

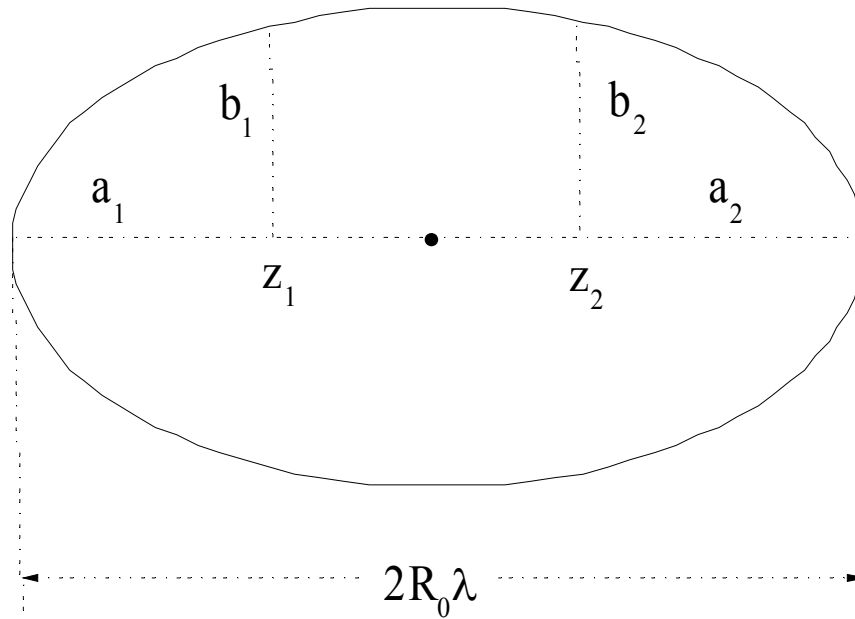
# Isomeric states in heaviest nuclei

1. Introduction
2. Parametrisation of nuclear shape with TCSM  
Comparison with other approaches
3. One- and two-quasiparticle isomers
4. Summary

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- Search of shells and subshells closure which crucially influence the evaporation residue cross sections
- Identification of heavy nuclei by  $\alpha$ -decay needs the analysis of isomer states
- Population of isomer states in reactions

$$\beta_i = a_i / b_i$$

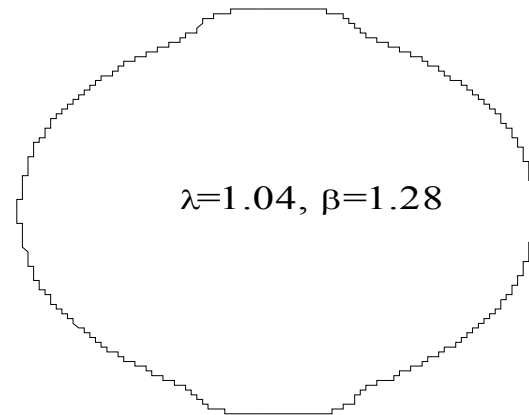
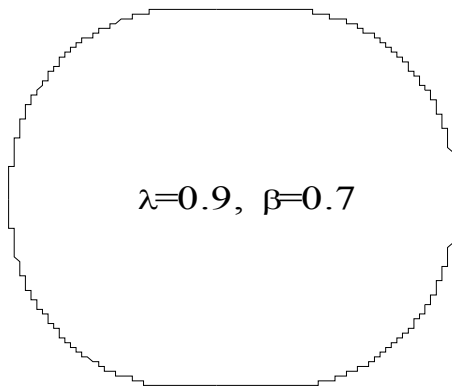
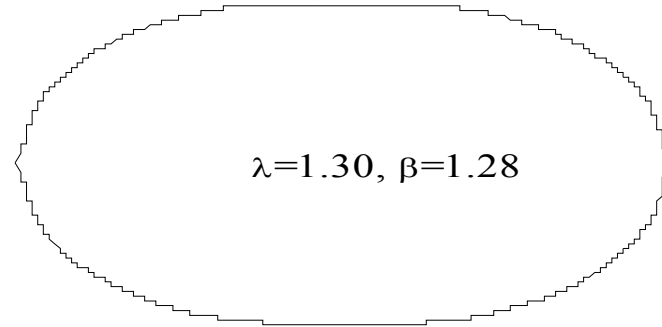
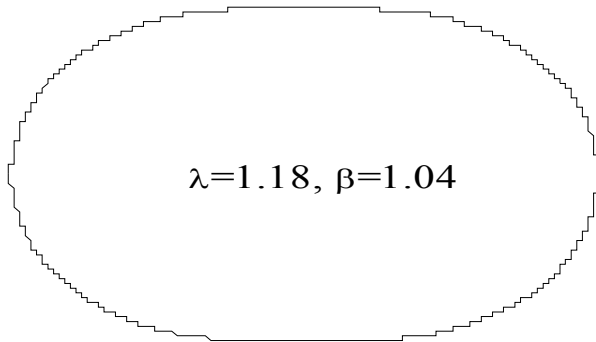


$\beta_1 = \beta_2$ , even multipolarities;       $\beta_1 \neq \beta_2$ , odd and even multipolarities

