Faddeev-type description of few-body nuclear reactions including core excitation

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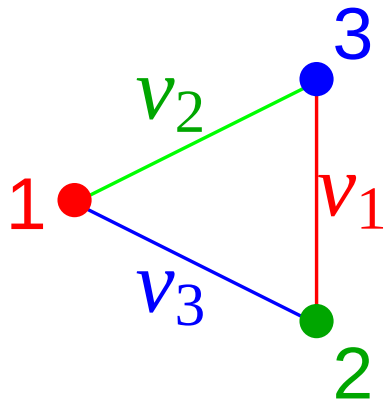
Few-body scattering

- 3-particle scattering equations
- extension: including core excitation
- 3-body nuclear reactions

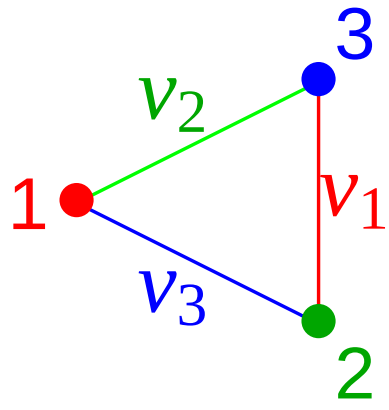
- 4-particle scattering equations
- 4N reactions
- 4-body nuclear reactions

Three-particle system

Hamiltonian $H_0 + \sum_{\alpha} v_{\alpha}$



Three-particle system



Hamiltonian $H_0 + \sum_\alpha v_\alpha$

- Faddeev equations

$$(E - H_0 - v_\alpha)|\psi_\alpha\rangle = v_\alpha \sum_\sigma \bar{\delta}_{\alpha\sigma}|\psi_\sigma\rangle$$

$$|\Psi\rangle = \sum_\alpha |\psi_\alpha\rangle$$

Alt, Grassberger, and Sandhas equations

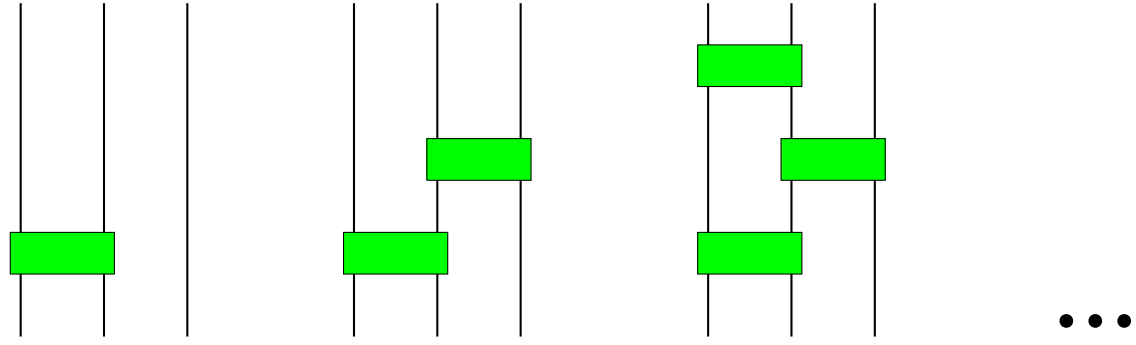
$$U_{\beta\alpha} = \overline{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \overline{\delta}_{\beta\sigma} T_\sigma G_0 U_{\sigma\alpha}$$

$$U_{0\alpha} = G_0^{-1} + \sum_{\sigma} T_\sigma G_0 U_{\sigma\alpha}$$

$$T_\sigma = v_\sigma + v_\sigma G_0 T_\sigma$$

$$G_0 = (E + i0 - H_0)^{-1}$$

channel states $\quad (E - H_0 - v_\alpha)|\phi_\alpha\rangle = 0$



...

AGS equations with 3BF

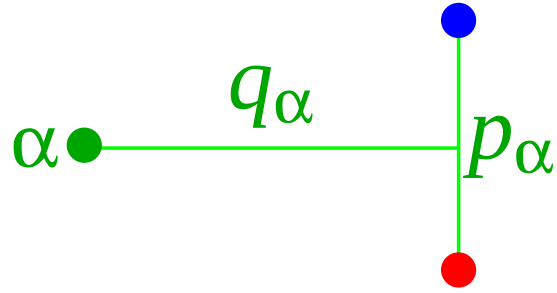
$$V_{3BF} = \sum_{\alpha=1}^{3} w_\alpha$$

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_\gamma \bar{\delta}_{\beta\gamma} T_\gamma G_0 U_{\gamma\alpha}$$

$$+ w_\alpha + \sum_\gamma w_\gamma G_0 (1 + T_\gamma G_0) U_{\gamma\alpha}$$

AGS equations: numerical solution

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_\sigma G_0 U_{\sigma\alpha}$$



- 3 sets of Jacobi momenta
- momentum-space partial wave basis
- set of coupled 2-variable integral equations
- integrable singularities in kernel
- Coulomb interaction: screening and renormalization

[PRC 71, 054005; PRC 72, 054004; PRC 76, 064001]

Application to 3-body nuclear reactions

$$\left.\begin{matrix} p+(nA) \\ d+A \end{matrix}\right\} \rightarrow \begin{cases} n+(pA) \\ p+(nA) \\ d+A \\ p+n+A \end{cases}$$

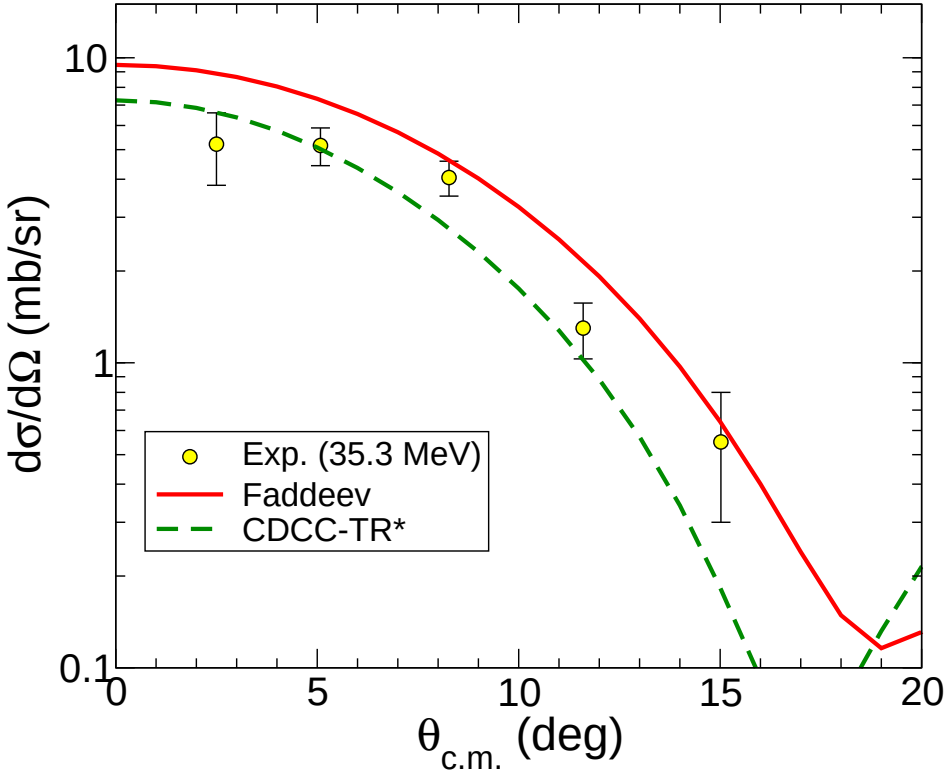
with $A = {}^{4}\mathrm{He},\ {}^{10}\mathrm{Be},\ {}^{12}\mathrm{C},\ {}^{14}\mathrm{C},\ {}^{16}\mathrm{O},\ {}^{28}\mathrm{Si},\ {}^{40}\mathrm{Ca},\ {}^{48}\mathrm{Ca},\ {}^{58}\mathrm{Ni},\ \ldots$

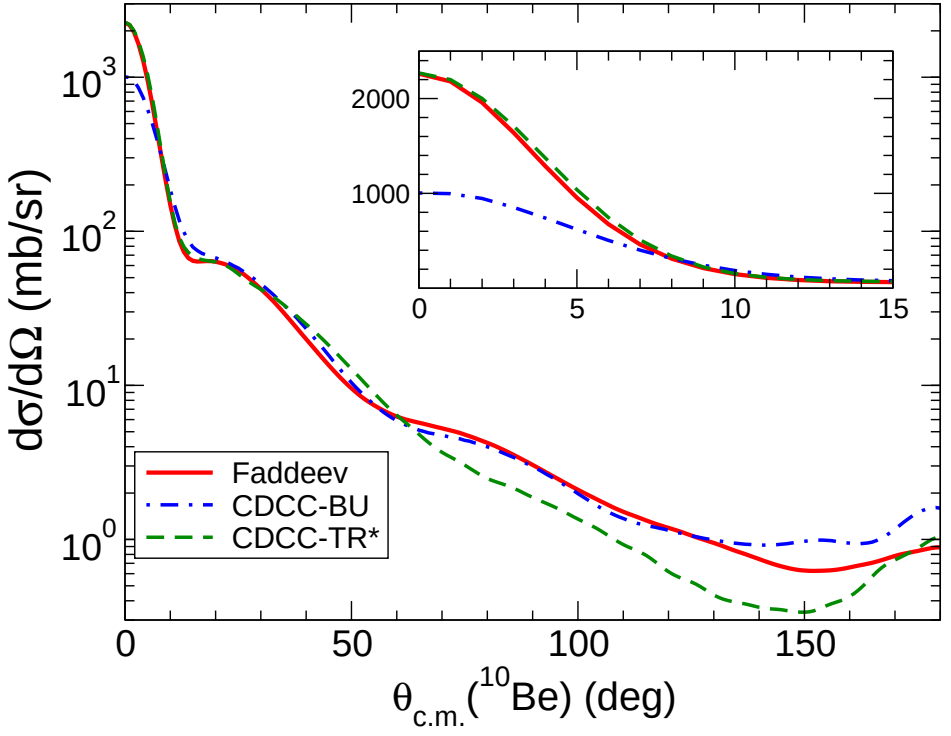
V_{np}: realistic NN potentials (CD Bonn)

V_{NA}: optical/binding NA potentials (CH89, KD, ...)

- validity test of approximate nuclear reaction methods: DWBA, ADWA, CDCC, ...
- novel dynamic input: core excitation, nonlocal potentials, ...

