Proton-neutron correlations in particle radioactivity: the limitations of the shell-model

Radioactive Decay width

In this talk I will describe the formation and radioactive decay of nuclear clusters by using a microscopic formalism. This is based in the shell model. I will show the advantages and the success of the standard shell model in this subject, but also its limitations and ways of improving it.

The starting point of all microscopic descriptions of cluster decay is by using the expression of the decay width formulated by Thomas in 1954. Thomas obtained his famous expression by evaluating the residues of the R-matrix in a profound and very difficult paper (Prog. Theor. Phys. 12 (1954) 253).
Since in the microscopic treatment I will present the Thomas expression is fundamental, it is important to understand all the elements that enter in it. I will therefore start by presenting a clear and easy derivation of the Thomas formulae by using simple quantum mechanics arguments.

The first feature to be noticed is that a decaying cluster feels only the centrifugal and Coulomb interactions outside the surface of the daughter nucleus. Therefore the corresponding (outgoing) wave function in that region has the form

$$r\psi_{lj}^{out}(r) = N_{lj}[G_{lj}(r) + iF_{lj}(r)]$$

At very large distances, where both the centrifugal interaction (depending upon $1/r^2$) and the Coulomb one ($1/r$), are negligible, the wave function is a plane wave, i. e.

.
$$\lim_{r\to\infty} | r\psi_{lj}^{out}(r) |^2 = | N_{lj} |^2$$

The detector of the decaying particle can be considered to be at that distance The probability rate per second that the particle goes through a detector surface element $dS=r^2 \sin\theta\, d\theta\, d\varphi$ is

$$P_{lj} = | \psi_{lj}^{out}(\vec{r}) |^2 \, v dS$$

$$v = \hbar k / \mu$$

Since

$$\lim_{r\to\infty} | r\psi_{lj}^{out}(r) |^2 = | N_{lj} |^2$$

integrating over the angles, the decay probability per second becomes

$$1/T = |N_{lj}|^2 v.$$

Matching the out and the inner solution at R one gets

$$R\Psi_{lj}(R) = N_{lj}[G_{lj}(R) + iF_{lj}(R)] \text{ and}$$

$$\Gamma_{lj}(R) = \hbar/T = \frac{\hbar^2 k}{\mu} \frac{R^2 |\psi_{lj}(R)|^2}{F_{lj}^2(R) + G_{lj}^2(R)}$$

$$\Gamma_{lj}(R) = \hbar/T = \frac{\hbar^2 k}{\mu} \frac{R^2|\psi_{lj}(R)|^2}{F_{lj}^2(R) + G_{lj}^2(R)}$$

This is the famous Thomas expression for the decay width, which he obtained as the residues of the R-matrix.

$\Psi_{lj}(R)$ is the cluster formation amplitude and $kR/(F^2_{lj}(R)+G^2_{lj}(R))$ is the penetrability through the centrifugal and Coulomb barriers.

It is important to notice that the width should NOT depend upon R if the calculation of the formation amplitude is properly performed.

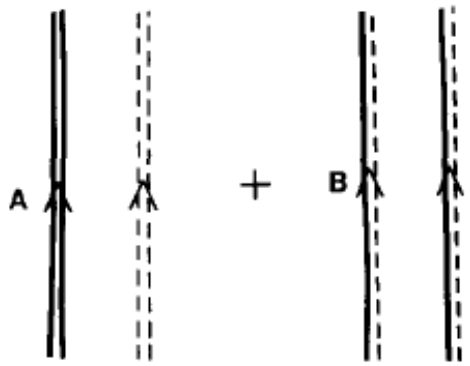
For the decay process B➔A+C the formation amplitude F is

$$F(R) = \int d\xi_A d\xi_C \psi_B^*(\xi_B)\psi_A(\xi_A)\psi_C(\xi_C)$$

Maglione,Ferreira, RL, PRL81, 538 (1998)

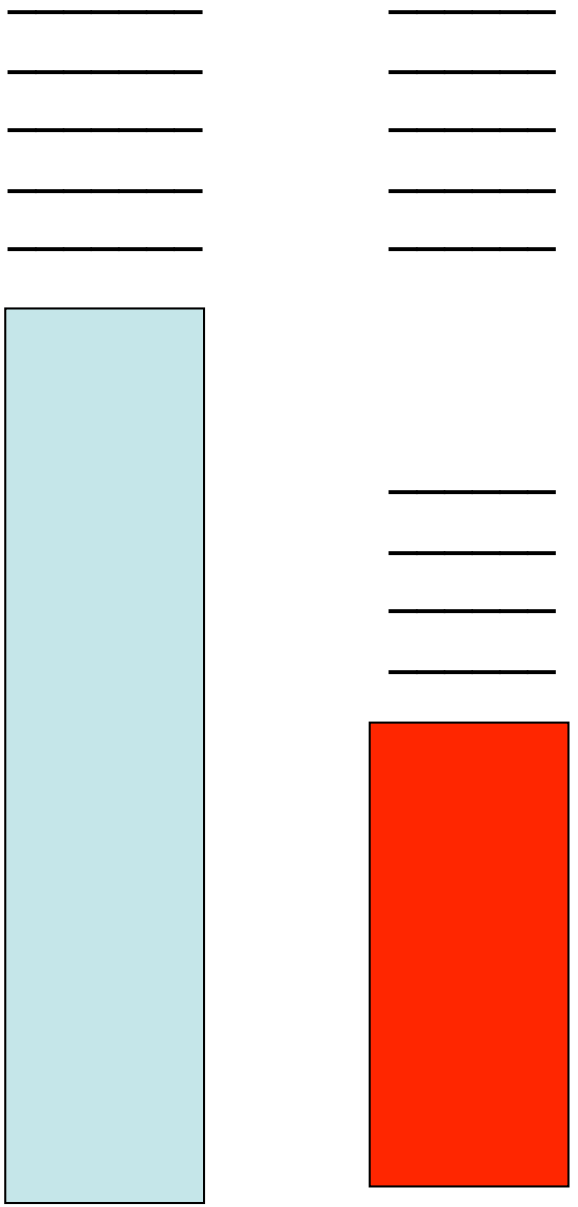
The analysis of the formation amplitude can provide important Information on the mother nucleus B. For instance in the decay ^{212}Po(gs) ➔ ^{208}Pb(gs)+α, one can write within the pairing vibration approximation

$$|^{212}Po(gs)>=|^{210}Po(gs)\otimes^{210}Pb(gs)>$$



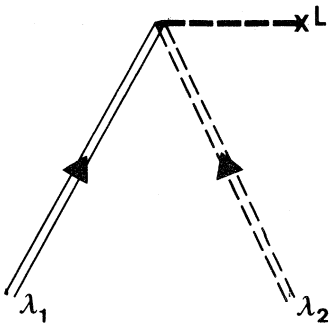
^{212}Po(gs)

