Alpha-like quartet correlations induced by realistic shell-model interactions

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outline

- pn pairing and the QCM/QM approaches
- realistic even-even and odd-odd N=Z nuclei in the sd shell
- heavy N=Z nuclei: the cases of ^{92}Pd and ^{96}Cd
- conclusions

in collaboration with N. Sandulescu (NIPNE, Bucharest)
and C.W. Johnson (San Diego, California)

pn pairing and the QCM/QM approaches

- The quartet condensate model (QCM) assumes that the ground state of an even-even N=Z nucleus has the form

$$|QCM\rangle = (Q^\dagger)^n|0\rangle$$

- The quartet model (QM) assumes instead that

$$|QM\rangle = \prod_{\rho=1}^{n} Q^\dagger_\rho|0\rangle$$

- Common features:
  - N,T,J are exactly preserved.
  - Quartets are constructed variationally for each nucleus.

isovector pairing

- The Hamiltonian:

$$H^{(iv)} = \sum_i \epsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ij} V_{ij} \sum_\tau P^\dagger_{i,\tau} P_{j,\tau}$$

$$P^\dagger_{i,\tau} = [a^\dagger_i a^\dagger_{\bar{i}}]^{T=1}_\tau$$

- The quartets:

$$Q^\dagger_\rho = \sum_{ij} c^{(\rho)}_{ij} [P^\dagger_i P^\dagger_j]^{T=0}$$

isovector pairing: results

Deformed single-particle basis (HF + Skyrme force SLy4)
V_{ij}=-24/A

	exact	QM	QCM
^{20}Ne	-6.5505	-6.5505	-6.539 (0.18%)
^{24}Mg	-8.4227	-8.4227	-8.388 (0.41%)
^{28}Si	-9.6610	-9.6610	-9.634 (0.28%)
^{32}S	-10.2629	-10.2629	-10.251 (0.12%)
^{44}Ti	-3.1466	-3.1466	-3.142 (0.15%)
^{48}Cr	-4.2484	-4.2484	-4.227 (0.50%)
^{52}Fe	-5.4532	-5.4531	-5.426 (0.50%)
^{104}Te	-1.0837	-1.0837	-1.082 (0.16%)
^{108}Xe	-1.8696	-1.8696	-1.863 (0.35%)
^{112}Ba	-2.7035	-2.7034	-2.688 (0.57%)

M.S. and N. Sandulescu, PRC 88, 061303(R) (2013)

like-particle pairing: exact and PBCS solutions

$$H = \sum_{i=1}^{\Omega} \epsilon_i \mathcal{N}_i - g \sum_{i,i'=1}^{\Omega} P_i^\dagger P_{i'}$$

$$\mathcal{N}_i = \sum_{\sigma} a_{i\sigma}^\dagger a_{i\sigma}, \qquad P_i^\dagger = a_{i+}^\dagger a_{i-}^\dagger, \qquad (P_i^\dagger)^\dagger = P_i$$

The exact ground state (Richardson, 1963)

$$|\Psi_{gs}\rangle = \prod_{\nu=1}^{N} B_\nu^\dagger |0\rangle, \qquad B_\nu^\dagger = \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_\nu} P_k^\dagger, \qquad E^{(\Psi)} = \sum_{\nu=1}^{N} E_\nu$$

The projected BCS ground state

$$|PBCS\rangle \propto (B^\dagger)^N |0\rangle, \qquad B^\dagger = \sum_{k=1}^{\Omega} x_k P_k^\dagger$$

beyond pn pairing

- Extending the quartet formalism to realistic even-even N=Z nuclei:
  - which quartets to involve?
  - how to construct them?
  - what to do with them?

^{24}Mg in a formalism of quartets

$^{24}_{12}\text{Mg}_{12}$ = 4 protons + 4 neutrons outside the ^{16}O core.