CDCC test: $^{11}\mathrm{Be}(p,d)^{10}\mathrm{Be}$ and $^{11}\mathrm{Be}(p,pn)^{10}\mathrm{Be}$

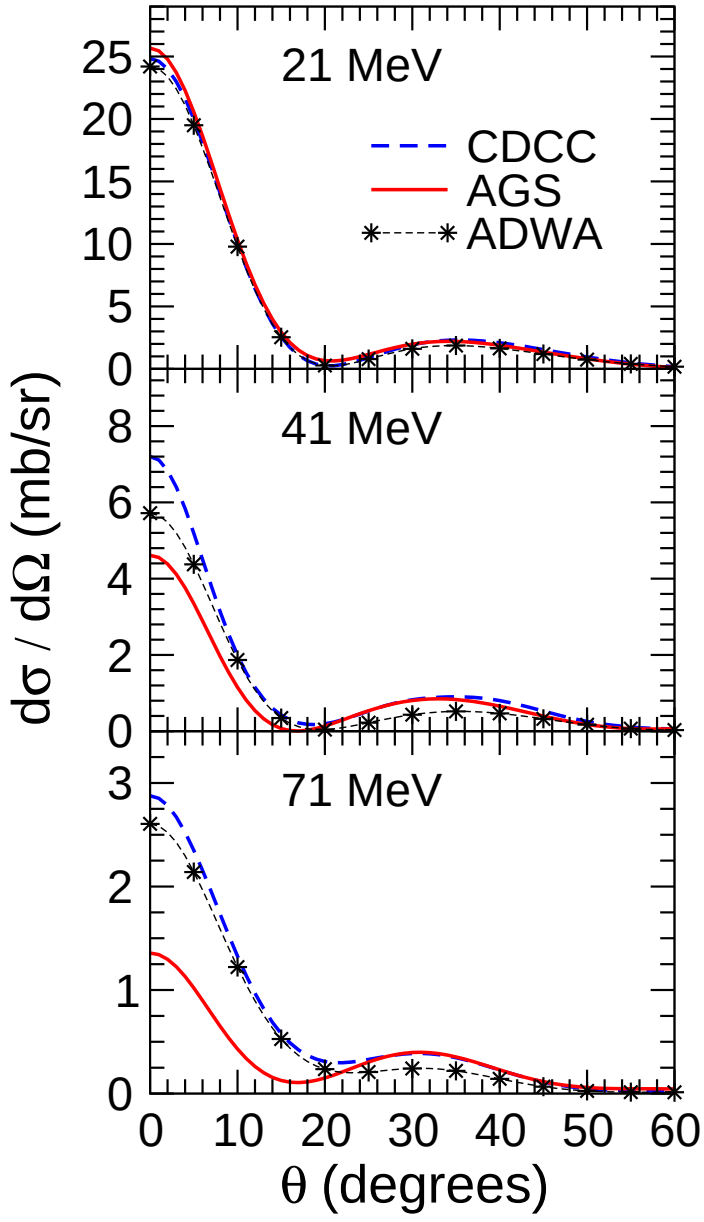


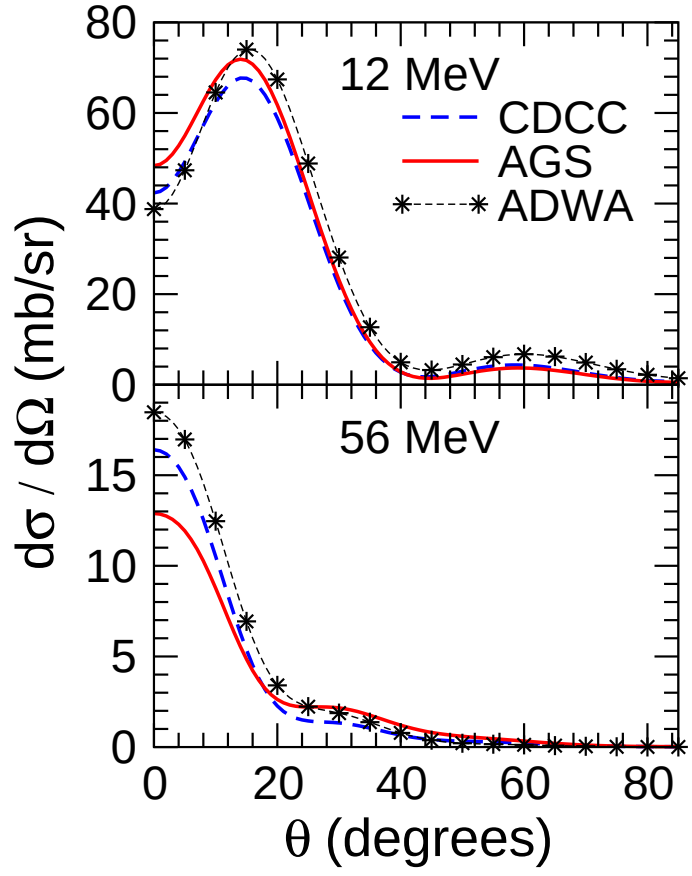


$E/A = 38$ MeV

CDCC: A. M. Moro, F. M. Nunes [PRC 76, 064602]

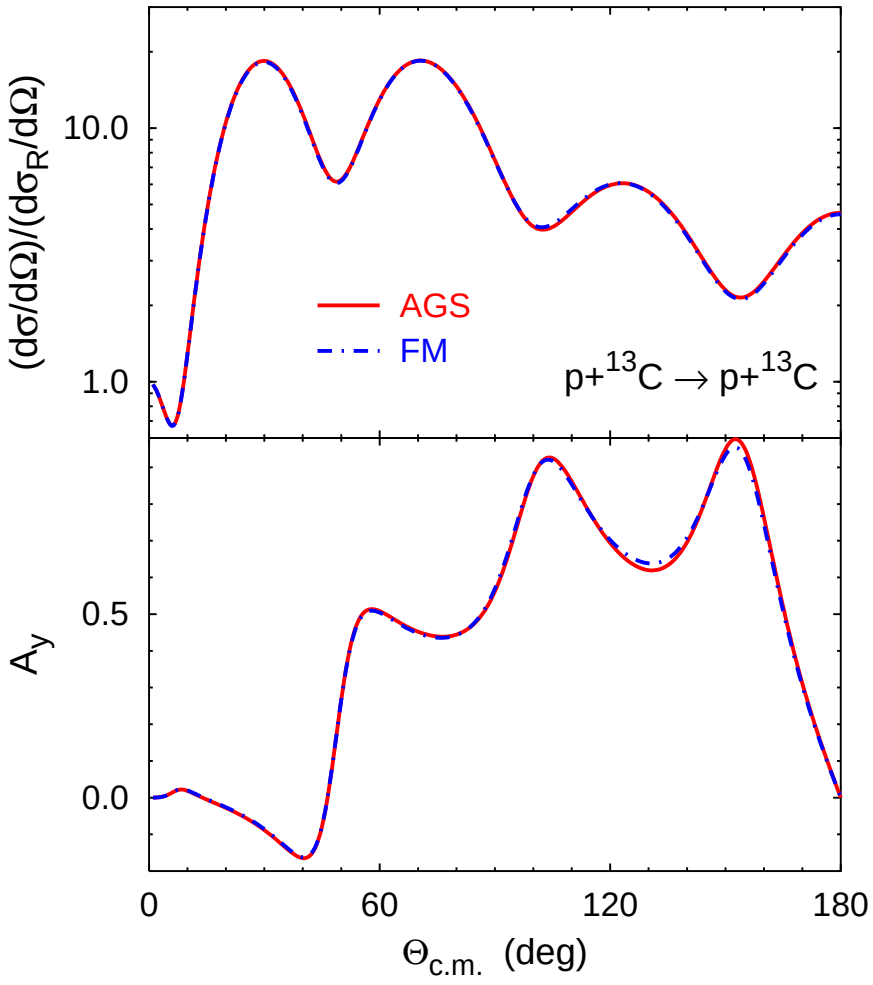
CDCC test: $^{10}\mathrm{Be}(d,p)^{11}\mathrm{Be}$ and $^{12}\mathrm{C}(d,p)^{13}\mathrm{C}$

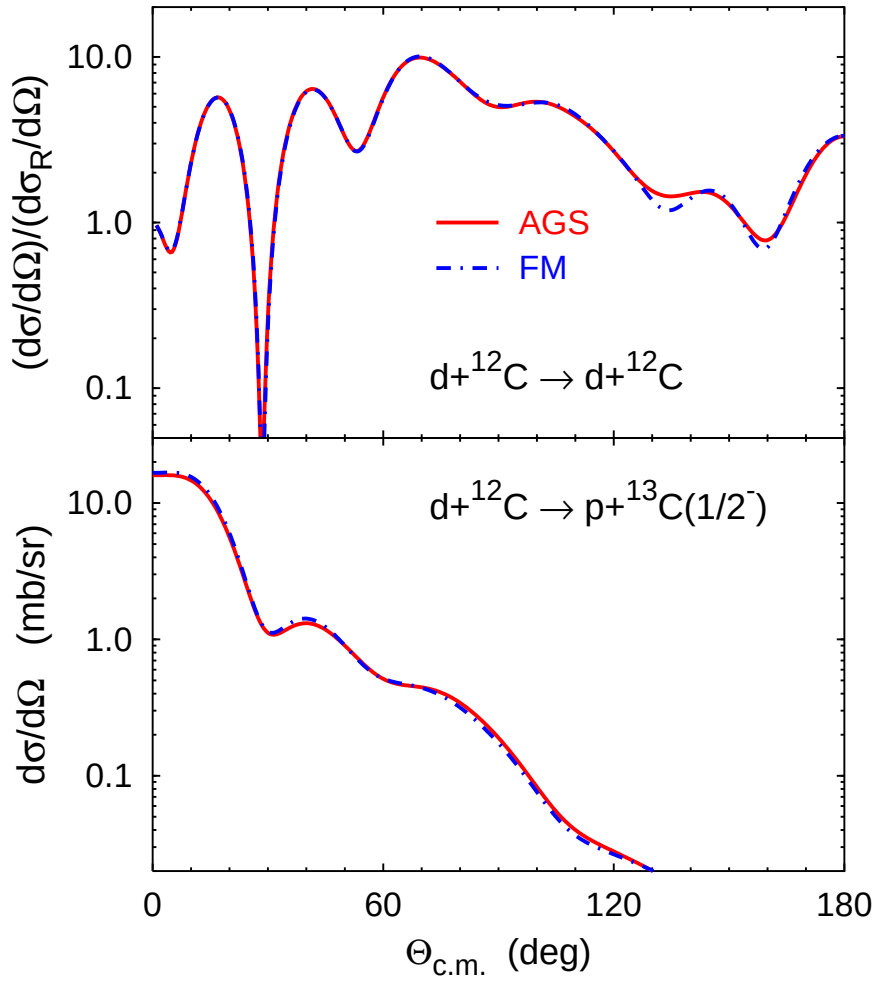




CDCC/ADWA: F. M. Nunes, N. Upadhyay [PRC 85, 054621]

Comparison with r-space FM results

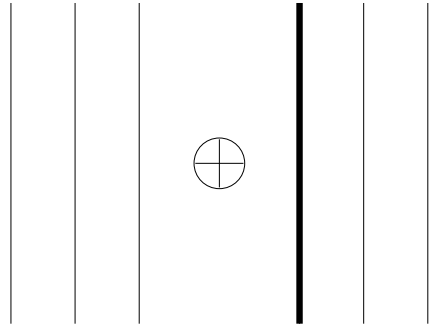




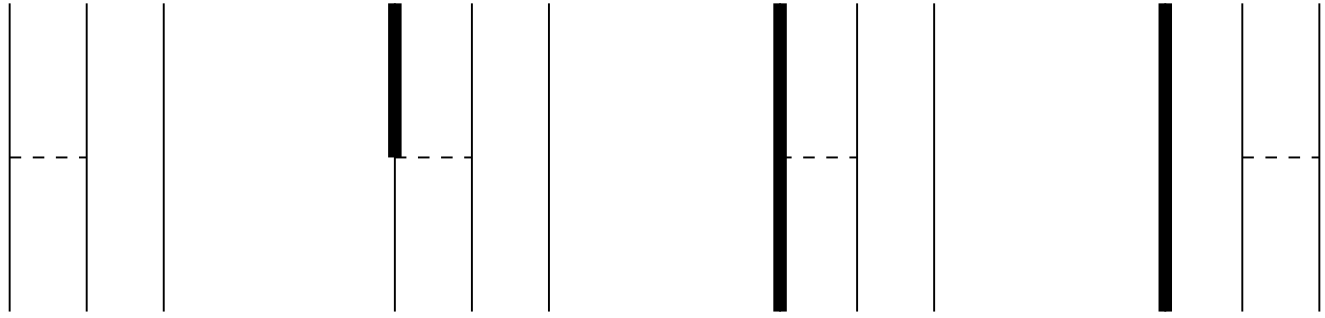
FM: R. Lazauskas [LNP 875, Clusters in Nuclei V.3]