$R_0$  is the radius of spherical nucleus

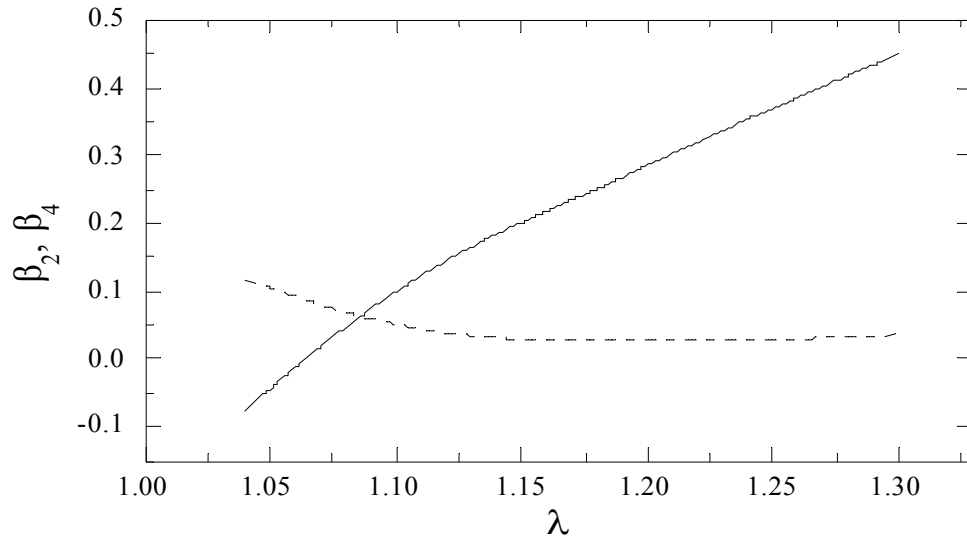
# Parametrisation of nuclear shape with TCSM

# $^{248}\text{Fm}$

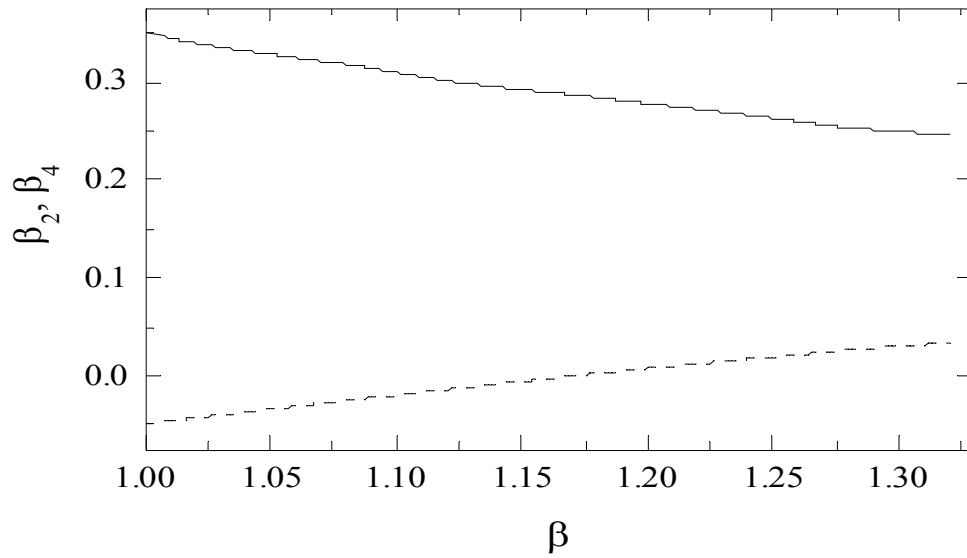


$^{248}\text{Fm}$

$\beta_2$  – solid \_ lines,  $\beta_4$  – dashed \_ lines



$\beta = 1.28$



$\lambda = 1.18$

$$H = T + V(\rho, z) + V_{LS} + V_{L^2}$$

$$V(\rho, z) = \begin{cases} \frac{1}{2} m \omega_z^2 (z - z_1)^2 + \frac{1}{2} m \omega_\rho^2 \rho^2, & z < z_1 \\ \frac{1}{2} m \omega_\rho^2 \rho^2, & z_1 < z < z_2 \\ \frac{1}{2} m \omega_z^2 (z - z_2)^2 + \frac{1}{2} m \omega_\rho^2 \rho^2, & z > z_2 \end{cases}$$

$$V_{LS} = -\frac{2\hbar \kappa_i}{m \omega_{0i}} (\nabla V \times \vec{p}) \vec{s}$$

$$V_{L^2} = -\hbar \omega_{0i} \kappa_i \mu_i \hat{l}^2 + \hbar \kappa_i \mu_i \omega_{0i} N_1 (N_1 + 3) / 2 \delta_{if}$$

$$\omega_{0i} = 41 \text{ MeV} / A_i^{1/3}, \quad A_i = a_i b_i^2 / 1.22^3, \quad \omega_\rho / \omega_z = a_i / b_i, \quad z_2 - z_1 = 2R_0 \lambda - a_1 - a_2$$

# Parameters

$$35 \leq N - Z \leq 64$$

*for neutrons*

$$\kappa_n = -0.076 + 0.0058(N - Z) - 6.53 \times 10^{-5}(N - Z)^2 + 0.002 A^{1/3},$$

$$\mu_n = 1.598 - 0.0295(N - Z) + 3.036 \times 10^{-4}(N - Z)^2 - 0.095 A^{1/3},$$

*for protons*

$$\kappa_p = 0.0383 + 0.00137(N - Z) - 1.22 \times 10^{-5}(N - Z)^2 - 0.003 A^{1/3},$$

$$\mu_p = 0.335 + 0.01(N - Z) - 9.367 \times 10^{-5}(N - Z)^2 + 0.003 A^{1/3},$$

The parts in front of the terms with  $A^{1/3}$  vary:

(0.05-0.053) for  $\kappa_n$  , (0.075-0.0768) for  $\kappa_p$  ,

(0.88-0.92) for  $\mu_n$  , (0.58-0.61) for  $\mu_p$

**Parameters were set so to describe spins & parities of g.-s. of known heavy nuclei**

## Potential energy

$$U(Z, A, \lambda, \beta) = U_{LDM}(Z, A, \lambda, \beta) + \delta U_{mic}(Z, A, \lambda, \beta)$$

## Binding energy

$$B(Z, A) = U(Z, A, \lambda_{gs}, \beta_{gs}) - a_v \left(1 - a_s \left(\frac{N-Z}{A}\right)^2\right) A$$

$$+ W \left|\frac{N-Z}{A}\right| + \delta + c[(N-Z) - 58]$$

$$a_s = 1.778, W = 30 \text{ MeV}, c = 0.25 \text{ MeV},$$

$$\delta = 4.8/N^{1/3} \delta_{N,odd} + 4.8/Z^{1/3} \delta_{Z,odd},$$

$$a_v = 15.703 \text{ MeV at } N - Z < 52; 15.715 \text{ MeV at } 54 \leq N - Z \leq 61,$$



# Comparison with other calculations

$^{248}\text{Fm}$  gs.:

$$\lambda=1.18, \beta=1.28 \rightarrow \beta_2=0.25, \beta_4=0.027$$

P.Möller et al.  $\beta_2=0.235, \beta_4=0.049$

For  $^{247,248,249}\text{Fm}$ , the microscopic corrections are  
-3.85, -3.88, and -4.3 MeV.

P.Möller et al.: -3.52, -3.57, and -3.97 MeV

The values of  $\Delta$  differ within 0.1 MeV.

# Comparison with other calculations

$^{270}\text{Hs}$  gs.:

$$\lambda=1.14, \beta=1.06 \rightarrow \beta_2=0.25, \beta_4=-0.03$$

P.Möller et al.:  $\beta_2=0.231, \beta_4=-0.086$

For  $^{268,269,270,271}\text{Hs}$ , the microscopic corrections are

$$-5.95, -6.38, -6.54, -6.64 \text{ MeV}$$

P.Möller et al.:  $-5.94, -6.37, -5.95, -5.86 \text{ MeV}$

A.Kuzmina et al., Eur. Phys. J. A 47 (2011) 145

Phys. Rev. C 85 (2012) 014319; 017302

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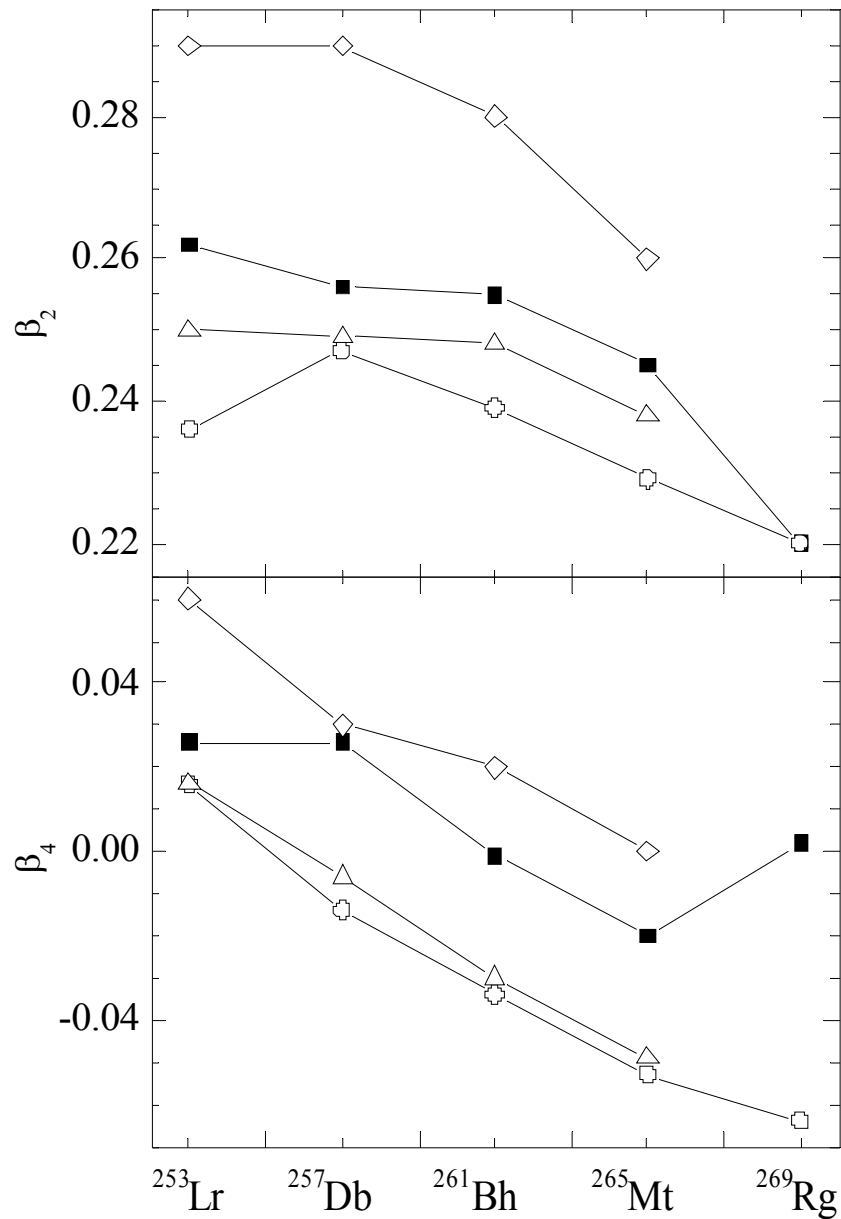
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- TCSM

- $\triangle$  A. Sobiczewski et al.