- We want to represent its states as linear superpositions of

$$[Q^+_{\alpha J_1 T_1} Q^+_{\beta J_2 T_2}]^{J,T=0}_{M,T_z=0}|0\rangle$$

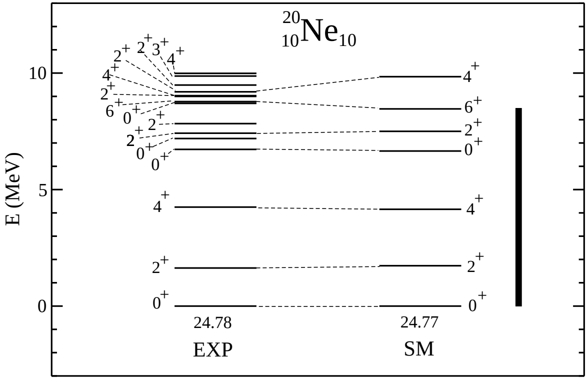
$$Q^+_{\alpha,JM,TT_z} = \sum_{i_1 j_1 J_1 T_1} \sum_{i_2 j_2 J_2 T_2} C^{(\alpha)}_{i_1 j_1 J_1 T_1, i_2 j_2 J_2 T_2} \times \left[[a^+_{i_1} a^+_{j_1}]^{J_1 T_1} [a^+_{i_2} a^+_{j_2}]^{J_2 T_2} \right]^{JT}_{MT_z}$$

- Quartets are chosen according to the following general criterion:

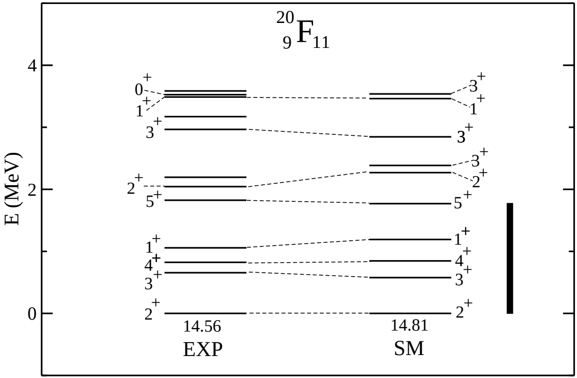
  the Q^+_{JT}'s are the quartets describing the low-lying states of nuclei with four active particles outside the inert core of reference.

- We perform a configuration interaction calculation in the quartet space.