Core excitation: extended Hilbert space

$$\mathcal{H} = \mathcal{H}_g \oplus \mathcal{H}_x$$



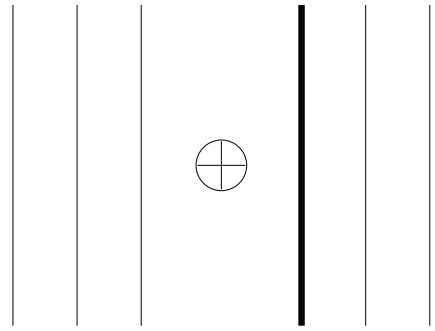
sector coupling by interaction



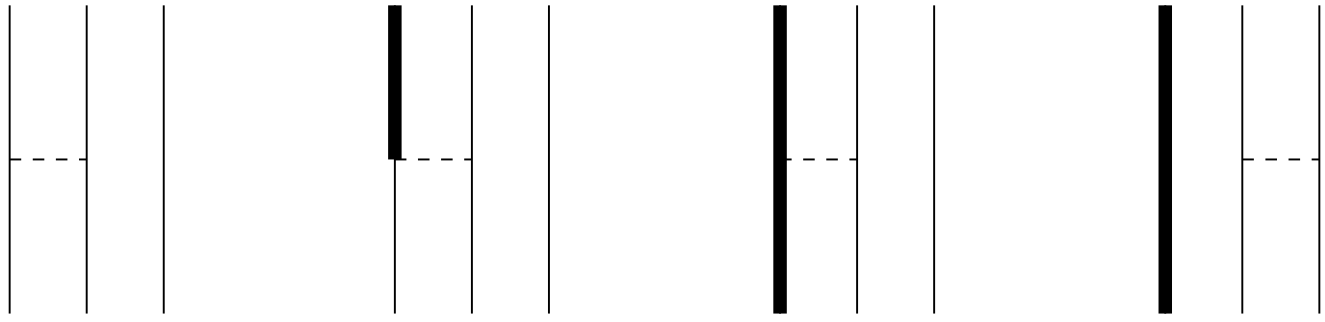
[PRC 88, 011601(R)]

Core excitation: extended Hilbert space

$$\mathcal{H} = \mathcal{H}_g \oplus \mathcal{H}_x$$



sector coupling by interaction



standard form of AGS eqs. with $H_0 \to H_0 + h_A^{\rm int}$

$$h_A^{\rm int}|\mathcal{H}_a\rangle = (m_{A^*} - m_A)\delta_{ax}|\mathcal{H}_a\rangle$$

[PRC 88, 011601(R)]

3-body AGS equations with core excitation

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_\sigma G_0 U_{\sigma\alpha}$$

$$U_{0\alpha} = G_0^{-1} + \sum_{\sigma} T_\sigma G_0 U_{\sigma\alpha}$$

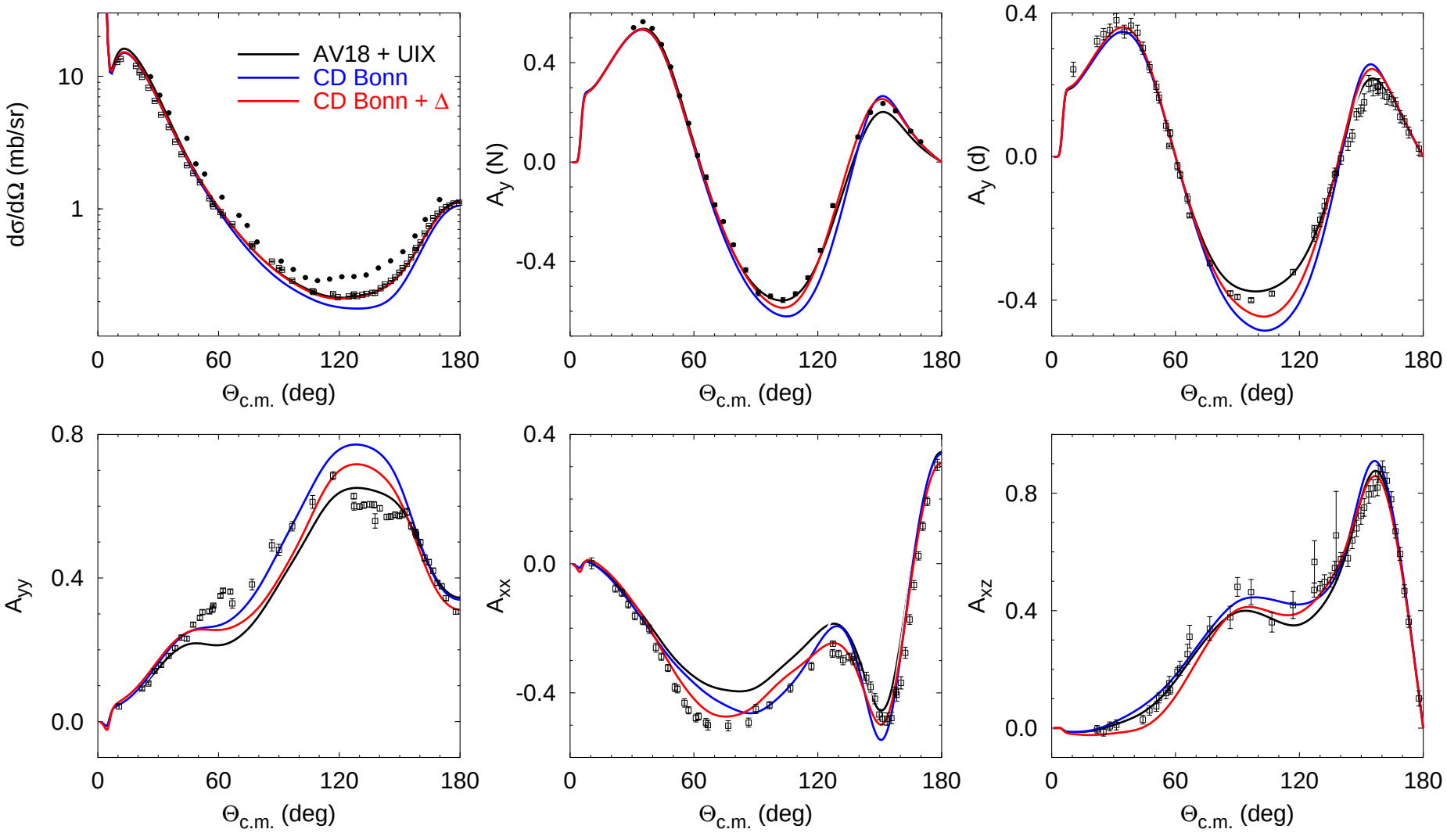
$$T_\sigma = v_\sigma + v_\sigma G_0 T_\sigma$$

$$G_0 = (E + i0 - H_0)^{-1}$$

channel states $\quad (E - H_0 - v_\alpha)|\phi_\alpha\rangle = 0$

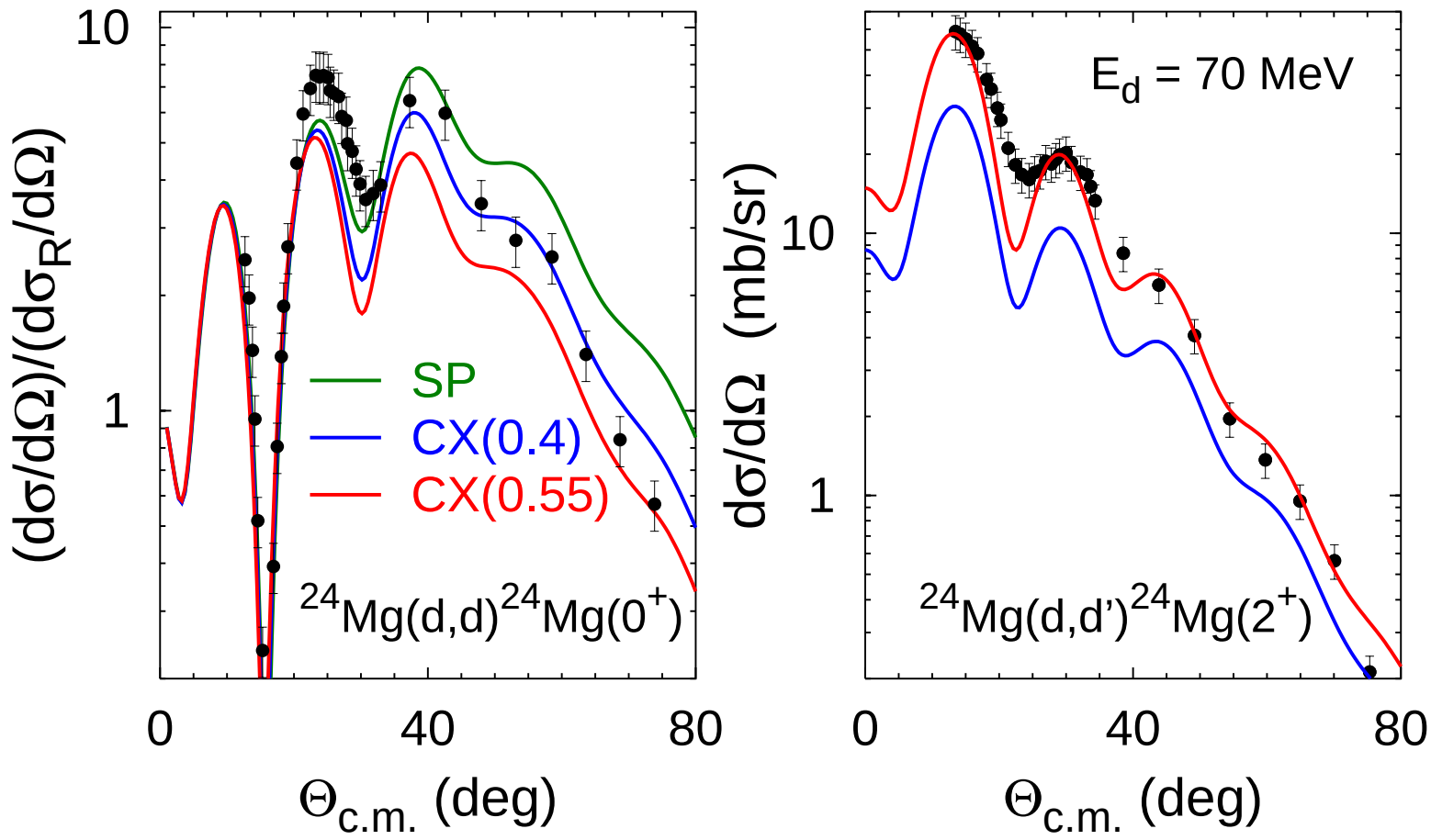
$$H_0|\mathbf{p}_\alpha \mathbf{q}_\alpha\rangle_a = [p_\alpha^2/2\mu_\alpha + q_\alpha^2/2M_\alpha + (m_{A^*} - m_A)\delta_{ax}]|\mathbf{p}_\alpha \mathbf{q}_\alpha\rangle_a$$