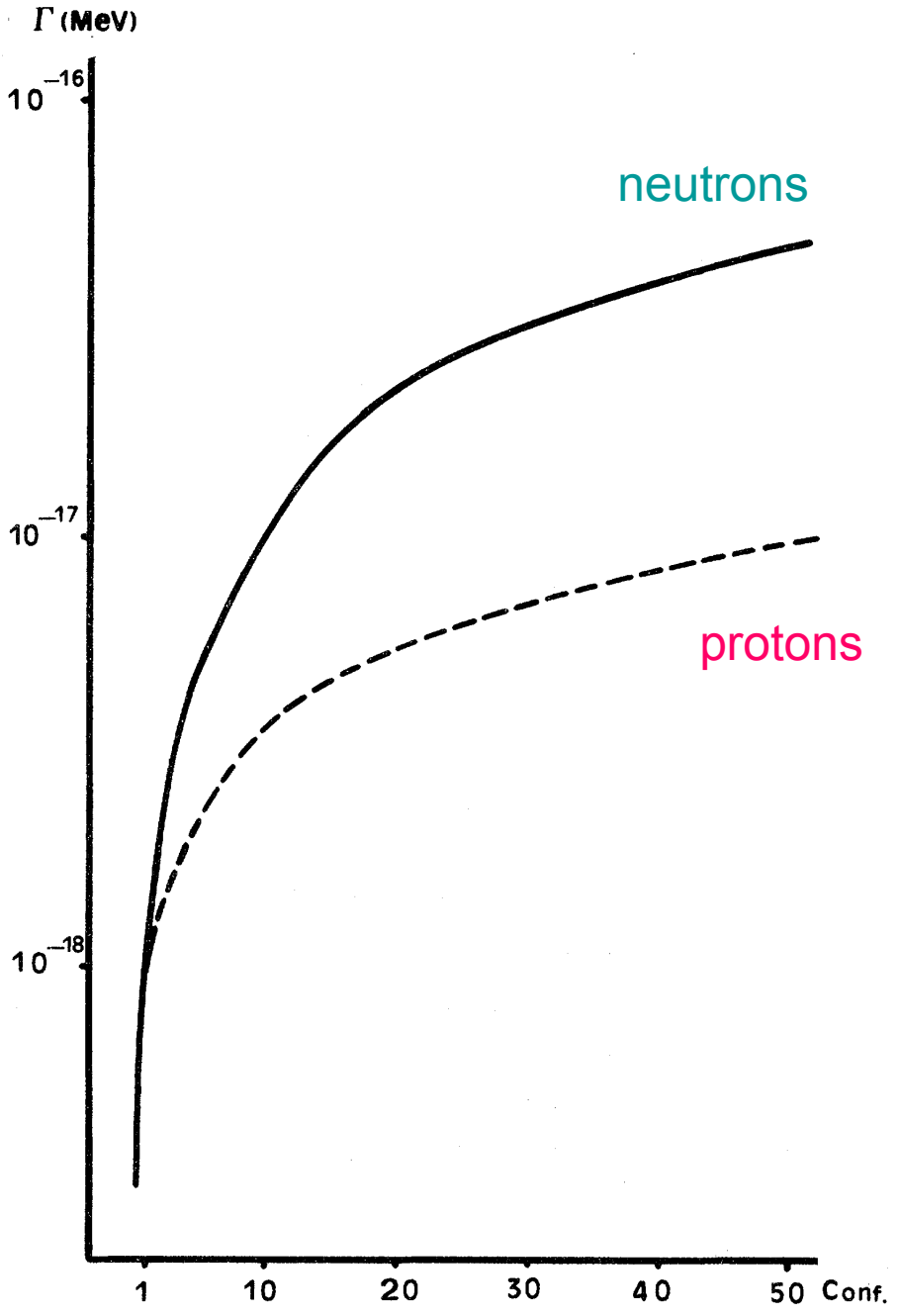
^{208}Pb(gs)



