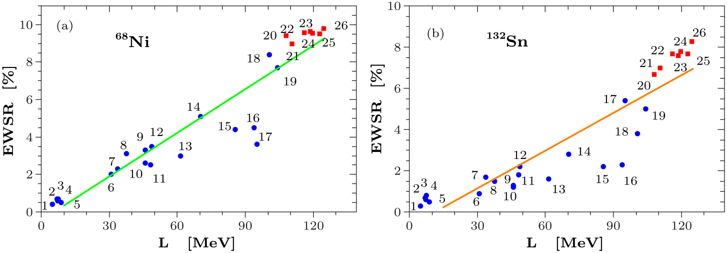



*Physical Review C* **77**, *061304 (2008)*

The larger the neutron skin of $^{208}$Pb, the faster the symmetry energy increases with density around saturation $\left[S(\rho_A) \approx J - L\frac{\rho_0 - \rho_A}{3\rho_0}\right]$, and the smaller the excitation energy of the Giant Dipole Resonance (GDR).

# The symmetry energy and the Pygmy Dipole Resonance

**(Pygmy: low-energy excited state appearing in the dipole response of N ≠ Z nuclei)**

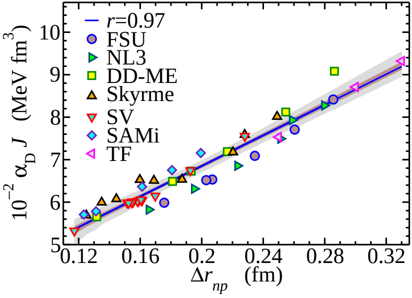



*Physical Review C* **81**, *041301 (2010)*

The larger the neutron skin in $^{208}$Pb, the faster the symmetry energy increases with density, the larger is the energy (E) times the probability (P) of exciting the Pygmy state (EWSR = E × P)

**WARNING:** we lack of a clear understanding of the physical reason for this correlation

# Dipole polarizability and the neutron skin in $^{208}$Pb




*Physical Review C* **85** *041302 (2012);* **88** *024316 (2013);* **92**, *064304 (2015)*

**Macroscopic model:**

→ Using the **dielectric theorem:** $m_{-1}$ moment can be computed from the expectation value of the Hamiltonian in the constrained (D dipole operator) ground state $\mathcal{H}' = \mathcal{H} + \lambda D$

→ Assuming the **Droplet Model** (heavy nucleus):

$$\alpha_D \approx \alpha_D^{bulk}\left[1 + \frac{1}{5}\frac{L}{J}\right] \text{ where}$$

$$\alpha_D^{bulk} \equiv \frac{\pi e^2}{54}\frac{A\langle r^2\rangle}{J} \text{ (Migdal first derived)}$$

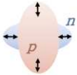
→ $$L \approx \frac{\alpha_D^{exp} - \alpha_D^{bulk}}{\alpha_D^{bulk}} 5J$$

By using the Droplet Model one can also find:

$$\alpha_D J \approx \frac{\pi e^2}{54} A\langle r^2\rangle\left[1 + \frac{5}{2}\frac{\Delta r_{np} - \Delta r_{np}^{coul} - \Delta r_{np}^{surf}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

For a fixed value of the symmetry energy at saturation, the larger the neutron skin in $^{208}$Pb, the larger the dipole polarizability.

# IV-IS GQRs and the neutron skin in $^{208}$Pb



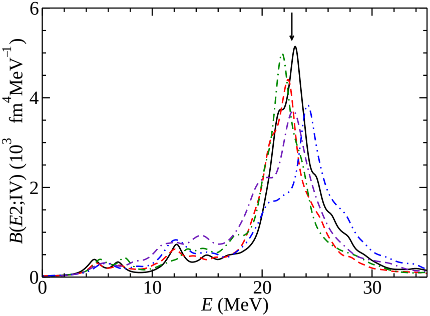

Within the Quantum Harmonic Oscillator approach

$$E_x^{IV} = 2\hbar\omega_0 \sqrt{1 + \frac{5}{4}\frac{\hbar^2}{2m}\frac{V_{sym}\langle r^2\rangle}{(\hbar\omega_0)^2\langle r^4\rangle}}$$

and EDF calculations, one can deduce

$$V_{sym} \approx 8(S(\rho_A) - S^{kin}(\rho_0))$$

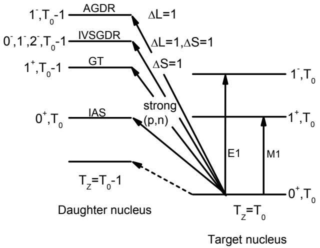
$$S^{kin}(\rho_0) \approx \varepsilon_{F_0}/3 \text{ (Non-Rel)}$$

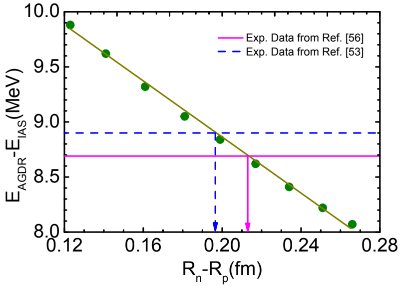



$$S(\rho_A) \approx J - L\frac{\rho_0 - \rho_A}{3\rho_0} \approx \frac{\varepsilon_{F_0}}{3}\left\{\frac{A^{2/3}}{8\varepsilon_{F_0}^2}\left[\left(E_x^{IV}\right)^2 - 2\left(E_x^{IS}\right)^2\right] + 1\right\}$$

The larger the neutron skin in $^{208}$Pb, the smallest the difference between the IS and IV excitation energies in GQRs.