p+d elastic scattering at 135 MeV with Δ excitation



[PRC 80, 064002]

d+^{24}Mg elastic and inelastic scattering



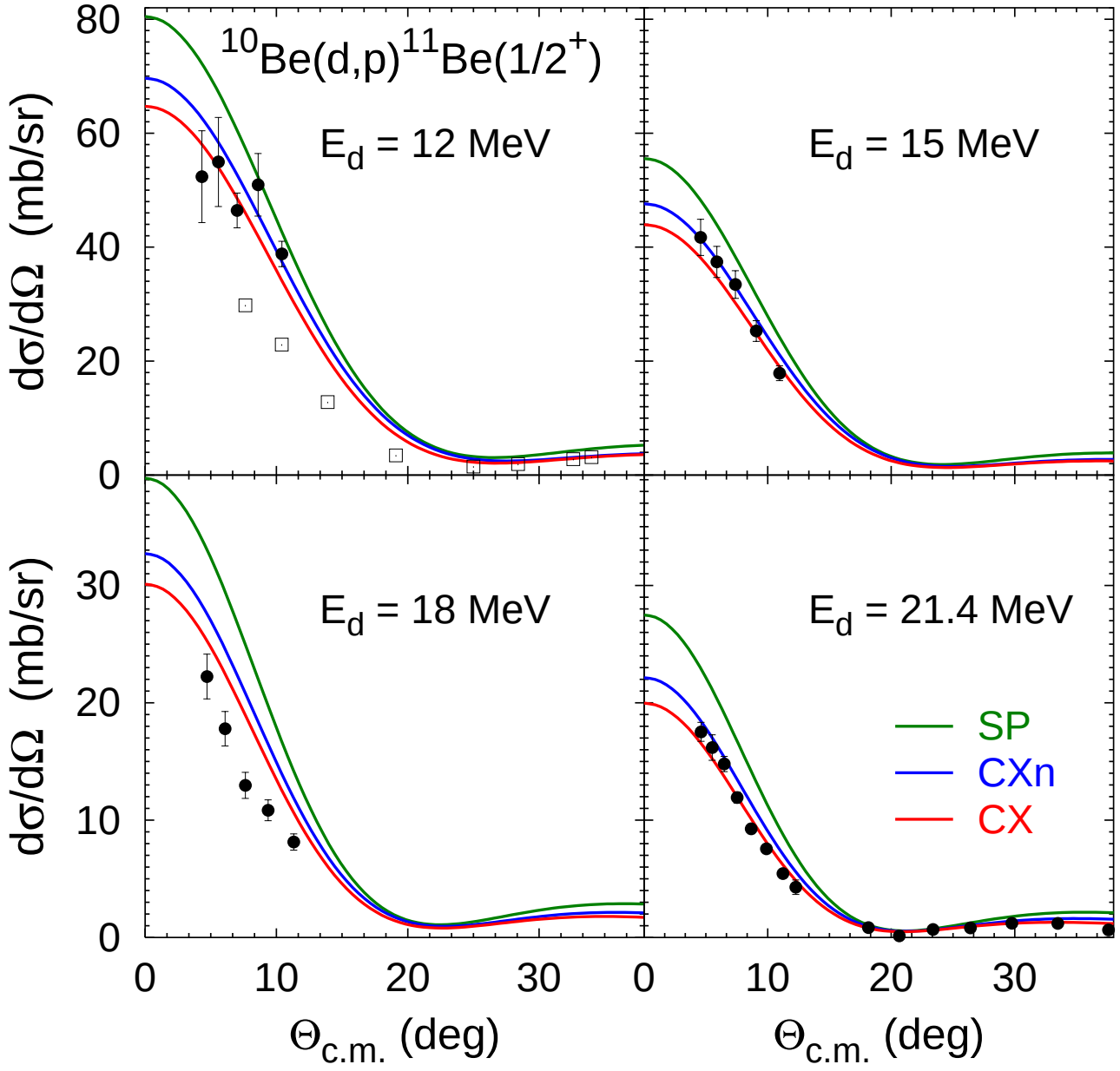
SP: without core excitation

CX: rotational model for V_{NA} with $\beta_2 = 0.55$ (0.4)

DWBA: $\beta_2 \sim 0.55\ (p, p'), \quad \beta_2 \sim 0.4\ (d, d')$

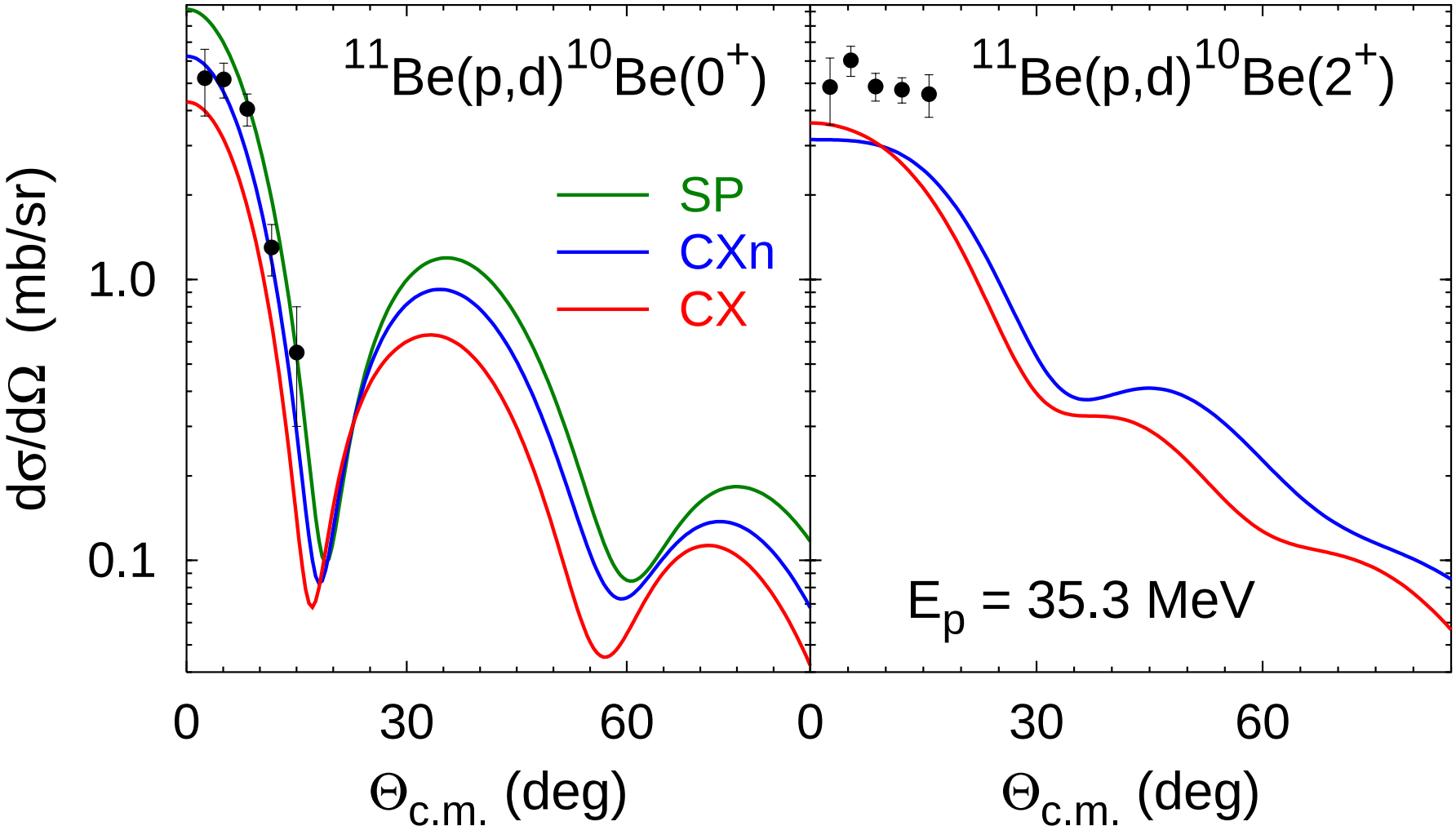
[PRC 88, 011601(R)]