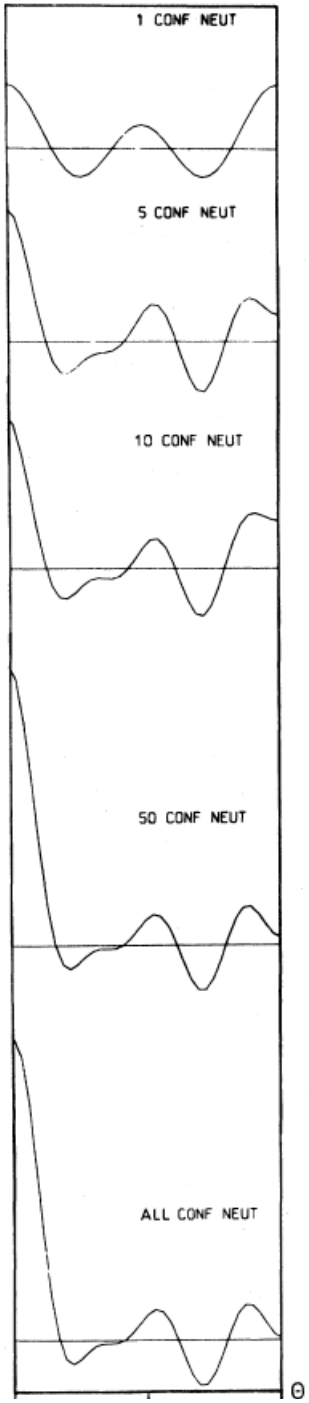
^{209}Pb ^{209}Bi

G. Dodig-Crnkovic, F. A. Janouch and RJL, PLB139, 143 (1984)



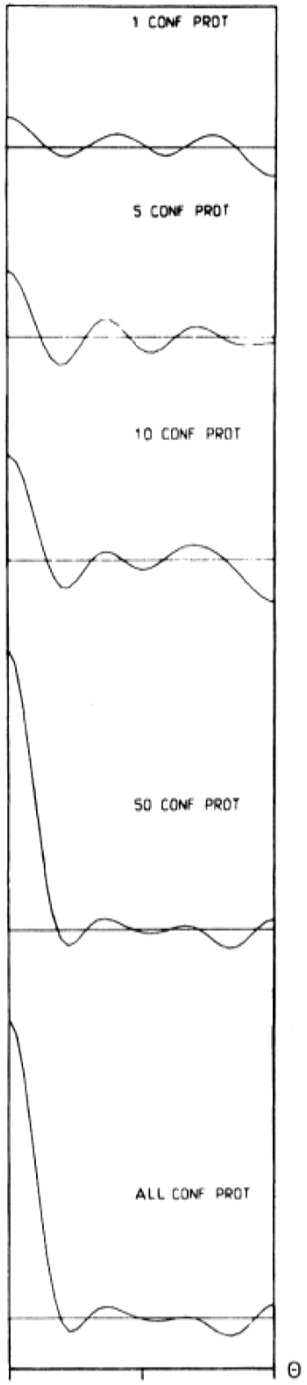


F. Janouch and RJL, PRC 25, 2123 (1982)



^{210}Pb(gs)

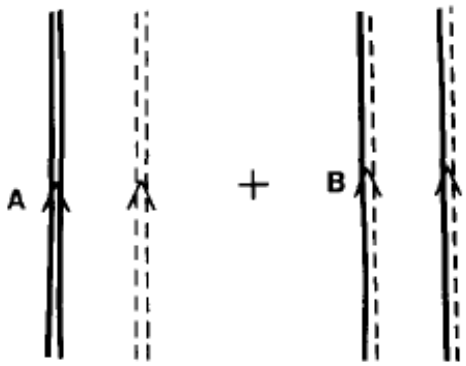
^{210}Po(gs)



$$^{212}Po \rightarrow \alpha + ^{208}Pb$$

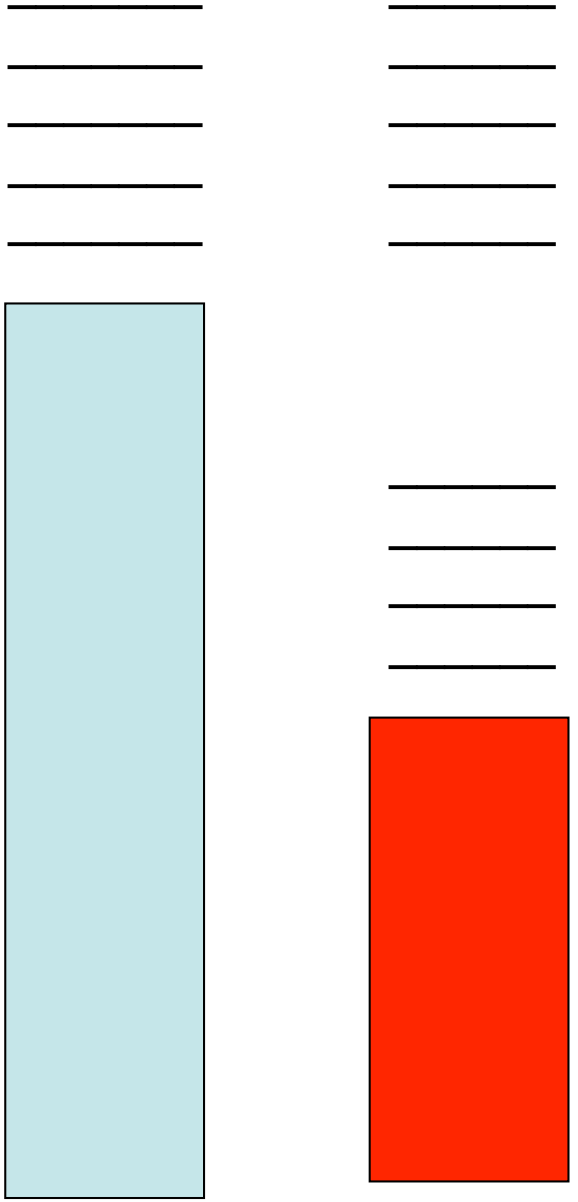
$$^{212}Po = ^{210}Pb \otimes ^{210}Po$$

F. Janouch and RJL, PRC 27, 896 (1983)



^{212}Po(gs)