# E1 transitions in CER and the neutron skin in $^{208}$Pb




*Phys. Rev. C* **92**, *034308 (2015)*

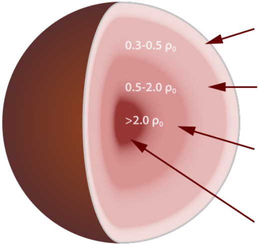
**AGDR ($\Delta J^{\pi} = 1^{-}$ with $\Delta L = 1$ and $\Delta S = 0$) is the $T_0 - 1$ component of the charge-exchange of the GDR.**

$$E_{AGDR} - E_{IAS} \approx 5\sqrt{\frac{5}{3}\frac{J}{I}\frac{1+\gamma}{\alpha_H Z}}\frac{\hbar c}{m\langle r^2\rangle^{1/2}}\left[\left(1-\frac{\varepsilon_{F_\infty}}{3J}\right)I - \frac{3}{2}\left(\frac{\Delta R_{np} - \Delta R_{np}^{surf}}{\langle r^2\rangle^{1/2}}\right) - \frac{3}{7}I_C\right]$$

$$E_{AGDR} - E_{IAS} \approx \frac{\varepsilon}{\Delta E_C}\left(E_{IVGDR} - \varepsilon\right)\frac{m_0^{AGDR}}{m_0^{IVGDR}}$$

The larger the neutron skin in $^{208}$Pb, the smaller the excitation energy of the IVGDR (as we have seen) and consistently the smaller the difference between the excitation energies of AGDR - IAS

# Relevance of the neutron star crust on the star evolution and dynamics (brief motivation)




→ The crust separates neutron star interior from the photosphere (X-ray radiation).

→ The thermal conductivity of the crust is relevant for determining the relation between observed X-ray flux and the temperature of the core.

→ Electrical resistivity of the crust might be important for the evolution of neutron star magnetic field.

→ Conductivity and resistivity depend on the structure and composition of the crust

→ Neutrino emission from the crust may significantly contribute to total neutrino losses from stellar interior (in some cooling stages).

→ A crystal lattice (solid crust) is needed for modelling pulsar glitches, enables the excitation of toroidal modes of oscillations, can suffer elastic stresses...

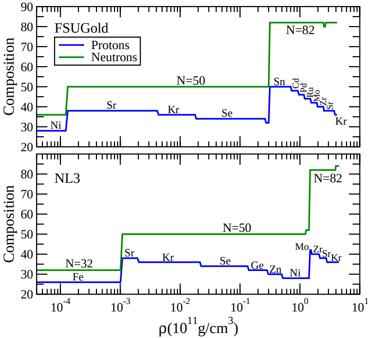
→ Mergers (binary systems that merge) may enrich the interstellar medium with heavy elements, created by a rapid neutron-capture process.

→ In accreting neutron stars, instabilities in the fusion light elements might be responsible for the phenomenon of X-ray bursts

*Source: Pawel Haensel 2001*

# The neutron skin in $^{208}$Pb and the structure and composition of a neutron star outer crust

- → span **7** orders of magnitude in **denisty** (from **ionization** $\sim 10^4$ g/cm to the **neutron drip** $\sim 10^{11}$ g/cm)
- → it is organized into a **Coulomb lattice** of neutron-rich nuclei (ions) embedded in a relativistic **uniform electron gas**
- → $T \sim 10^6$ K $\sim 0.1$ keV → one can treat **nuclei and electrons at $T = 0$ K**
- → At the **lowest densities**, the electronic contribution is negligible so the Coulomb lattice is populated by **$^{56}$Fe** nuclei.
- → As the **density increases**, the electronic contribution becomes important, it is energetically advantageous to lower its electron fraction by $e^- + (N, Z) \rightarrow (N+1, Z-1) + \nu_e$ and therefore $Z\downarrow$ with constant (approx) number of N
- → As the **density continues to increase, penalty energy from the symmetry energy** due to the neutron excess changes the composition to a dif ferent $N-$*plateau*
  $\frac{Z}{A} \approx \frac{Z_0}{A_0} - \frac{p_{F_e}}{8 a_{sym}}$ where $(A_0, Z_0) = {}^{56}\mathrm{Fe}_{26}$
- → The Coulomb lattice is made of more and more neutron-rich nuclei until the critical **neutron-drip density is reached ($\mu_{drip} = m_n$)**.
  $[M(N, Z) + m_n < M(N+1, Z)]$




*Physical Review C* **78**, *025807 (2008)*

The larger the neutron skin of $^{208}$Pb ($L\uparrow$), the more exotic the composition of the outer crust.

