Role of structure of heaviest nuclei in their production

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The experiments with the actinde-based complete fusion reactions $^{48}Ca + {}^{238}U, {}^{237}Np, {}^{242,244}Pu, {}^{243}Am, {}^{245,248}Cm, {}^{249}Bk, {}^{249}Cf$ -->112 - 118 were carried out at JINR(Dubna),GSI(Darmstadt),LBNL (Berkeley) in order to approach to «island of stability» of superheavy elements (SHE) predicted at charge number Z=114 and neutron number N=184 with microscopic-macroscopic models.

Experimental data on SHE:

The found experimental trend of the nuclear properties [Q(alpha)-values and half-lives] of the SHE indicates

the importance of* N=184 *shell
but
***a small influence of the proton shell at* Z=114.**

No discontinuity is observed when the proton number 114 is crossed at N=172 - 176!

Experimental data on production cross sections:

The measured production cross sections in reactions ^{48}Ca+Actinide do not depend strongly on atomic number Z of SHE and are on the picobarn level.

The fusion cross section strongly decreases with increasing $Z_1 \times Z_2$!

Production cross section=
=(Fusion cross section) x (Survival probability)

Dynamics of fusion in the dinuclear system model

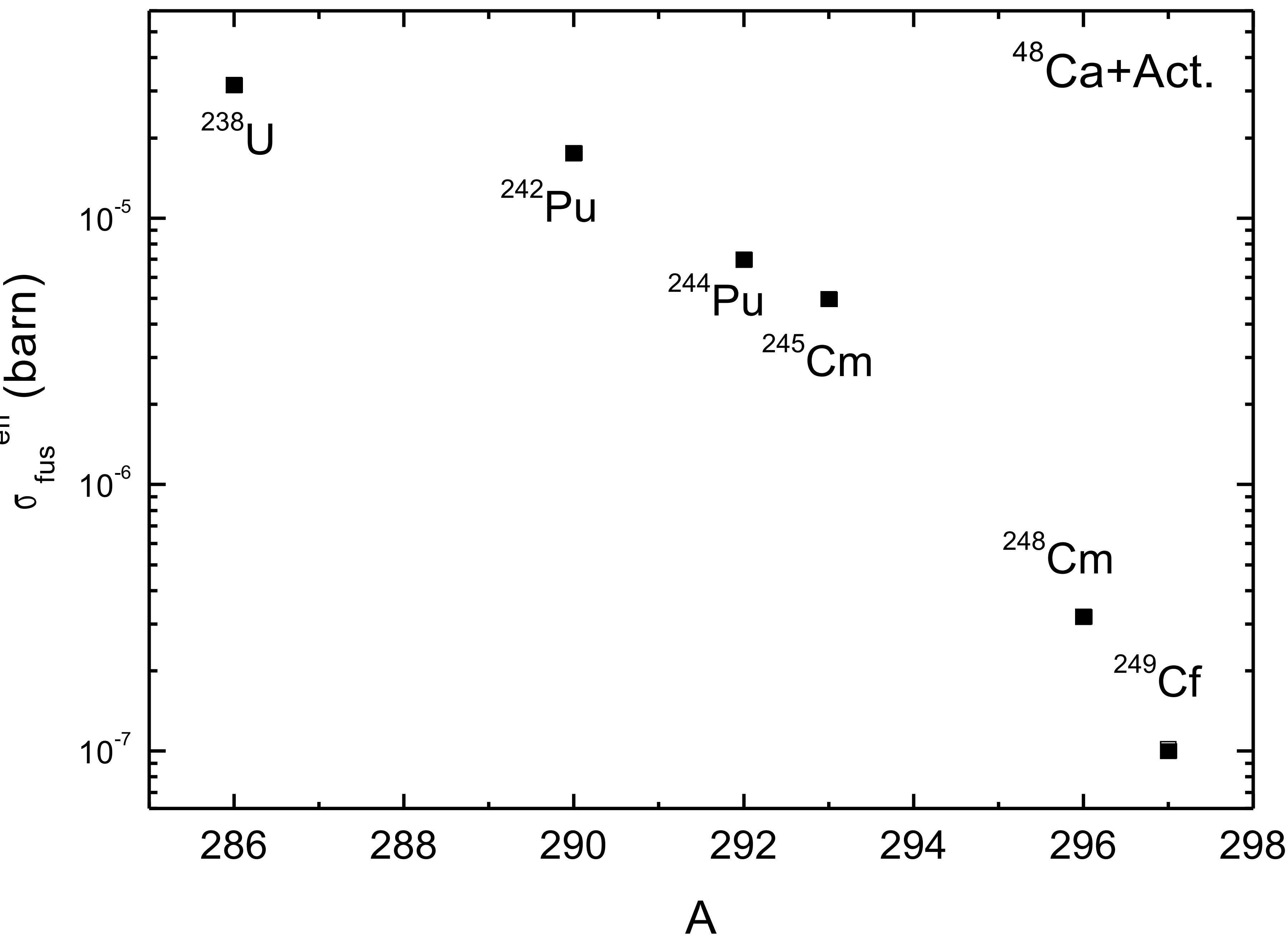
Evaporation residue cross section for the production of superheavy nuclei:

$$\sigma^{s}_{ER}(E_{c.m.}) = \sum_{J=0} \sigma_c(E_{c.m.}, J) P_{CN}(E_{c.m.}, J) W^{s}_{sur}(E_{c.m.}, J)$$

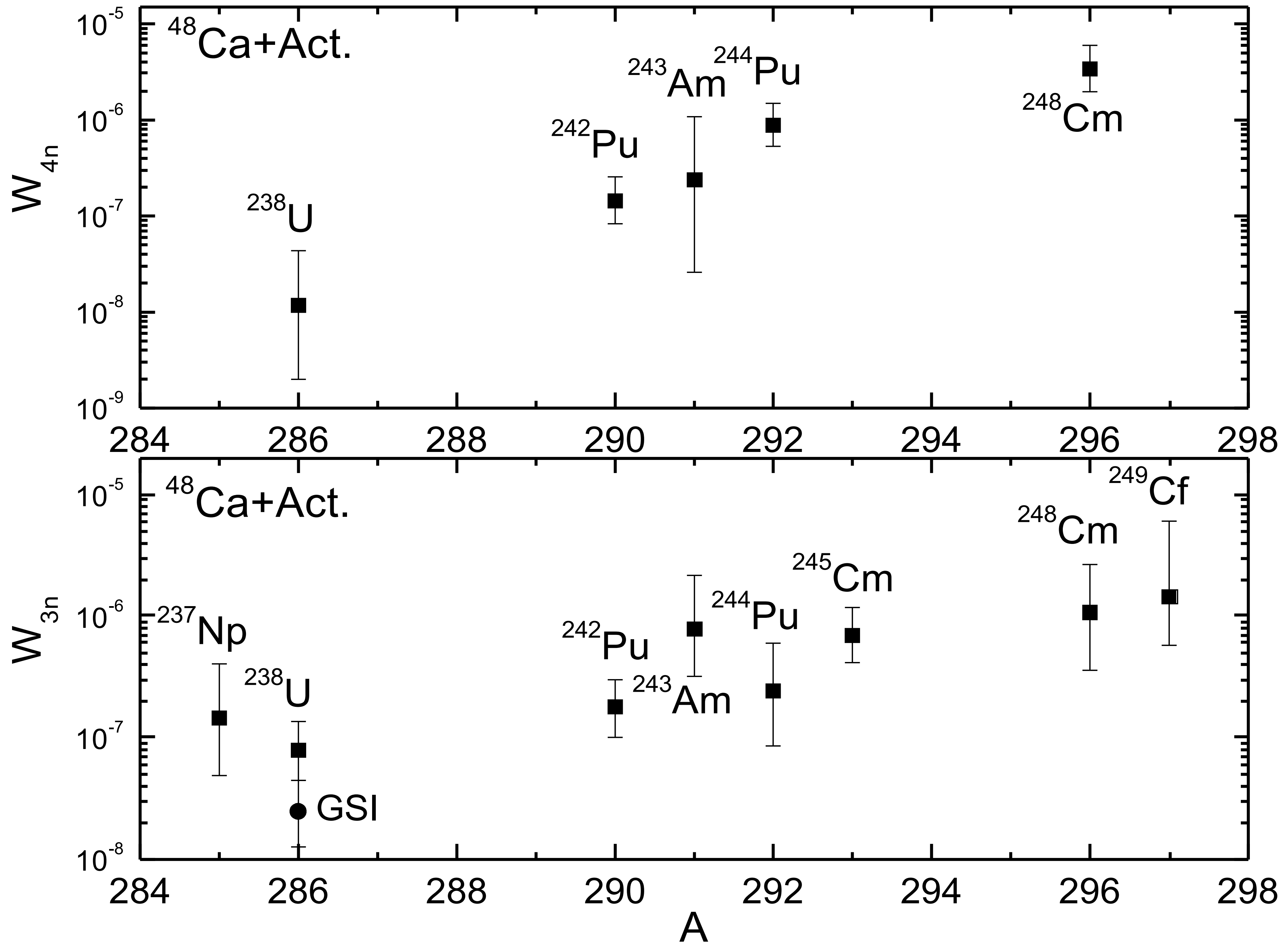
$$\sigma_c(E_{c.m.}, J) = \pi \lambda\!\!\!^{-}{}^2 (2J+1) T(E_{c.m.}, J)$$

$$\sigma^{s}_{ER}(E_{c.m.}) \approx P_{CN}(E_{c.m.}, J=0) \sum_{J} \sigma_c(E_{c.m.}, J) W^{s}_{sur}(E_{c.m.}, J)$$

$$\approx \sigma_c(E_{c.m.}) P_{CN}(E_{c.m.}) W^{s}_{sur}(E_{c.m.})$$



48Ca+Act.
238U
242Pu
244Pu
245Cm
248Cm
249Cf
σ fus eff (barn)
10-5
10-6
10-7
286
288
290
292
294
296
298
A


48Ca+Act.
243Am
244Pu
248Cm
242Pu
238U
W4n
10-5
10-6
10-7
10-8
10-9
284
286
288
290
292
294
296
298
48Ca+Act.
249Cf
248Cm
245Cm
244Pu
237Np
242Pu
243Am
238U
GSI
W3n
10-5
10-6
10-7
10-8
284
286
288
290
292
294
296
298
A

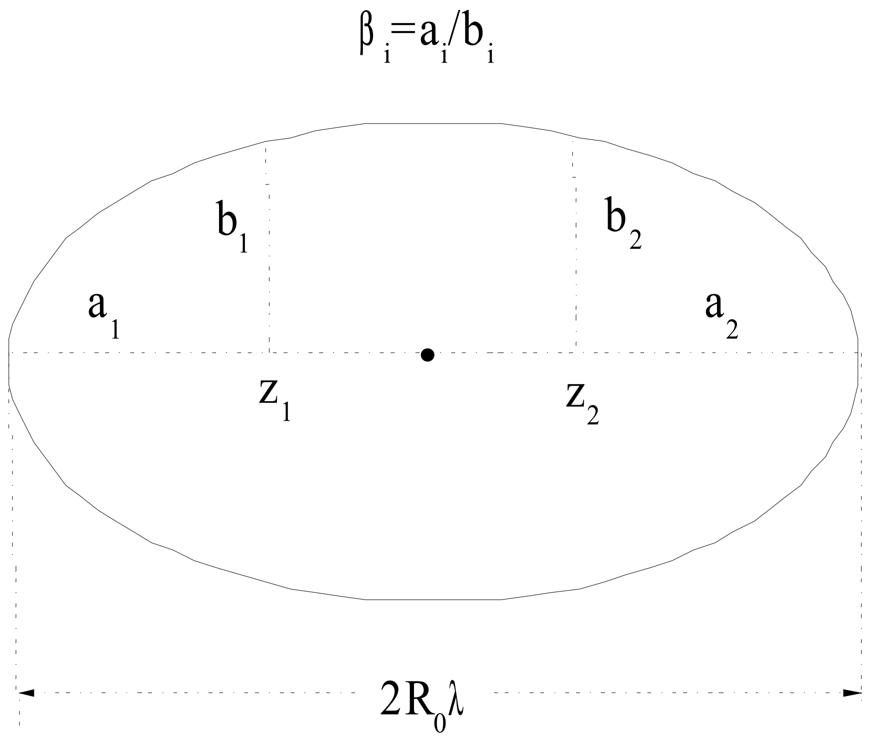
Thus, the present experimental data point out that magic proton shell is located at Z >118 !

This observation is an accordance with predictions of mean field models where island of stability is near nucleus with Z=120 – 126 and N=184.

All microscopic-macroscopic approaches based on various shape parametrizations provide the closed proton shell Z=114 ?

Microscopic-macroscopic approach based on the two-center shell model (TCSM) [J.Maruhn and W.Greiner] was used.

Parameters were set so to describe spins & parities of g.-s. of known heavy nuclei.