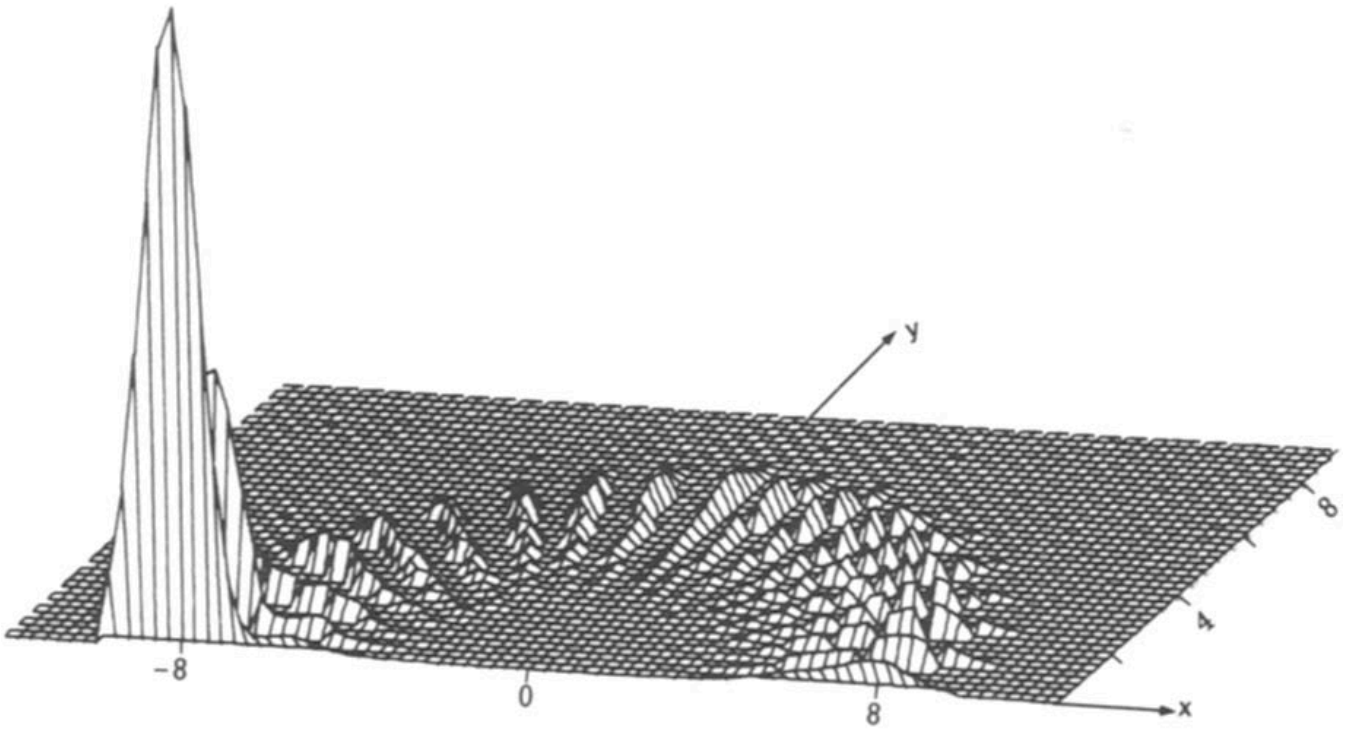
^{208}Pb(gs)



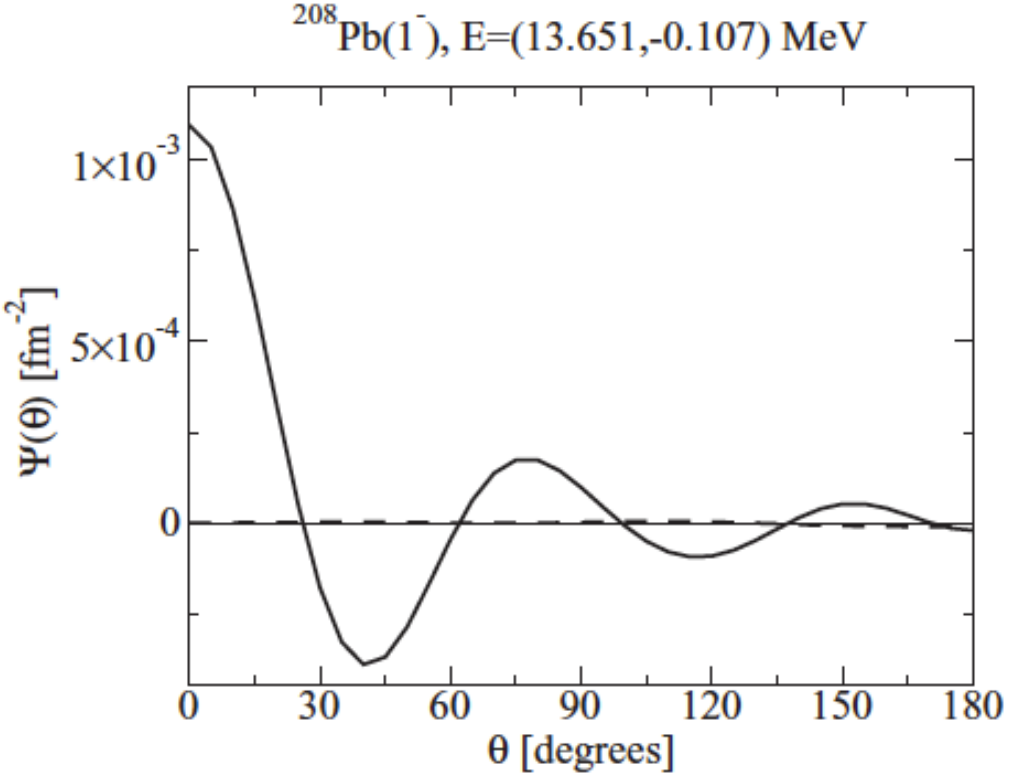
^{209}Pb

^{209}Bi

$$|^{212}Po(gs)> = A|^{210}Pb(gs)\otimes^{210}Po(gs)> + B\,|^{210}Bi(0_1^+)\otimes^{210}Bi(0_1^+)>$$