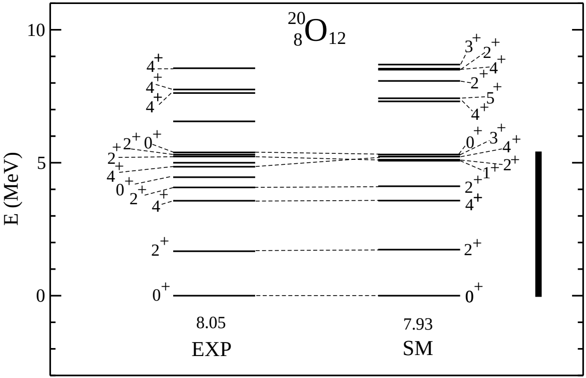
^{20}Ne: T=0 quartets



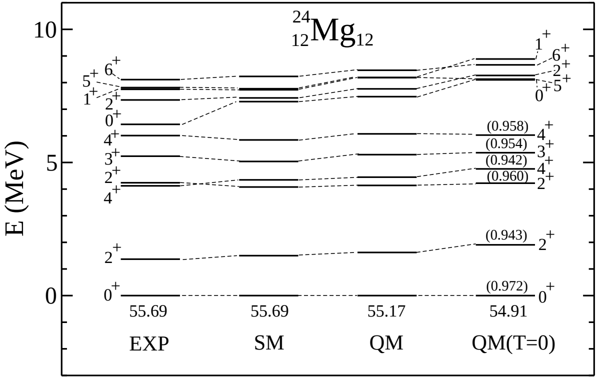
^{20}F: T=1 quartets



^{20}O: T=2 quartets

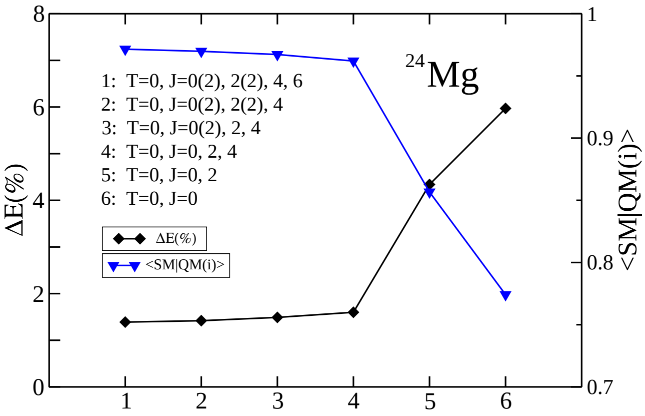


^{24}Mg: the spectrum

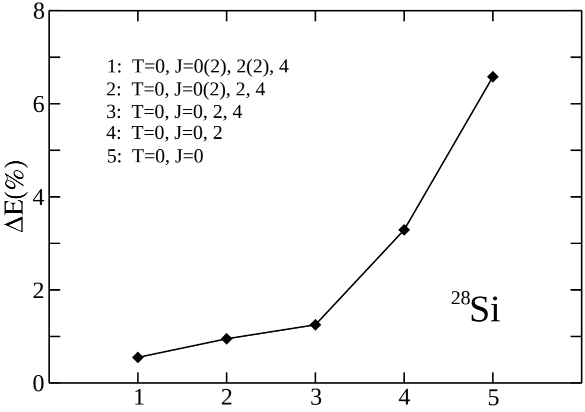


M.S. and N. Sandulescu, PRL 115, 112501 (2015)

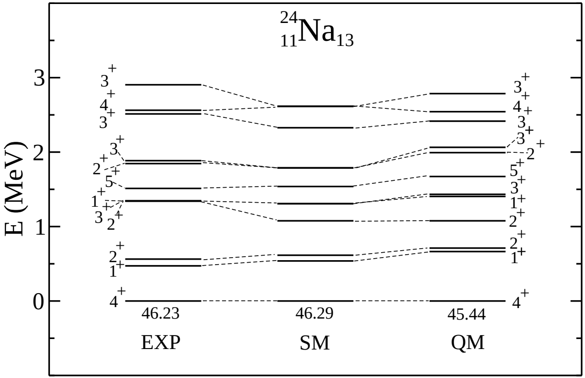
^{24}Mg: ground state correlation energy



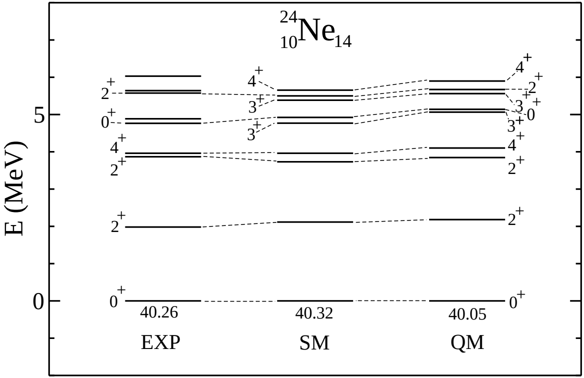
^{28}Si: ground state correlation energy