$\beta_1 = \beta_2$, even multipolarities; $\beta_1 \neq \beta_2$, odd and even multipolarities

R_0 is the radius of spherical nucleus

Parametrisation of nuclear shape with TCSM

$$H = T + V(\rho, z) + V_{LS} + V_{L^2}$$

$$V(\rho, z) = \begin{cases} \frac{1}{2} m \omega_z^2 (z - z_1)^2 + \frac{1}{2} m \omega_\rho^2 \rho^2, & z < z_1 \\ \frac{1}{2} m \omega_\rho^2 \rho^2, & z_1 < z < z_2 \\ \frac{1}{2} m \omega_z^2 (z - z_2)^2 + \frac{1}{2} m \omega_\rho^2 \rho^2, & z > z_2 \end{cases}$$

$$V_{LS} = -\frac{2\hbar \kappa_i}{m \omega_{0\mathrm{i}}} (\nabla V \times \vec{p}) \vec{s}$$

$$V_{L^2} = -\hbar \omega_{0\mathrm{i}} \kappa_i \mu_i \hat{l}^2 + \hbar \kappa_i \mu_i \omega_{0\mathrm{i}} N_1 (N_1 + 3)/2 \, \delta_{if}$$

$$\omega_{0\mathrm{i}} = 41\, MeV / A_i^{1/3}, \; A_i = a_i b_i^2 / 1.22^3, \; \omega_\rho / \omega_z = a_i / b_i, \; z_2 - z_1 = 2 \mathrm{R}_0 \lambda - a_1 - a_2$$

Parameters

$$35 \le N - Z \le 64$$

$$\textit{for neutrons}$$

$$\kappa_n = -0.076 + 0.0058(N-Z) - 6.53\times10^{-5}(N-Z)^2 + 0.002\,A^{1/3},$$

$$\mu_n = 1.598 - 0.0295(N-Z) + 3.036\times10^{-4}(N-Z)^2 - 0.095\,A^{1/3},$$

$$\textit{for protons}$$

$$\kappa_p = 0.0383 + 0.00137(N-Z) - 1.22\times10^{-5}(N-Z)^2 - 0.003\,A^{1/3},$$

$$\mu_p = 0.335 + 0.01(N-Z) - 9.367\times10^{-5}(N-Z)^2 + 0.003\,A^{1/3},$$

Parameters were set so to describe spins & parities of g.-s. of known heavy nuclei

Potential energy

$$U(Z, A, \lambda, \beta) = U_{LDM}(Z, A, \lambda, \beta) + \delta U_{mic}(Z, A, \lambda, \beta)$$

Binding energy

$$B(Z, A) = U(Z, A, \lambda_{gs}, \beta_{gs}) - a_v \left(1 - a_s \left(\frac{N-Z}{A}\right)^2\right) A$$

$$+ W \left|\frac{N-Z}{A}\right| + \delta + c[(N-Z) - 58]$$

$$a_s = 1.778, W = 30\ MeV, c = 0.25\ MeV,$$

$$\delta = 4.8/N^{1/3}\delta_{N,odd} + 4.8/Z^{1/3}\delta_{Z,odd},$$

$$a_v = 15.703\ MeV\ at\ N-Z < 52; 15.715\ MeV\ at\ 54 \leqslant N-Z \leqslant 61,$$

Comparison with other calculations

^{270}Hs gs.:

λ=1.14, β=1.06 -> β_2=0.25, β_4=-0.03

P.Möller et al.: β_2=0.231, β_4=-0.086

For ^{268,269,270,271}Hs, the microscopic corrections are

-5.95, -6.38, -6.54, -6.64 MeV

P.Möller et al.: -5.94, -6.37, -5.95, -5.86 MeV

A.Kuzmina et al., Eur. Phys. J. A 47 (2011) 145

Phys. Rev. C 85 (2012) 014319; 017302

Strength parameters of paring interaction

$$G_{\substack{n\\p}} = \left(19.2 \mp 7.4\,\frac{N-Z}{A}\right) A^{-1}\ MeV$$

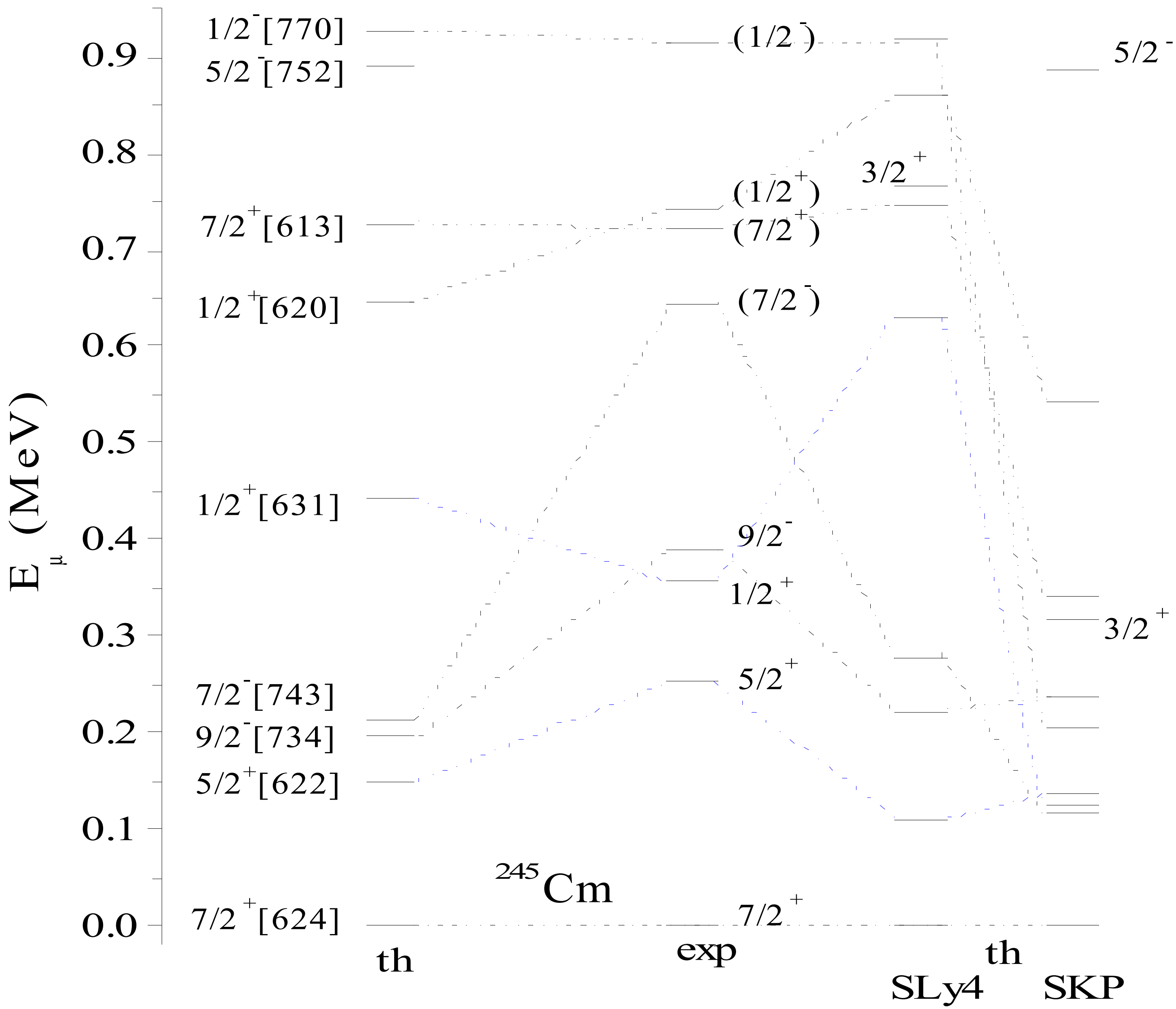
$$A \approx 250 \rightarrow G_n \approx 0.075\ MeV, G_p \approx 0.085\ MeV$$

One-quasiparticle excitations

$$E_\mu = \sqrt{\left(e_\mu - e_F\right)^2 + \Delta^2} - \sqrt{\left(e_\mu{}' - e_F\right)^2 + \Delta^2}$$

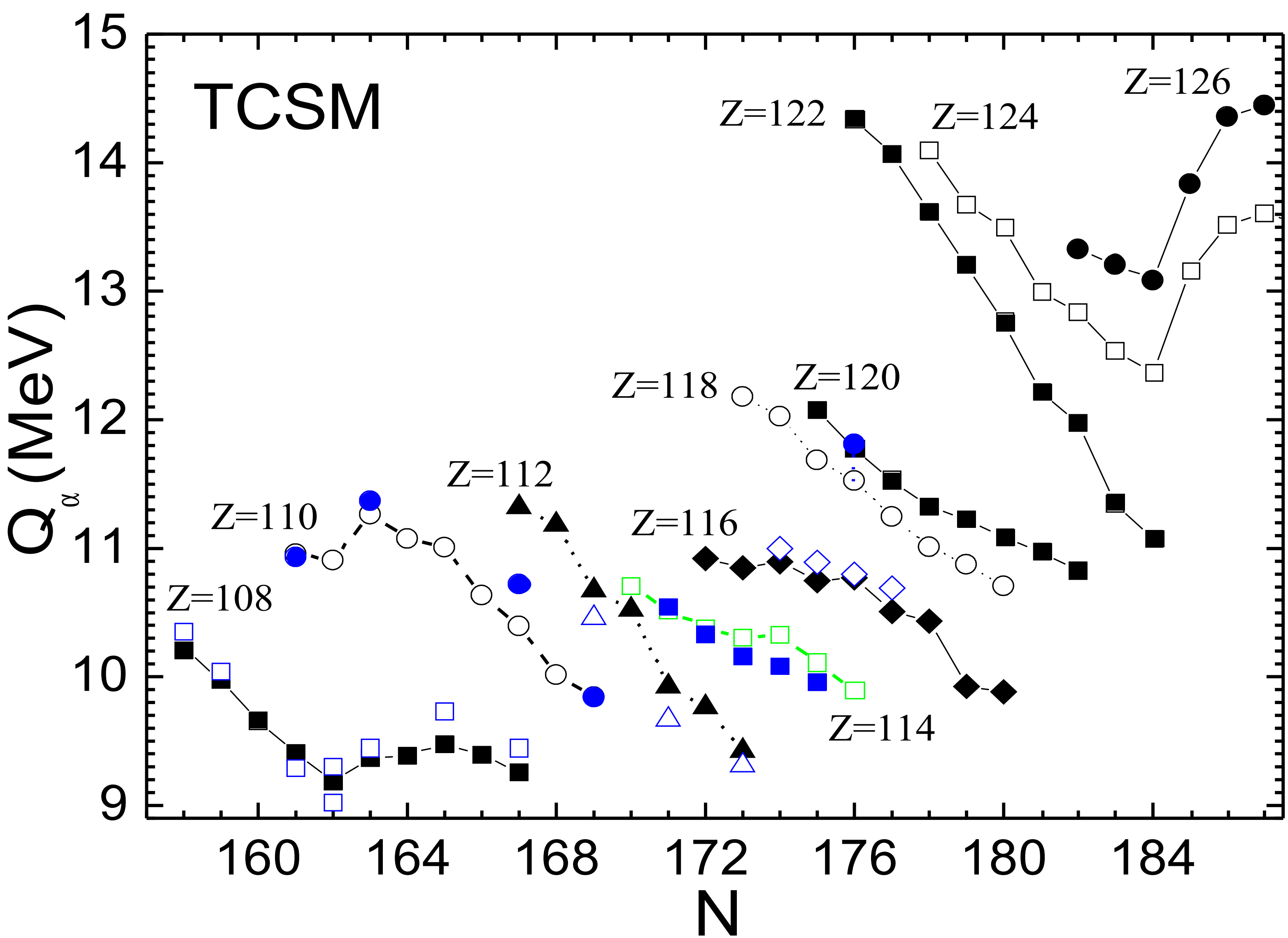
Two-quasiparticle excitations

$$E_\mu = \sqrt{\left(e_\mu - e_F\right)^2 + \Delta^2} + \sqrt{\left(e_\mu{}' - e_F\right)^2 + \Delta^2}$$

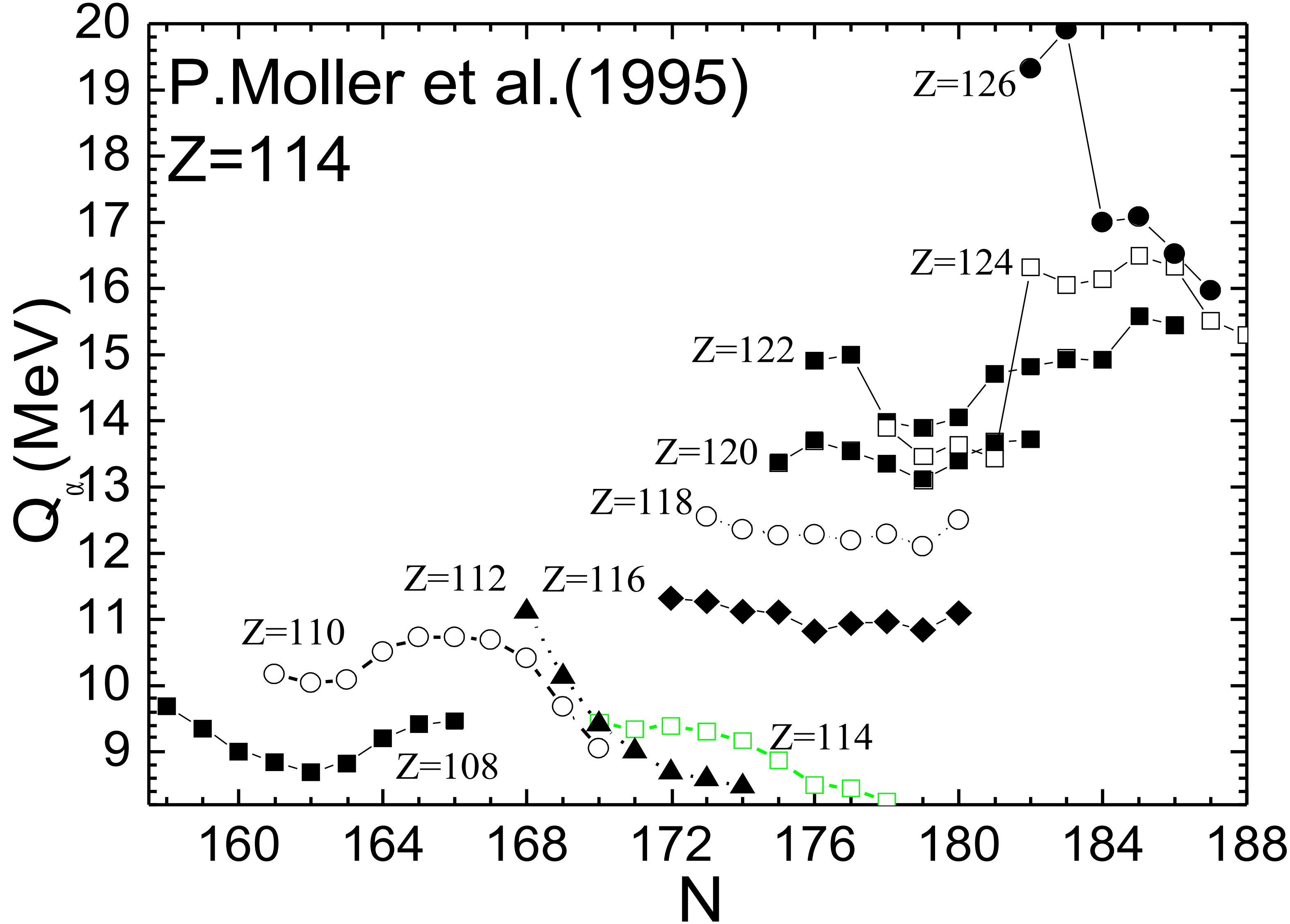


1/2-[770]
5/2-[752]
7/2+[613]
1/2+[620]
1/2+[631]
7/2-[743]
9/2-[734]
5/2+[622]
7/2+[624]
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Eμ (MeV)
(1/2-)
(1/2+)
(7/2+)
(7/2-)
9/2-
1/2+
5/2+
7/2+
3/2+
5/2-
3/2+
245Cm
th
exp
th
SLy4
SKP