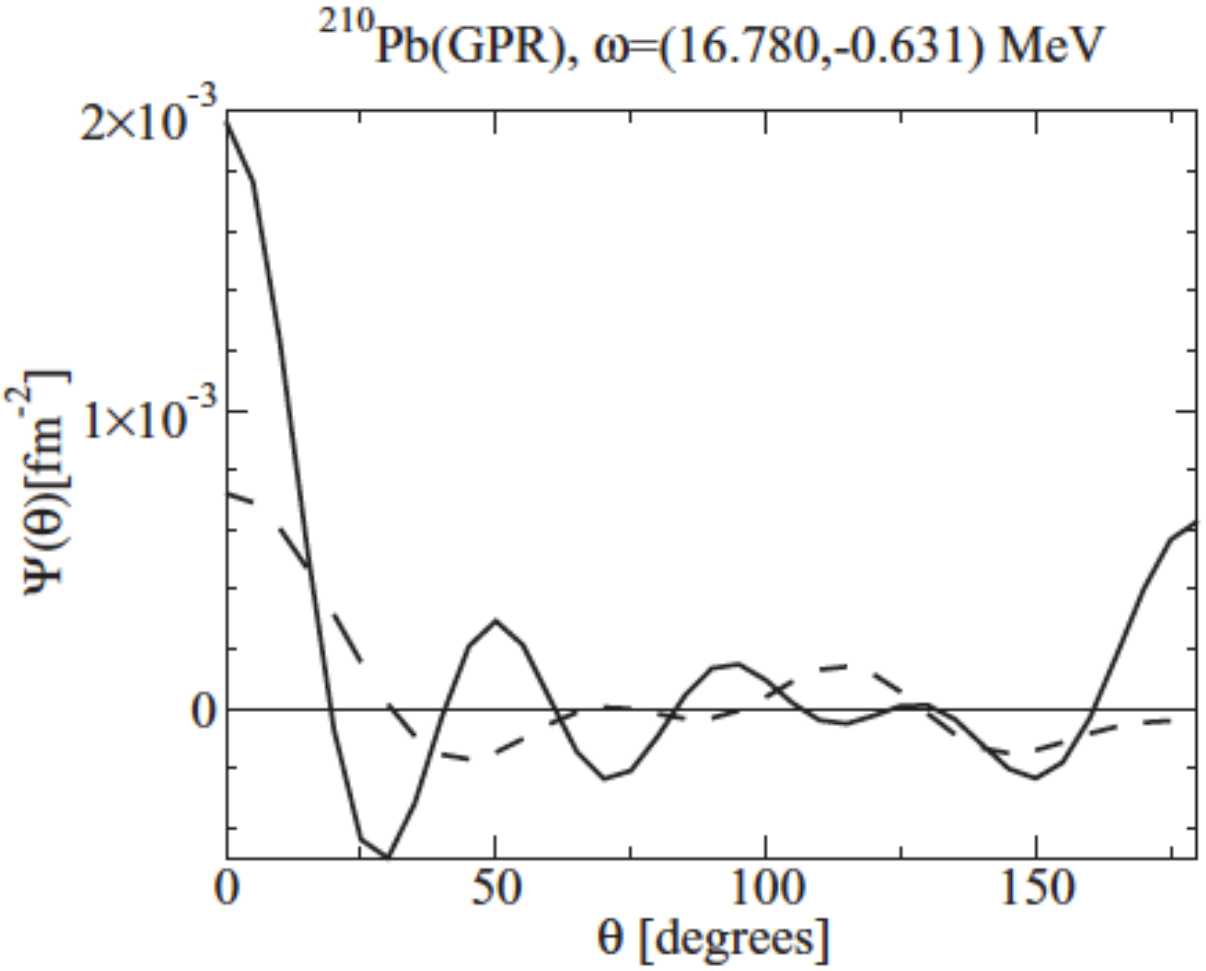


To understand why the giant pairing resonance was not seen in ^{210}Pb the Berggren representation to evaluate ph and pp states was used. It was found that the ph GDR was well explained by the Berggren representation. In particular the ph clustering shows practically only real values for the wave function, as seen below, where the dashed line indicate the imaginary part of the wave function,