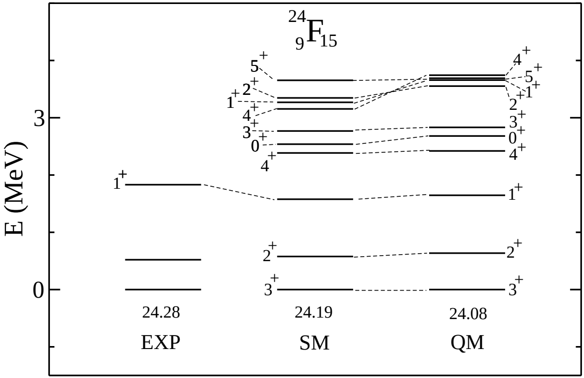
^{24}Na

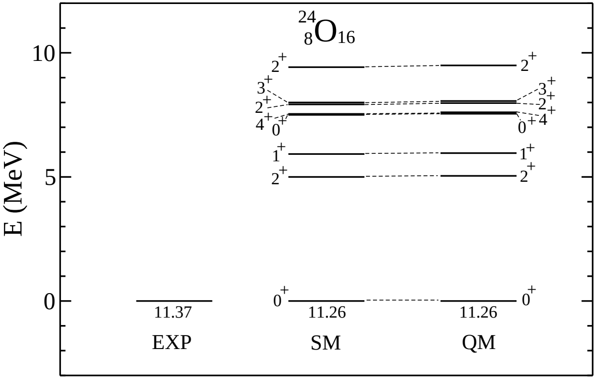


^{24}Ne



^{24}F




$^{24}_{8}O_{16}$
E (MeV)
10
5
0
2^+
3^+
2^+
4^+
0^+
1^+
2^+
0^+
11.37
11.26
11.26
EXP
SM
QM

like-particle pairing and quartets

$$|\Psi\rangle = \prod_{\lambda=1}^{N/2} B^\dagger_{2\lambda-1} B^\dagger_{2\lambda} |0\rangle$$

$$B_i^\dagger = \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_i} P_k^\dagger, \qquad E_{2\lambda-1} = \xi_\lambda - i\eta_\lambda, \qquad E_{2\lambda} = \xi_\lambda + i\eta_\lambda$$

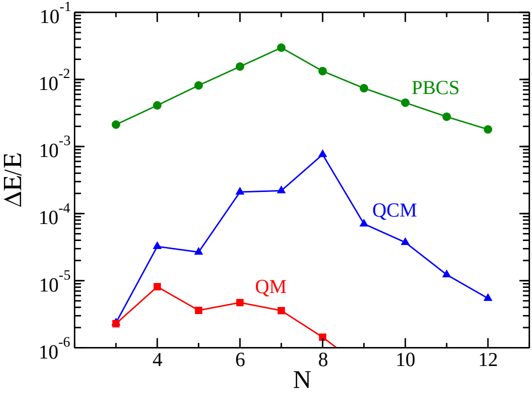
It follows that

$$B^\dagger_{2\lambda-1} B^\dagger_{2\lambda} = \sum_{k,k'=1}^{\Omega} \gamma_{kk'}^{(\lambda)} P_k^\dagger P_{k'}^\dagger, \qquad \gamma_{kk'}^{(\lambda)} \in R$$

$$|\Psi\rangle = \prod_{\lambda=1}^{N/2} Q_\lambda^\dagger |0\rangle, \qquad Q_\lambda^\dagger \equiv B^\dagger_{2\lambda-1} B^\dagger_{2\lambda}$$

M.S., PRC 75, 054314 (2007)

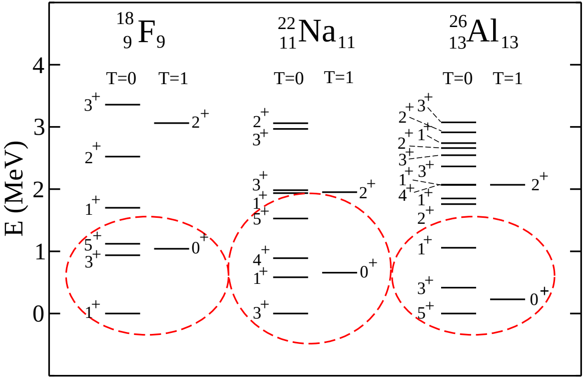
Sn isotopes: ground state correlation energies