# Some available constraints on J and L from terrestrial experiments and astrofisical observations

B.-A. Li, X. Han / Physics Letters B 727 (2013) 276–281

**Table 1**
Constrained values of $E_{sym}(\rho_0)$ and $L(\rho_0)$ from 28 analyses of terrestrial nuclear experiments and astrophysical observations.

| Analysis | $E_{sym}(\rho_0)$ | $L(\rho_0)$ |
|---|---|---|
| Thomas–Fermi model analysis of masses (Myers 1996) | 32.65 | 50 |
| Atomic masses (Liu 2010) | $31.1 \pm 1.7$ | $66 \pm 13$ |
| Liquid drop model analysis of atomic masses (Lattimer 2012) | $29.6 \pm 3.$ | $46.6 \pm 37$ |
| FRDM analysis of atomic masses (Moller 2012) | $32.5 \pm 0.5$ | $70 \pm 15$ |
| Atomic masses and n-skin of Sn isotopes (Chen 2011) | $30.5 \pm 3$ | $52.5 \pm 20$ |
| Atomic masses and n-skin in an empirical approach (Agrawal 2012) | 32.1 | $64 \pm 5$ |
| IAS + n-skin (Danielewicz and Lee 2013) | $31.95 \pm 1.75$ | $52.5 \pm 17.5$ |
| SHF + n-skin (Chen 2010) | $30.5 \pm 5.5$ | $41 \pm 41$ |
| Droplet Model + n-skin (Centelles and Warda 2009) | $31.5 \pm 3.5$ | $55 \pm 25$ |
| IBUU04 analysis of isospin diffusion at 50 MeV/A (Chen and Li 2005) | 31.6 | $86 \pm 25$ |
| IQMD analysis of isospin diffusion at 50 MeV/A (Tsang 2009) | $32.5 \pm 2.5$ | $77.5 \pm 32.5$ |
| IQMD analysis of isospin diffusion at 35 MeV/A (Sun 2010) | 30.1 | 52 |
| Isoscaling analysis of fragments (Shetty 2007) | 31.6 | 65 |
| Global nucleon optical potential (Xu 2010) | $31.3 \pm 4.5$ | $52.7 \pm 22.5$ |
| Pygmy dipole resonances (Klimkiewicz 2007) | $32 \pm 1.8$ | $43 \pm 15$ |
| Pygmy dipole resonances (Carbone 2010) | $32 \pm 1.3$ | $65 \pm 16$ |
| AMD analysis of transverse flow (Kohley 2010) | 30.5 | 65 |
| $\alpha$-decay energy (Dong 2013) | $31.6 \pm 2.2$ | $61 \pm 22$ |
| $\beta$-decay energy (Dong 2013) | $32.3 \pm 1.3$ | $50 \pm 15$ |
| Mass differences and n-skin (Zhang 2013) | $32.3 \pm 1.0$ | $45.2 \pm 10$ |
| Dipole polarizability of $^{208}$Pb (Tamii 2013) | $30.9 \pm 1.5$ | $46 \pm 15$ |
| r-mode instability of neutron stars (Vidana 2012) | $30. \pm 5$ | $\geqslant 50$ |
| r-mode instability of neutron stars (Wen 2012) | $32.5 \pm 7.5$ | $\leqslant 65$ |
| Mass-radius of neutron stars-analysis 1 (Steiner 2010) | $31 \pm 3$ | $50 \pm 10$ |
| Mass-radius of neutron stars-analysis 2 (Steiner 2012) | $33 \pm 1.6$ | $46 \pm 10$ |
| Torsional crust oscillation of neutron stars (Gearheart 2011) | $32.5 \pm 7.5$ | $\leqslant 50$ |
| Torsional crust oscillation of neutron stars (Sotani 2012) | $32.5 \pm 7.5$ | $115 \pm 15$ |
| Binding energy of neutron stars (Newton 2009) | $32.5 \pm 7.5$ | $\leqslant 70$ |

# CONCLUSIONS

# Conclusions: EoS around saturation

- → The **isovector channel** of the nuclear effective interaction is **not well constrained by current experimental information.**
- → Many **observables available in current laboratories** are sensitive to the symmetry energy. **Problems**: **accuracy and model dependent analysis**. **Systematic experiments** may help.
- → **Exotic nuclei more sensitive** to the isovector properties (due to larger neutron excess). **Problems**: more **difficult to measure, accuracy and model dependent analysis**. **Systematic experiments** may help.
- → The most promising observables to constraint the symmetry energy are the **neutron skin thickness** and the dipole polarizability in **medium and heavy nuclei**.

# Thank you for your attention!