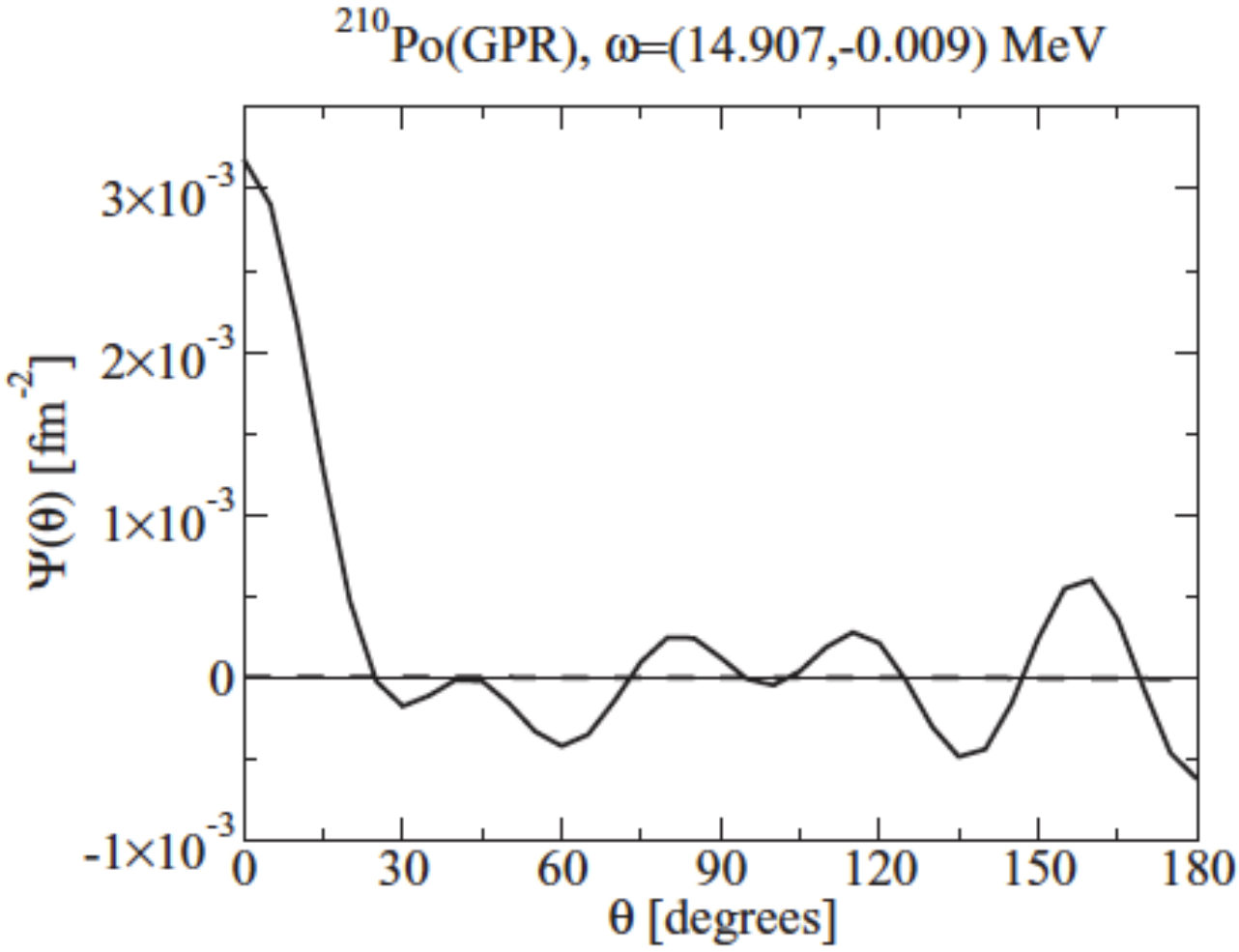


G. G. Dussel et. al., PRC 80, 064311 (2009)

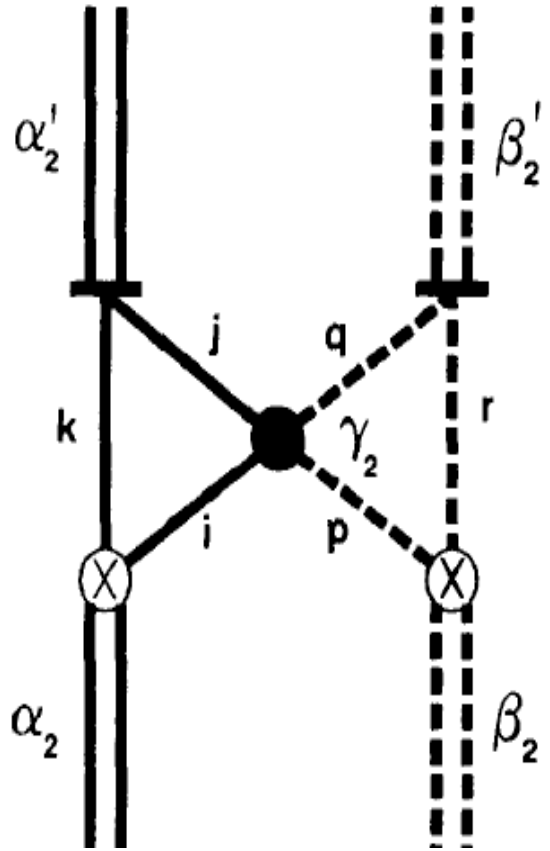
In the case of the GPR evaluated within the Berggren representation it was found that the corresponding energy is very high, at 16.78 Mev, and very wide, with a width of 1.26 MeV. Besides the imaginary part of the wave function is large.



Instead in ^{210}Po the GPR is very narrow and the wave function practically real.



Going back to the pn interaction, the proper shell-model treatment corresponds to the MSM diagram.



but including this contribution with high lying single-particle states the decay width was still one order of magnitude too small.

The corresponding equation is given by

$$\langle \alpha_2 \beta_2; \alpha_4 \mid V_{pn} \mid \alpha_2' \beta_2'; \alpha_4 \rangle = (-1)^{\beta_2+\beta_2'+\alpha_4}$$

$$[\alpha_2]^{1/2}[\alpha_2']^{1/2}[\beta_2]^{1/2}[\beta_2']^{1/2} \sum_{ijk} \sum_{pqr} \sum_{\lambda} (-1)^{\lambda+q+i-j+k}[\lambda]$$

$$\sum_{l}[l] \begin{Bmatrix} \alpha_2 & \alpha_2' & l \\ k & i & j \end{Bmatrix} \begin{Bmatrix} p & i & \lambda \\ k & r & l \end{Bmatrix} \begin{Bmatrix} \beta_2 & \beta_2' & l \\ r & p & q \end{Bmatrix} \begin{Bmatrix} \beta_2 & \beta_2' & l \\ \alpha_2' & \alpha_2 & \alpha_4 \end{Bmatrix}$$

$$Y(ij;\alpha_2)Y(pq;\beta_2)Y(kj;\alpha_2')Y(rq;\beta_2') \langle ip; \lambda \mid V_{pn} \mid kr; \lambda \rangle$$

