
השמים מספרים כבוד אל
ומעשה ידיו מגיד הרקיע

T=0 interaction - Is it Pairing?

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ECT Trento

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My Happy History of N-P Pairing at Trento

On my first visit to Trento, Alan Goodman patiently explained N-P pairing to me

On my second visit to Trento, Piet van Isacker and I started working on a model to test approximate calculational methods for N-P pairing. This turned out nicely and I will discuss some of those results.

On this, my third visit -let's see what happens

Outline

- I) Generalized Pairing Interaction
  - A-Hamiltonian
  - B-Trial Wavefunction
- II. Successes of the Model
  - A - Provides a simple explanation for the Wigner Energy Anomaly
  - B - Explains low-lying 1^+ states in odd-odd N=Z nuclei
  - C - Provides simple explanation for difference between Woods-Saxon shell spacings and spacings obtained from measured binding energies
- III Variational Configuration Interaction Method
  - A - Explanation of the Method

- B - Comparison with Exact Solutions
  - 1 - Like Particle Pairing
  - 2 - N-P Pairing

- IV. Failures of the Model
  - A - Fails to explain weak N-P Pair Transfer Amplitude to 1^+ states in odd-odd N=Z nuclei
  - B -Failure to explain low-lying high J -state in odd-odd N=Z nuclei; e.g. in ^{42}Sc, there is a 7^+ degenerate with the 1^+ state

- V. Attempts to Improve the Model
  - A - Larger Diagonal Matrix Elements
  - B - Adding Deformation to the Interaction

- VI. Conclusions

N-P Pairing

$$H = \sum_{k>0} \varepsilon_k \left(a_k^\dagger a_k + a_{-k}^\dagger a_{-k} + b_k^\dagger b_k + b_{-k}^\dagger b_{-k} \right)$$
$$- \sum_{i,j} G_{i,j}^{T=1} \left[A_i^\dagger A_j + B_i^\dagger B_j + C_i^\dagger C_j \right]$$
$$- \sum_{i,j} G_{i,j}^{T=0} \left[D_i^\dagger D_j + (M_i^\dagger M_j + N_i^\dagger N_j) \delta(\Omega_{i,j}) \right] \quad (1)$$

indices i, j and k denote values of J_z for spherical nuclei or the projection of angular momentum on the nuclear symmetry axis for deformed nuclei Ω_k

$a_k^\dagger \left(b_k^\dagger \right)$ denotes a neutron (proton) creation operator;

T=1 Pair Operators

$$A_i{}^\dagger = (a_i^\dagger a_{-i}^\dagger)$$

$$B_i{}^\dagger = (b_i^\dagger b_{-i}^\dagger)$$

$$C_i{}^\dagger = \frac{1}{\sqrt{2}} \left[a_i^\dagger b_{-i}^\dagger - a_{-i}^\dagger b_i^\dagger \right]$$

T=0 Pair Operators

$$D_i^\dagger = \frac{i}{\sqrt{2}}\left[a_i^\dagger b_{-i}^\dagger + a_{-i}^\dagger b_i^\dagger\right].$$

$$M_i^\dagger = i(a_i^\dagger b_i^\dagger)$$

$$N_i^\dagger = i(a_{-i}^\dagger b_{-i}^\dagger)$$

$M_i^\dagger$ and $N_i^\dagger$ terms do not usually lead to collective correlations, because Ω_i (jz_i) changes from level to level but their diagonal matrix elements are important.

Our variational wave-function is a product form

$$\Theta \;=\; \mathcal{P}\prod^{k}\Psi_k\prod^{m}\Phi_m|0\rangle \tag{2}$$

where the index k runs over the unblocked orbitals, the index m runs over all blocked orbitals (odd number of nucleons), and $|0\rangle$ is the physical vacuum. Each level accommodates up to two neutrons and two protons,$\pm j_z$

For unblocked orbitals, Ψ_k is given by

$$\begin{aligned}\Psi_k \;=\; &\Big[1 + U(1,k)A_k^\dagger + U(2,k)B_k^\dagger \\ &+U(3,k)C_k^\dagger + U(4,k)D_k^\dagger + U(5,k)W_k^\dagger\Big]\,,\end{aligned} \tag{3}$$

where $U(i,k)$ are variational amplitudes. $W_k^\dagger$ denotes the configuration in which level k is occupied by two neutrons and two protons. We give it an independent amplitude.The ordering of creation operators in $W_k^\dagger$ is $A_k^\dagger B_k^\dagger$.

For blocked orbitals, Φ_m is given by

$$\begin{aligned}\Phi_m \;=\; &\Big[T(1,m)a_m^\dagger + T(2,m)b_m^\dagger \\ &+T(3,m)A_m^\dagger b_m^\dagger + T(4,m)a_m^\dagger B_m^\dagger\Big]\,,\end{aligned} \tag{4}$$

Here $\mathcal{P}$ the projection operator projects neutron number, proton number and the number parity of T=1 n-p pairs, which is conserved by the interaction. Projection is done before the variation.

Consider a two level system - we want solutions with 2 neutrons 2 protons and T=1 even and odd number parity configurations

$$\Theta = (1 + A_1^\dagger + B_1^\dagger + C_1^\dagger + D_1^\dagger + W_1^\dagger)(1 + A_2^\dagger + B_2^\dagger + C_2^\dagger + D_2^\dagger + W_2^\dagger)$$

Particle Number Projection

- Even Number of T=1 Configurations

  $$A_1^\dagger B_2^\dagger + A_2^\dagger B_1^\dagger + C_1^\dagger C_2^\dagger + D_1^\dagger D_2^\dagger + W_1^\dagger + W_2^\dagger$$

- Odd Number of T=1 configurations

  $$C_1^\dagger D_2^\dagger + D_1^\dagger C_2^\dagger$$

What is the point of a product wavefunction?

A product form gives wavefunctions with many configurations using very few amplitudes!

A particle number projected BCS wavefunction describes (L!/(P!(L-P)!) different configurations with just L parameters; where L is the number of doubly degenerate levels and P is the number of pairs.

$$\Psi = \prod^{k}(1 + U_k A_k^{\dagger})|0\rangle$$

In a typical calculation, one deals with 30 levels and 15 pairs. In this case, $1.5 * 10^8$ independent configurations are described by 30 parameters.

Product wavefunctions for n-p pairing

For the n-p pairing Hamiltonian considered here, again 30 levels with 15 proton and 15 neutron pairs are typical numbers. In this case, the number of variational amplitudes is 150.

There are $1.745 * 10^{21}$ number projected configurations

Unlike shell model calculations, the running time when using product wavefunctions has a weak dependence on the number of nucleons in the system and depends mostly on the number of levels and how they are grouped.

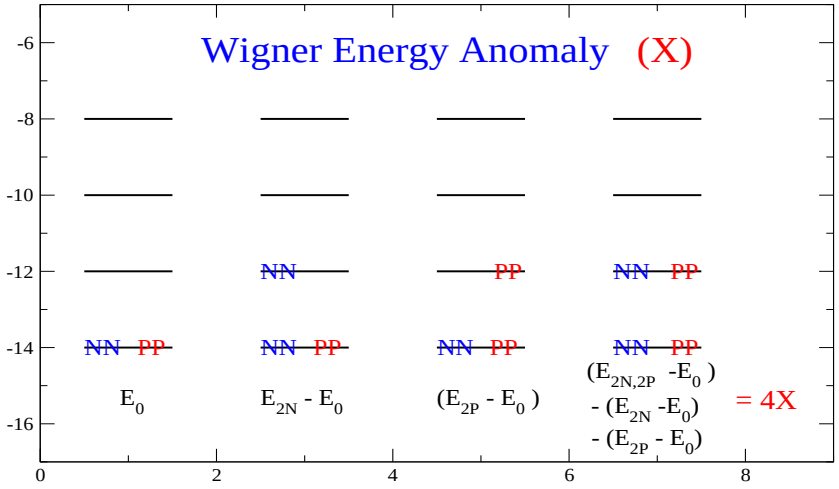
Three Problems

What is the source of the Wigner Energy Anomaly? $\approx 2.5 MeV$ in ^{20}Ne and $\approx 1 MeV$ in ^{56}Ni

Why are 'observed' shell spacings so much larger than the shell spacings calculated with a Woods Saxon potential, when the W.S. potential agrees so well with observation within a shell?

How does n-p pairing affect the excitation spectrum of $N = Z$ nuclides?