- $\circ$  P. Möller et al.

- $\diamond$  S. Goriely et al.

$Q_\alpha$  energy

$$Q_\alpha(Z, A) = B(Z, A) + 28.296 - B(Z - 2, A - 4)$$

Alpha decay half-lives  $T_\alpha$  (A. Sobiczewski et al.)

$$\log_{10} T_\alpha(Z, A) = 1.5372 Z Q_\alpha^{-1/2} - 0.1607 Z - 36.573$$

# Strength parameters of pairing interaction

$$G_{\begin{smallmatrix} n \\ p \end{smallmatrix}} = \left( 19.2 \mp 7.4 \frac{N-Z}{A} \right) A^{-1} \text{ MeV}$$

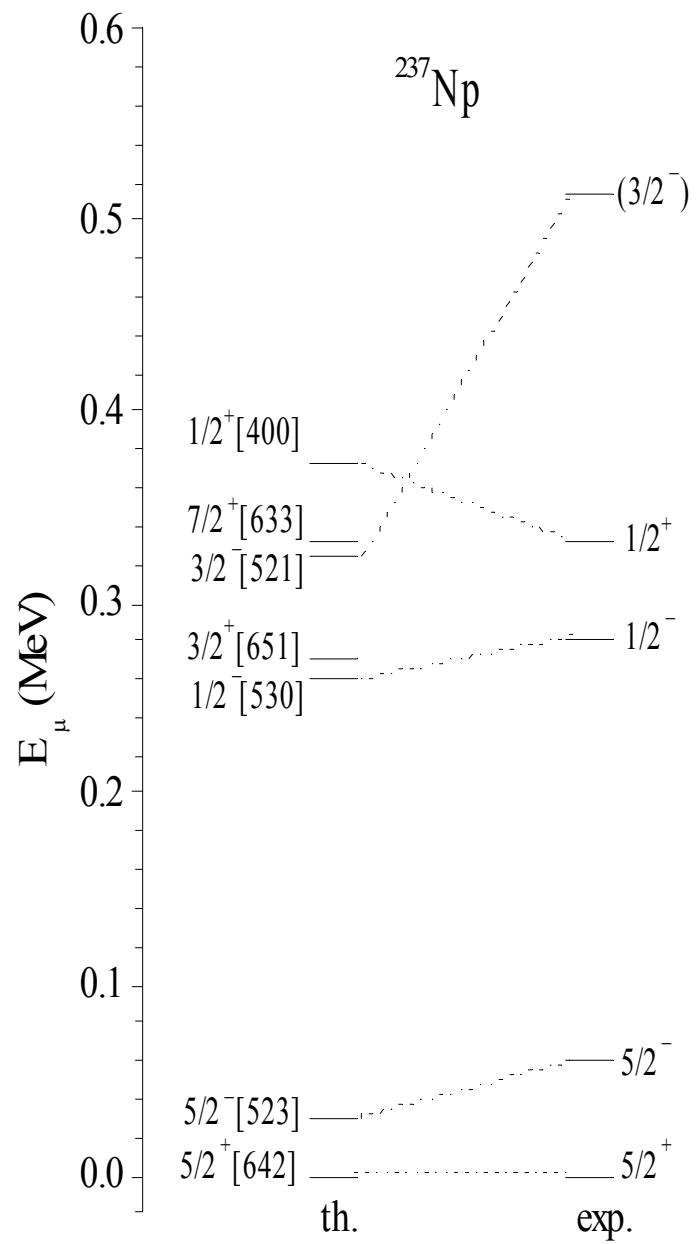
$$A \approx 250 \rightarrow G_n \approx 0.075 \text{ MeV}, G_p \approx 0.085 \text{ MeV}$$