(ϵ_i and $g_{ii'}$ from Zelevinsky & Volya, Phys. of Atomic Nuclei, 66, 1781 (2003))

M.S. and N. Sandulescu, J. Phys. G: Nucl. Part. Phys. 40, 055107 (2013)

odd-odd self-conjugate nuclei in the sd shell



M.S. and N. Sandulescu, arXiv:1608.01105

odd-odd formalism

We want to represent the states of ^{22}Na as linear superpositions of

$$Q^+_{J_1 M_1} P^+_{JM,T0}|0\rangle$$

and the states of ^{26}Al as linear superpositions of

$$Q^+_{J_1 M_1} Q^+_{J_2 M_2} P^+_{JM,T0}|0\rangle$$

with

$$Q^+_{JM} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q_{i_1 j_1 J_1, i_2 j_2 J_2, T'} [[a^+_{i_1} a^+_{j_1}]^{J_1 T'} [a^+_{i_2} a^+_{j_2}]^{J_2 T'}]^{JT=0}_M$$

and

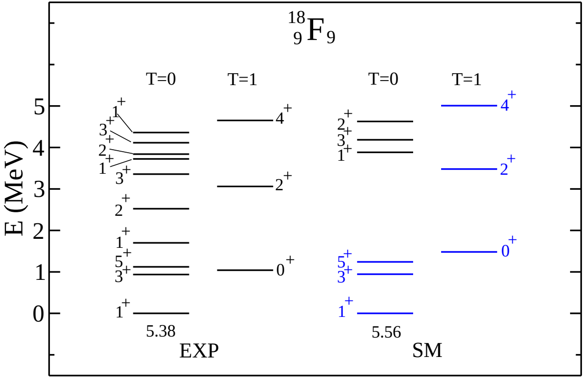
$$P^+_{JM,TT_z} = \sum_{ij} p_{ij} [a^+_i a^+_j]^{JT}_{MT_z}$$

Which Q^+_{JM} and $P^+_{JM,T0}$?

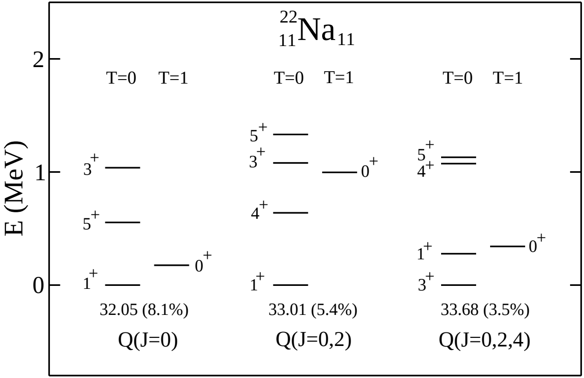
choice of quartets and pairs

- As T=0 quartets we assume those describing the lowest J=0,2,4 states of ^{20}Ne
- As T=0 (T=1) pairs we assume those describing the lowest J=1,3,5 (J=0,2,4) states of ^{18}F