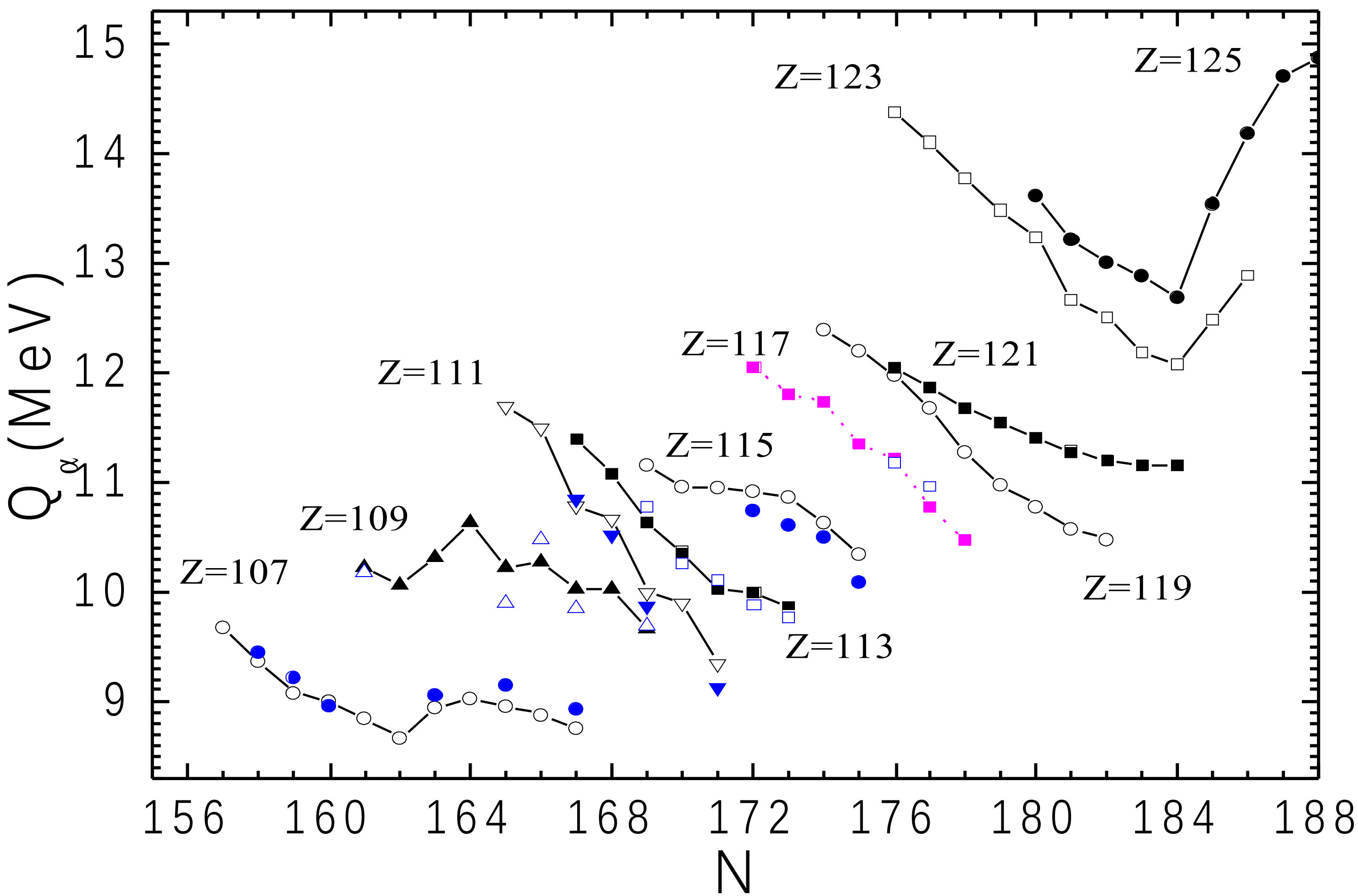
B.N. Lu, S.G. Zhou



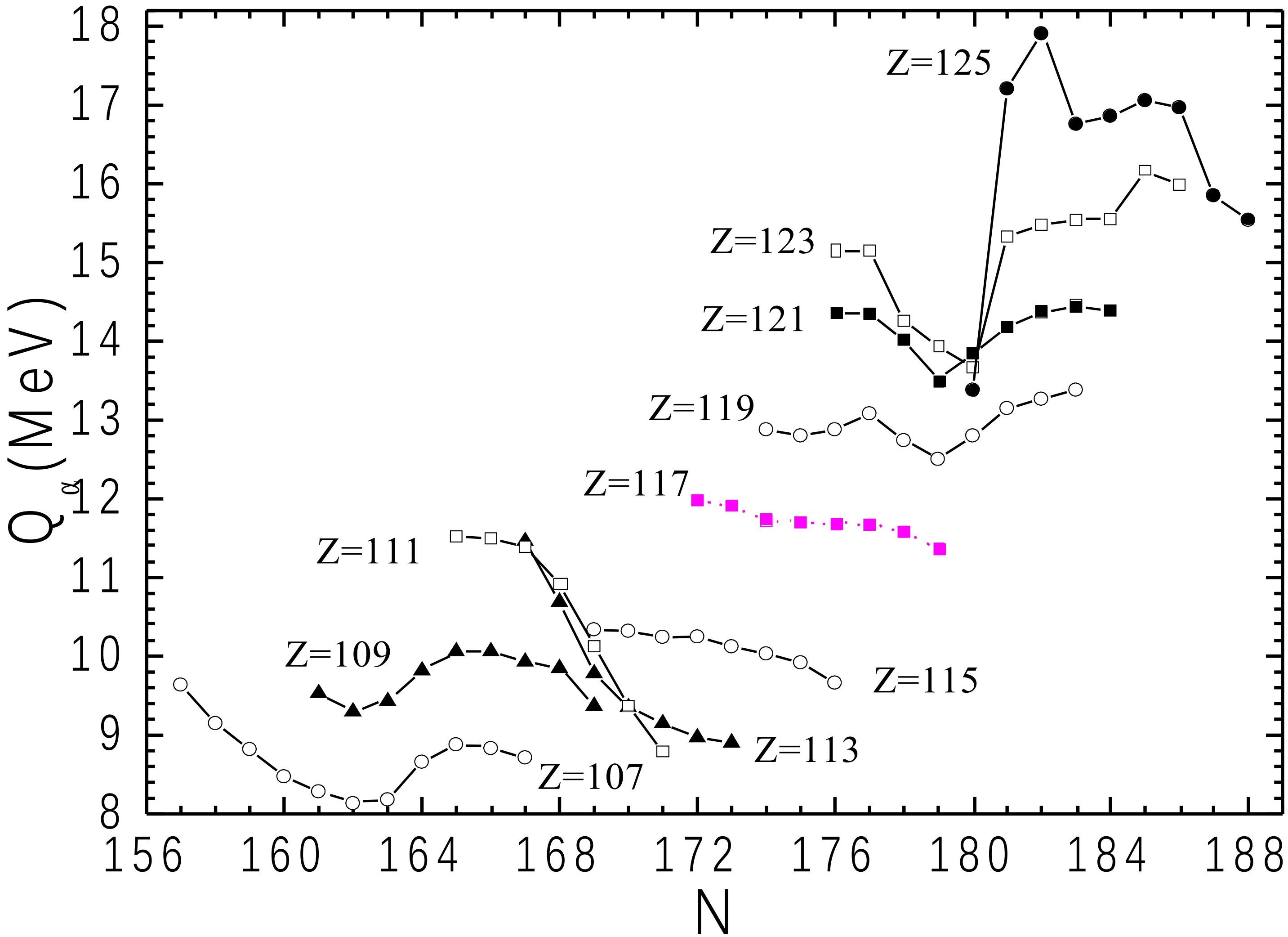
TCSM
Z=108
Z=110
Z=112
Z=114
Z=116
Z=118
Z=120
Z=122
Z=124
Z=126
Q_α (MeV)
N
15
14
13
12
11
10
9
160
164
168
172
176
180
184



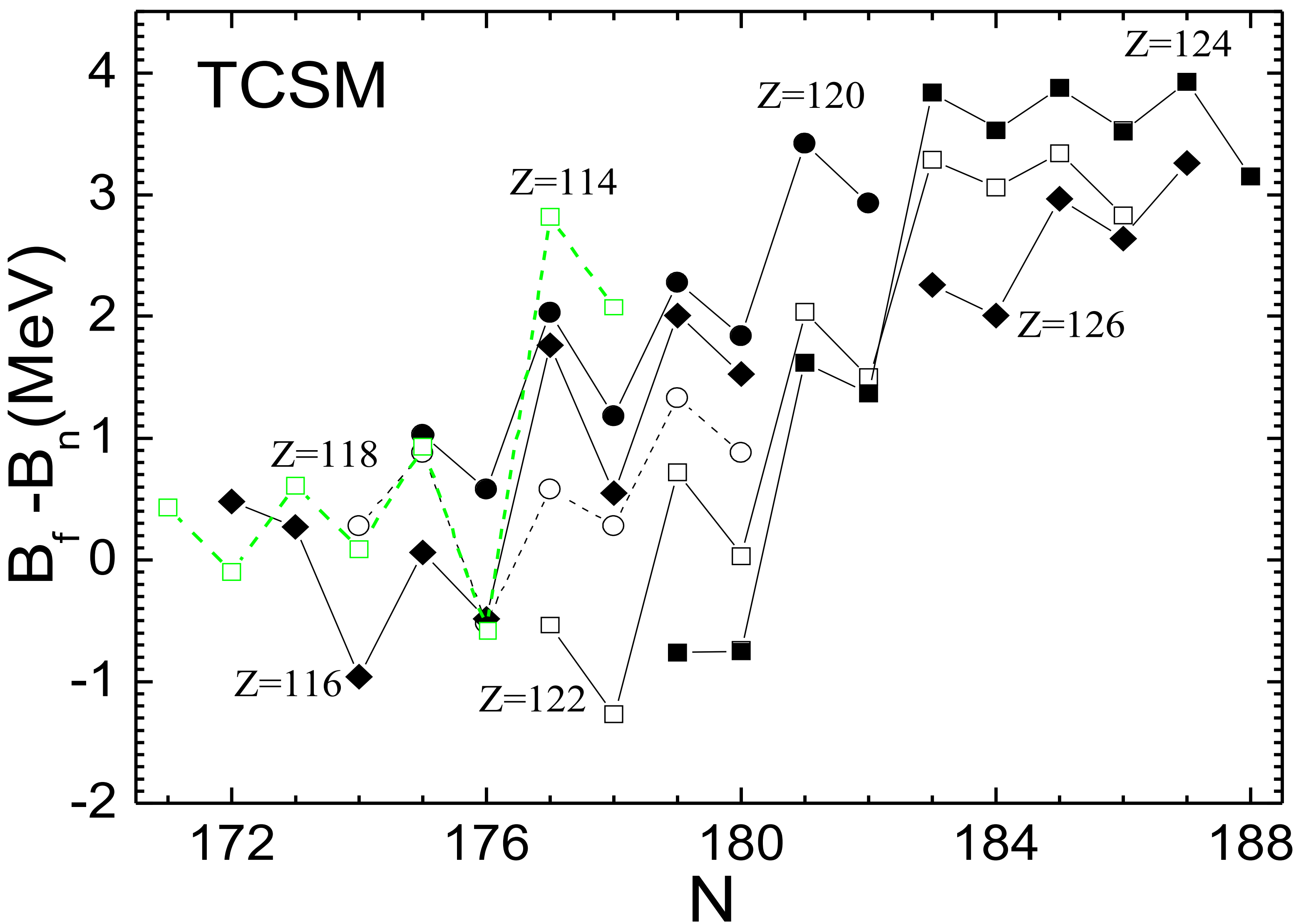
P.Moller et al.(1995)
Z=114
Z=126
Z=124
Z=122
Z=120
Z=118
Z=116
Z=112
Z=110
Z=108
Z=114
Q_α (MeV)
N
20
19
18
17
16
15
14
13
12
11
10
9
160
164
168
172
176
180
184
188