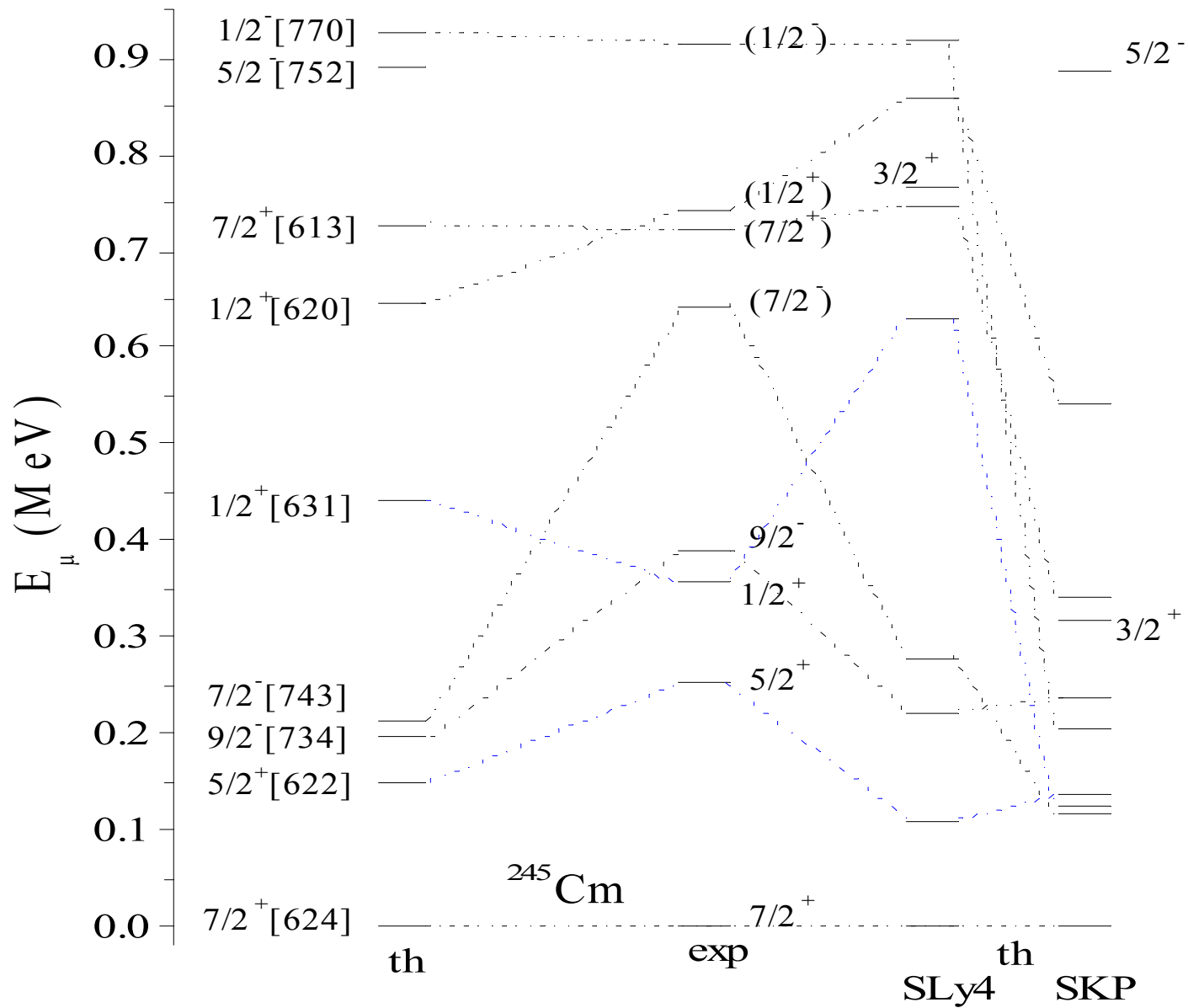
## One-quasiparticle excitations

$$E_{\mu} = \sqrt{(e_{\mu} - e_F)^2 + \Delta^2} - \sqrt{(e_{\mu}' - e_F)^2 + \Delta^2}$$