^{18}F

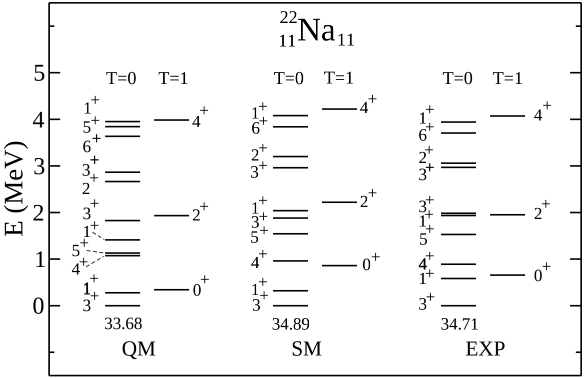


^{22}Na

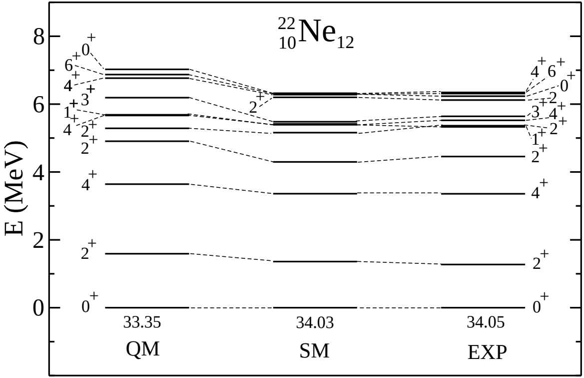


$^{22}_{11}Na_{11}$
E (MeV)
T=0
T=1
3^+
5^+
1^+
0^+
4^+
32.05 (8.1%)
33.01 (5.4%)
33.68 (3.5%)
Q(J=0)
Q(J=0,2)
Q(J=0,2,4)

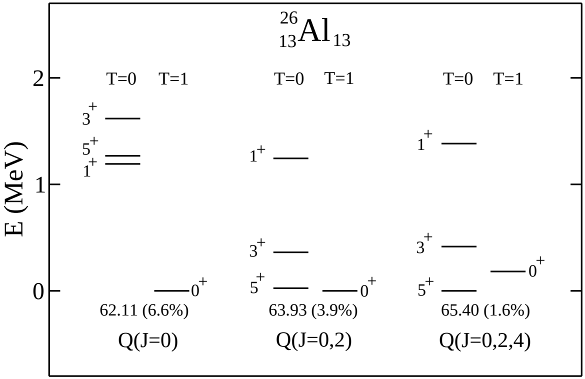
^{22}Na



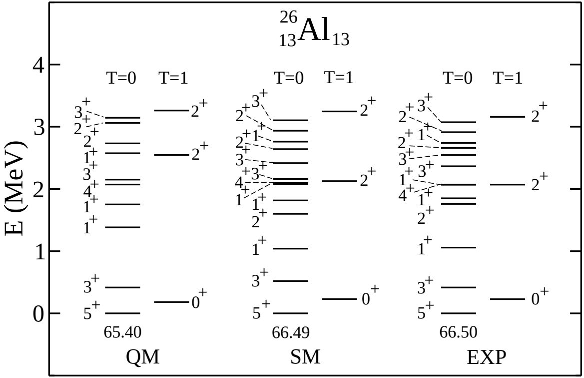
^{22}Ne



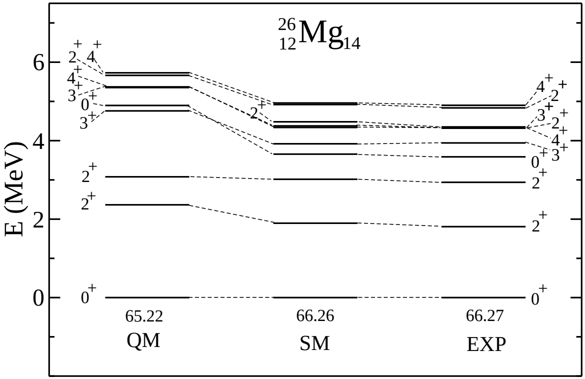
^{26}Al


$^{26}_{13}Al_{13}$
E (MeV)
0
1
2
T=0
T=1
3^+
5^+
1^+
0^+
62.11 (6.6%)
Q(J=0)
T=0
T=1
1^+
3^+
5^+
0^+
63.93 (3.9%)
Q(J=0,2)
T=0
T=1
1^+
3^+
5^+
0^+
65.40 (1.6%)
Q(J=0,2,4)