^{10}Be(d,p)^{11}Be



CX(CXn): rotational model for V_{NA} (V_{nA} only), $\beta_2 = 0.67$

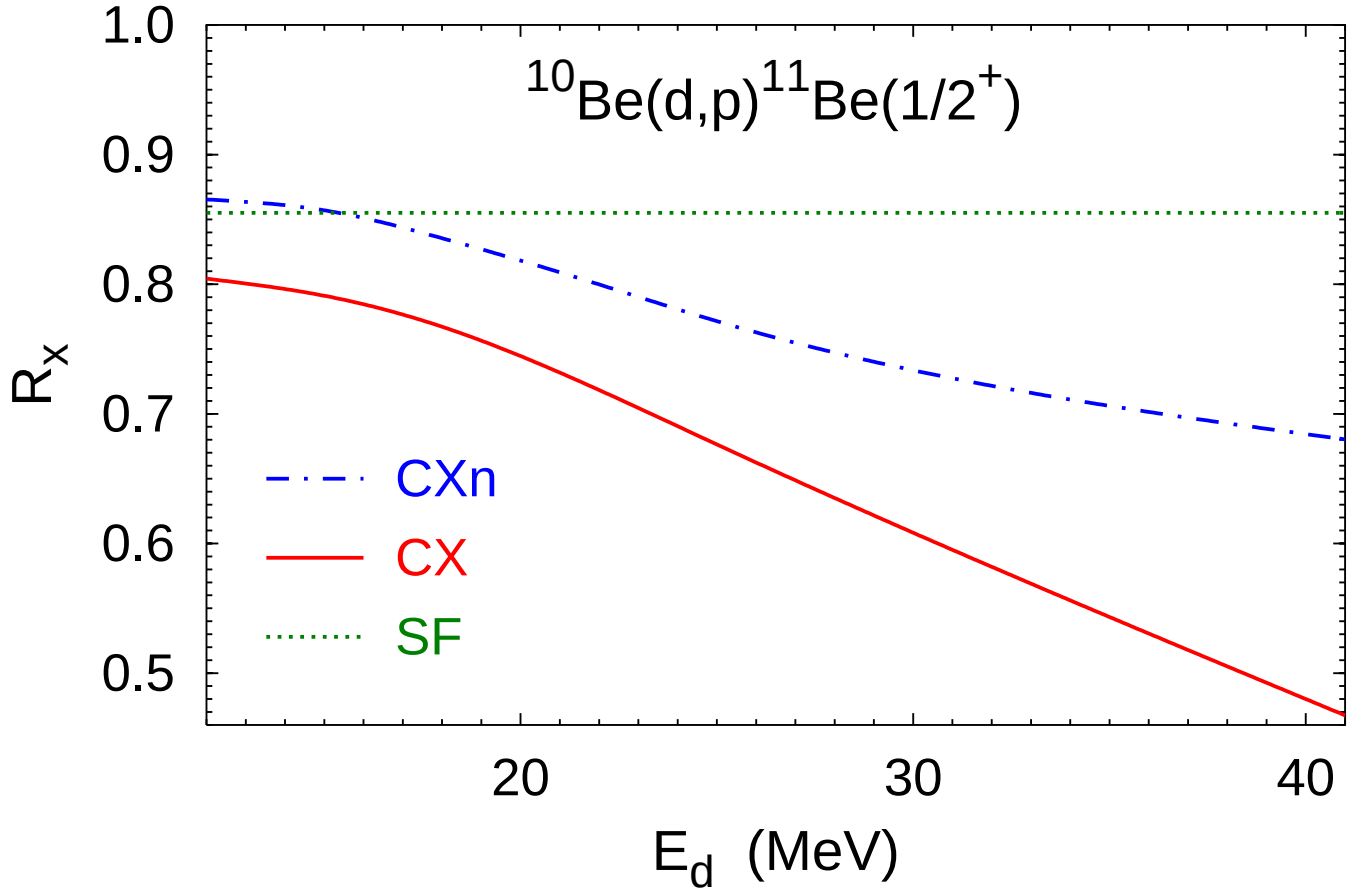
^{11}Be(p,d)^{10}Be



CX(CXn): rotational model for V_{NA} (V_{nA} only), $\beta_2 = 0.67$

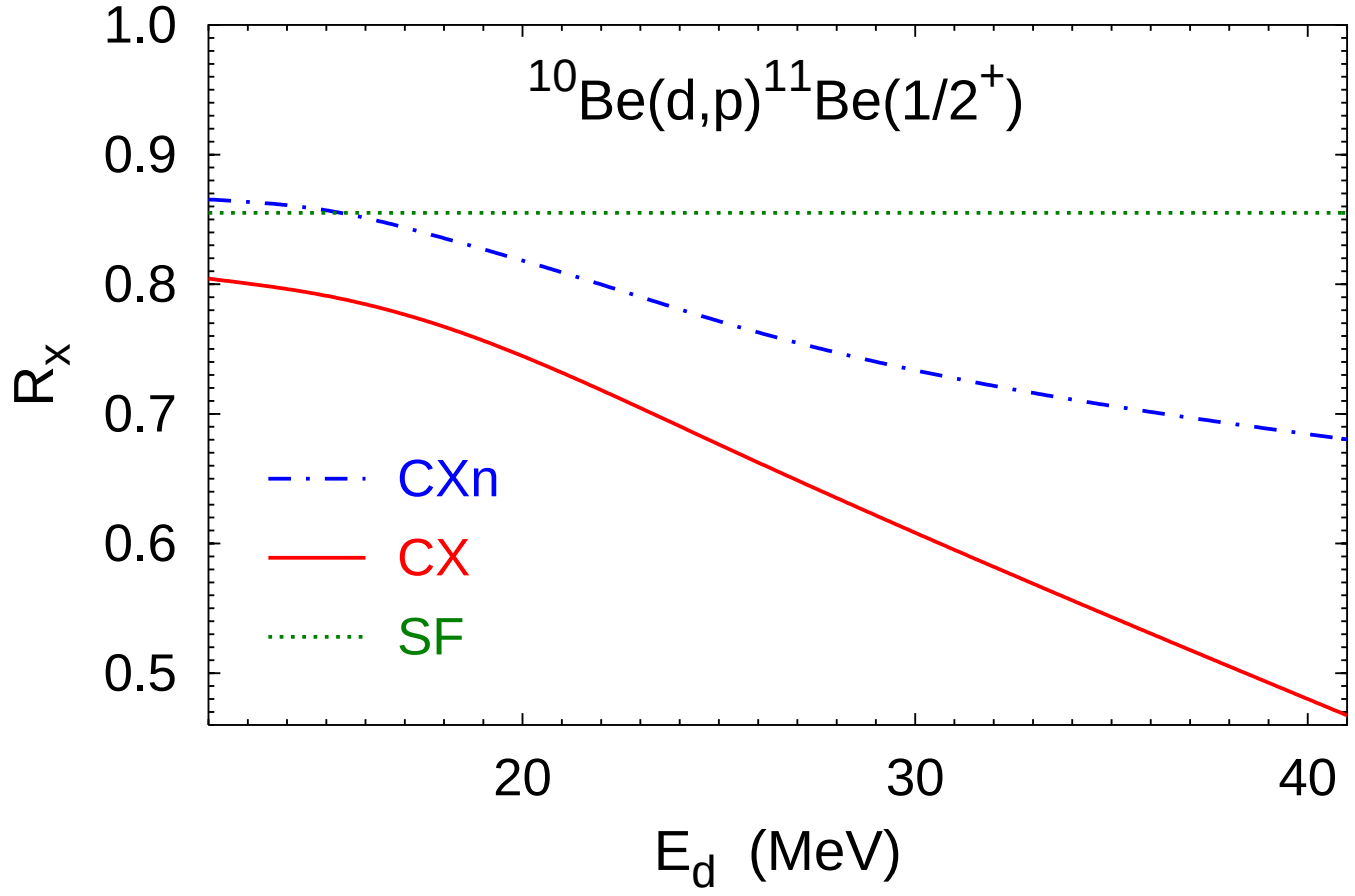
[PRC 88, 011601(R)]

^{10}Be(d,p)^{11}Be: core excitation effect



$$R_x = (d\sigma/d\Omega)_x/(d\sigma/d\Omega)_{\mathrm{SP}}\big|_{\Theta=0^\circ}$$

^{10}Be(d,p)^{11}Be: core excitation effect

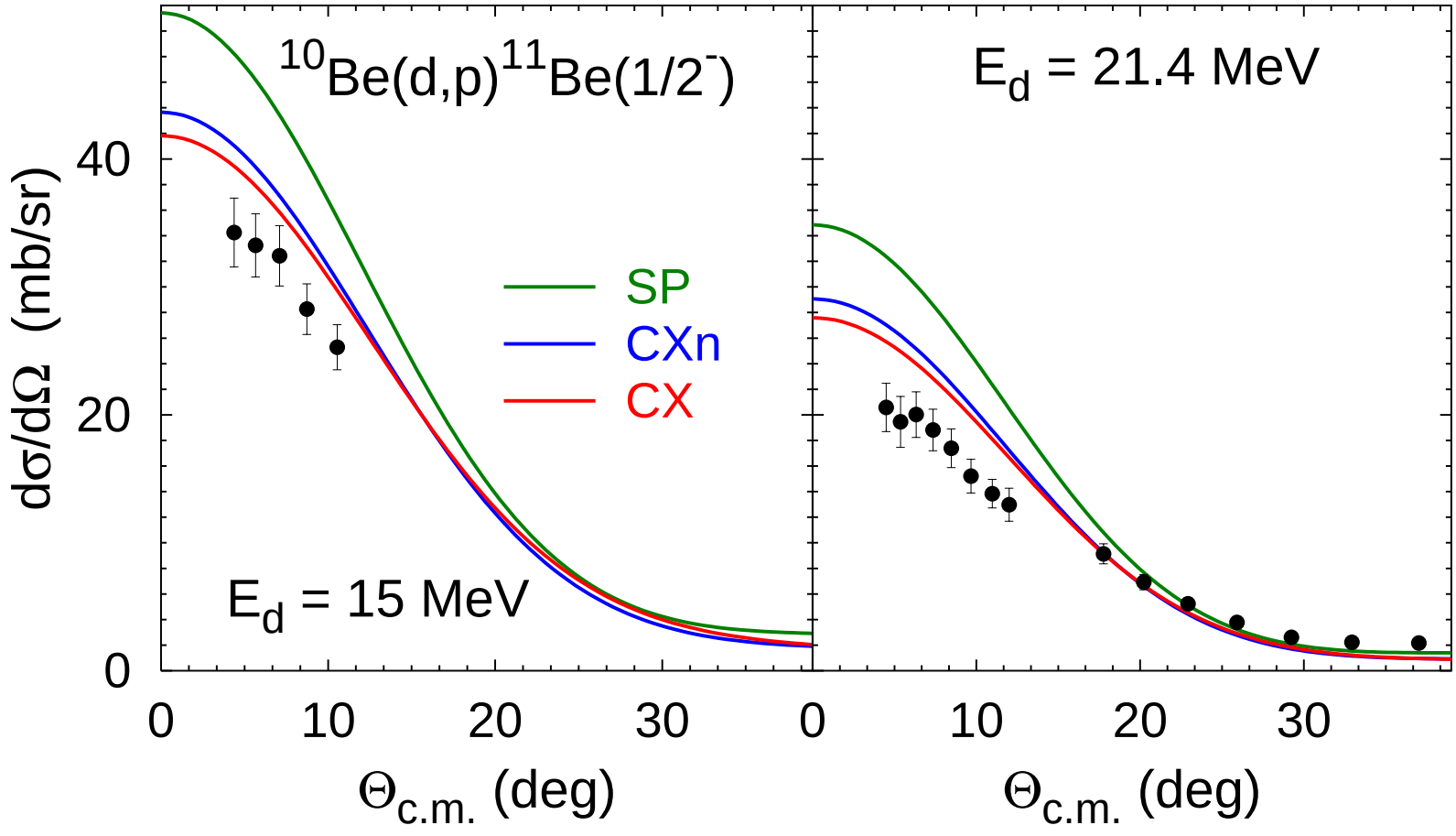


$$R_x = (d\sigma/d\Omega)_x/(d\sigma/d\Omega)_{\rm SP}\big|_{\Theta=0^\circ}$$

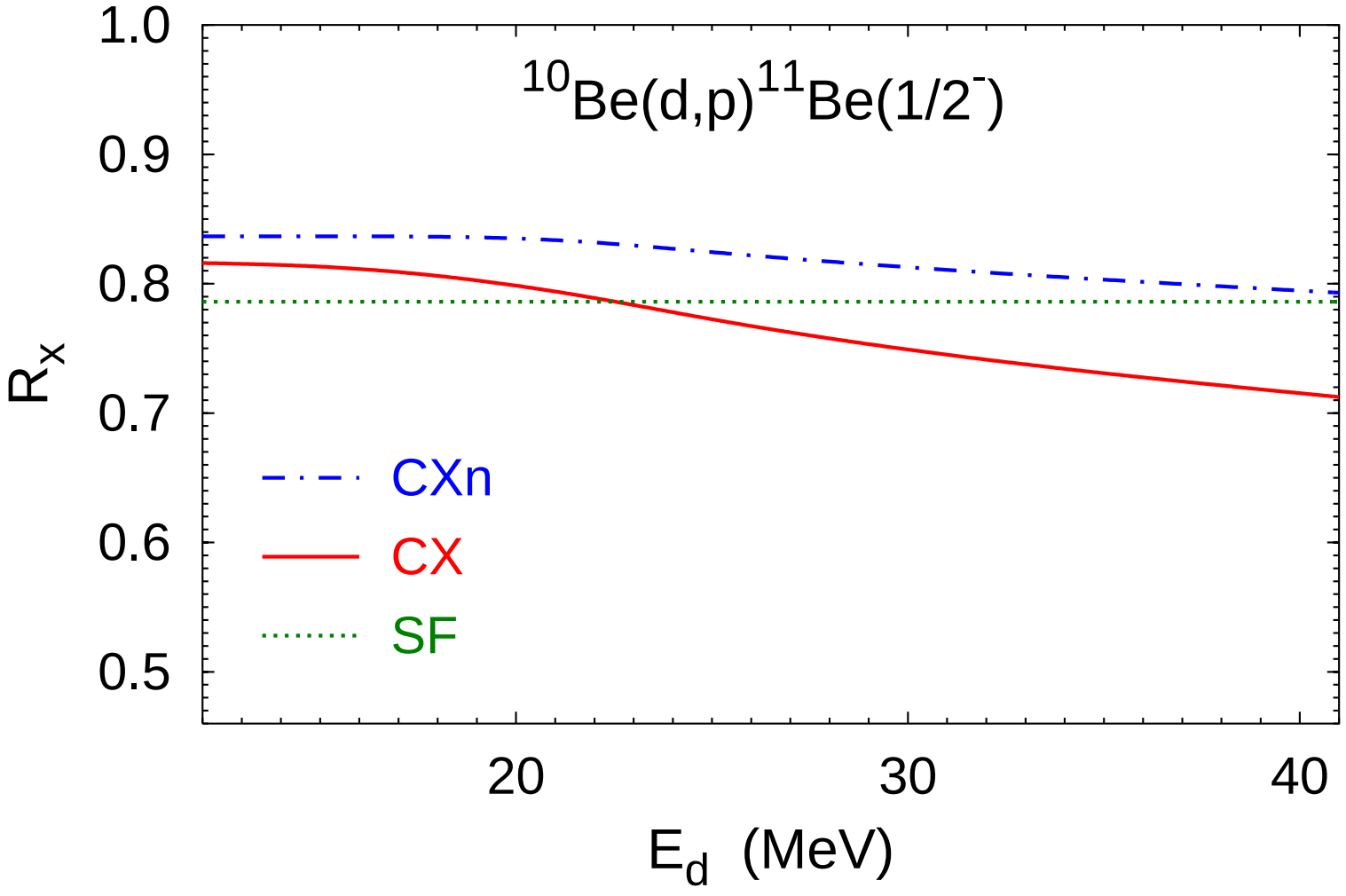
DWBA/ADWA: SF= R_{exp} unreliable !