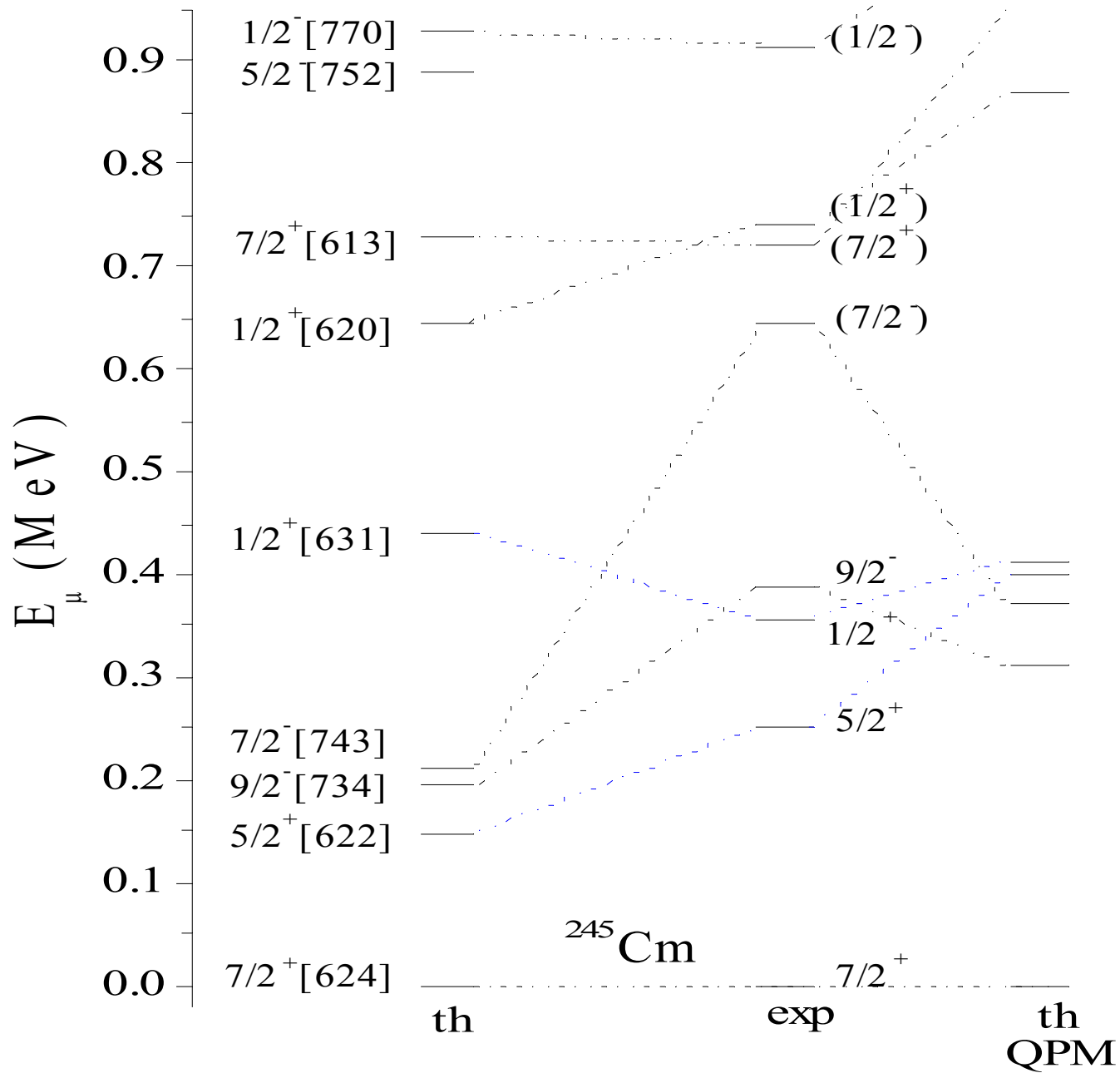
## Two-quasiparticle excitations

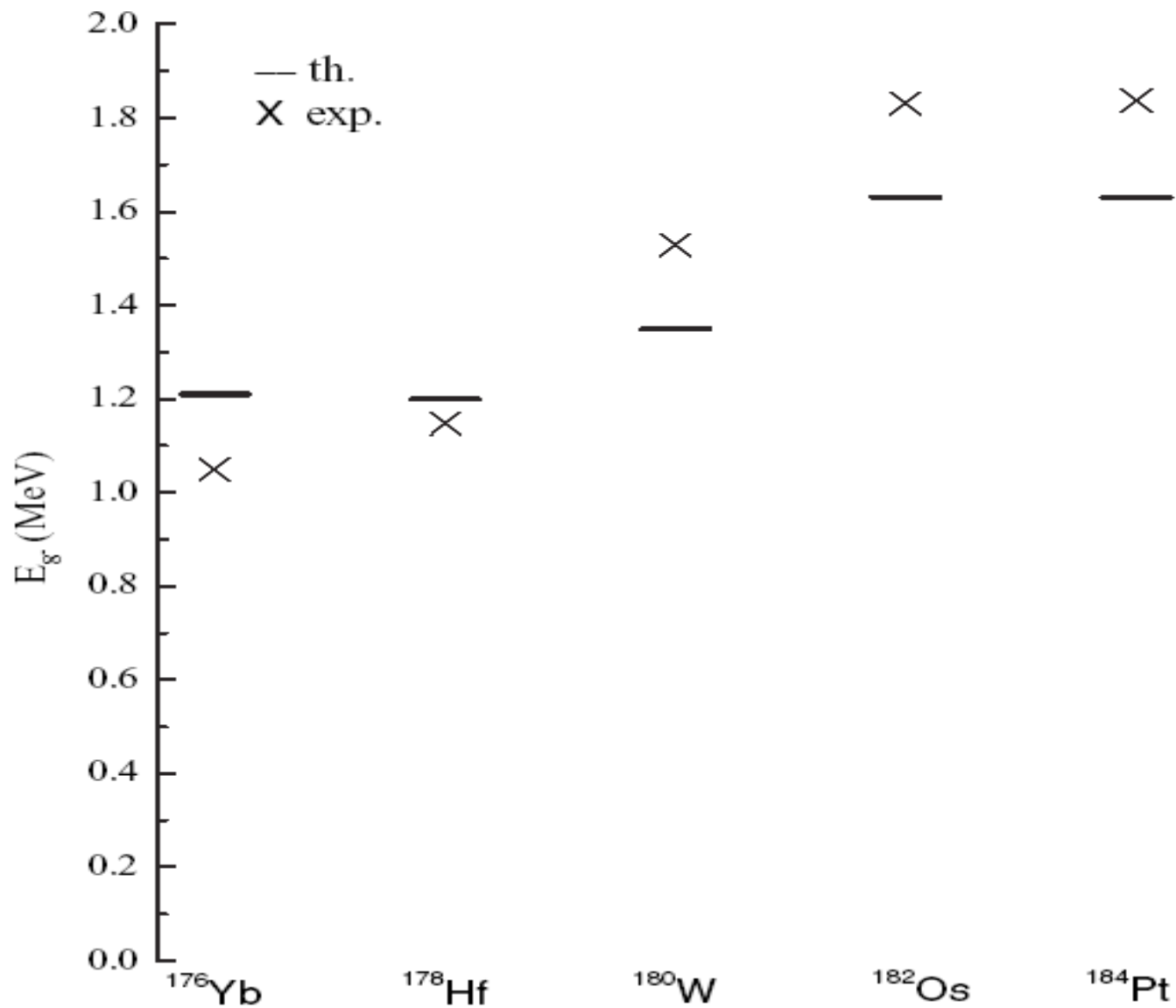
$$E_{\mu} = \sqrt{(e_{\mu} - e_F)^2 + \Delta^2} + \sqrt{(e_{\mu}' - e_F)^2 + \Delta^2}$$