Wigner Energy Anomaly



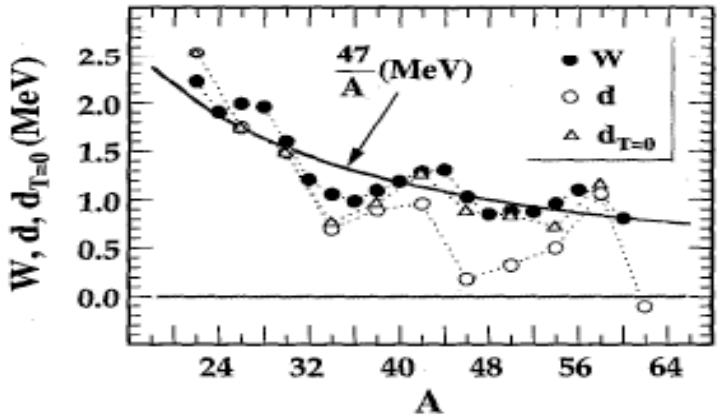
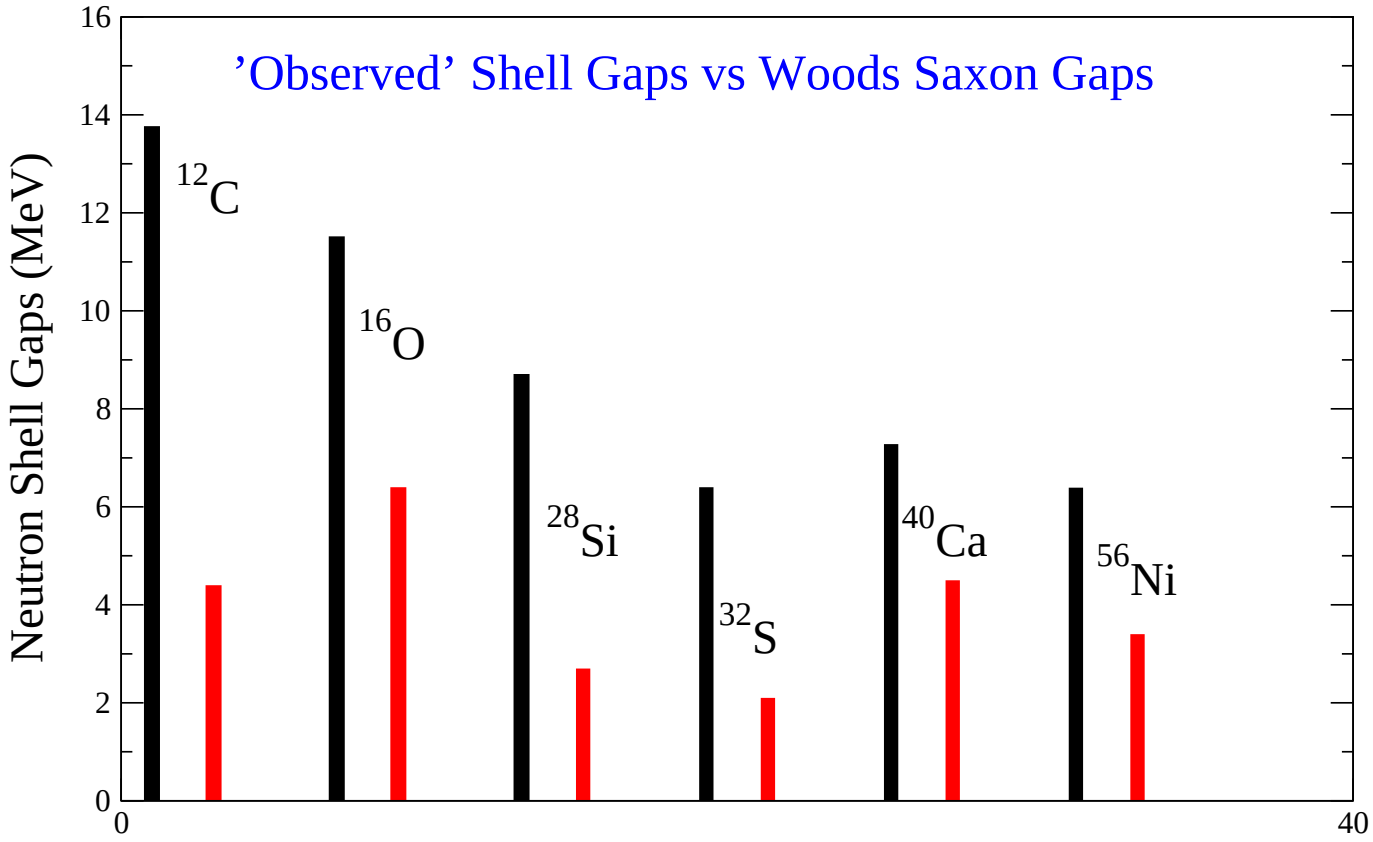


Fig. 1. Experimental values of W (filled circles) and d [Eq. (6), open circles] in $N = Z$ nuclei extracted from measured binding energies [24]. The values of W were obtained using the indicators given by Eqs. (4) and (5). The triangles mark the values of d calculated with Eq. (6) using experimental binding energies of the lowest $T = 0$ states in odd-odd nuclei. The solid line represents the average value $W_{av} = 47/A$ MeV (see text).

Satula et al. PLB407,103(1997)


'Observed' Shell Gaps vs Woods Saxon Gaps
Neutron Shell Gaps (MeV)
16
14
12
10
8
6
4
2
0
0
40
^{12}C
^{16}O
^{28}Si
^{32}S
^{40}Ca
^{56}Ni

To extract these correlation energies from experimental data, we utilize the function $\delta V(Z,N)$

P. Van Isacker, D.D. Warner and D.S. Brenner, Phys. Rev.Lett. **74**, 4607 (1995)

W. Satula et al., Phys. Lett. B **407**, 103 (1997).

$$\delta V(Z,N) = B(Z,N) - B(Z-2,N) \\ -B(Z,N-2) + B(Z-2,N-2) \qquad (5)$$

for e-e nuclei.

Note that $\delta V(Z,N)$ can be rewritten as three terms

$$\begin{aligned}\delta V(Z,N) &= [B(Z,N) - B(Z-2,N-2)] \\ &-[B(Z,N-2) - B(Z-2,N-2)] \\ &-[B(Z-2,N) - B(Z-2,N-2)]\end{aligned} \tag{6}$$

The first term is the binding energy gained by adding two protons and two neutrons to the nuclide (Z-2,N-2) and the last two terms are the binding energies gained by adding a neutron pair and a proton pair separately. $\delta V(Z,N)$ gives the extra binding due to the four nucleon correlation.

$$\delta V(Z,N) = [B(Z,N) - B(Z-2,N-2)]$$

$$4\,\varepsilon - 3[G^{T=0}_{i,i} + G^{T=1}_{i,i}] \;\; (W^\dagger)$$

$$-[B(Z,N-2) - B(Z-2,N-2)]$$

$$2\,\varepsilon - G^{T=1}_{i,i} \;\; (A^\dagger)$$

$$-[B(Z-2,N) - B(Z-2,N-2)]$$

$$2\,\varepsilon - G^{T=1}_{i,i} \;\; (B^\dagger)$$

$$\delta V(Z,N) = -[3\,G^{T=0}_{i,i} + G^{T=1}_{i,i}]$$

$\delta V(Z,N)$ = 4 x Wigner Correlation Energy

$G_{i,j}^{T=1} = (19/A)MeV$

$G_{i,i}^{T=1} = 2\ G_{i,j}^{T=1}$ for δ or density- dependent δ Interaction

$G_{i,i}^{T=1} = 2.4\ G_{i,j}^{T=1}$ for Gogny Interaction

(calculated with program from J.L. Egido and L.M. Robledo)

Taking $G_{i,i}^{T=1} = G_{i,i}^{T=0}$, this gives $45.5/A$ MeV for the Wigner Correlation Energy in the Slater approximation. This is in extraordinarily good agreement with the empirically determined (Satula '97) smooth dependence of $47/A$ MeV.

Our approach explains the 1/A dependence of the Wigner energy in a very simple way; it is just the $1/A$ dependence of the pairing force.

Just as we define $\delta V(A/2, A/2)$ for even-even nuclides, we define a similar quantity for odd mass nuclei, e.g. the nuclide with $Z = (A/2 - 1)$ and $N = A/2$,

$$\delta K(Z-1, N) = B(Z-1, N) - B(Z-1, N-2)$$
$$-B(Z-2, N) + B(Z-2, N-2) \quad (7)$$

$\delta K(Z-1, N)$ gives the difference in binding energy gained by adding a proton and two neutrons together, as compared to adding the proton and neutron pair separately.

$$\delta\tilde{V}(Z, N) = -[3G_{i,i}^{T=0} + G_{i,i}^{T=1}]$$

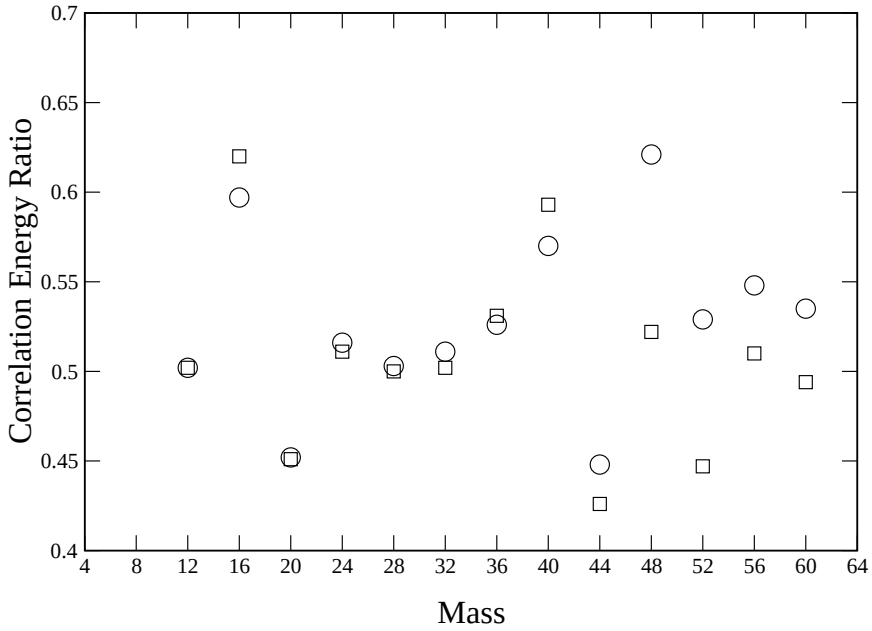
$$\delta\tilde{K}(Z, N-1) = -1/2[3G_{i,i}^{T=0} + G_{i,i}^{T=1}]$$

Zero-th order Prediction $\delta\tilde{K}/\delta\tilde{V}$ = 0.5

Ratio Smoothed Odd/Even Correlation Energies Function of Mass

Expected Ratio is 0.5 in zero-th order

$\delta\tilde{K}(A/2-1,A/2)/\delta\tilde{V}(A/2,A/2)$
squares (odd proton)
$\delta\tilde{K}(A/2,A/2-1)/\delta\tilde{V}(A/2,A/2)$
circles (odd neutron)