^{26}Al



^{26}Mg



Model spaces

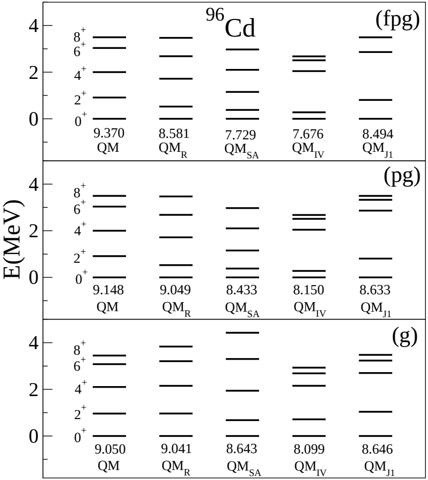
- g space: $0g_{9/2}$ (renormalized SLGT0)
- pg space: $1p_{1/2},\ 0g_{9/2}$ (F-FIT)
- fpg space: $0f_{5/2},\ 1p_{1/2},\ 1p_{3/2},\ 0g_{9/2}$ (JUN45)

^{96}Cd and ^{92}Pd: approximation schemes

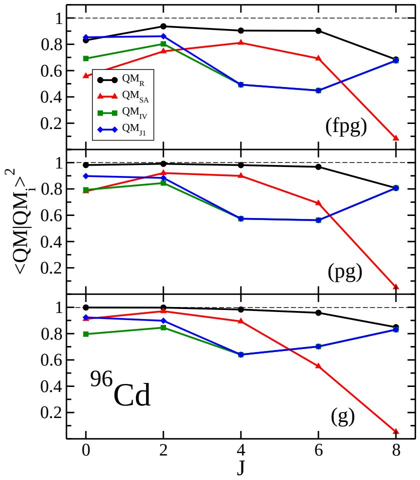
We have explored four different approximation schemes:

- QM$_{SA}$: $$\left[[a^+_{i_1} a^+_{j_1}]^{J_1=9\,T_1=0}[a^+_{i_2} a^+_{j_2}]^{J_2=9\,T_2=0}\right]^{JT=0}$$
- QM$_{IV}$: $$\left[[a^+_{i_1} a^+_{j_1}]^{J_1=0\,T_1=1}[a^+_{i_2} a^+_{j_2}]^{J_2=J\,T_2=1}\right]^{JT=0}$$
- QM$_{R}$: $$\mathrm{QM}_{IV} + \mathrm{QM}_{SA}$$
- QM$_{J1}$: $$\mathrm{QM}_{IV} + \left[[a^+_{i_1} a^+_{j_1}]^{J_1=1\,T_1=0}[a^+_{i_2} a^+_{j_2}]^{J_2=1\,T_2=0}\right]^{JT=0}$$

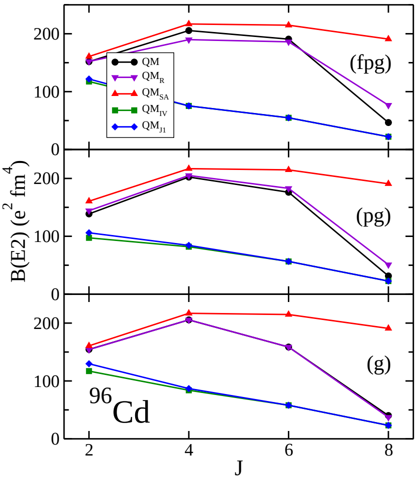
^{96}Cd: energies



^{96}Cd: overlaps



^{96}Cd: B(E2; J $\rightarrow$ J-2)



^{92}Pd in a formalism of quartets

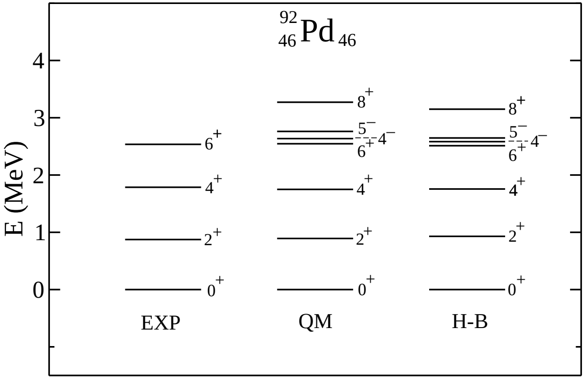
$^{92}_{46}$Pd$_{46}$ = 4 protons holes + 4 neutrons holes in the ^{100}Sn core.