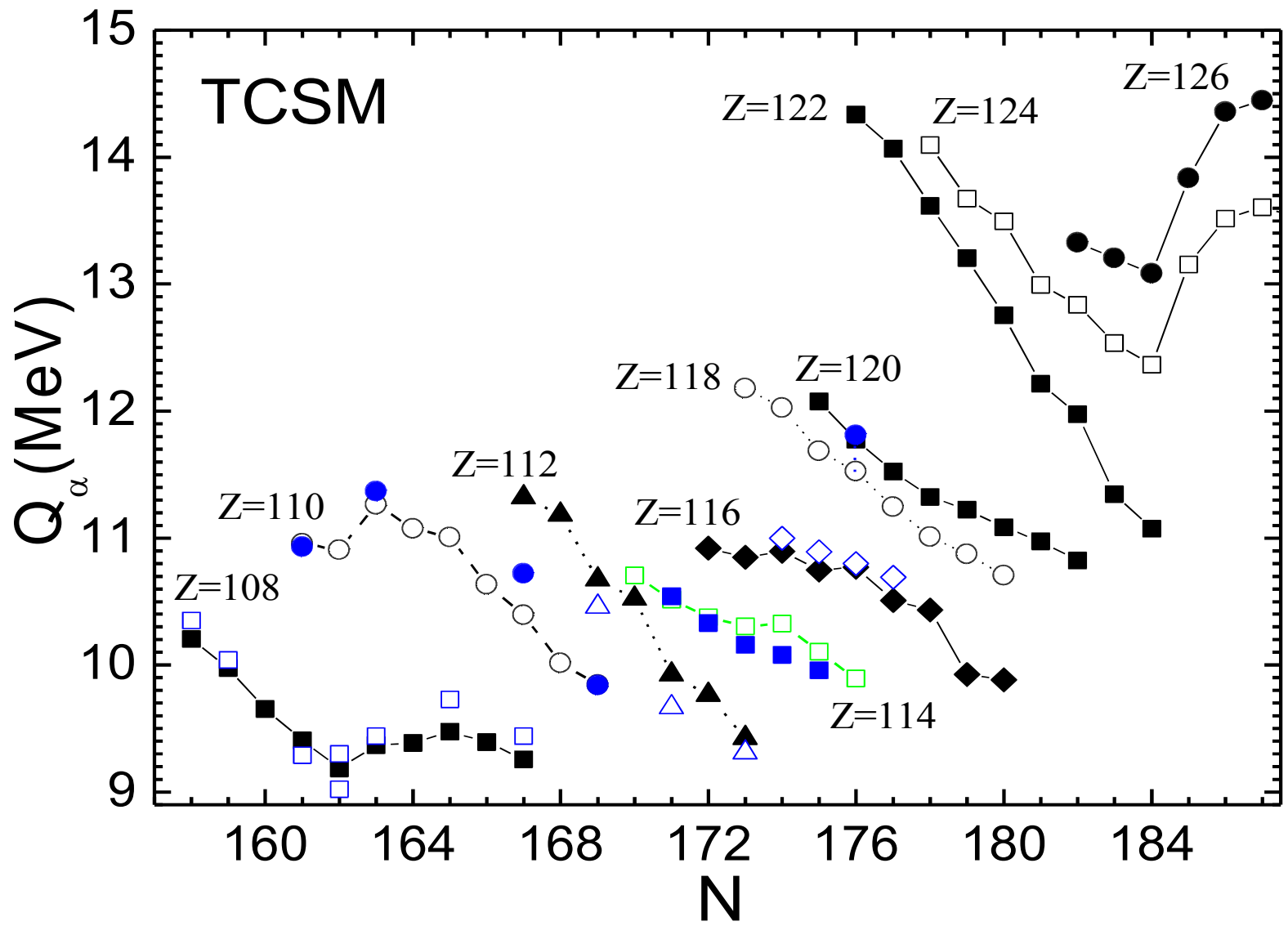


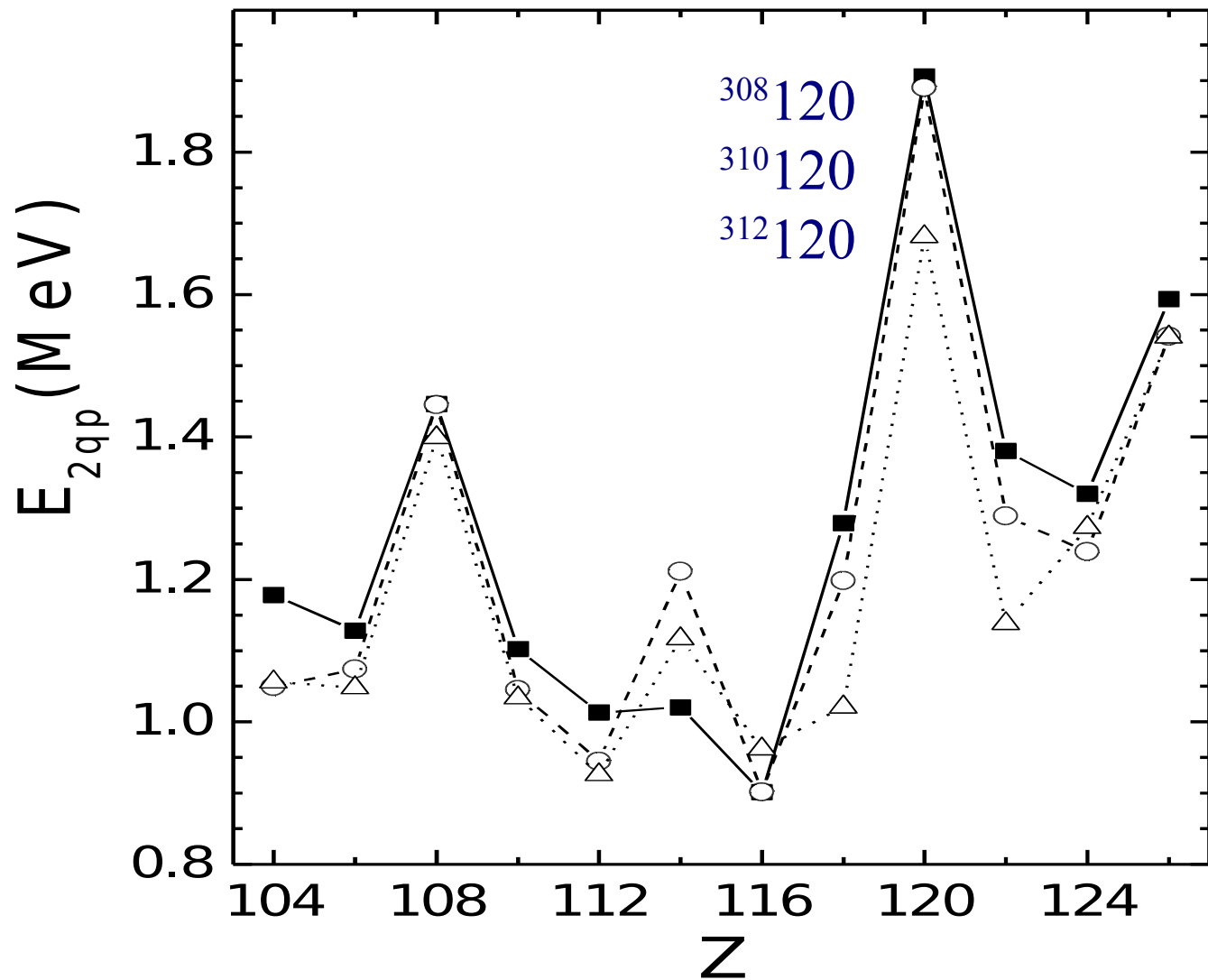