The main wave function components in ^{212}Po(gs) are

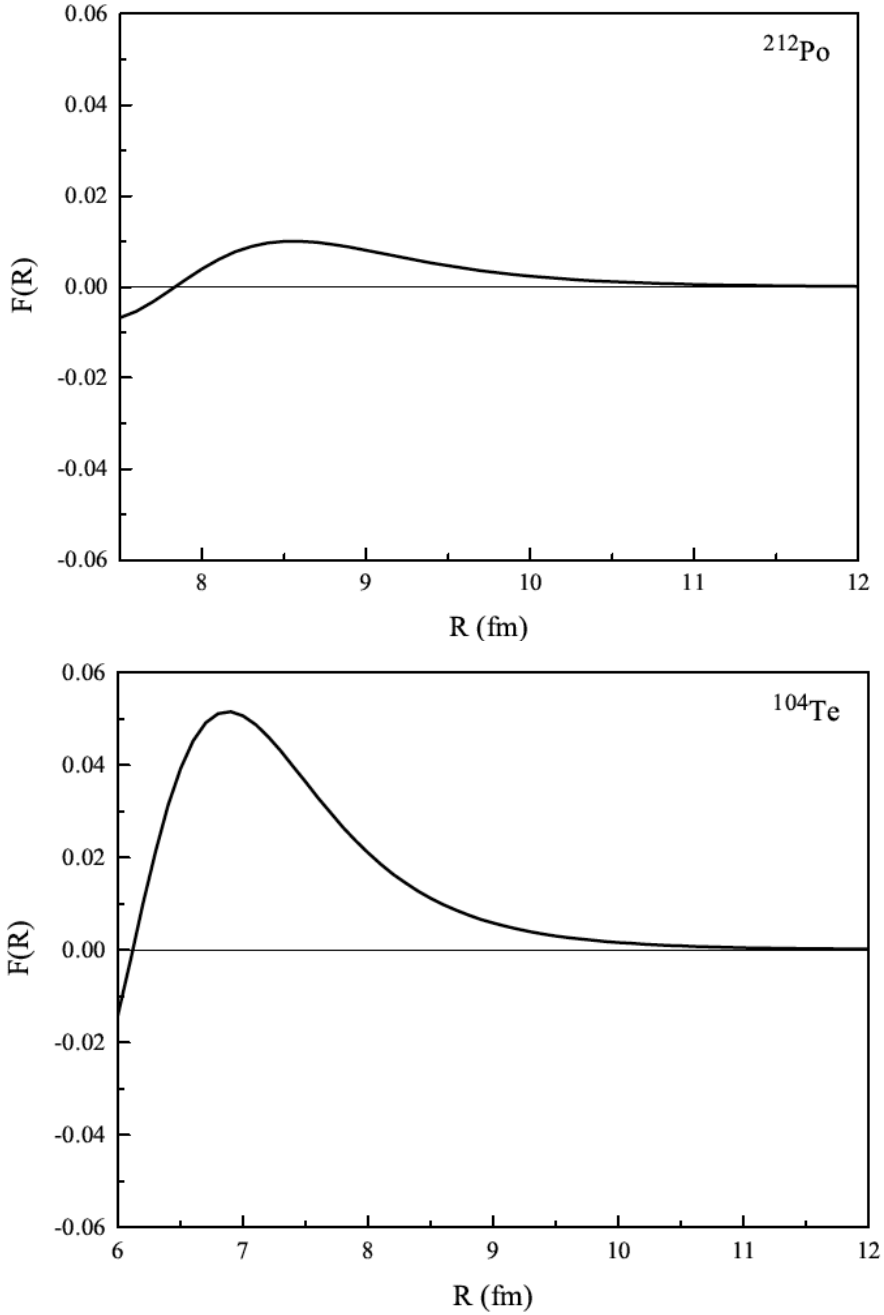
$^{210}\mathrm{Po}(\alpha_2)$	$^{210}\mathrm{Pb}(\beta_2)$	$X(\alpha_2\beta_2;\alpha_4)$
0_1^+	0_1^+	0.913
2_1^+	2_1^+	-0.253
4_1^+	4_1^+	0.122
6_1^+	6_1^+	0.064
8_1^+	8_1^+	0.030

While in ^{104}Te it is,

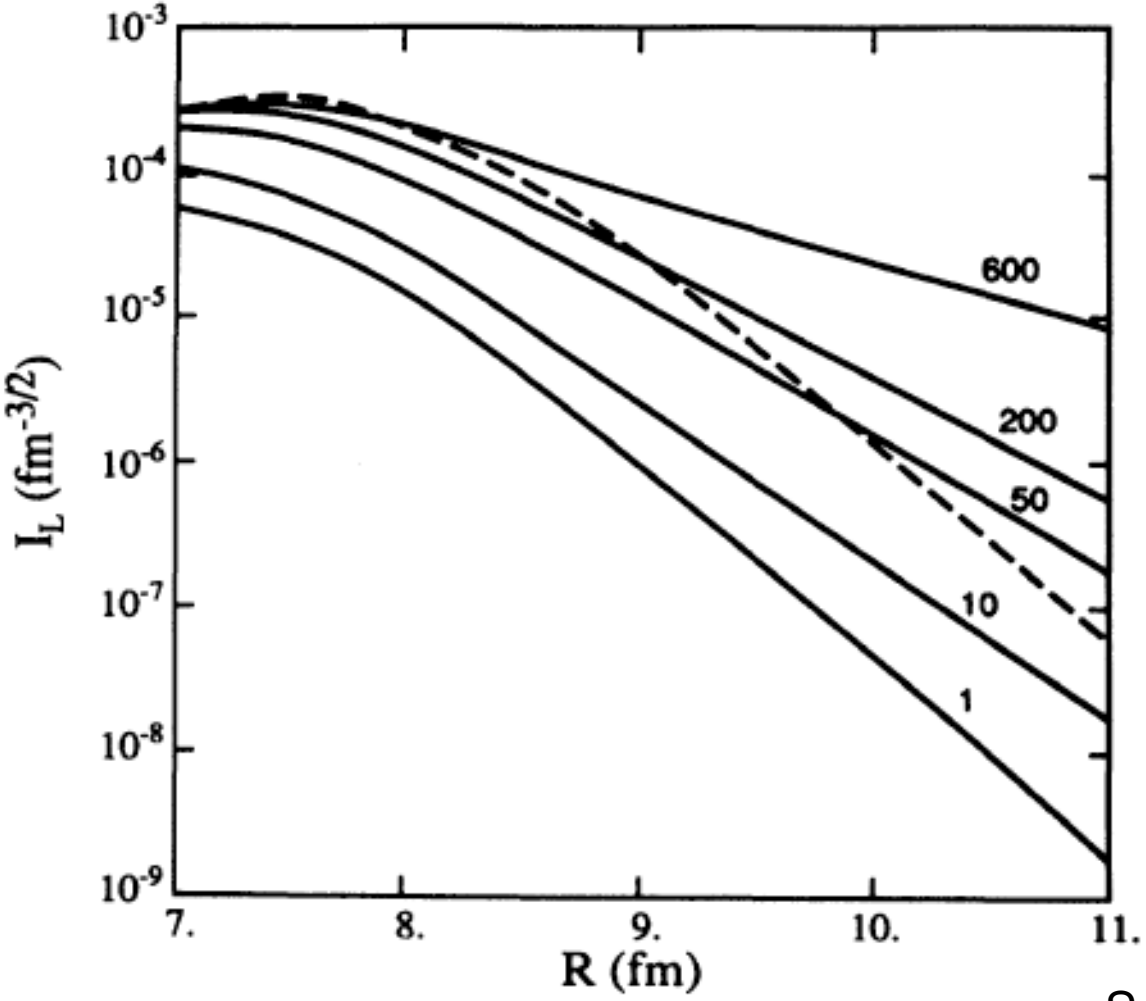
$^{102}\mathrm{Te}(\alpha_2)$	$^{102}\mathrm{Sn}(\beta_2)$	$X(\alpha_2\beta_2;\alpha_4)$
0_1^+	0_1^+	0.544
2_1^+	2_1^+	0.483
4_1^+	4_1^+	0.318
6_1^+	6_1^+	0.228

M. Patial, RL, R. Wyss, PRC 93,054326 (2016)

The formation amplitudes become



One may think that the reason why the decay width is not well explained by using standard shell model representations is due to a wrong asymptotic behaviour. Using a Berggren representation, with wave functions which behave as outgoing waves, one gets similar results, as shown below, where the solid lines correspond to the Berggren representation.