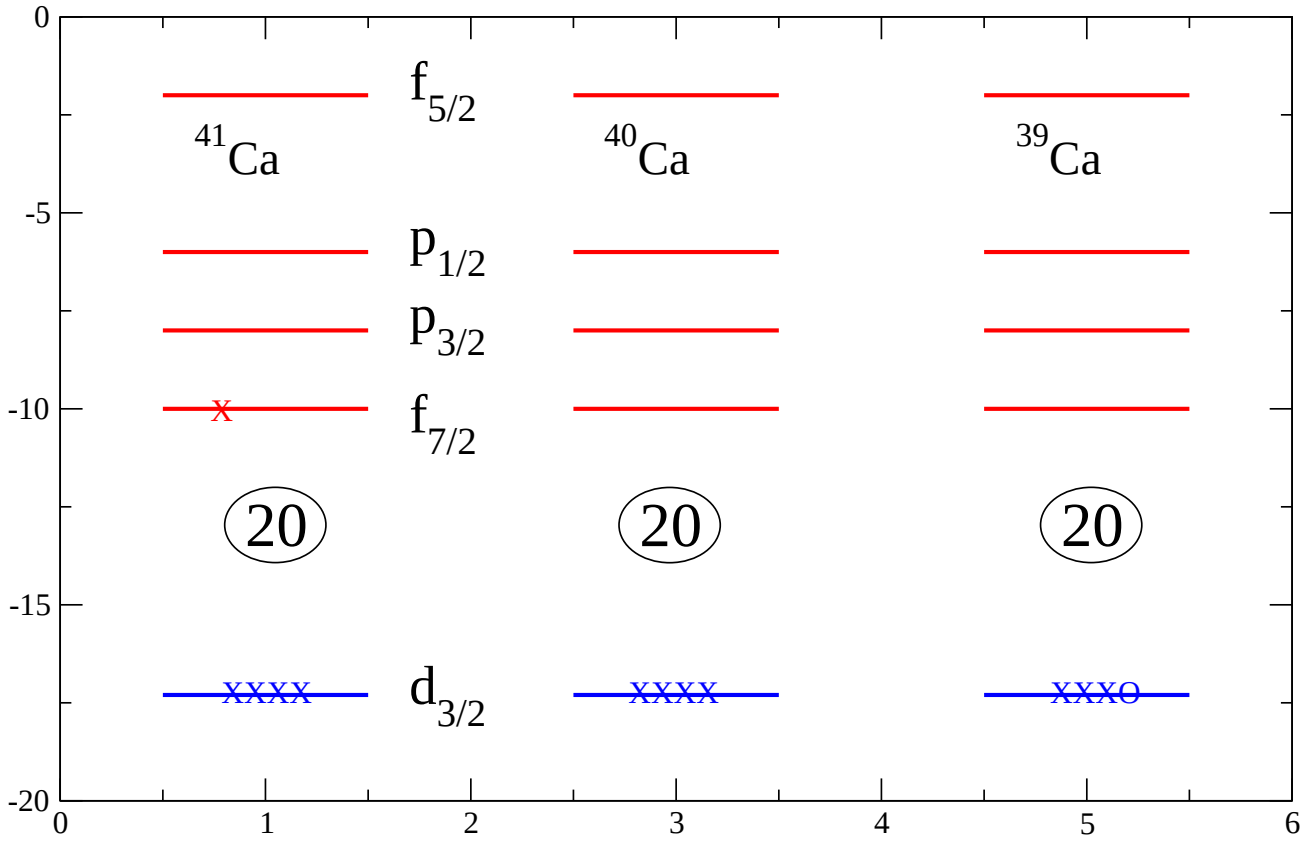
The puzzle of inter shell level spacings

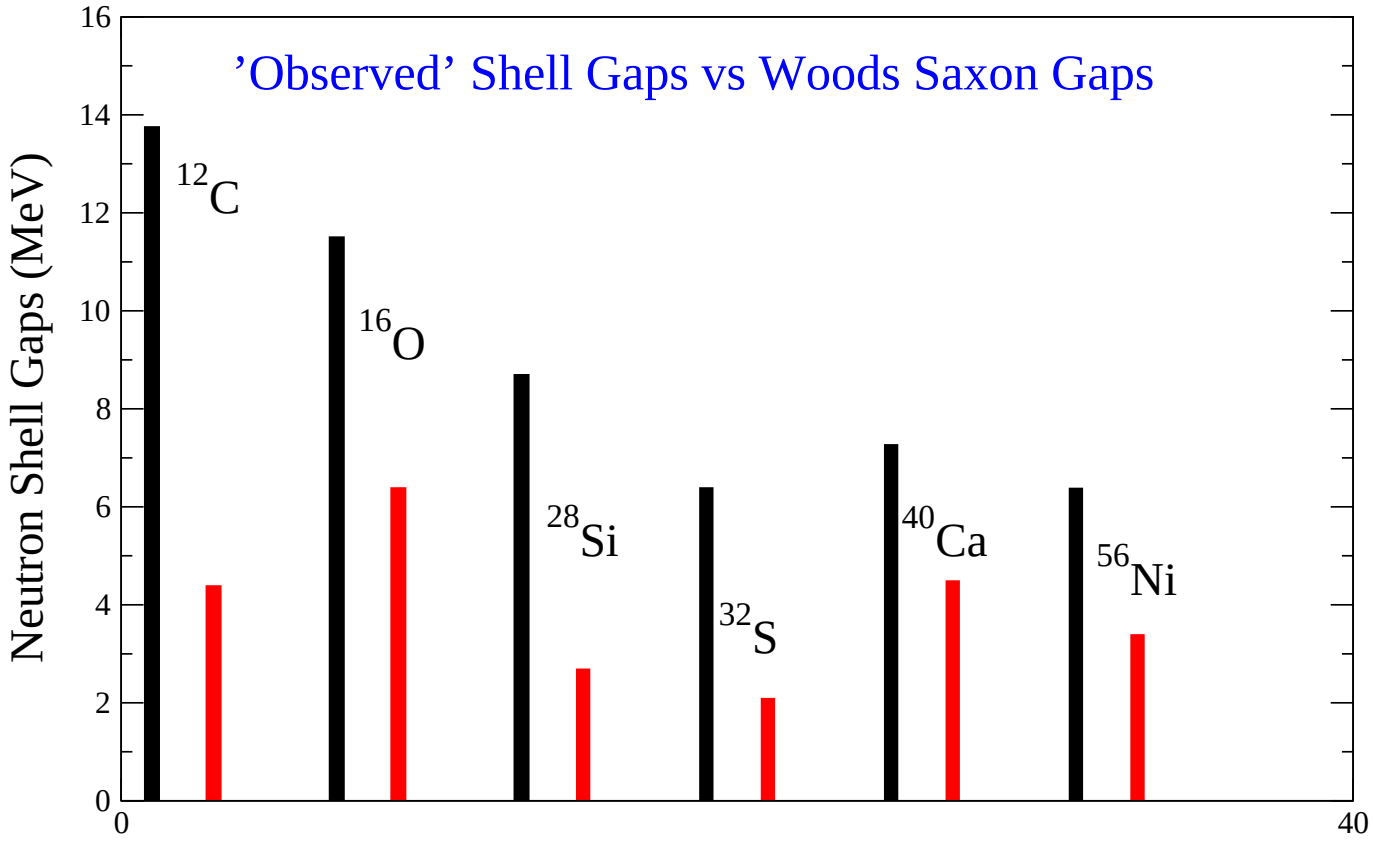
The standard method for calculating the single particle level spacings between shells is to take binding energy differences. Using ^{40}Ca as an example, the energy of the $f_{7/2}$ level, just above the N=20 gap, is defined as the difference in ground-state mass of ^{41}Ca and ^{40}Ca. Similarly, the energy of the $d_{3/2}$ level, just below the N=20 gap, is defined as the difference in mass of ^{40}Ca and ^{39}Ca.

$$B.E.(^{41}Ca) = B.E.(^{40}Ca) + \varepsilon_{f_{7/2}}$$

$$B.E.(^{40}Ca) = B.E.(^{39}Ca) + \varepsilon_{d_{3/2}}$$

Single Particle States

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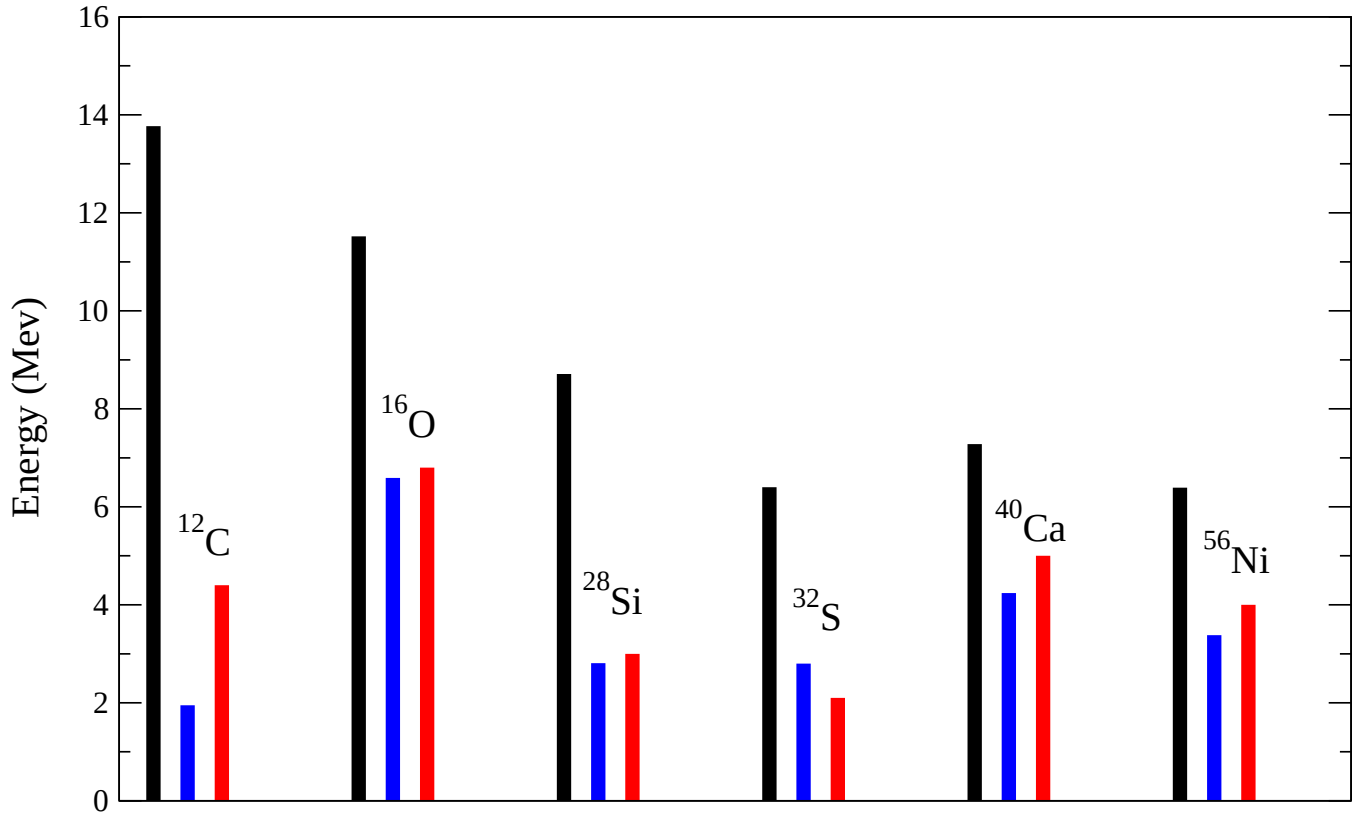
^{56}Ni