Faddeev/AGS: (V_{NA} - SF - data) compatibility check

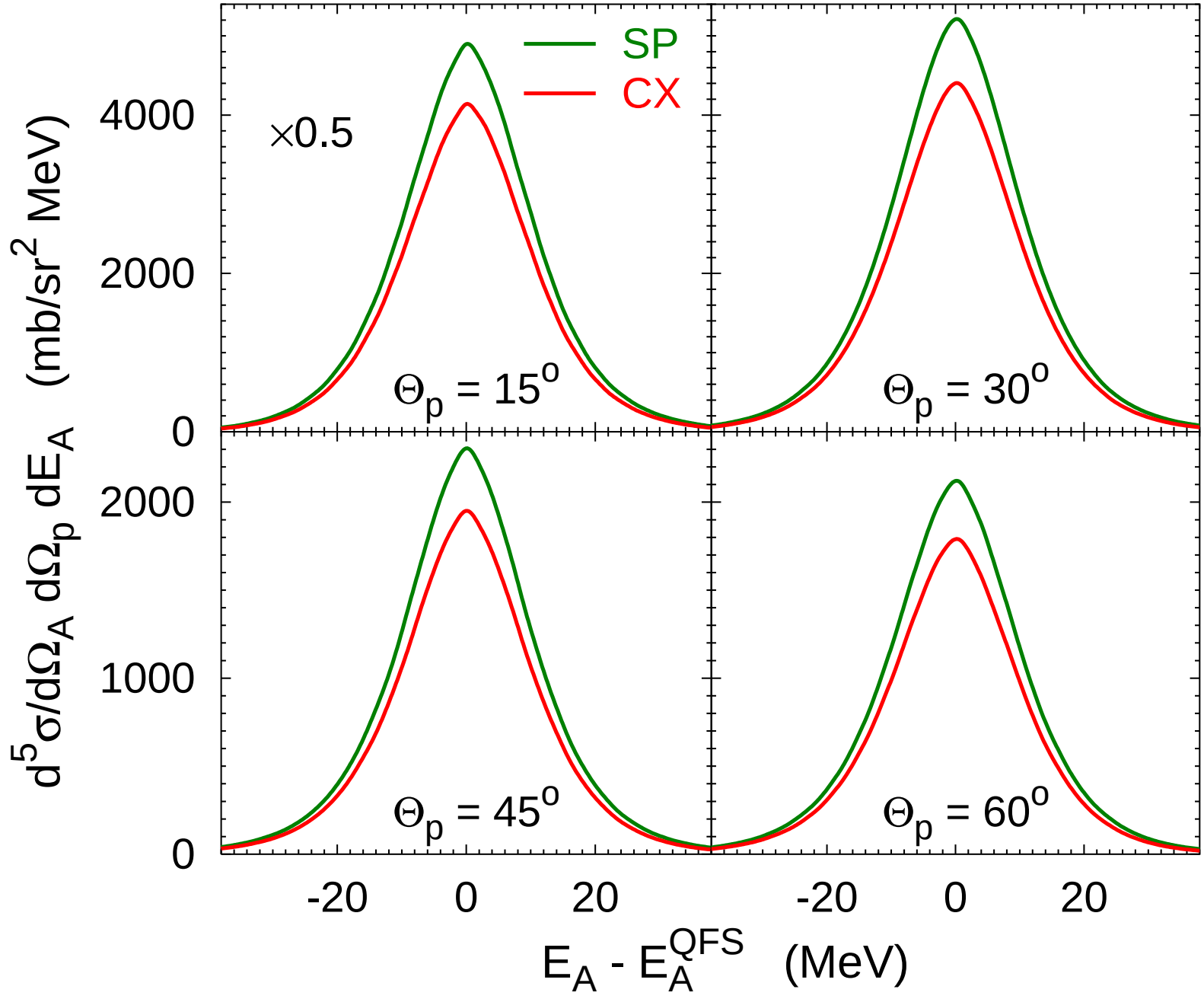
^{10}Be(d,p)^{11}Be


10Be(d,p)11Be(1/2-)
Ed = 15 MeV
Ed = 21.4 MeV
SP
CXn
CX
dσ/dΩ (mb/sr)
Θc.m. (deg)
Θc.m. (deg)
0
20
40
0
10
20
30
0
10
20
30

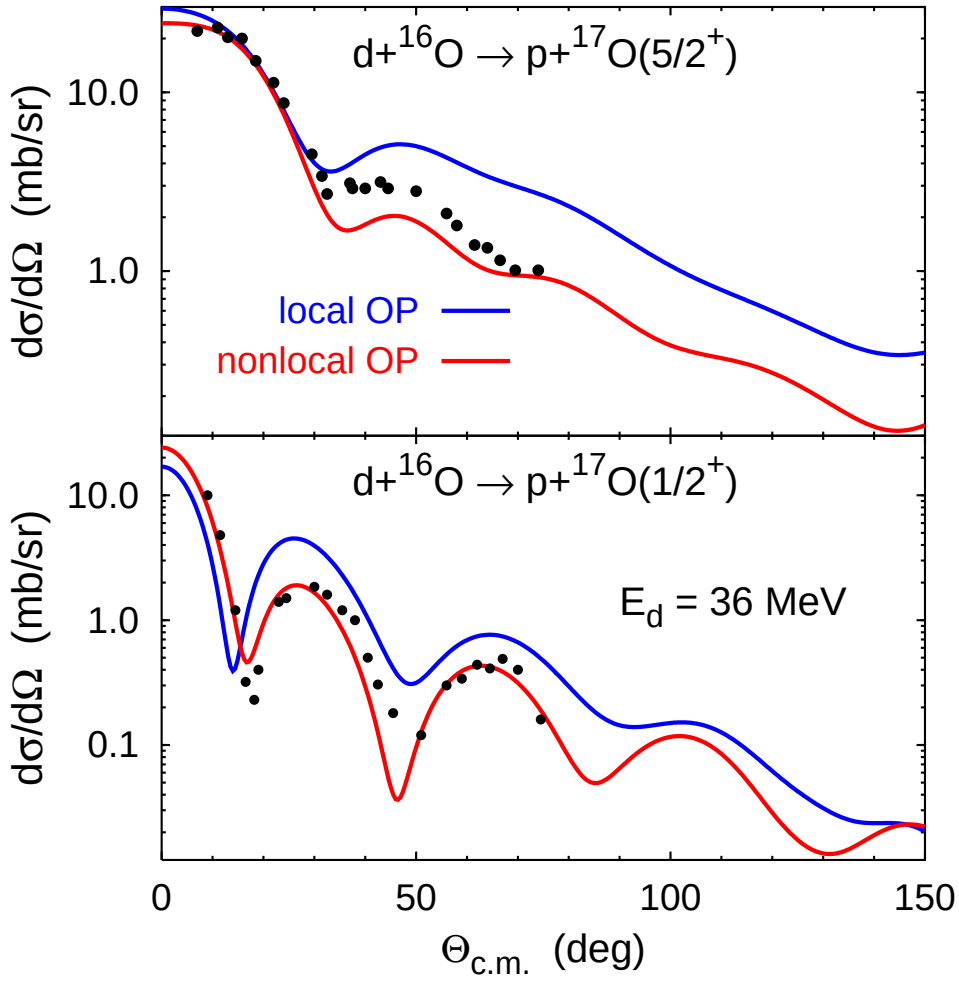
^{10}Be(d,p)^{11}Be

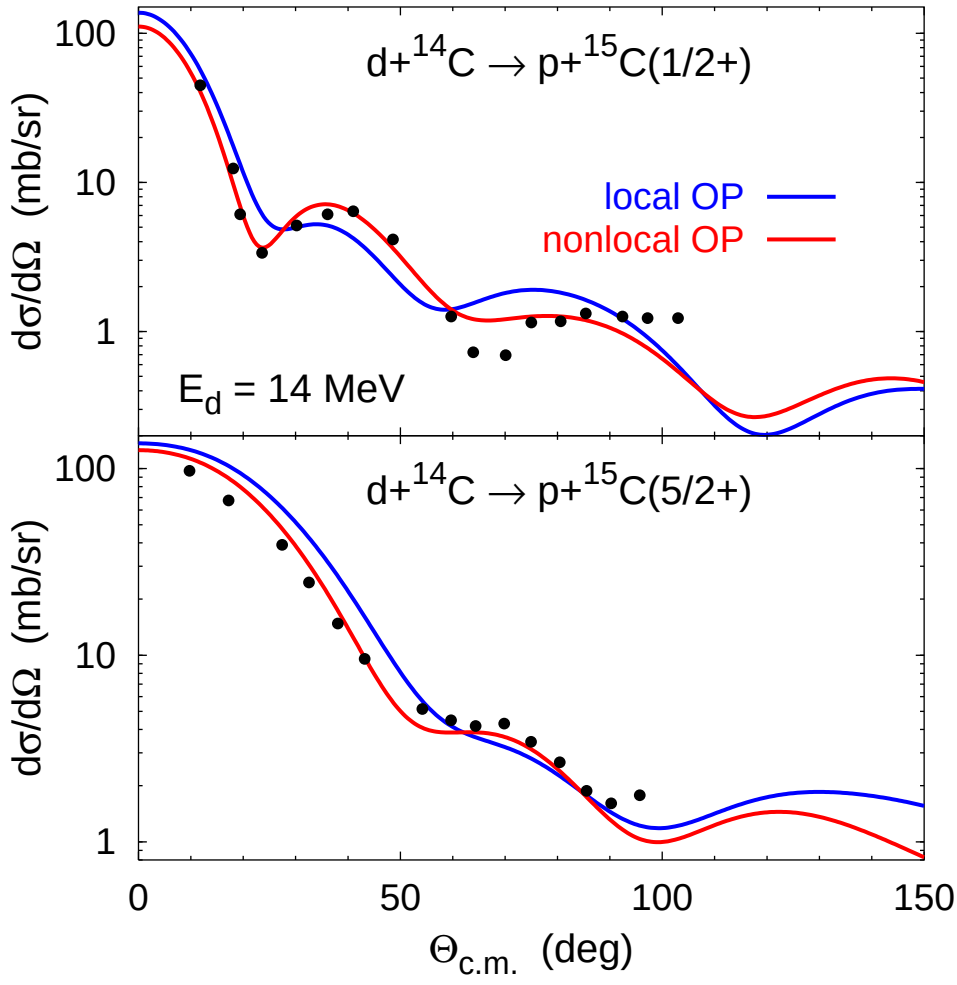


^{11}Be(p,pn)^{10}Be at 200 MeV/u near np QFS ($\Theta_A = 0^\circ$)