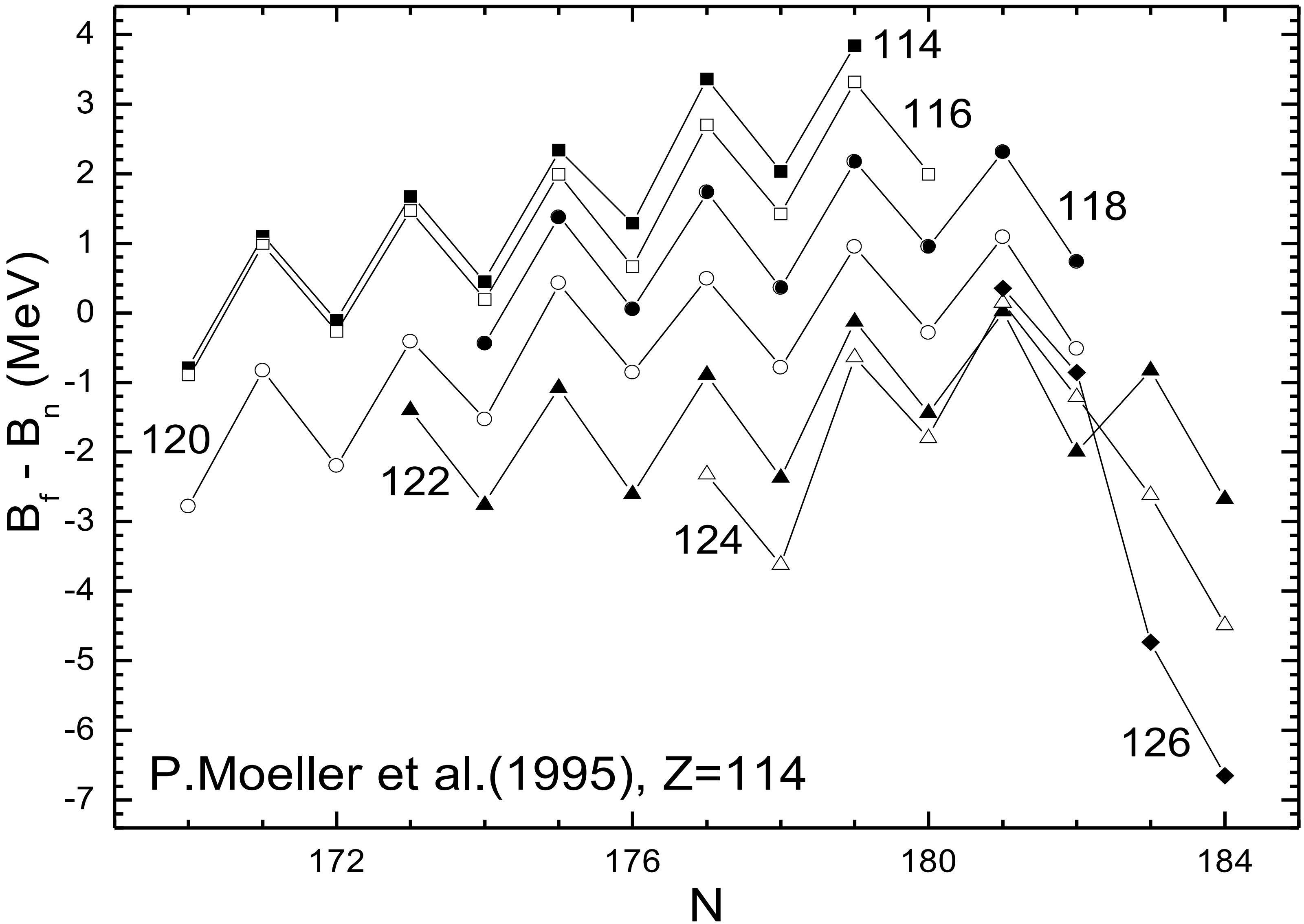
Z=107
Z=109
Z=111
Z=113
Z=115
Z=117
Z=119
Z=121
Z=123
Z=125
Q_α (MeV)
N



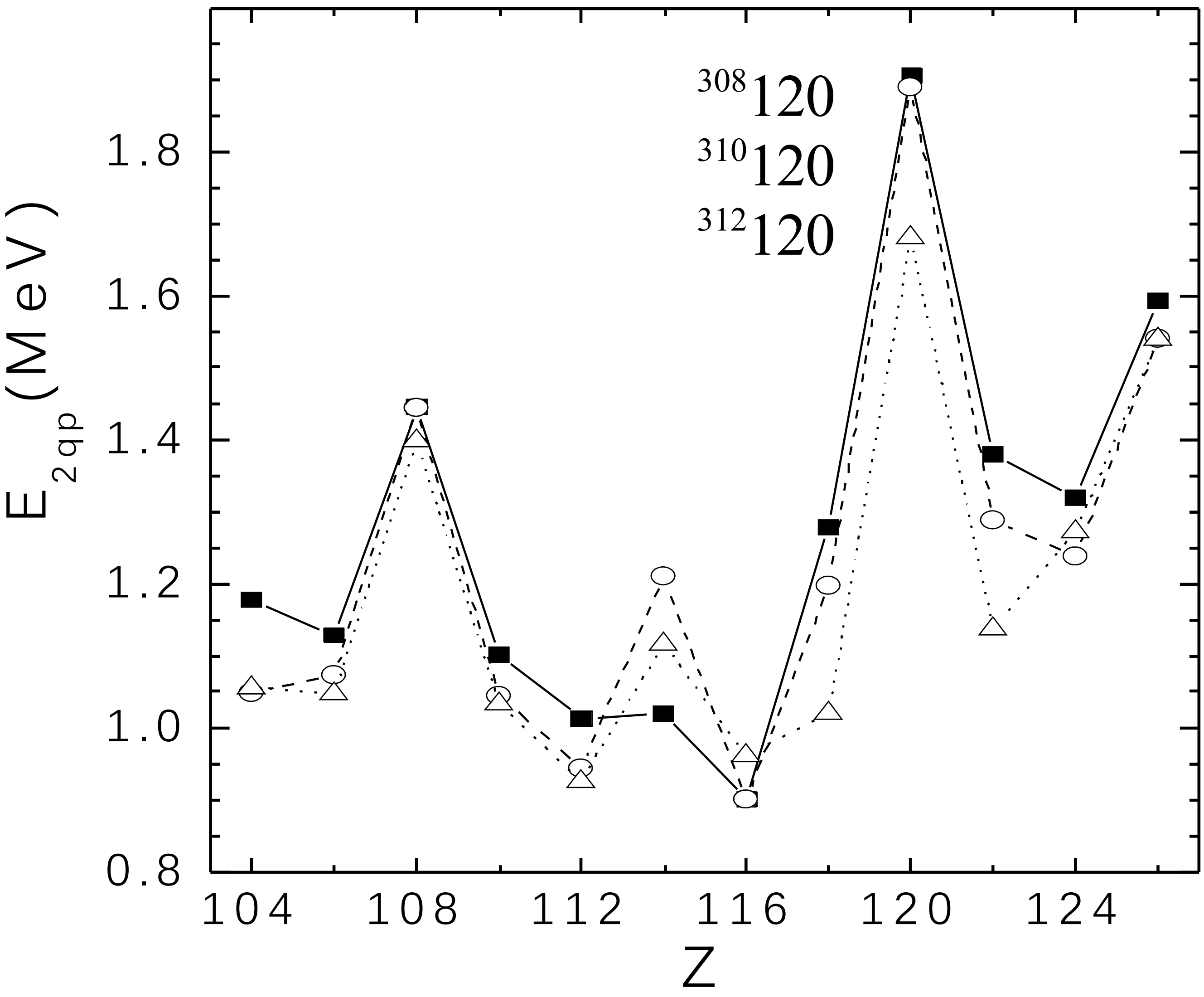
Z=125
Z=123
Z=121
Z=119
Z=117
Z=111
Z=109
Z=115
Z=113
Z=107
Q_{α} (MeV)
N
8
9
10
11
12
13
14
15
16
17
18
156
160
164
168
172
176
180
184
188



TCSM
Z=114
Z=116
Z=118
Z=120
Z=122
Z=124
Z=126
$B_f - B_n$(MeV)
N
4
3
2
1
0
-1
-2
172
176
180
184
188



4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
B_f - B_n (MeV)
172
176
180
184
N
114
116
118
120
122
124
126
P.Moeller et al.(1995), Z=114



Energies of two-quasi-proton states in alpha-decay chains containing indicated nuclei with Z=120

Parameter of level density

$$\rho = \frac{\exp[S(\beta, \lambda_Z, \lambda_N)]}{(2\pi)^{3/2}\sqrt{D}},$$

where S is the entropy, $\beta = T^{-1}$ is the inverse temperature, λ_Z and λ_N are the chemical potentials for protons and neutrons, respectively, and D is the determinant of the matrix comprised of the second derivatives of the entropy

$$D = \begin{vmatrix} \frac{\partial^2 S}{\partial \beta^2} & \frac{\partial^2 S}{\partial \beta \partial \mu_Z} & \frac{\partial^2 S}{\partial \beta \partial \mu_N} \\ \frac{\partial^2 S}{\partial \beta \partial \mu_Z} & \frac{\partial^2 S}{\partial \mu_Z^2} & 0 \\ \frac{\partial^2 S}{\partial \beta \partial \mu_N} & 0 & \frac{\partial^2 S}{\partial \mu_N^2} \end{vmatrix},$$

where $\mu_k = \beta\lambda_k$, $(k = N, Z)$.

$$S = 2\sum_{k=Z,N}\sum_{\nu}\left\{\ln[1+\exp(-\beta E_{\nu k})] + \frac{\beta E_{\nu k}}{1+\exp(\beta E_{\nu k})}\right\},$$

$$E_{\nu k} = \sqrt{(\varepsilon_{\nu k} - \lambda_k)^2 + \Delta_k^2}$$

$$Z = \sum_{\nu}\left(1 - \frac{\varepsilon_{\nu Z} - \lambda_Z}{E_{\nu Z}}\tanh[\frac{1}{2}\beta E_{\nu Z}]\right),$$

$$N = \sum_{\nu}\left(1 - \frac{\varepsilon_{\nu N} - \lambda_N}{E_{\nu N}}\tanh[\frac{1}{2}\beta E_{\nu N}]\right),$$

$$\frac{2}{G_Z} = \sum_{\nu}\frac{\tanh[\beta E_{\nu Z}/2]}{E_{\nu Z}},$$

$$\frac{2}{G_N} = \sum_{\nu}\frac{\tanh[\beta E_{\nu N}/2]}{E_{\nu N}},$$

where G_Z and G_N are the constants of pairing interaction.

The total E and excitation U energies of the nucleus at temperature T are calculated as

$$E(T) = \sum_{k=Z,N} \left\{ \sum_{\nu} \varepsilon_{\nu k} \left(1 - \frac{\varepsilon_{\nu k} - \lambda_k}{E_{\nu k}} \tanh \frac{1}{2} \beta E_{\nu k} \right) - \frac{\Delta_k^2}{G_k} \right\},$$

$$U = E(T) - E(0).$$

Level-density parameter

$$\rho_{FG}(U) = \frac{\sqrt{\pi}}{12 a^{1/4} E^{*5/4}} \exp 2\sqrt{a E^*},$$

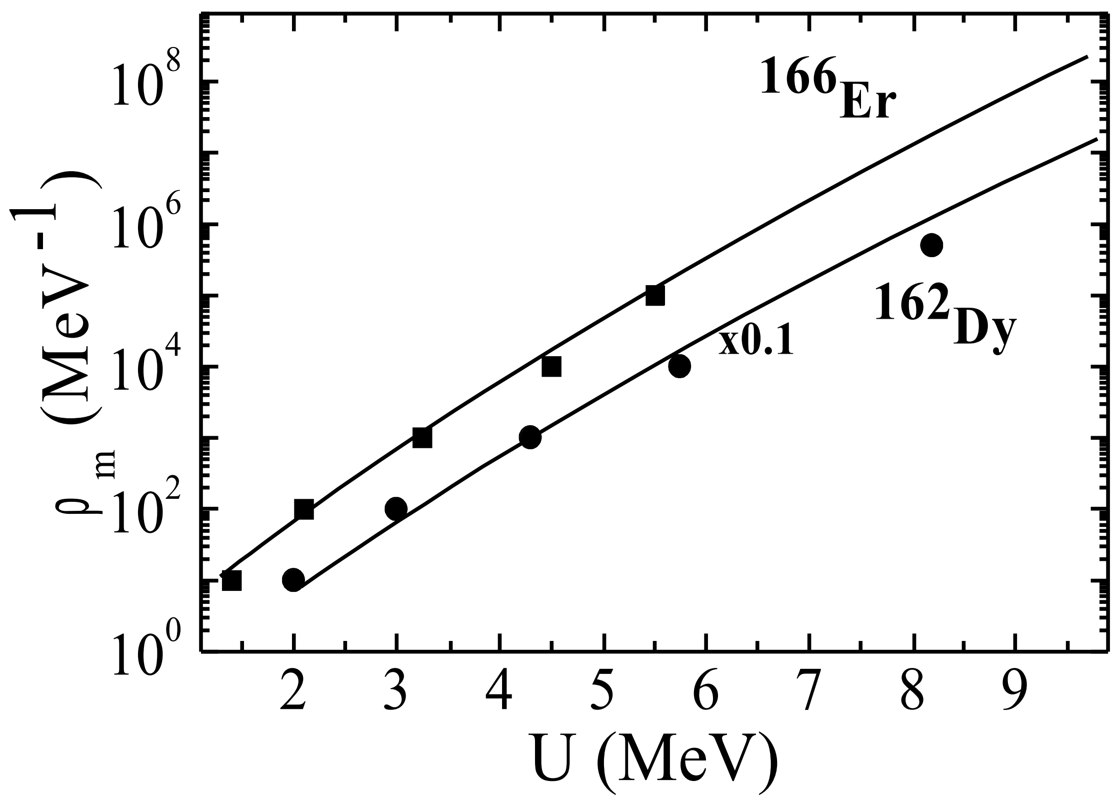
where $E^* = U - \Delta_Z - \Delta_N$ is the excitation energy back-shifted