Denoting the energy of the $N = Z = A/2$ nucleus as E_0^0, the single-particle energy of the last occupied orbital as ϵ_0 and the energy of the first unoccupied orbital as ϵ_1

$$B(A/2, A/2+1) - B(A/2, A/2) = \varepsilon_1 - E_0^0 \tag{8}$$

where $B(Z, N)$ denotes a binding energy. However, the neutron removal energy has a diagonal correlation contribution

$$B(A/2, A/2) - B(A/2, A/2-1) = E_0^0 - [\varepsilon_0 - \frac{3}{2}[G_{i,i}^{T=0} + G_{i,i}^{T=1}]] \tag{9}$$


Energy (Mev)
0
2
4
6
8
10
12
14
16
12C
16O
28Si
32S
40Ca
56Ni

Summary - Successes

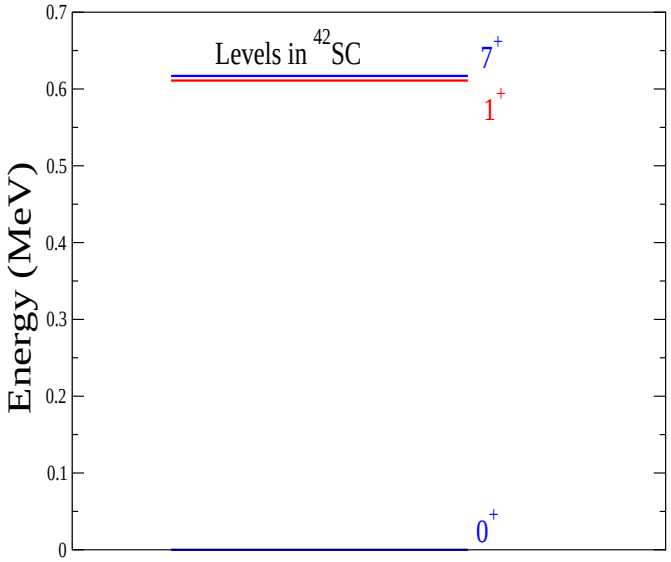
zero-th order picture

- This picture provides a simple understanding of the heretofore mysterious Wigner correlation energy in (A/2,A/2) nuclei and the equivalent correlation energy in (A/2-1,A/2) and (A/2,A/2 -1) nuclei.

- It gives the ratio of these correlation energies correctly.

- It accounts for near degeneracy of 0^+ and 1^+ in odd-odd $N = Z$ nuclides

- After extracting pairing correlation effects from the experimental data, binding energies of nuclides near N=Z provide a description of inter shell and inter subshell spacings that agrees reasonably well with Woods Saxon calculations.



Failures

Why is the pair transfer probability to 1^+ generally so weak compared to the transfer probability to the 0^+ ground state in N=Z odd-odd nuclei if the two states are equally collective? (Augusto's research)

Why is 7^+ level so close to the 1^+ in ^{42}Sc, if $T = 0$ pairing is pulling down the 1^+ state. What's pulling down the 7^+?

There are two approaches that should be examined to deal with these problems.

- Perhaps the T=0 interaction is not at all like the short range pairing interaction?
- Perhaps the 7^+ state is deformed

If the $T = 0$ interaction is long range, the ratio of diagonal to off-diagonal matrix elements would be larger than the factor of 2.4 that we have been using.

G. Bertsch has suggested (private communication) that the ratio of diagonal to off-diagonal matrix elemrents should be 1.8:1 for pairing rather than 2.4:1.

The successes that we have discussed above depend, by and large, on the sum of the diagonal matrix elements $[G^{T=0}_{i,i} + G^{T=1}_{i,i}]$.

Decreasing the $G^{T=1}_{i,i}$ to $1.8\times(19/A)$ and increasing [$G^{T=0}_{i,i}$ to $3.0\times(19/A)$ preserves the successes discussed above and helps with our two problems.(It does increase the estimate of the Wigner energy to [51.2/A] which is still quite good.)

It brings down the energy of the 7^+ and the 1^+ levels equally without introducing any collectivity into the 1^+ state. This is in the spirit of the Modified Surface Delta interaction (P.W.M. Glaudemans et al., Nuclear Physics A102(1967)). That change is easy to implement in our code.

Deformation

2) It might be possible to explain the lowering of the 7^+ in ^{42}Sc as due to deformation.That's not so easy to implement.

Before getting to those results, let me explain my procedure for calculating energies and wavefunctions. There is some confusion in the literature about the conservation of isospin in my approach (I do conserve isospin) and the nature of my variational method.

Beyond the Particle Number and J_z Projected Solution

Variational Configuration-Interaction (VCI) Method

- Variational optimization of configuration-interaction wavefunctions
- Works extremely well for ground states and for excited states
- The VCI method is quite general in that it does not require any restrictions on energies (e.g. degeneracies) or on matrix elements (e.g. equality).
- The starting point is the projected product wavefunction described above which we denote as Θ_1

The variational trial function, Ξ_m^{n+1} is

$$\Xi_m^{n+1} = \Phi_m^n + \Theta_{n+1}. \tag{10}$$

with the starting wave function Φ_m^n, where n denotes the number of basis states and m denotes the specific state that we are approximating, e.g. ground state or excited state.

$$\Phi_m^n = \sum_{i=1,n} t_{i,m}^n \Theta_i. \tag{11}$$

All states Θ_i are identical in form, but with different coefficients. The amplitudes $t_{i,m}^n$ are determined by diagonalizing the Hamiltonian with normalized basis states Θ_i. States Θ_j and Θ_k are generally not orthogonal (taken into account).

We then determine a new basis state Θ_{n+1}, variationally, to improve the approximation Φ_m^n. After Θ_{n+1} is determined variationally, and then normalized, we do a diagonalization to obtain a new set of amplitudes $t_{i,m}^{n+1}$ for the $(n+1)$ basis state approximate wave function Φ_m^{n+1}. We then calculate the next basis state Θ_{n+2} variationally. This procedure can be continued indefinitely. At some point, the improvements are minimal.

The energy expression is

$$\langle E\rangle = \langle H\rangle/\langle N\rangle \tag{12}$$

with the normalization $\langle N\rangle$,

$$\langle N\rangle \;=\; 1 + 2\sum_{i=1,n} t^n_{i,m}\langle\Theta_i|\Theta_{n+1}\rangle + \langle\Theta_{n+1}|\Theta_{n+1}\rangle \tag{13}$$

where Θ_{n+1}, the variational wave function, is not normalized.

$$\begin{aligned}\langle H\rangle \;=\; & E^n_m + 2\sum_{i=1,n} t^n_{i,m}\langle\Theta_i|H|\Theta_{n+1}\rangle \\ & +\langle\Theta_{n+1}|H|\Theta_{n+1}\rangle\end{aligned} \tag{14}$$

where E^n_m is the m-th eigenvalue calculated in the n-basis state diagonalization procedure and $t^n_{i,m}$ is the expansion amplitude.

The variational parameters $U_{n+1}(k)$ are determined by iteratively solving the set of coupled algebraic equations

$$\partial\langle E\rangle/\partial U_{n+1}(k) = 0. \tag{15}$$

In general, the VCI solutions do not look like mean field solutions, (e.g. GCM solutions) they can even have negative amplitudes allowing for a rich variety of solutions. One might think of the GCM wavefunctions as a shotgun approach, while the VCI method is more like a rifle.

When m is not the lowest state for a given set of quantum numbers, we can avoid collapse of the (n+1)-th trial function to the ground state by adding a constraint term to the Hamiltonian

$$\langle \Delta H \rangle \;=\; +\eta \sum_{j=1,m-1} \langle \Phi_j^n | \Theta_{n+1} \rangle^2 \tag{16}$$

with $\eta \approx 3(E_m \;-\; E_0)$

or just stop iterating when

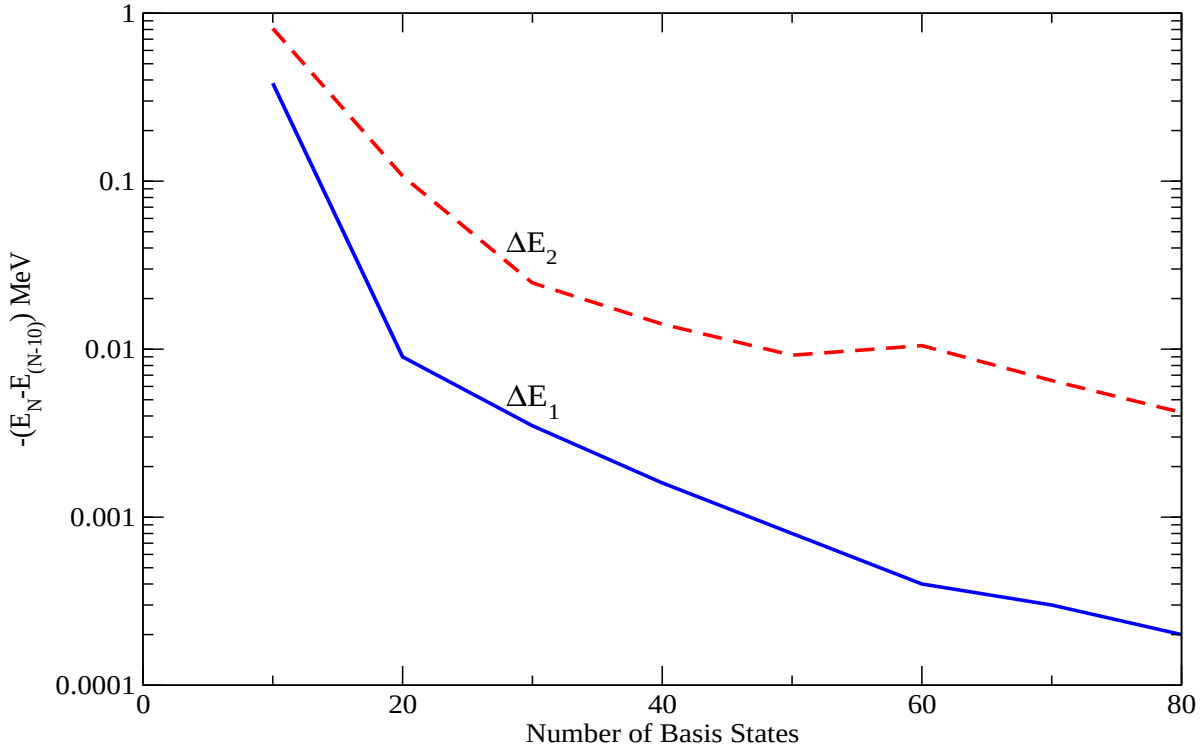
$\langle \Theta_k | \Theta_{n+1} \rangle^2$ exceeds some predetermined value

We have tested this method for n-n pairing for which there is an exact solution in the literature R.W. Richardson Phys. Rev. 141,949(1966)

and for the n-p pairing interaction for which there were no exact solution published at the time that we did this work.