- We represent its states as linear superpositions of

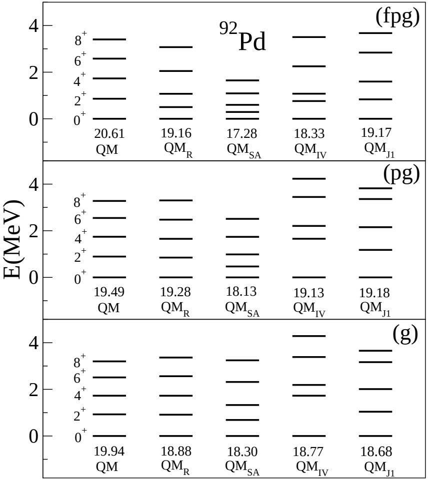
$$[Q^+_{\alpha J_1 T_1} Q^+_{\beta J_2 T_2}]^{J,T=0}_{M,T_z=0}|0\rangle$$

- As quartets we adopt those associated with the positive-parity yrast states of ^{98}Cd up to $J = 8$. These are all T=0 quartets.
- We explore the same approximation schemes employed for ^{96}Cd.
- We carry out configuration interaction calculations in the quartet model space.

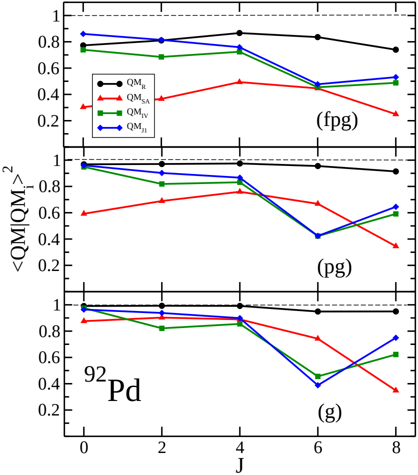
^{92}Pd: testing QM



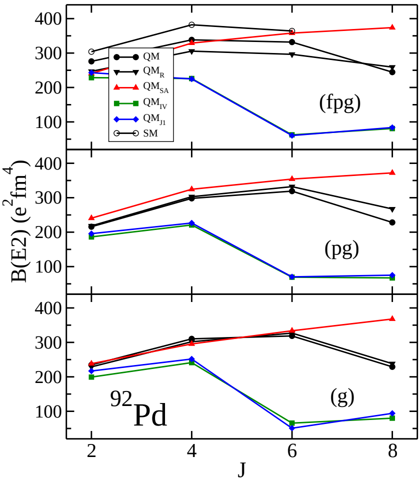
^{92}Pd: energies



^{92}Pd: overlaps



^{92}Pd: B(E2;J $\rightarrow$ J-2)



conclusions

- T=0 quartets play a major role in the structure of the low-lying states of both even-even and odd-odd self-conjugate nuclei.
- T=0, J=0,2,4 quartets account to a very large extent for the ground state correlation energy in even-even nuclei, the J=0 quartet being by far the most relevant one.
- T=0, J=0,2,4 quartets in N=Z nuclei appear to play a role analogous to that of S,D,G pairs in ordinary nuclei.
- T=0 (T=1) spectra of odd-odd spectra in the sd shell are well described in terms of T=0, J=0,2,4 quartets and T=0, J=1,3,5 (T=1, J=0,2,4) pairs.