$$a = U/T^2$$

or

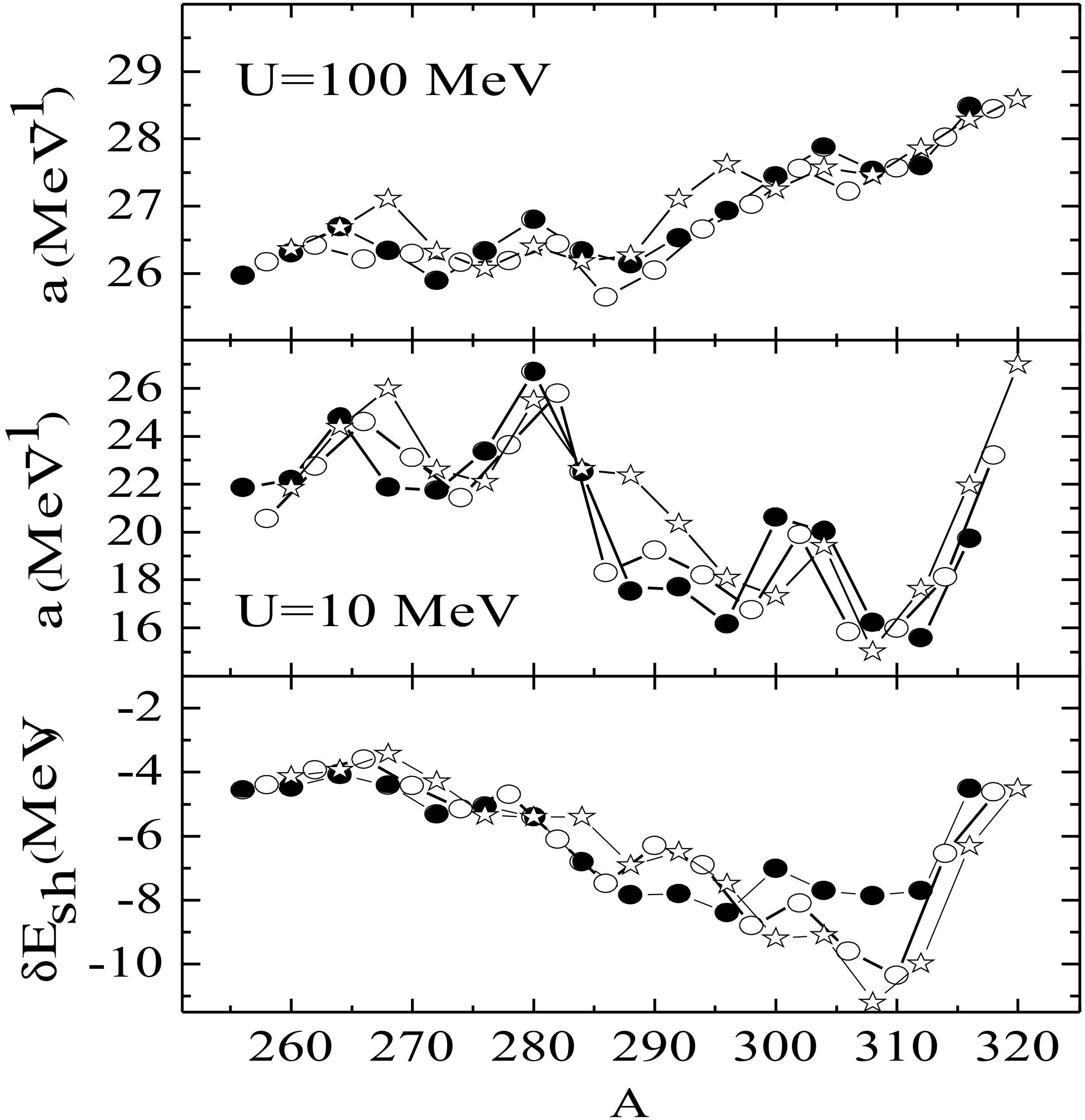
$$a = \frac{\pi^2}{6T} \sum_{k=Z,N} \sum_{\nu} \overline{n}_{k\nu}(1 - \overline{n}_{k\nu})$$

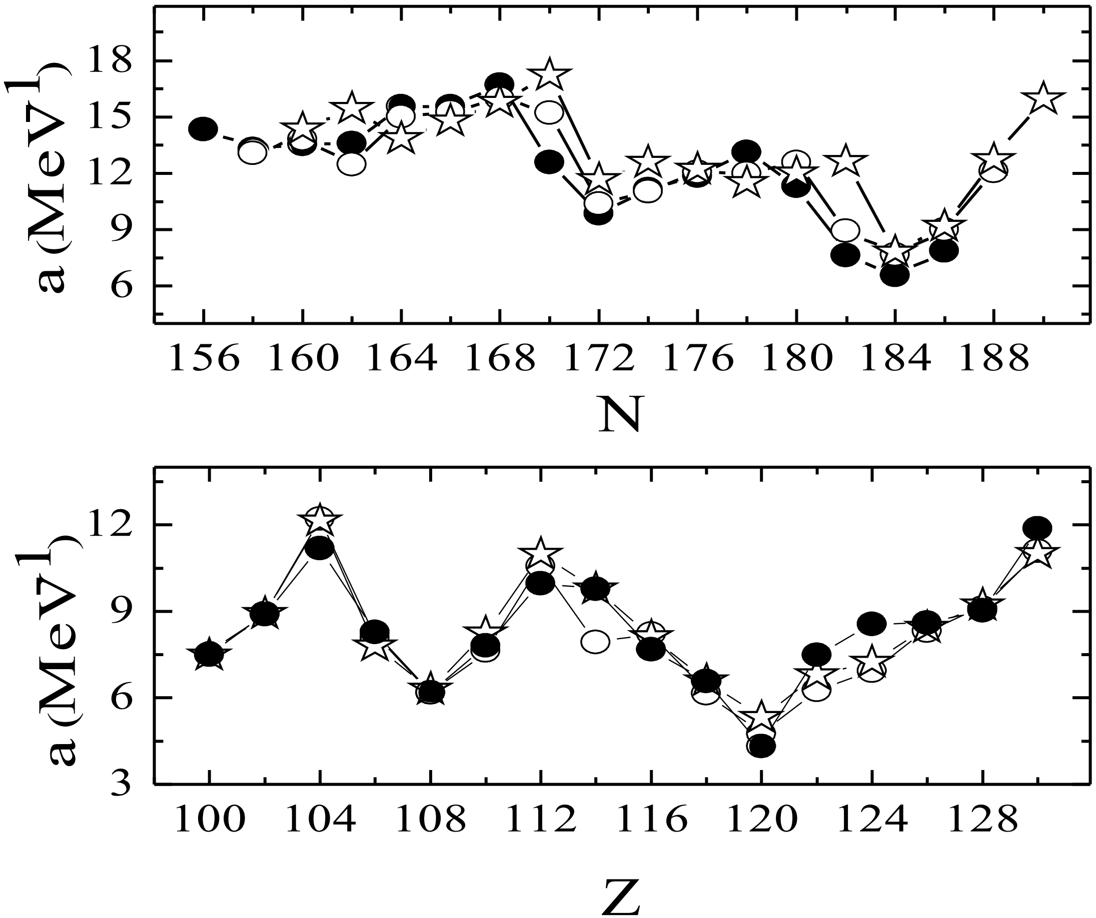
$$\overline{n}_{k\nu} = \left[1 + \exp\left(\beta E_{k\nu}\right)\right]^{-1}$$



E. Melby, M. Guttormsen, J. Rekstad, A. Schiller, and S. Siem, Phys. Rev. C **63**, 044309 (2001).

$^{296,298,300}120$





$$a(A,U) = \tilde{a}(A)\left[1 + \frac{1 - \exp\{-E^*/E'_D\}}{E^*}\delta E_{sh}\right]$$

E'_D=27 MeV

$$\tilde{a}(A) = \alpha A + \beta A^2$$

α=0.118 MeV^{-1} and β=-0.53$\times 10^{-4}$ MeV^{-1}

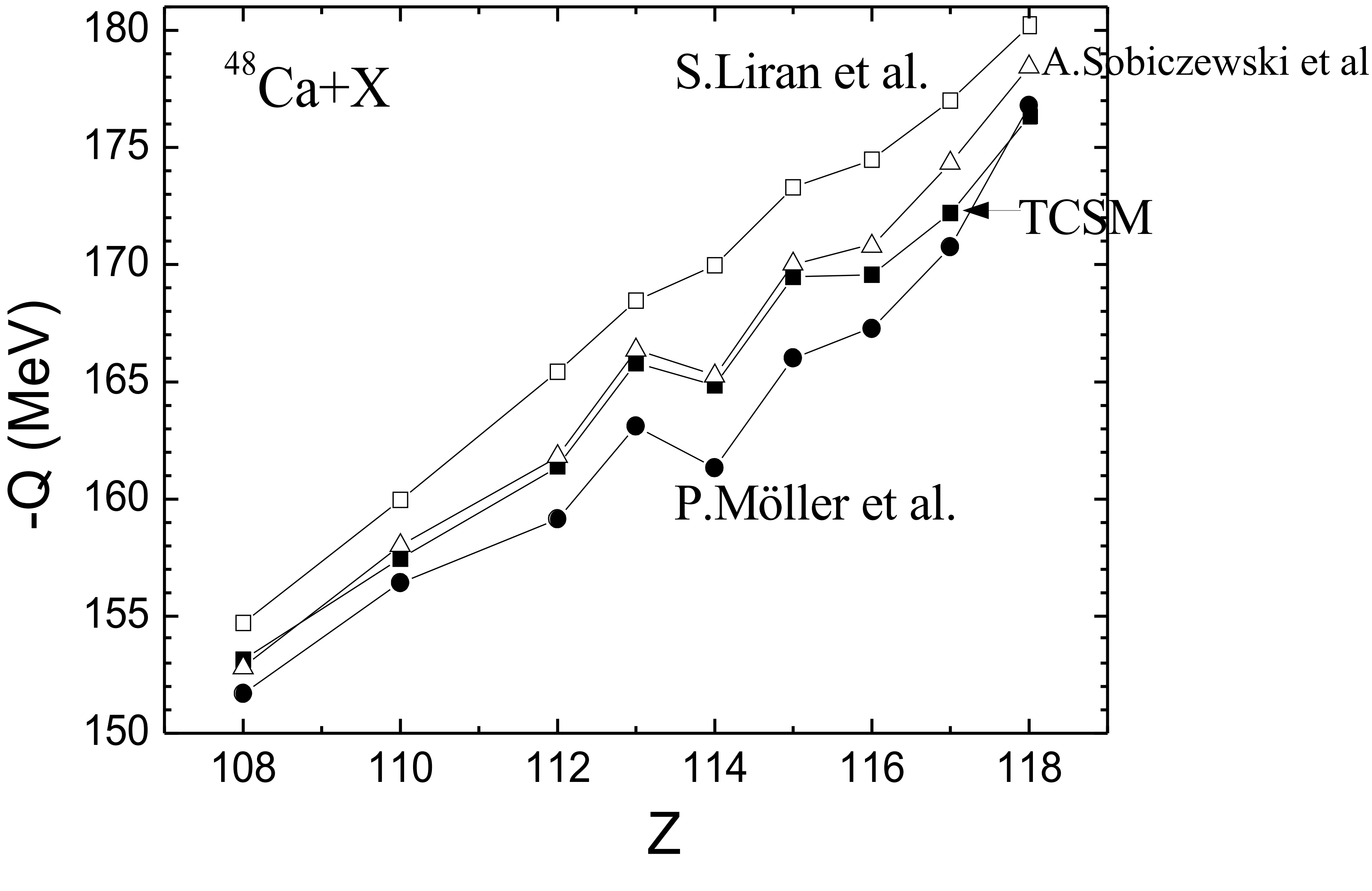
Dependence of the calculated evaporation residue cross sections on the predicted shell structure and magic numbers of SHE

Within the dinuclear system model we analysed the production of SHE in various actinide-based complete fusion reactions with projectiles heavier than ^{48}Ca.

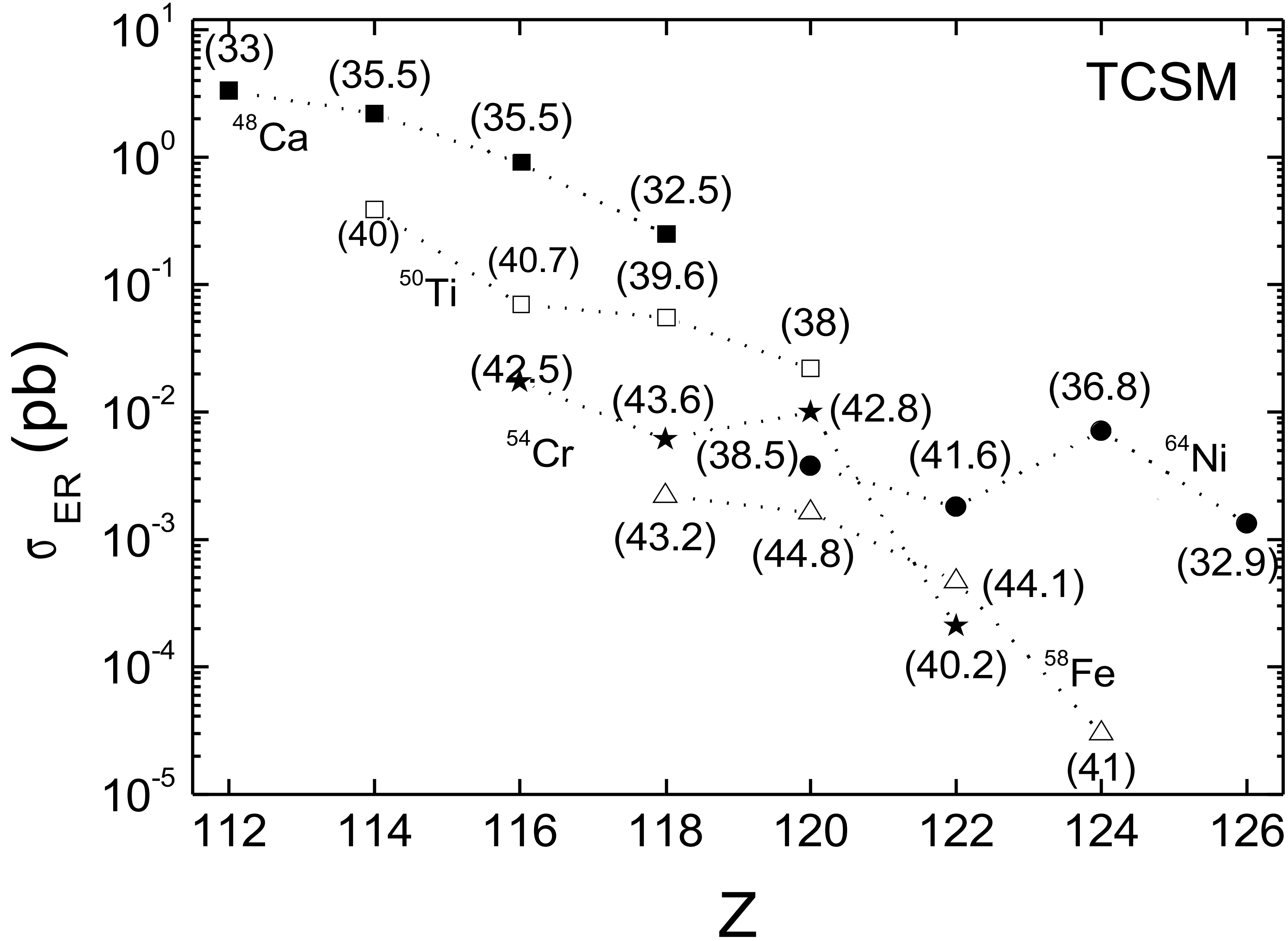
Different predictions of the properties of heaviest nuclei were used:

1) mass table from TCSM (2011), Z=120-126;

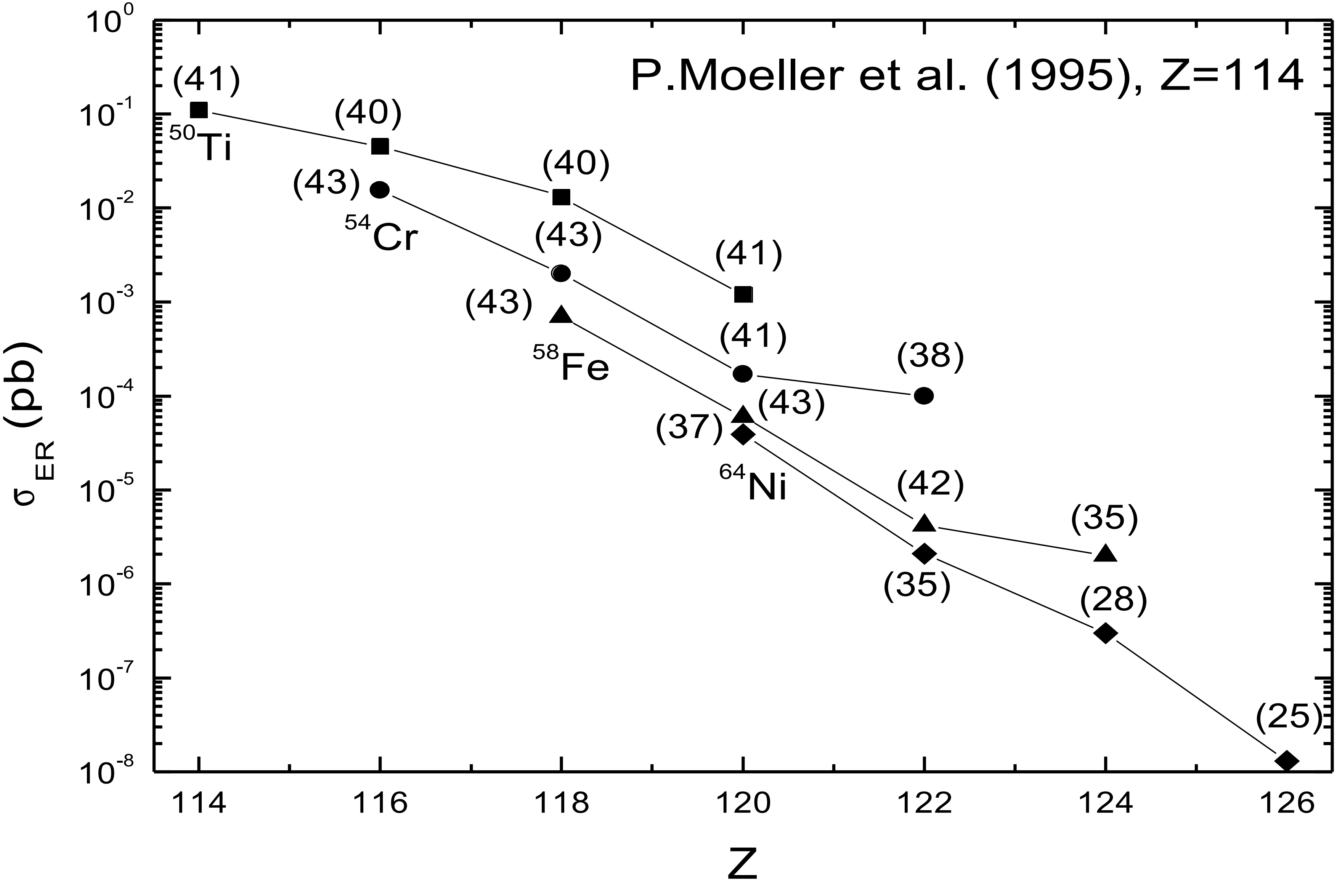
2) mass table by P. Moeller et al.(1995), Z=114;



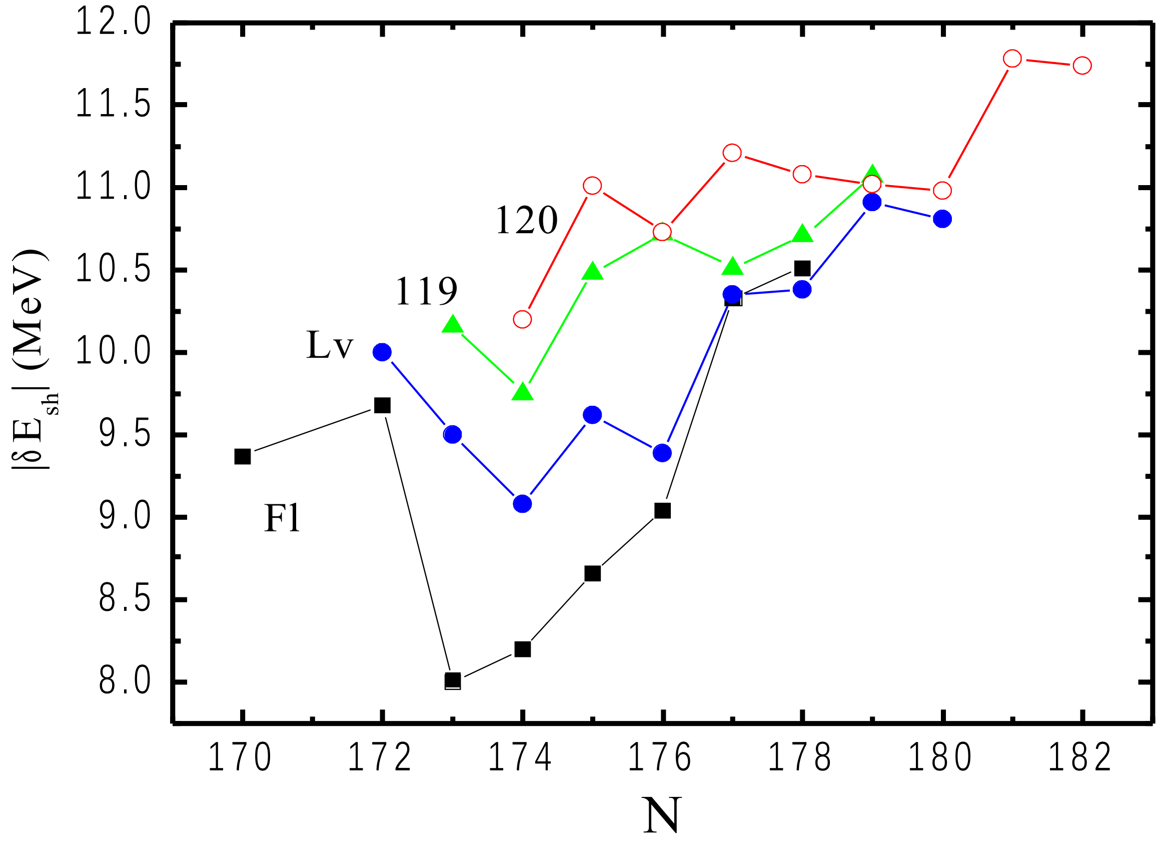
^{48}Ca+X
S.Liran et al.
A.Sobiczewski et al.
TCSM
P.Möller et al.
-Q (MeV)
Z
180
175
170
165
160
155
150
108
110
112
114
116
118



TCSM
σ_{ER} (pb)
Z
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
112
114
116
118
120
122
124
126
^{48}Ca
^{50}Ti
^{54}Cr
^{58}Fe
^{64}Ni
(33)
(35.5)
(35.5)
(32.5)
(40)
(40.7)
(39.6)
(38)
(42.5)
(43.6)
(42.8)
(38.5)
(41.6)
(36.8)
(43.2)
(44.8)
(32.9)
(44.1)
(40.2)
(41)

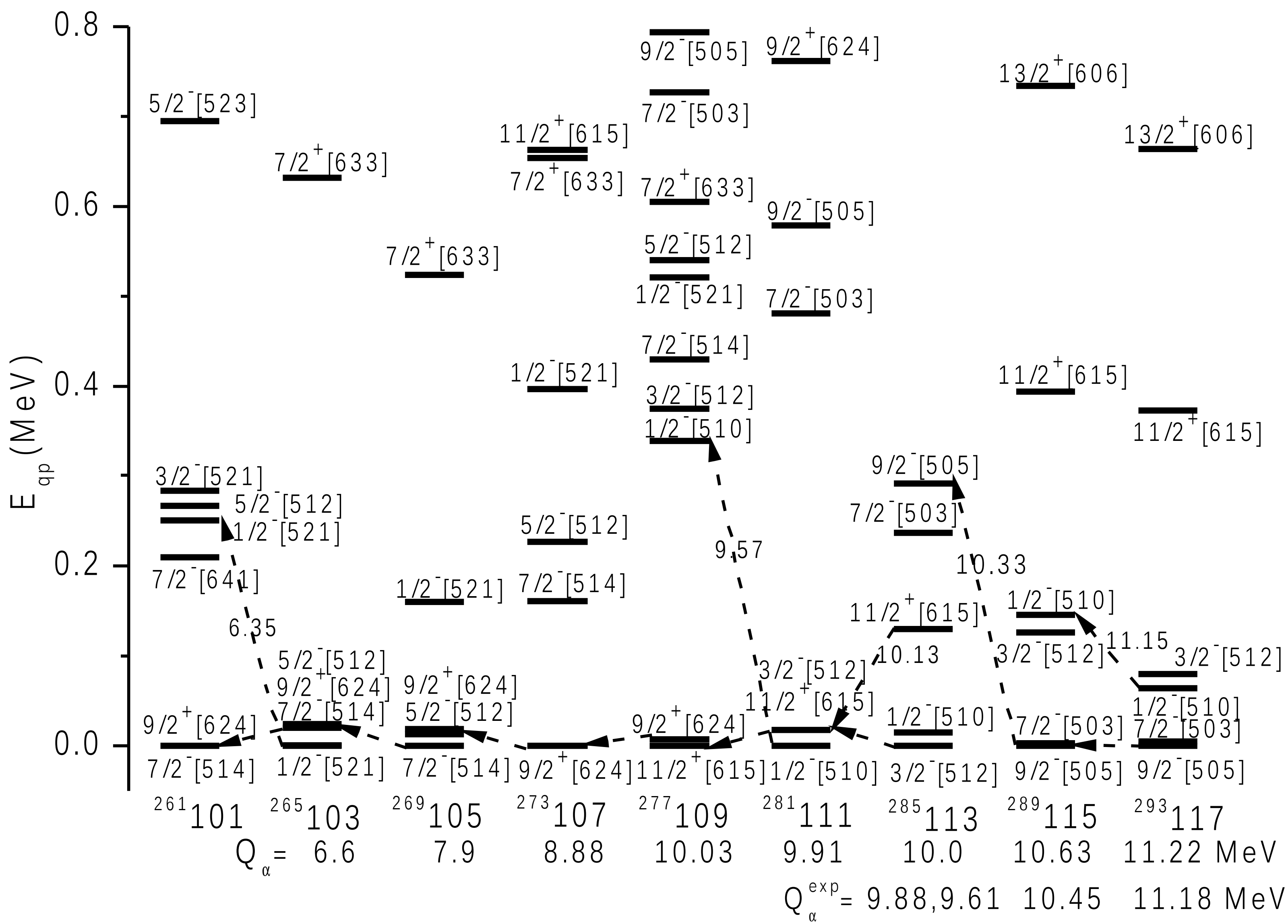


P.Moeller et al. (1995), Z=114
σ_{ER} (pb)
Z
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
114
116
118
120
122
124
126
^{50}Ti
^{54}Cr
^{58}Fe
^{64}Ni
(41)
(40)
(40)
(41)
(43)
(43)
(41)
(38)
(43)
(43)
(42)
(35)
(37)
(35)
(28)
(25)



Woods-Saxon potential with the depth

$$V_{\frac{Z}{N}} = 52 \pm 28.8 \frac{N-Z}{A}$$



0.8
0.6
0.4
0.2
0.0
E_qp (MeV)
5/2-[523]
3/2-[521]
5/2-[512]
1/2-[521]
7/2-[641]
6.35
9/2+[624]
7/2-[514]
7/2+[633]
5/2-[512]
9/2+[624]
7/2-[514]
1/2-[521]
7/2+[633]
1/2-[521]
9/2+[624]
5/2-[512]
7/2-[514]
11/2+[615]
7/2+[633]
1/2-[521]
5/2-[512]
7/2-[514]
9/2+[624]
9/2-[505]
7/2-[503]
7/2+[633]
5/2-[512]
1/2-[521]
7/2-[514]
3/2-[512]
1/2-[510]
9.57
9/2+[624]
11/2+[615]
9/2+[624]
9/2-[505]
7/2-[503]
3/2-[512]
11/2+[615]
1/2-[510]
9/2-[505]
7/2-[503]
10.33
11/2+[615]
10.13
1/2-[510]
3/2-[512]
13/2+[606]
11/2+[615]
1/2-[510]
3/2-[512]
7/2-[503]
9/2-[505]
11.15
13/2+[606]
11/2+[615]
3/2-[512]
1/2-[510]
7/2-[503]
9/2-[505]
261 101
265 103
269 105
273 107
277 109
281 111
285 113
289 115
293 117
Qα = 6.6 7.9 8.88 10.03 9.91 10.0 10.63 11.22 MeV
Qα exp = 9.88,9.61 10.45 11.18 MeV