S. Lenzi et. al., PRC 48, 1463(1993)

There have been successful microscopic attemps to describe alpha decay. But in these treatments the mother nucleus was described in terms of shell model plus cluster components, i. e. the wave function of the decaying nucleus was written as (K. Varga, R. Lovas, RJL, PRL **69**, 37 (1992))

$$\Phi = \Phi_{SM} + \Phi_{clus}$$

The important feature here is that the cluster component is assumed to take care of the high lying configurations and therefore the shell model component is evaluated within a major shell only. The cluster component is in this approach written in terms of shifted Gaussian functions.

Fig. 1 of K. Varga, R. G, Lovas, RJL, PRL 69, 37 (1992) shows the importance of the cluster component in the formation amplitude, i. e.,

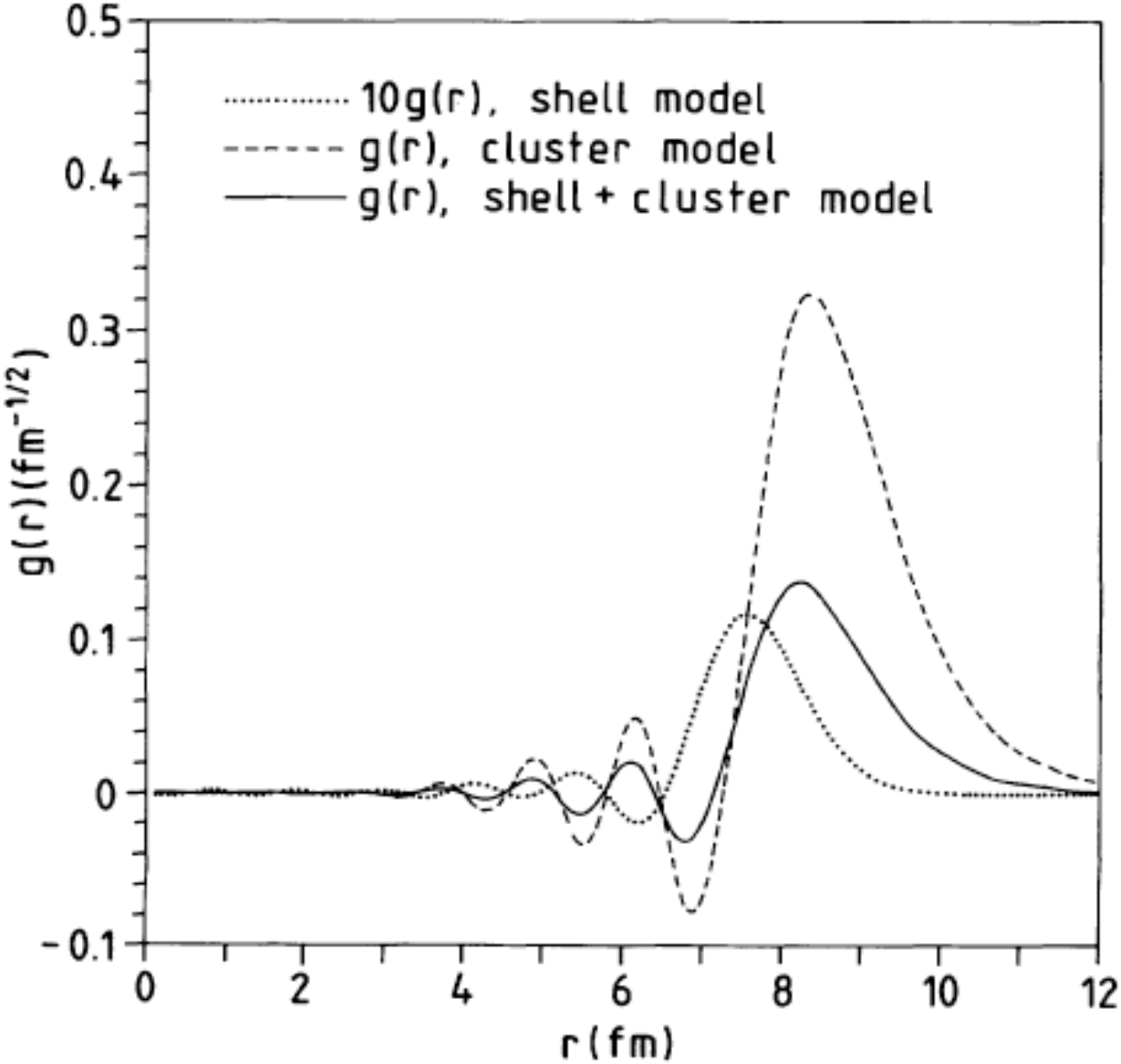


FIG. 1. Formation amplitudes in the pure shell model (multiplied by 10, dotted line), in the pure cluster model (dashed line), and in the combined model (solid line).

This method was applied to describe simultaneously anomalous small B(E2) values and alpha-decay half lives in transitions from ^{212}Po. By using a shifted Gaussian component in the single-particle wave functions it was possible to describe the alpha decay half life, while the shell model component describes the B(E2).

(D. S. Delion, RJL, P. Schuck, A. Astier, M. G. Porquet, PRC **85, 064306 (2012)).**