Nonlocal OP in transfer reactions





[PRC 79, 021602, PRC 79, 054603]

4-body scattering: AGS equations

4-body transition operators

$$t_i = v_i + v_i G_0 t_i$$

$$U_\gamma^{jk} = G_0^{-1}\bar{\delta}_{jk} + \sum_i \bar{\delta}_{ji} t_i G_0 U_\gamma^{ik}$$

$$\mathcal{U}_{\beta\alpha}^{ji} = (G_0 t_i G_0)^{-1}\bar{\delta}_{\beta\alpha}\delta_{ji} + \sum_{\gamma k} \bar{\delta}_{\beta\gamma} U_\gamma^{jk} G_0 t_k G_0 \mathcal{U}_{\gamma\alpha}^{ki}$$

i, j, k: pairs ($\equiv$ three-cluster (2+1+1) partitions)
α, β, γ: two-cluster (1+3 or 2+2) partitions

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i, j, k: pairs ($\equiv$ three-cluster (2+1+1) partitions)
α, β, γ: two-cluster (1+3 or 2+2) partitions

wave function

$$|\Psi_\alpha\rangle = |\Phi_\alpha\rangle + \sum_{\gamma jki} G_0 t_j G_0 U_\gamma^{jk} G_0 t_k G_0 \mathcal{U}_{\gamma\alpha}^{ki} |\phi_\alpha^i\rangle$$

$$|\Phi_\alpha\rangle = \sum_i |\phi_\alpha^i\rangle, \qquad |\phi_\alpha^i\rangle = G_0 \sum_j \bar{\delta}_{ij} t_j |\phi_\alpha^j\rangle$$

4-body scattering amplitudes

two-cluster reactions:

$$\langle \Phi_\beta | T_{\beta\alpha} | \Phi_\alpha \rangle = \sum_{ji} \langle \phi_\beta^j | \mathcal{U}_{\beta\alpha}^{ji} | \phi_\alpha^i \rangle$$

three-cluster breakup:

$$\langle \Phi^j | T_\alpha^j | \Phi_\alpha \rangle = \sum_{\beta ki} \langle \Phi^j | U_\beta^{jk} G_0 \, t_k \, G_0 \, \mathcal{U}_{\beta\alpha}^{ki} | \phi_\alpha^i \rangle$$

four-cluster breakup:

$$\langle \Phi_0 | T_{0\alpha} | \Phi_\alpha \rangle = \sum_{\beta jki} \langle \Phi_0 | t_j \, G_0 \, U_\beta^{jk} G_0 \, t_k \, G_0 \, \mathcal{U}_{\beta\alpha}^{ki} | \phi_\alpha^i \rangle$$

[PRC 75, 014005; PRA 85, 012708]

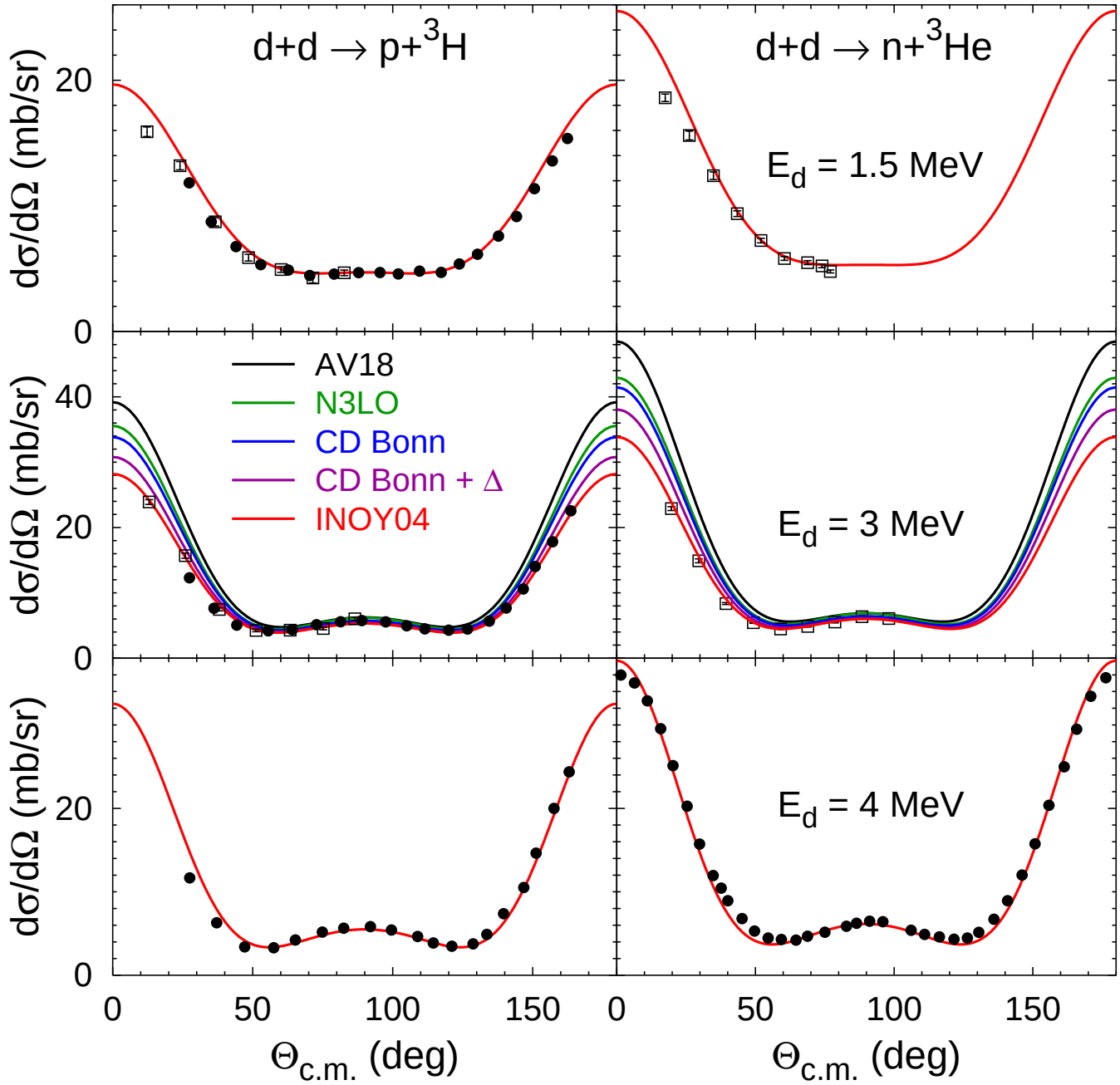
Applications: 4N scattering and 4-body nuclear reactions

$$\left.\begin{matrix} N+(NNN) \\ d+d \end{matrix}\right\} \rightarrow \begin{cases} N+(NNN) \\ d+d \\ d+N+N \\ N+N+N+N \end{cases}$$

$$p+(Ann) \rightarrow \begin{cases} p+n+n+A \\ p+n+(An) \\ p+(Ann) \\ d+(An) \\ t+A \end{cases}$$

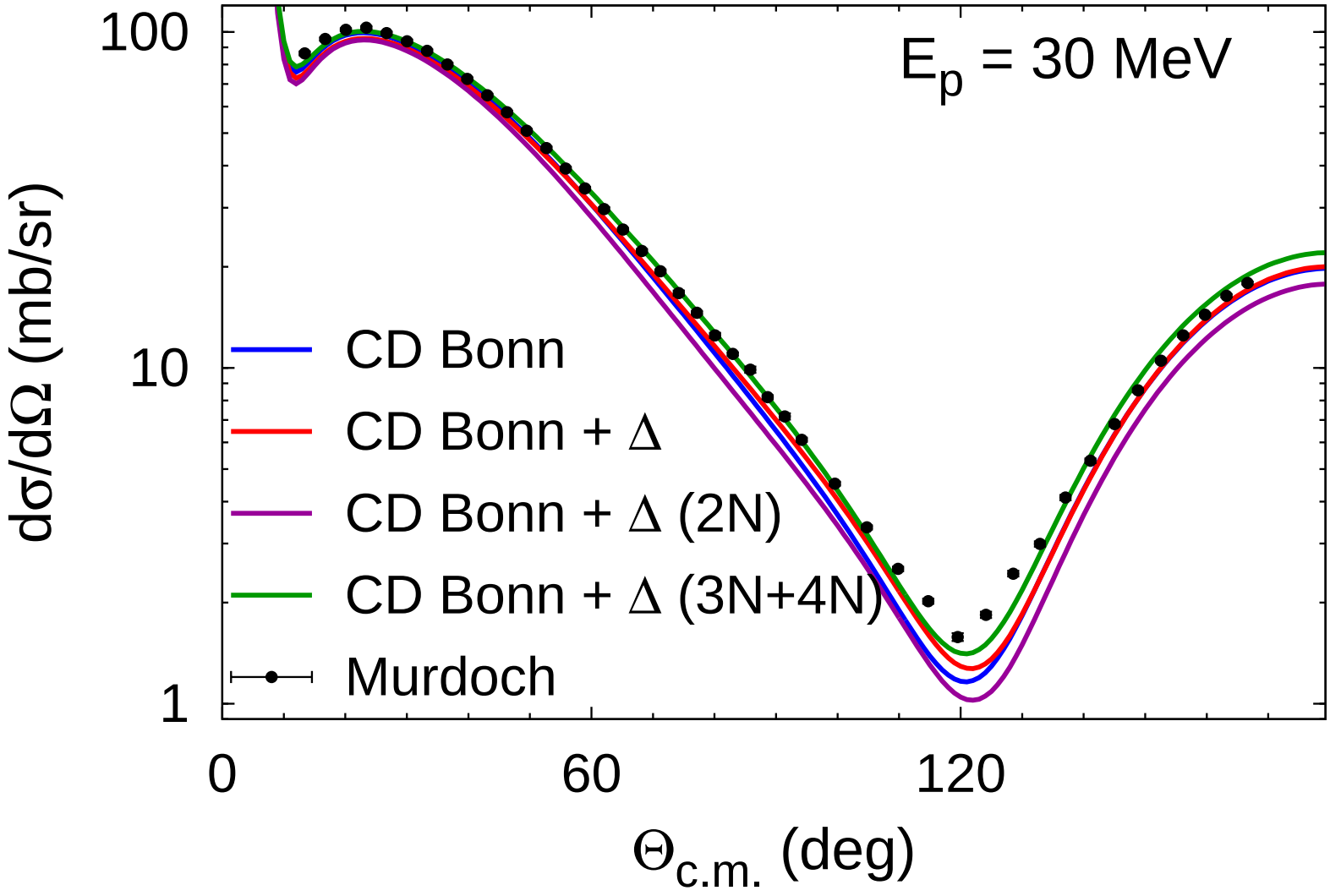
with $(Ann) = {}^{6}\mathrm{He},\ {}^{11}\mathrm{Li},\ {}^{12,14}\mathrm{Be},\ {}^{16,20,22}\mathrm{C},\ {}^{24}\mathrm{O},\ \ldots$