Summary

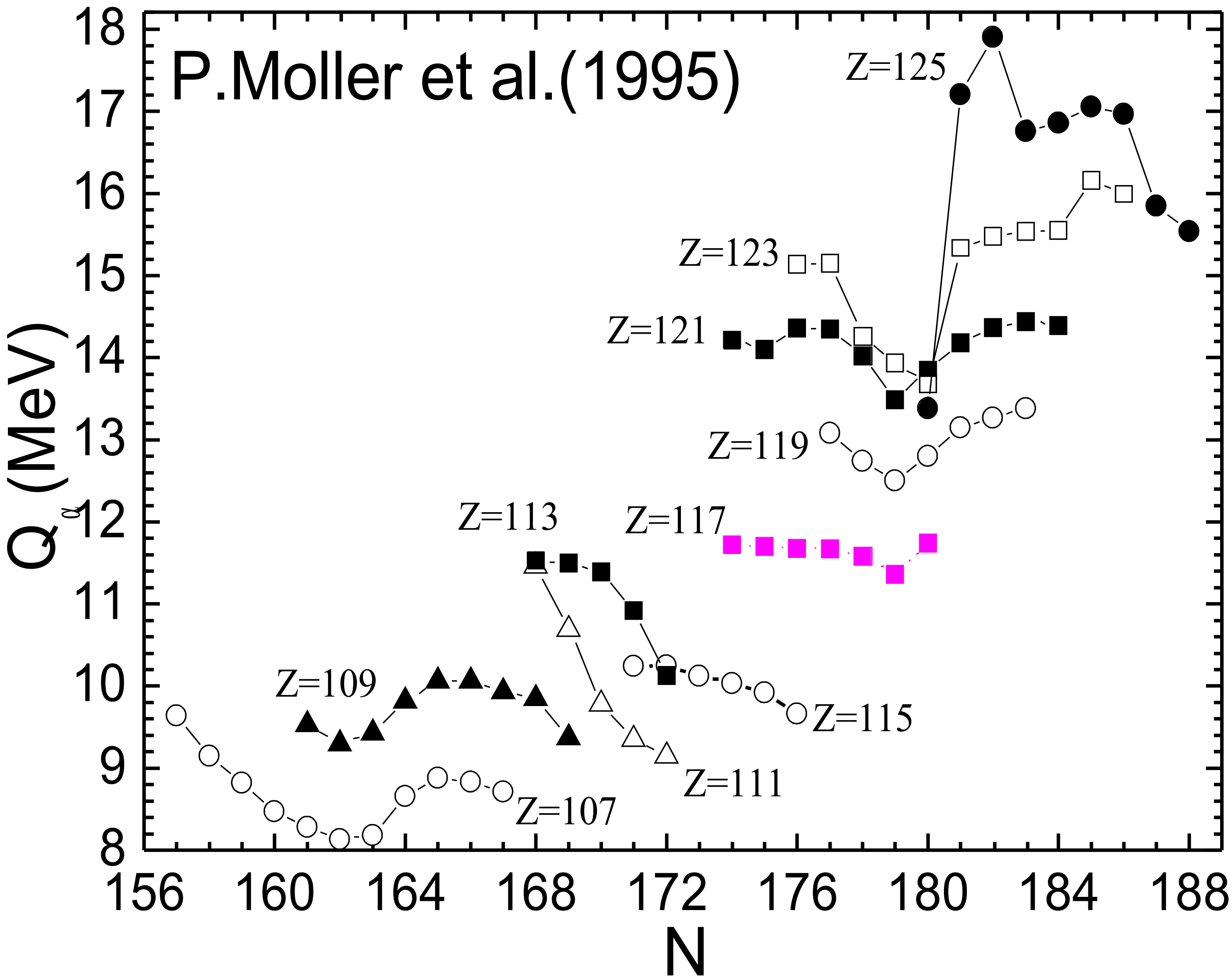
The calculations performed with the modified **TCSM** reveal quite strong shell effects at **Z=120-126** & **N=184** as in the self-consistent mean-field treatments.

If our prediction of the structure of heaviest nuclei is correct, than one can expect the production of **Z=120** in the reactions **$^{50}Ti+^{249}Cf$** and **$^{54}Cr+^{248}Cm$** with the cross sections **23** and **10 fb**, respectively.

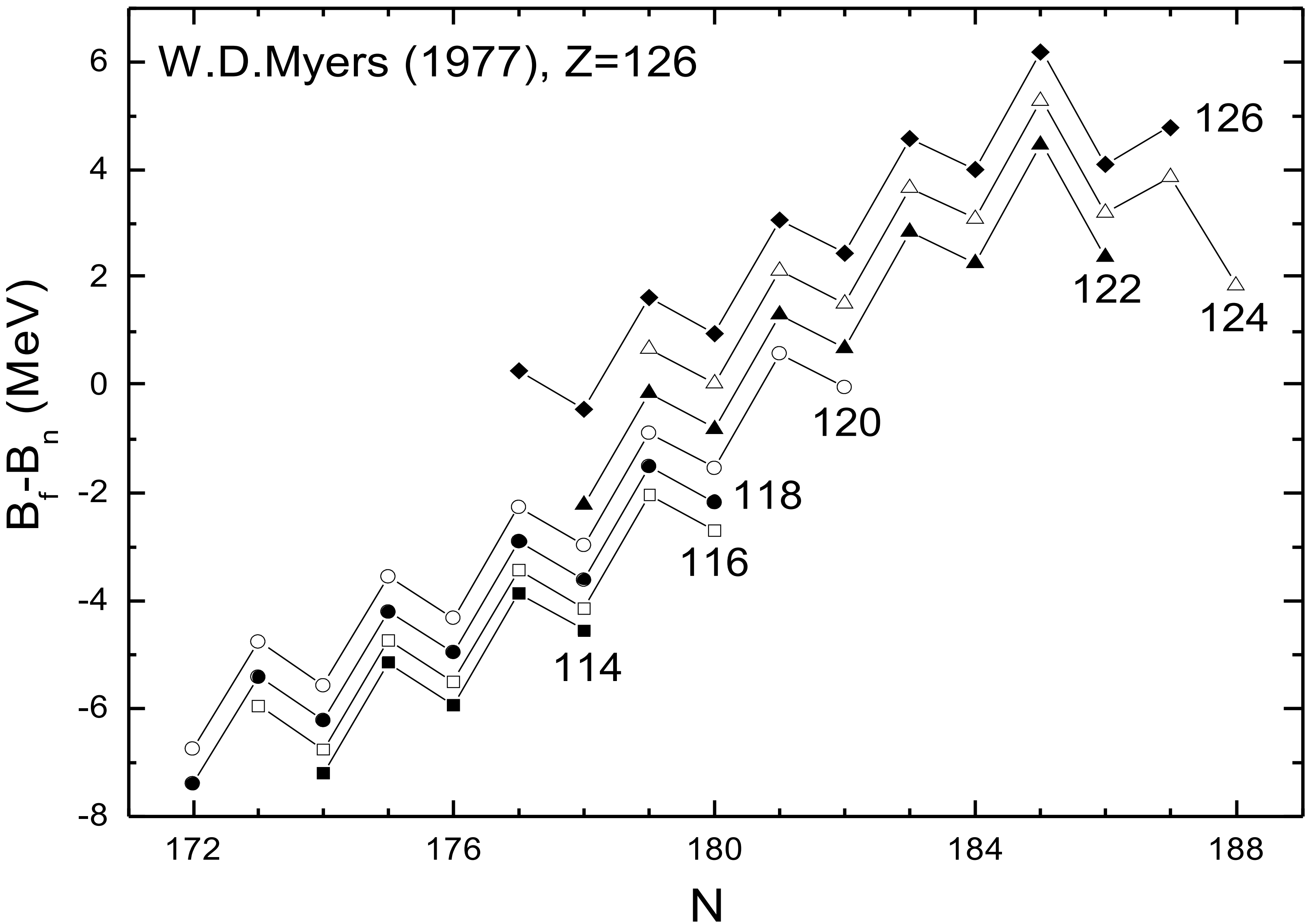
Z=120 nuclei with **N=175-179** are expected to have **Q_α** about **12.1 - 11.2 MeV** and lifetimes **1.7 ms - 0.16 s** in accordance with our predictions. These **Q_α** are in fair agreement with **S.Liran et al.(2000)** and about **2 MeV** smaller than those from **P.Möller et al.(1995) & A.Sobiczewski et al.(2003)**.

Experimental measurement of **Q_α** for at least one isotope of **Z=120** nucleus would help us to set proper shell model for the **SHE** with **Z>118**.

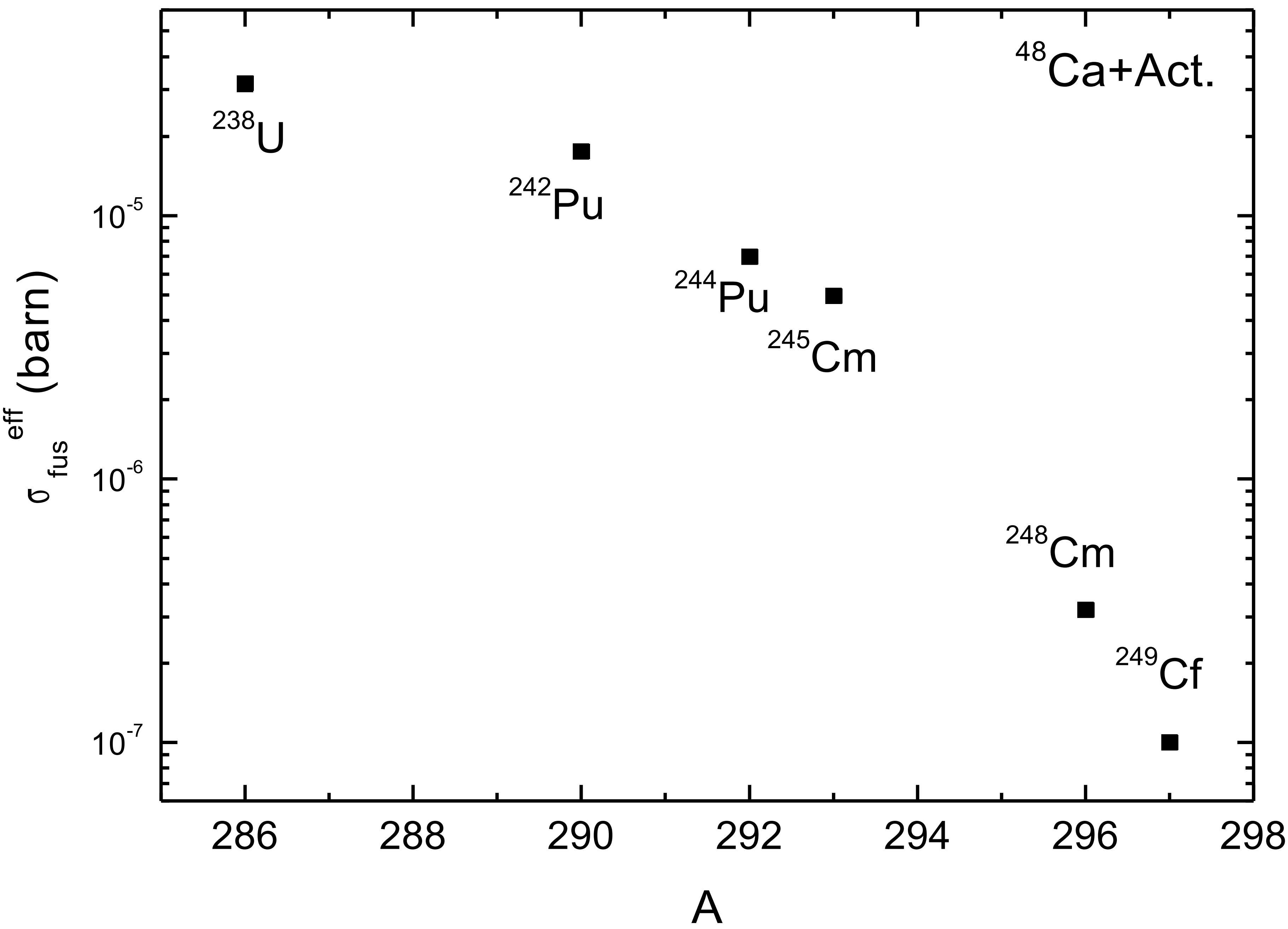
Thank you.