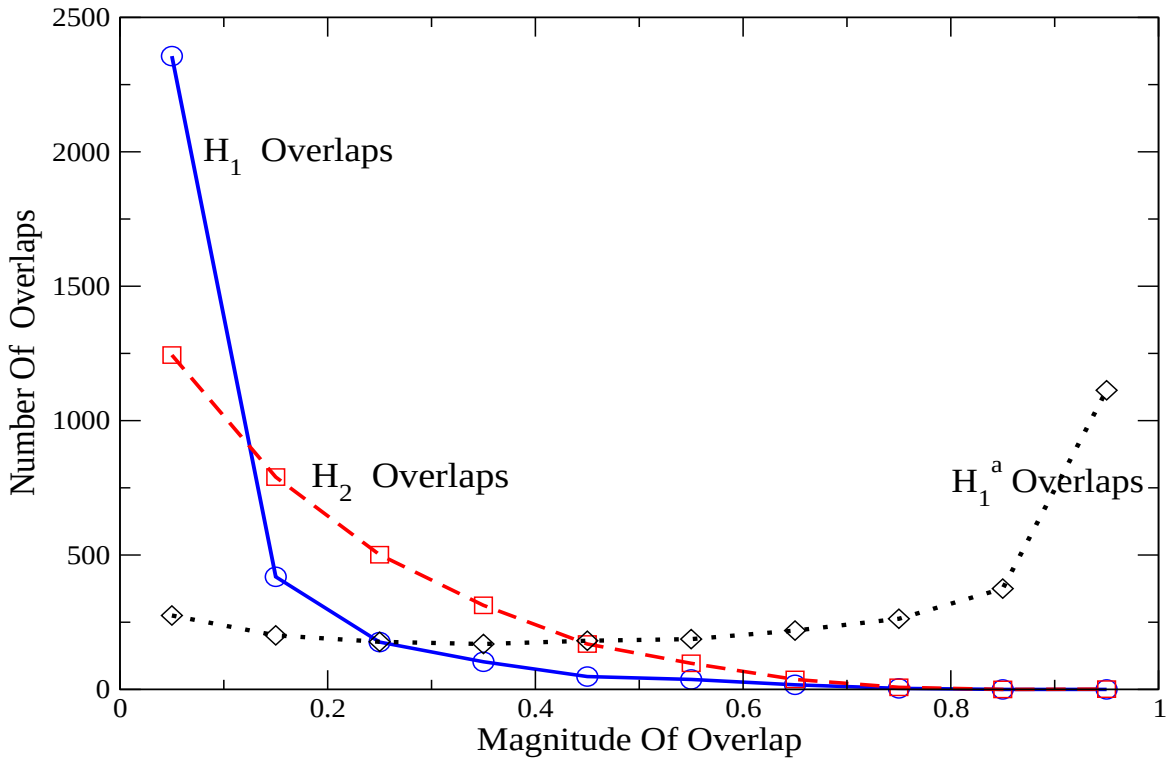
In both cases, the model space consisted of equally spaced Nilsson levels, 32 levels for the n-n pairing case and 30 for the n-p pairing case.

Ground State Energy Improvement as function of number basis states



H_1 Standard n-n pairing; Exact solution available H_2 n-p pairing Hamiltonian; no exact solutions available at that time.

Pairing energy E_1 agrees with exact energy to 1 KeV.



Because variational functions are not constrained to be monotonic, overlaps between basis functions are much smaller than with GCM basis functions.

Pairing Vibrations

Pair-vibrational states occur when nucleons are collectively excited across a gap in the single-particle spectrum. The conditions for the existence of such states are level degeneracies just above and below the gap and an appropriate value for the pairing interaction strength.

Given these conditions, there exist excited (pair-vibrational) states with energies less than the energy required to promote a pair of nucleons across the gap. In such cases the correlation energy in the excited state is comparable to that in the ground state.

Test of VCI - Comparison with Exact Ground State and Excited State Solutions

Model Space Study - with P. van Isacker

Setting $G^{T=1} = G^{T=0}$ and $G_{i,i} = G_{i,j}$, one can use group theoretical techniques, together with diagonalizing large matrices, to obtain exact solutions for the n-p pairing problem. The Hamiltonian must also conserve angular momentum. The exact solutions provide a needed test of the VCI method for a Hamiltonian more complicated than simple pairing for both ground states and excited states.

Satisfying these criteria led us to a model consisting of two groups of degenerate levels (degeneracy 8) separated by 1 MeV (each sublevel accomodating 2 neutrons and 2 protons). All levels have the same value of J , J=1/2. In this case, the $M^\dagger = i(a_k^\dagger b_k^\dagger)$ $(j_z = 1)$and $N^\dagger = i(a_{-k}^\dagger b_{-k}^\dagger)$ $(j_z = -1)$ modes are on the same footing as all other modes.

$$\Theta_i \;=\; \mathcal{P}\prod^{k}\psi_{i,k}|0\rangle \tag{17}$$

$\mathcal{P}$ is a projection operator that now includes J_z in addition to proton number, neutron number, and isospin number parity.

$$\begin{aligned}\psi_k \;=\; &\Big[1 + U(1,k)A_k^\dagger + U(2,k)B_k^\dagger \\ &+U(3,k)C_k^\dagger + U(4,k)D_k^\dagger + U(5,k)M_k^\dagger \\ &+U(6,k)N_k^\dagger + U(7,k)W_k^\dagger\Big]\end{aligned} \tag{18}$$

$M_k^\dagger = i(a_k^\dagger b_k^\dagger)$; $N_k^\dagger = i(a_{-k}^\dagger b_{-k}^\dagger)$

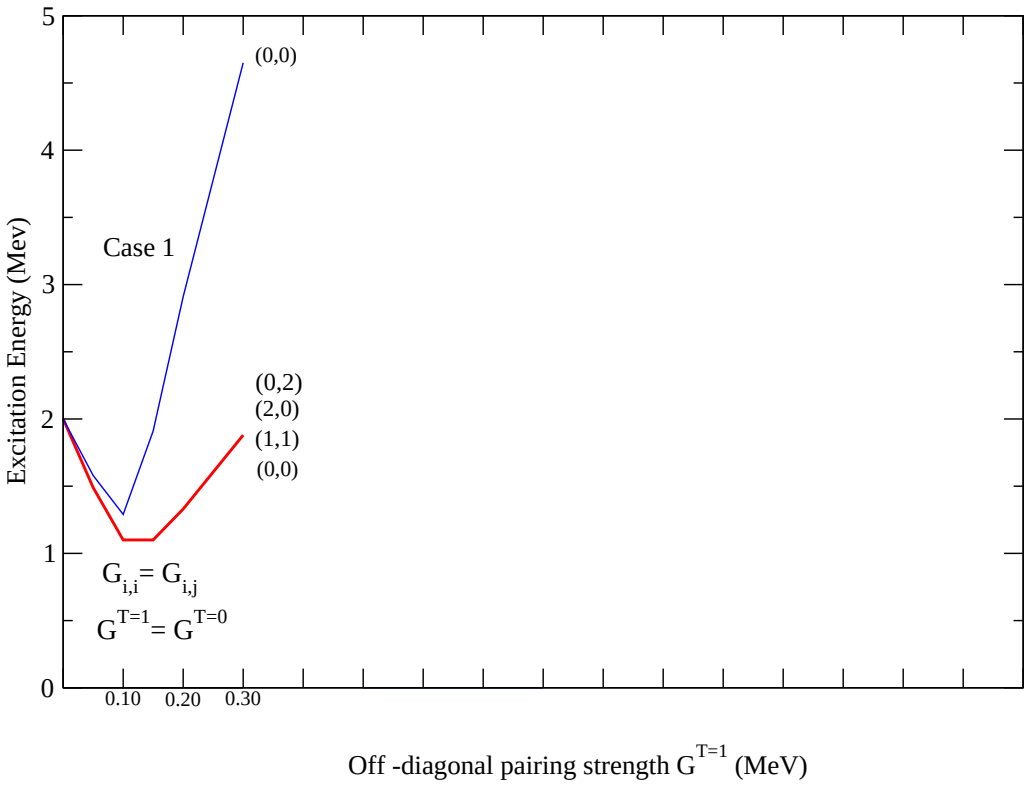
The solutions of this model Hamiltonian include a low-lying quadruplet of excited states.