^{2}H(d,p)^{3}H and ^{2}H(d,n)^{3}He



[PRC 81, 054002]

p+^{3}He elastic scattering: Δ effects



[PRC 87, 054002]

Single-scattering approximation

may be reasonable for elastic scattering

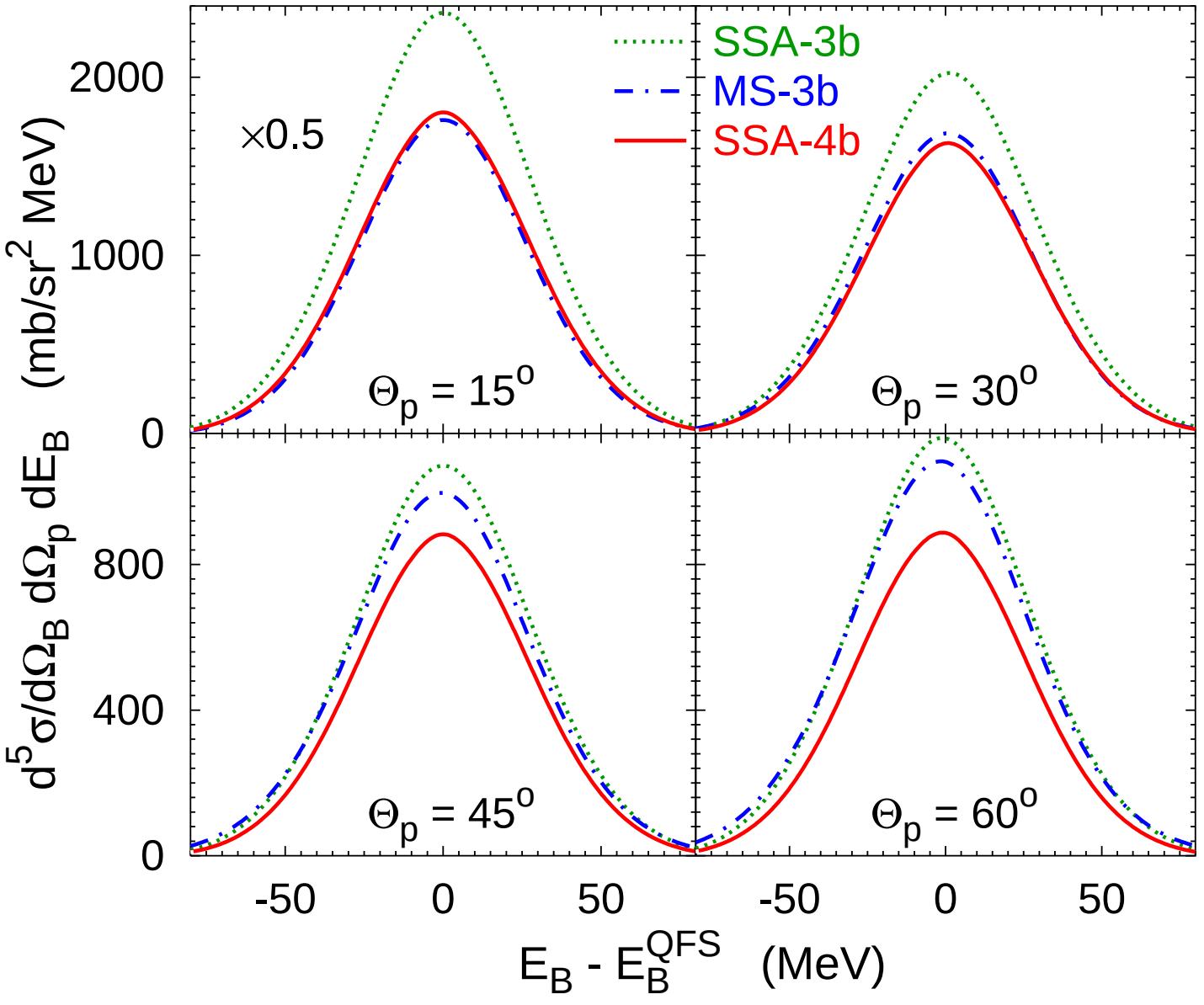
$$\langle \Phi_\alpha | T_{\alpha\alpha}^{SSA} | \Phi_\alpha \rangle = \sum_k \langle \Phi_\alpha | \overline{\delta}_{k\alpha} t_k | \Phi_\alpha \rangle$$

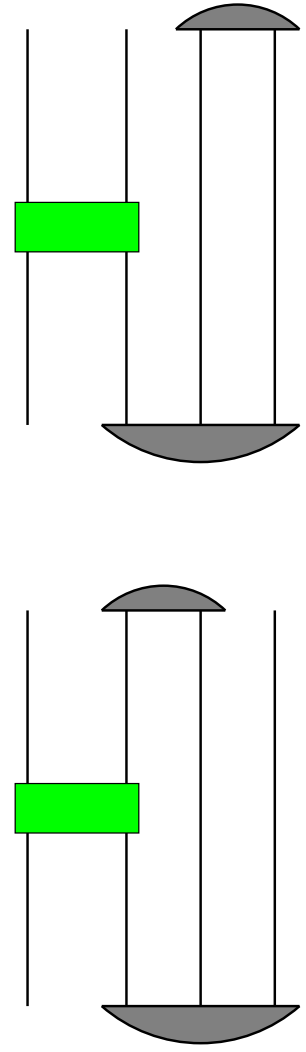
and breakup

$$\langle \Phi^j | T_\alpha^{jSSA} | \Phi_\alpha \rangle = \sum_k \langle \Phi^j | \overline{\delta}_{k\alpha} t_k | \Phi_\alpha \rangle$$

$$\langle \Phi_0 | T_{0\alpha}^{SSA} | \Phi_\alpha \rangle = \sum_k \langle \Phi_0 | \overline{\delta}_{k\alpha} t_k | \Phi_\alpha \rangle$$

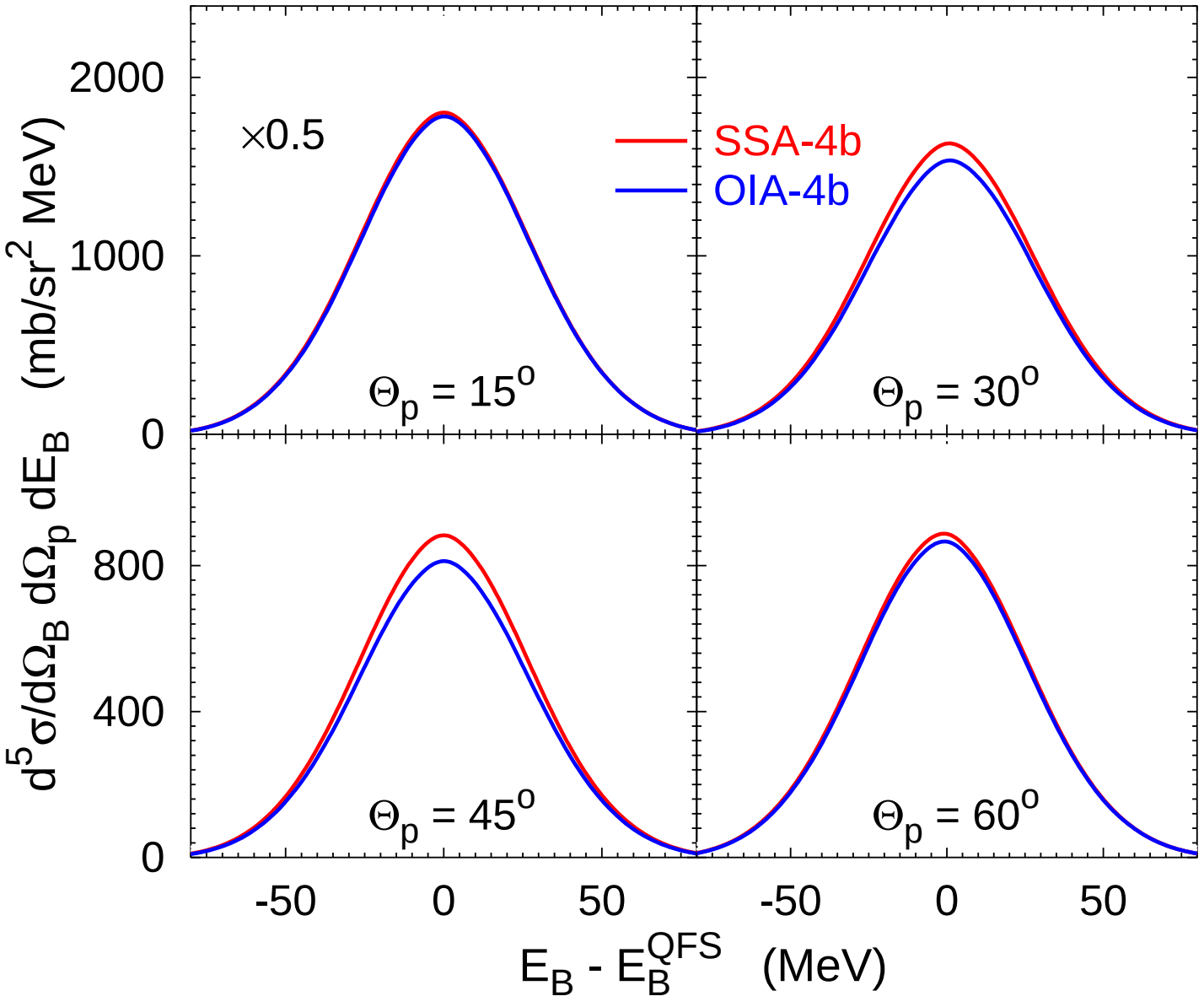
^{16}C(p,pn)^{15}C near np QFS at 300 MeV/u ($\Theta_B = 0°$)

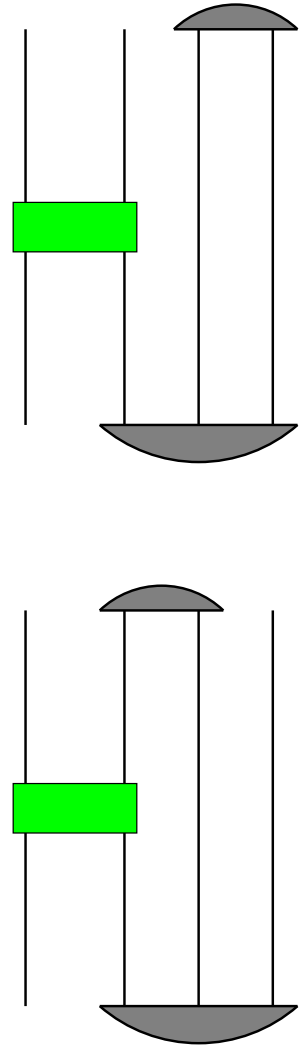




[PRC 87, 034609]

^{16}C(p,pn)^{15}C: overlap integral approximation





[PRC 87, 034609]

Summary

- 3- and 4-particle AGS equations in momentum space
- 3-body nuclear reactions including core excitation
- transfer reactions:
  complicated core excitation effects,
  no simple relation to SF
- 4N reactions
- 4-body nuclear reactions in SSA