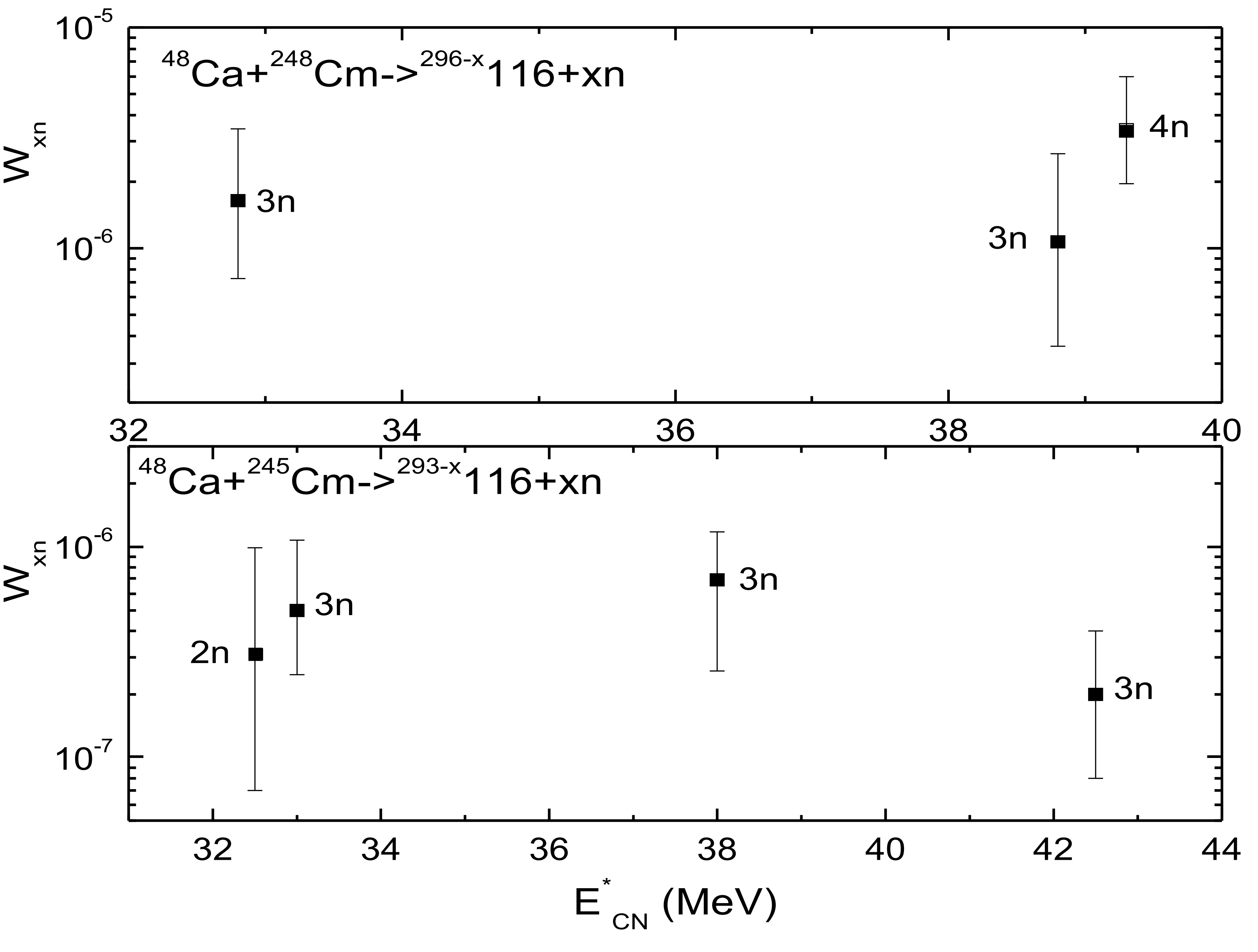
P.Moller et al.(1995)
Z=125
Z=123
Z=121
Z=119
Z=117
Z=113
Z=115
Z=111
Z=109
Z=107
Q_α (MeV)
N



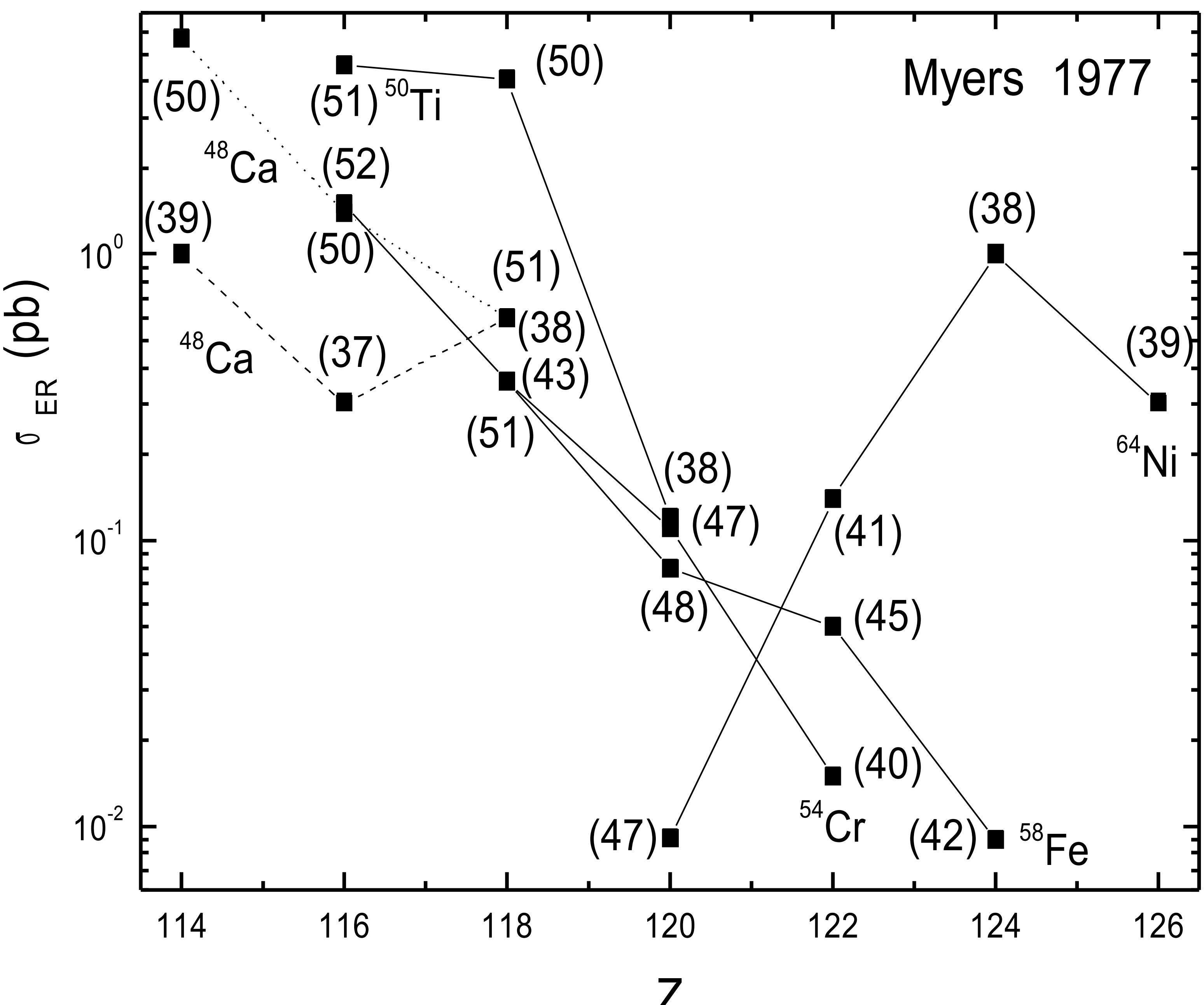
W.D.Myers (1977), Z=126
126
122
124
120
118
116
114
B_f-B_n (MeV)
N
6
4
2
0
-2
-4
-6
-8
172
176
180
184
188

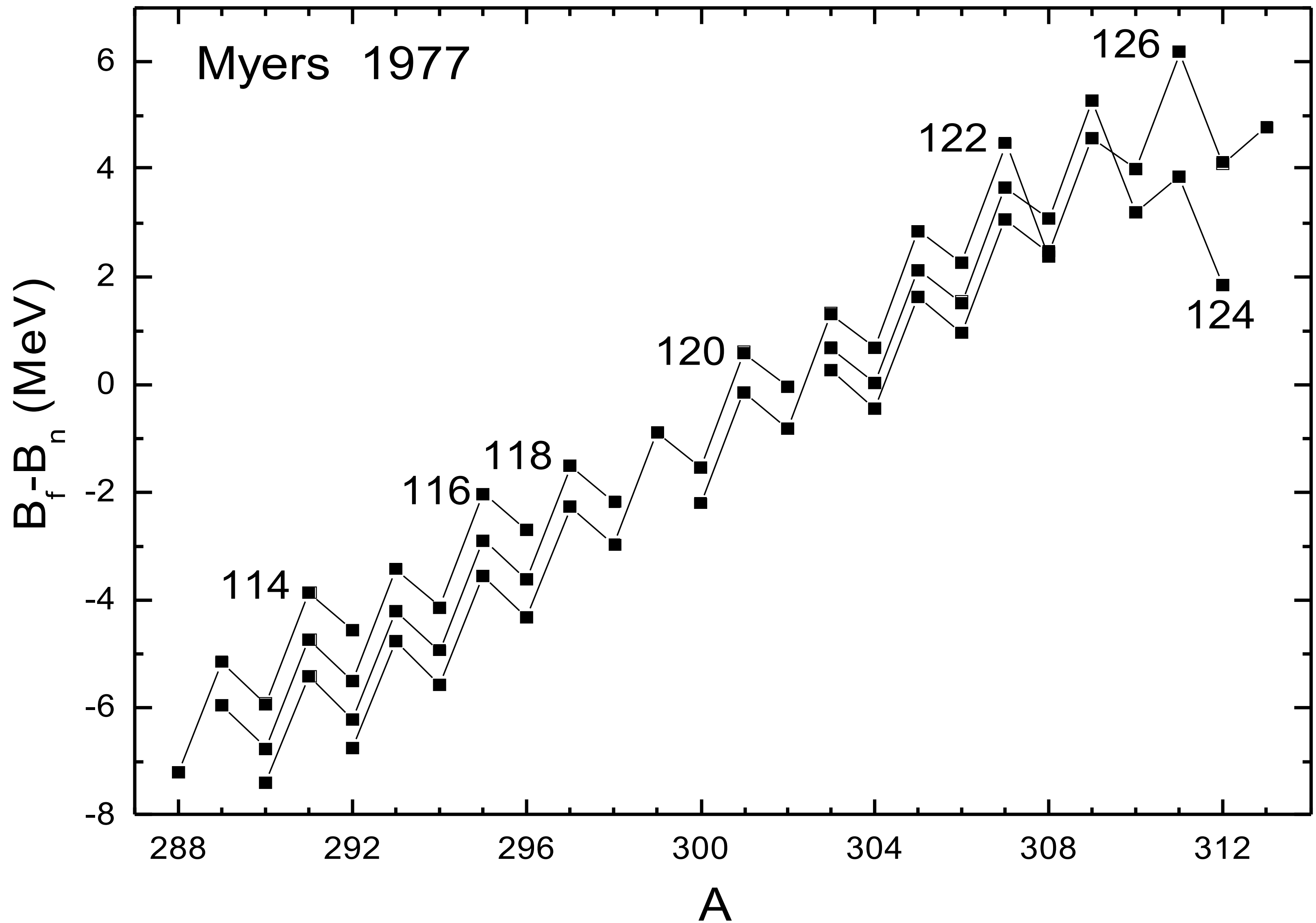


^{48}Ca+Act.
^{238}U
^{242}Pu
^{244}Pu
^{245}Cm
^{248}Cm
^{249}Cf
10^{-5}
10^{-6}
10^{-7}
σ_{fus}^{eff} (barn)
286
288
290
292
294
296
298
A



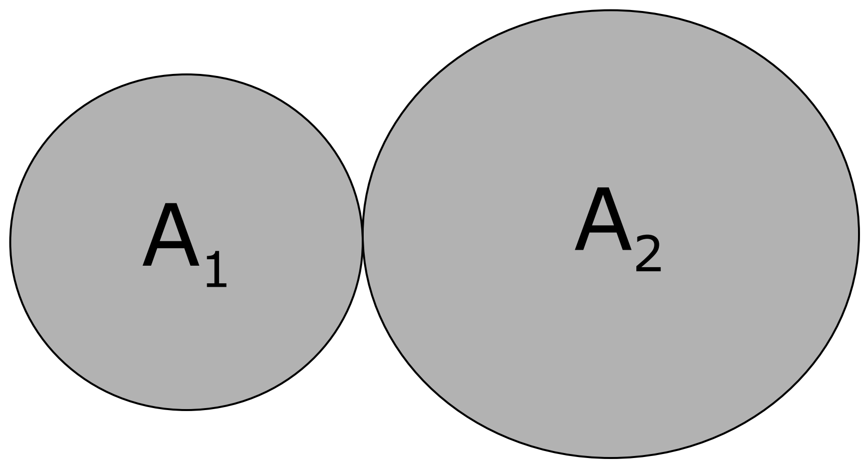
^{48}Ca+^{248}Cm->$^{296-x}$116+xn
W_{xn}
10^{-5}
10^{-6}
3n
3n
4n
32
34
36
38
40
^{48}Ca+^{245}Cm->$^{293-x}$116+xn
W_{xn}
10^{-6}
10^{-7}
2n
3n
3n
3n
32
34
36
38
40
42
44
E^{*}_{CN} (MeV)


Myers 1977
σ_{ER} (pb)
Z
114
116
118
120
122
124
126
10^0
10^{-1}
10^{-2}
^{48}Ca
^{48}Ca
^{50}Ti
^{54}Cr
^{58}Fe
^{64}Ni
(50)
(51)
(50)
(52)
(39)
(50)
(51)
(38)
(37)
(43)
(51)
(38)
(47)
(48)
(41)
(45)
(40)
(47)
(42)
(38)
(39)


Myers 1977
114
116
118
120
122
124
126
6
4
2
0
-2
-4
-6
-8
288
292
296
300
304
308
312
A
B_f-B_n (MeV)

Mass asymmetry coordinate

$$\eta = \frac{A_1 - A_2}{A_1 + A_2}$$



$\eta = 0$ for $A_1 = A_2$, $\eta = \pm 1$ for A_1 or $A_2 = 0$

Potential energy

$$U(Z,A,\lambda,\beta)=U_{LDM}(Z,A,\lambda,\beta)+\delta U_{mic}(Z,A,\lambda,\beta)$$

Binding energy

$$B(Z,A)=U(Z,A,\lambda_{gs},\beta_{gs})-a_v(1-a_s(\frac{N-Z}{A})^2)A$$

$$+W\left|\frac{N-Z}{A}\right|+\delta+c[(N-Z)-58]$$

$$a_s=1.778, W=30\ MeV, c=0.25\ MeV,$$

$$\delta=4.8/N^{1/3}\delta_{N,odd}+4.8/Z^{1/3}\delta_{Z,odd},$$

$$a_v=15.703\ MeV\ at\ N-Z<52; 15.715\ MeV\ at\ 54\leqslant N-Z\leqslant 61,$$