The four states are (J,T) = (0,0),(1,1),(0,2),(2,0)

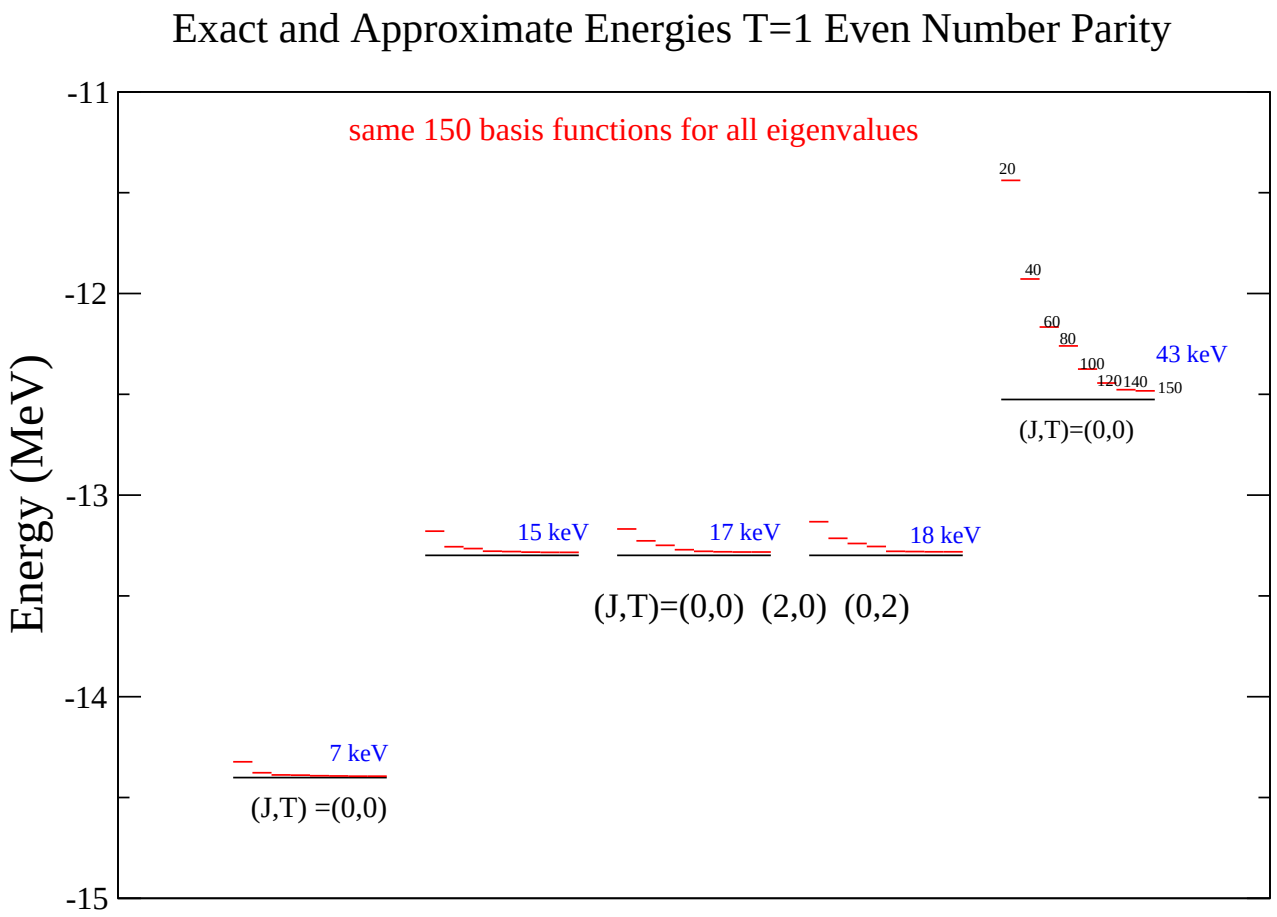
Using our approximation method, we determine the the ground state and the lowest T=1 state to an accuracy of a few keV. The next three even T excited states are accurate to 10-20 kev. The fourth state, which is the second excited 0^+ is calculated to 40-50 keV accuracy for the largest values of the interaction strength. This state looks like the classical pairing vibration.

Excited State Spectrum

S. P. Levels (8 levels $\epsilon = 0$; 8 levels $\epsilon = 1$)

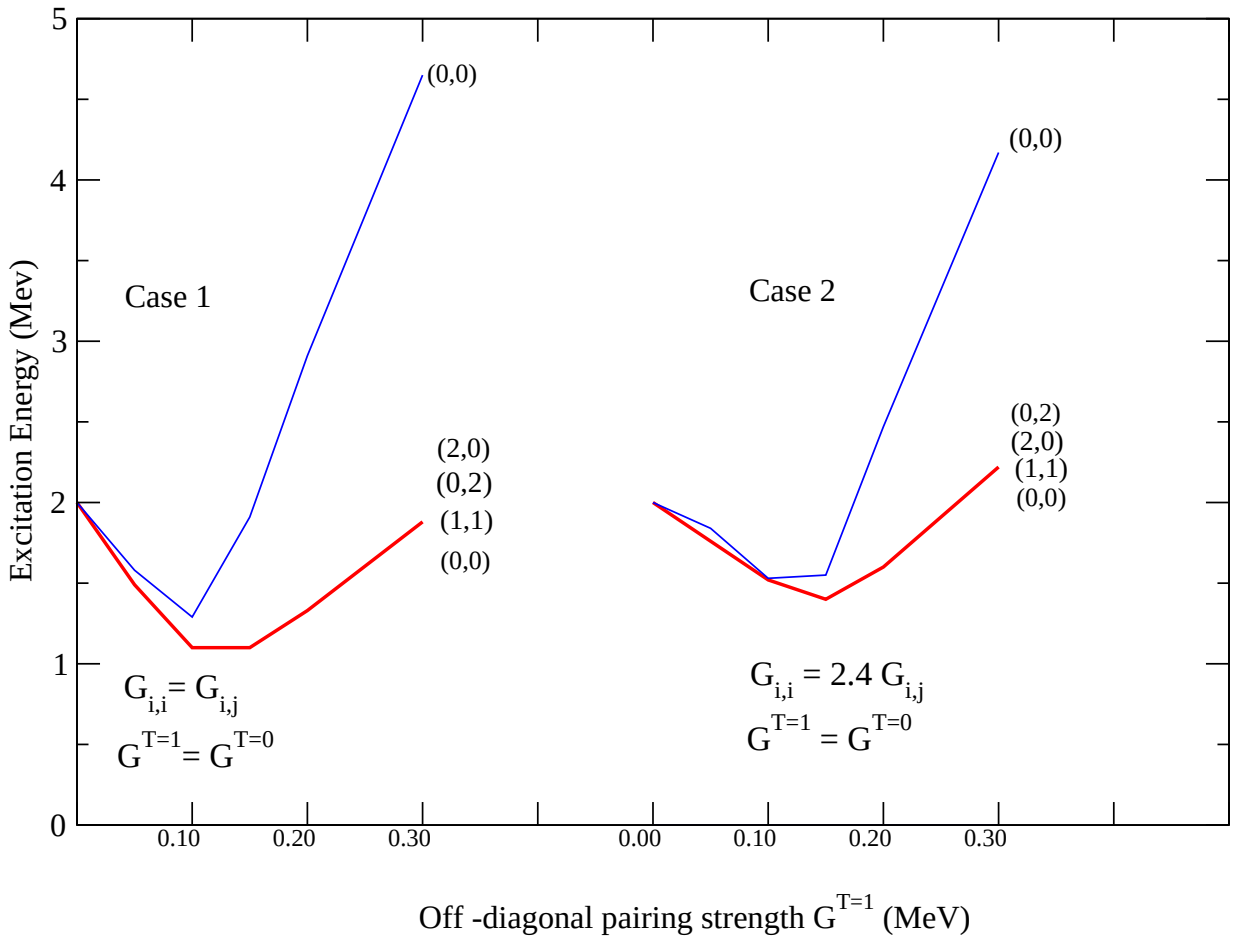


Approximate Energies as a function of Basis Size G=0.15 MeV



S. P. Levels (8 levels $\epsilon = 0$; 8 levels $\epsilon = 1$)

Note large change in $E_{J=2} - E_0$


Excitation Energy (Mev)
5
4
3
2
1
0
Case 1
Case 2
(0,0)
(2,0)
(0,2)
(1,1)
(0,0)
(0,0)
(0,2)
(2,0)
(1,1)
(0,0)
$G_{i,i} = G_{i,j}$
$G^{T=1} = G^{T=0}$
$G_{i,i} = 2.4\ G_{i,j}$
$G^{T=1} = G^{T=0}$
0.10
0.20
0.30
0.00
0.10
0.20
0.30
Off -diagonal pairing strength $G^{T=1}$ (MeV)

Pair vibrations in the presence of n-p pairing

- Comparisons with exact calculations show that the VCI method works extremely well in describing pairing vibrations in the presence of n-p pairing.

- There are lots and lots of pairing vibrational states along the N=Z line.When there is just like particle pairing, there are just 2 pairing vibrational states. Here there are 5.

N-P Pairing +'Nilsson'

$$H = \sum_{k>0} \varepsilon_k \left(a_k^\dagger a_k + a_{-k}^\dagger a_{-k} + b_k^\dagger b_k + b_{-k}^\dagger b_{-k} \right)$$

$$- \sum_{i,j} G_{i,j}^{T=1} \left[A_i^\dagger A_j + B_i^\dagger B_j + C_i^\dagger C_j \right]$$

$$- \sum_{i,j} G_{i,j}^{T=0} \left[D_i^\dagger D_j + (M_i^\dagger M_j + N_i^\dagger N_j) \delta(\Omega_{i,j}) \right] \quad (19)$$

$$- \sum_{i \le j, k \le l} V_{i,j,k,l} \left[\alpha_i^\dagger \alpha_j^\dagger \alpha_l \alpha_k \right] \quad (20)$$

- $\alpha^\dagger = a^\dagger$ or $b^\dagger$ with the condition that there be equal numbers of protons (neutrons) destroyed as created.
- $V_{n,n} = V_{p,p} = 0.2 V_{n,p}$
- $J_z(i) = J_z(k) \qquad J_z(j) = J_z(l)$
- Matrix elements $V_{i,j,k,l}$ are used as numbers so any potential can be used. (Not restricted to multipole - multipole)

Wavefunctions for 'n-p pairing + Nilsson'

For a spherical system, J_z is a good quantum number. For a cylindrically symmetric system the projection of angular momentum of single particle states on the symmetry axis, Ω, is the constant of motion. This suggests a trial wavefunction, a many-body Nilsson wavefunction

$$\Theta_i = \mathcal{P} \prod^{|j_z|} \mathcal{P}'(\mathcal{J}) \sum_i \psi_i^{|j_z|} |0\rangle \tag{21}$$

$\mathcal{P}$ projects configurations with the desired values of total proton number and neutron number. Each of the $|j_z|$ subgroups has only configurations with a fixed value of J_z

$\mathcal{P}'(\mathcal{J}) \sum_i \psi_i^{|j_z|}$ is a sum over all configurations that can be constructed from all of the proton and neutron orbitals having the desired value of j_z and π within each $|j_z|$ subgroup. Within the subgroup, the number of protons and neutrons is not constrained, however only those configurations with the desired value of J_z are chosen. We do projection before variation.

Assume that we have just one $7/2$ level, say $f_{7/2,7/2}$ then $\sum_i \psi_i^{|j_z|}$ would just be for the $J_z = 0$ configurations

$$\begin{aligned} \Psi_k &= \left[1 + U(1)A_k^\dagger + U(2)B_k^\dagger \right. \\ &\quad \left. + U(3)C_k^\dagger + U(4)D_k^\dagger + U(5)W_k^\dagger\right], \end{aligned} \tag{22}$$

If, however, there were two $5/2$ levels say $f_{7/2,5/2}$ and $f_{5/2,5/2}$ then things start to get interesting (complicated)

$$
\begin{aligned}
\psi_{k,r} \;=\; & \Big[1 + A_k^\dagger + B_k^\dagger + C_k^\dagger + D_k^\dagger + W_k^\dagger \\
& + M_k^\dagger + N_k^\dagger \\
& + a_k^\dagger + a_{-k}^\dagger + b_k^\dagger + b_{-k}^\dagger \\
& + a_k^\dagger a_{-k}^\dagger b_k^\dagger + a_k^\dagger a_{-k}^\dagger b_{-k}^\dagger \\
& + a_k^\dagger b_k^\dagger b_{-k}^\dagger + a_{-k}^\dagger b_k^\dagger b_{-k}^\dagger \Big] \\
& \times \\
& \Big[1 + A_r^\dagger + B_r^\dagger + C_r^\dagger + D_r^\dagger + W_r^\dagger \\
& + M_r^\dagger + N_r^\dagger \\
& + a_r^\dagger + a_{-r}^\dagger + b_r^\dagger + b_{-r}^\dagger \\
& + a_r^\dagger a_{-r}^\dagger b_r^\dagger + a_r^\dagger a_{-r}^\dagger b_{-r}^\dagger \\
& + a_r^\dagger b_r^\dagger b_{-r}^\dagger + a_{-r}^\dagger b_r^\dagger b_{-r}^\dagger \Big]
\end{aligned}
\tag{23}
$$

We then project all combinations with total $J_Z = 0$ (or any other desired value) and assign an independent amplitude to each of the projected configurations. All amplitudes are calculated using the VCI method discussed above.

In quasi-particle language, these configurations include all 2 quasi-particle configurations in an even-even system.

Whenever we add another level the number of independent amplitudes increases by a factor of ≈ 10 for $J_z = 0$

For four levels,that can contain two protons and two neutrons there are $\approx 10^4$ independent amplitudes.

Looking at Deformation in ^{40}Ca and ^{42}Sc

In order to make a reasonable estimate of the magnitude of the multipole-multipole strength to use in ^{42}Sc, we fitted, as best we could, the 0^+ excited states in ^{40}Ca which are thought to be deformed.

Carrying out these calculations, we used a quadrupole-quadrupole + hexadecapole-hexadecapole +coulomb interaction.For the quadrupole term,

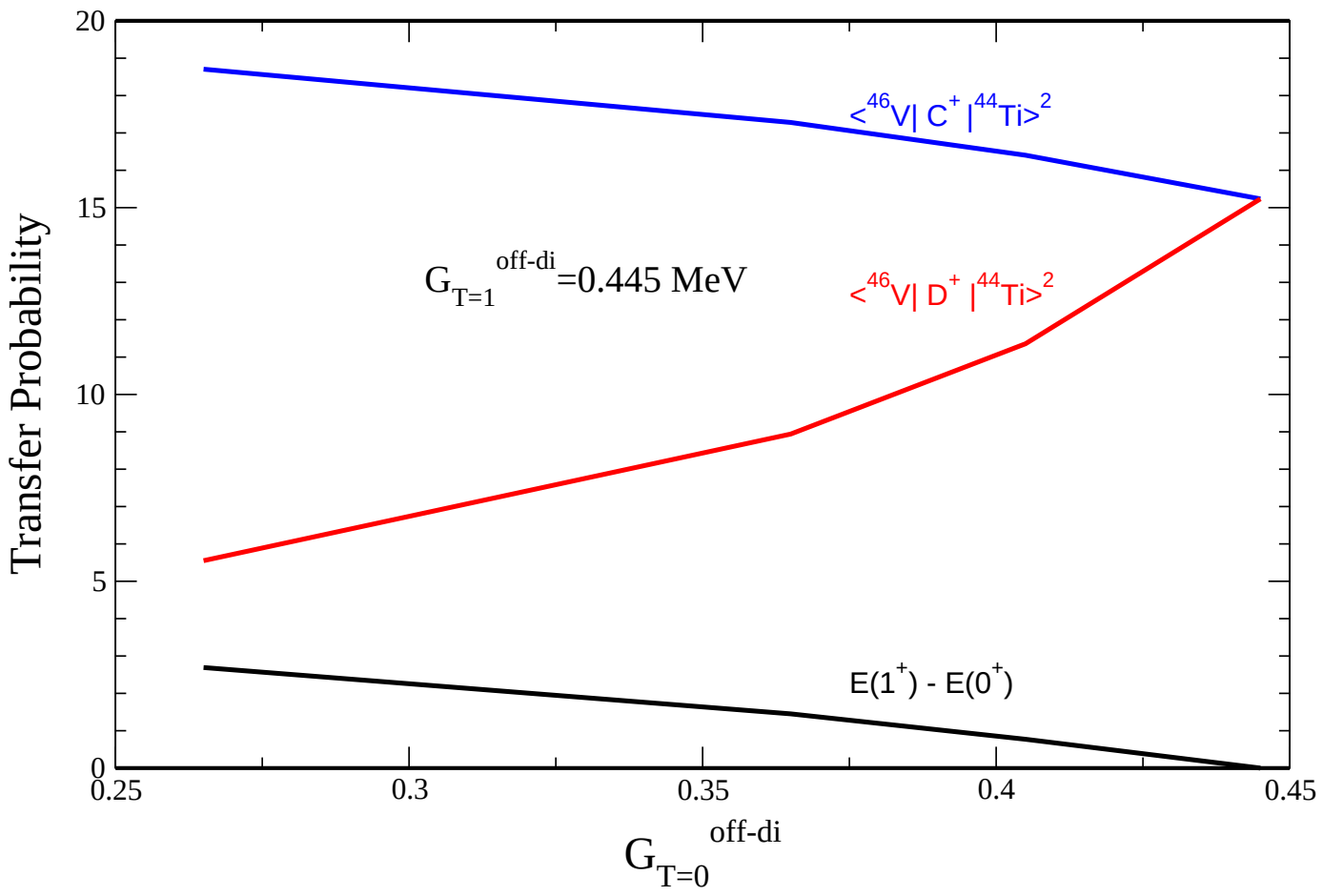
$-V_q \times \sum_{i \leq j, k \leq l} [Q_{i,k} Q_{j,l} - Q_{i,l} Q_{j,k}]$

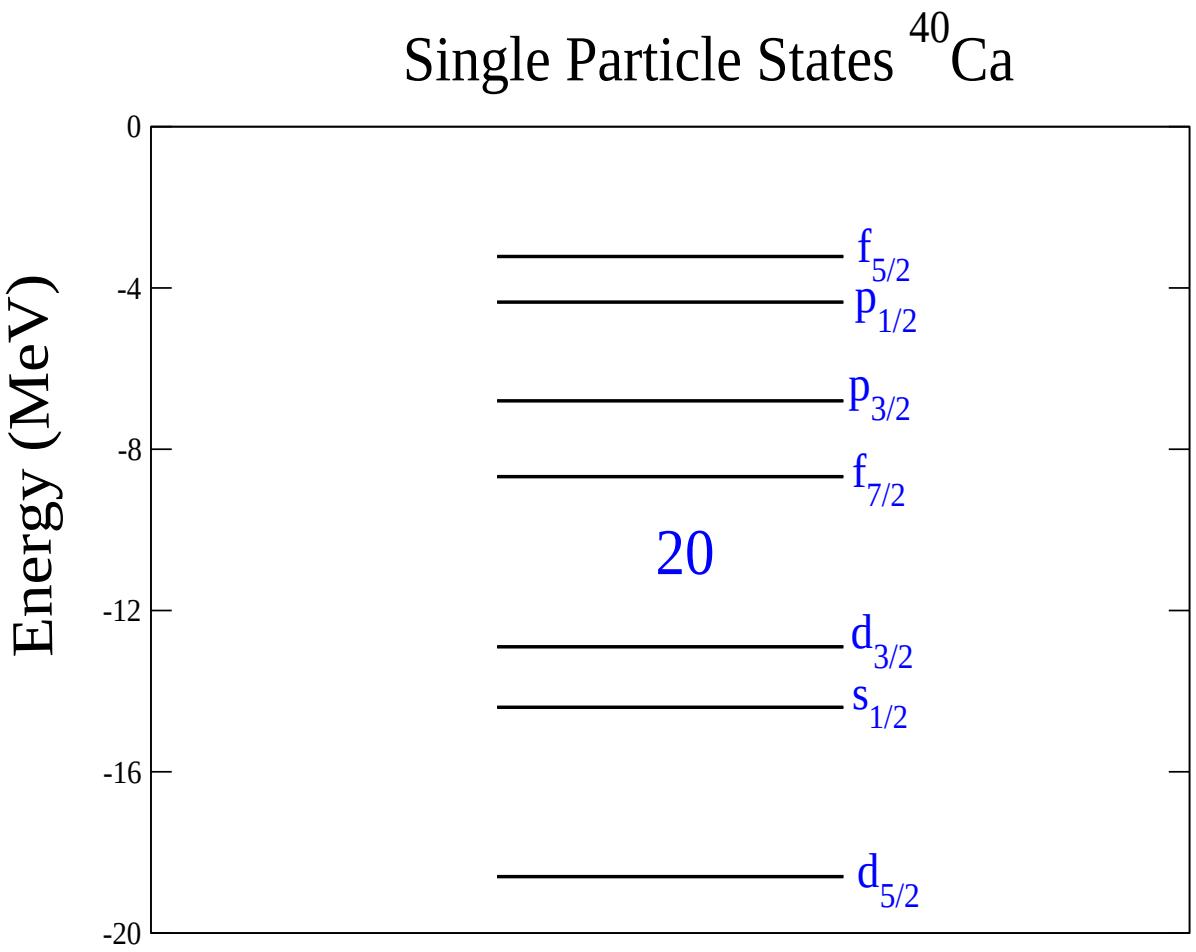
where $Q_{i,k} = [r^2 Y(2,0)]_{i,k} / [r^2_{i,i} \times r^2_{k,k}]^{0.5}$

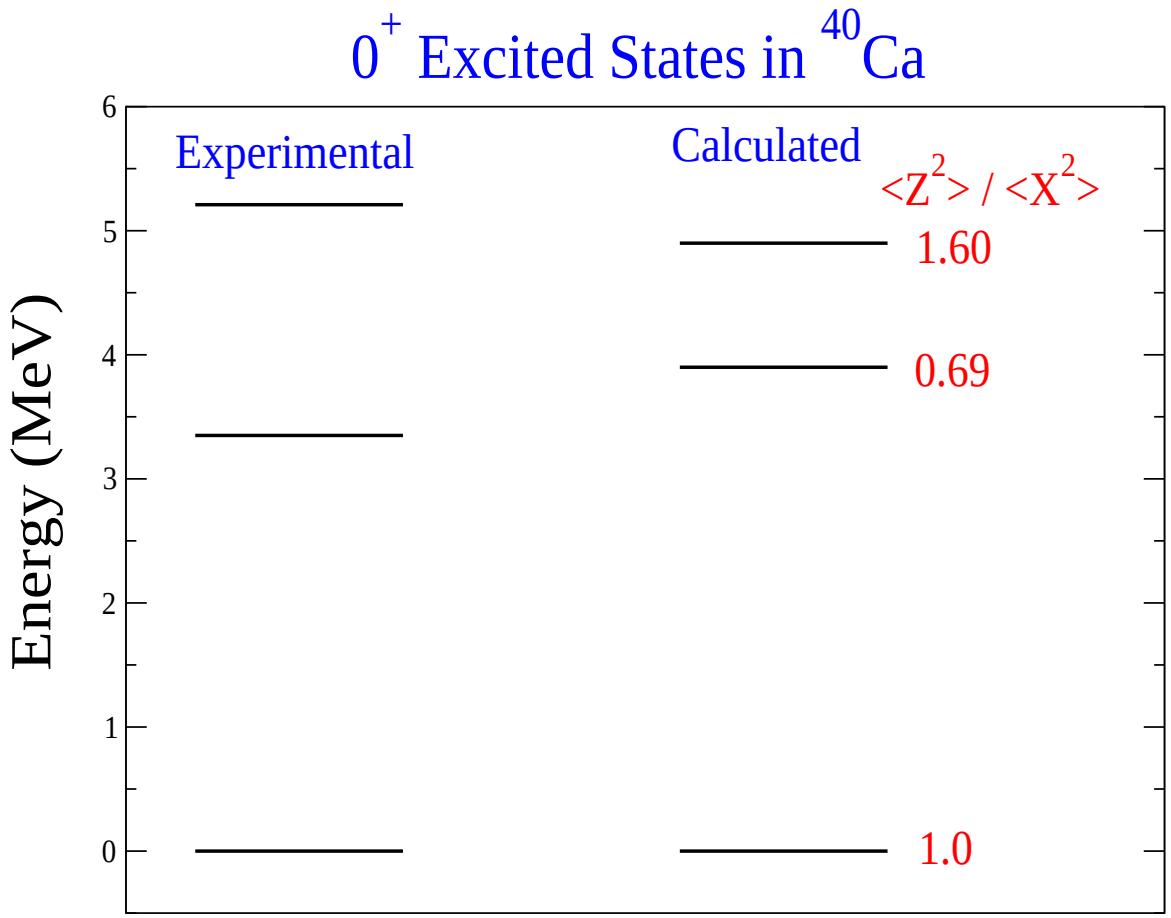
The exchange term is non-zero only when all orbitals have same j_z

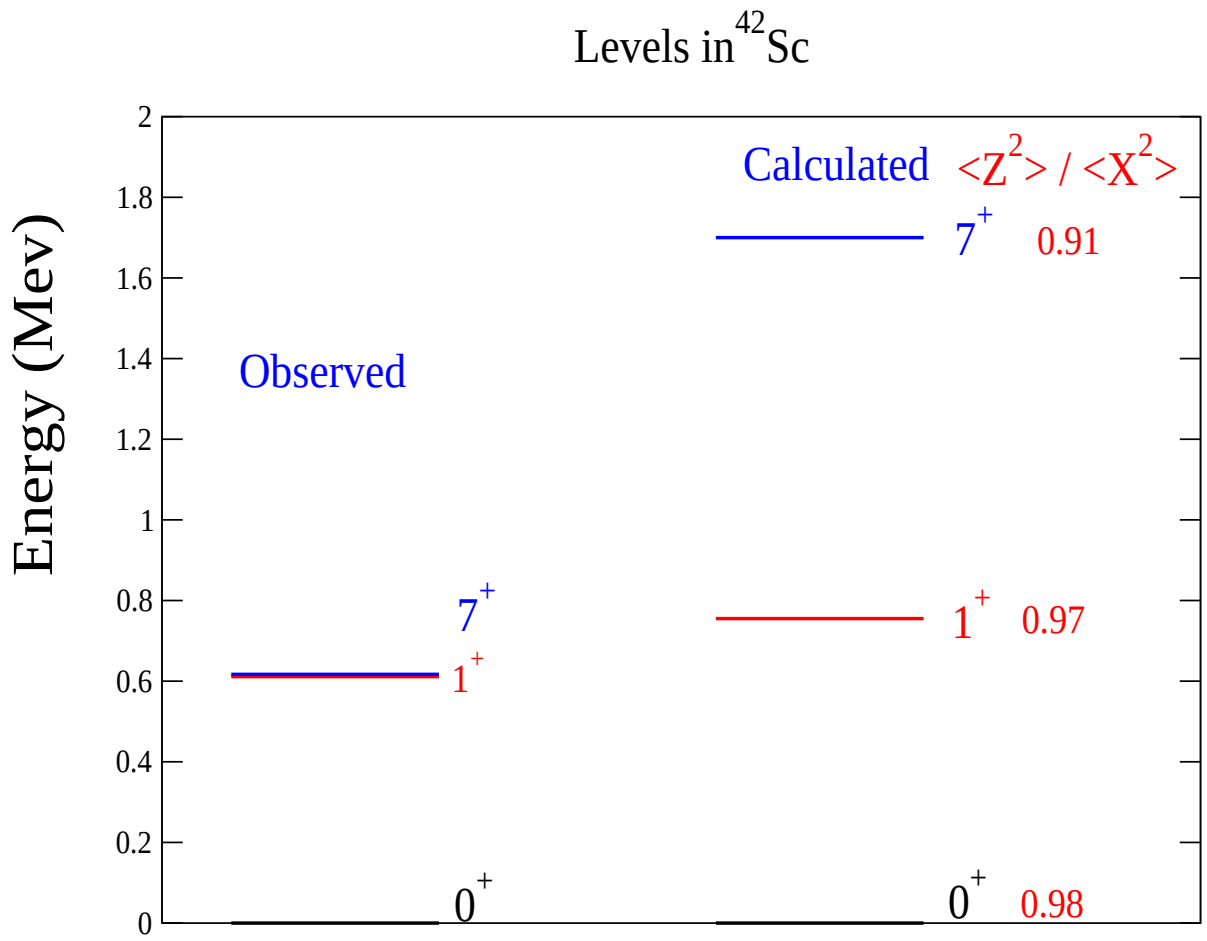
The coulomb term was added in the hope that it might improve things - it didn't

We added 0.5 MeV arbitrarily to the $T = 0$ diagonal matrix elements.


N-P T=0 and T=1 Pair Transfer
Transfer Probability
20
15
10
5
0
0.25
0.3
0.35
0.4
0.45
$G_{T=0}^{\text{off-di}}$
$G_{T=1}^{\text{off-di}}$=0.445 MeV
$<^{46}V|\ C^{+}\ |^{44}Ti>^{2}$
$<^{46}V|\ D^{+}\ |^{44}Ti>^{2}$
$E(1^{+}) - E(0^{+})$


Single Particle States ^{40}Ca
Energy (MeV)
0
-4
-8
-12
-16
-20
$f_{5/2}$
$p_{1/2}$
$p_{3/2}$
$f_{7/2}$
20
$d_{3/2}$
$s_{1/2}$
$d_{5/2}$


0+ Excited States in 40Ca
Experimental
Calculated
<Z2> / <X2>
1.60
0.69
1.0
Energy (MeV)
0
1
2
3
4
5
6


Levels in 42Sc
Energy (Mev)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Calculated
<Z2> / <X2>
7+
0.91
Observed
7+
1+
1+
0.97
0+
0+
0.98

At this point, it seems that the only way to explain the low-lying 7^+ state is to ascribe a huge diagonal matrix element, $\approx 3MeV$ to it.

After a little thought, one can understand (rationalize) this by considering the $J_z = 7$ state in terms of a density dependent long range interaction. The matrix element is maximized if it is peaked in in the nuclear surface region. Consider the $f_{7/2,7/2}$ orbital. It only has one component - it is $L = 3, S = 1/2$ making up the state. The wavefunction is ideal, it is nodeless - going as $z^3 \times exp{-\alpha r^2}$ and has a large component in the region of low nuclear density.

If ^{82}Nb were spherical, one might hope to see a low-lying 9^+ state there. Unfortunately ^{82}Nb seems to be strongly deformed near ground.

Conclusions

- $T = 1 + T = 0$ pairing provides a simple explanation of phenomena not previously understood
- A close look at the successes of the model show that it is the quantity $[G^{T=0}_{i,i} + G^{T=1}_{i,i}]$ that really counts
- Increasing $G^{T=0}_{i,i}$ and decreasing $G^{T=1}_{i,i}$ helps
- Large $G^{T=0}_{i,i}$ matrix elements suggest $T = 0$ interaction is long range
- Remains to be seen in what sense the off-diagonal $T = 0$ matrix elements are collective.
- VCI method applied to product wavefunctions is a powerful technique and should be further developed.
- Thanks to the organizers of this workshop.


השמים מספרים כבוד אל
ומעשה ידיו מגיד הרקיע