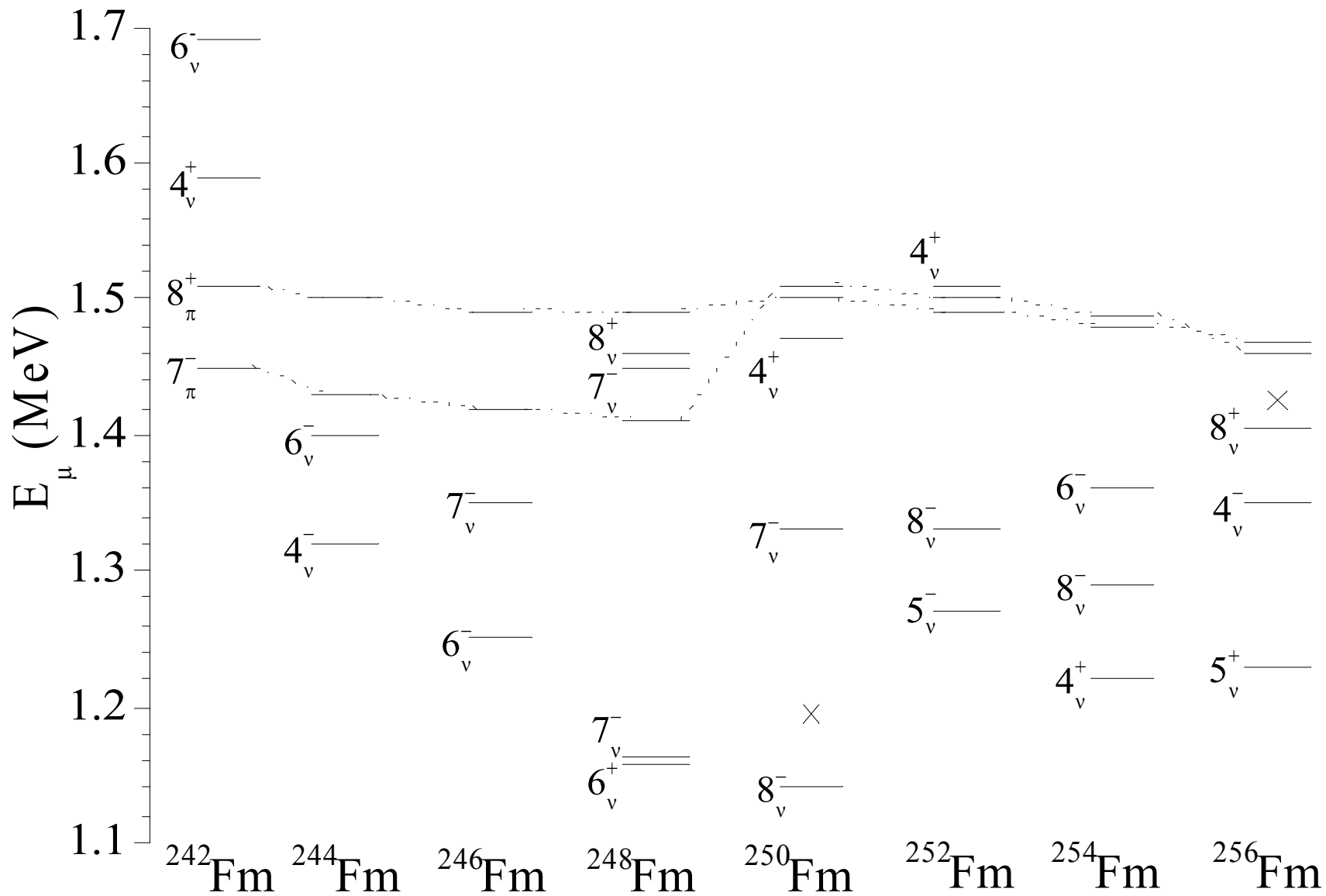


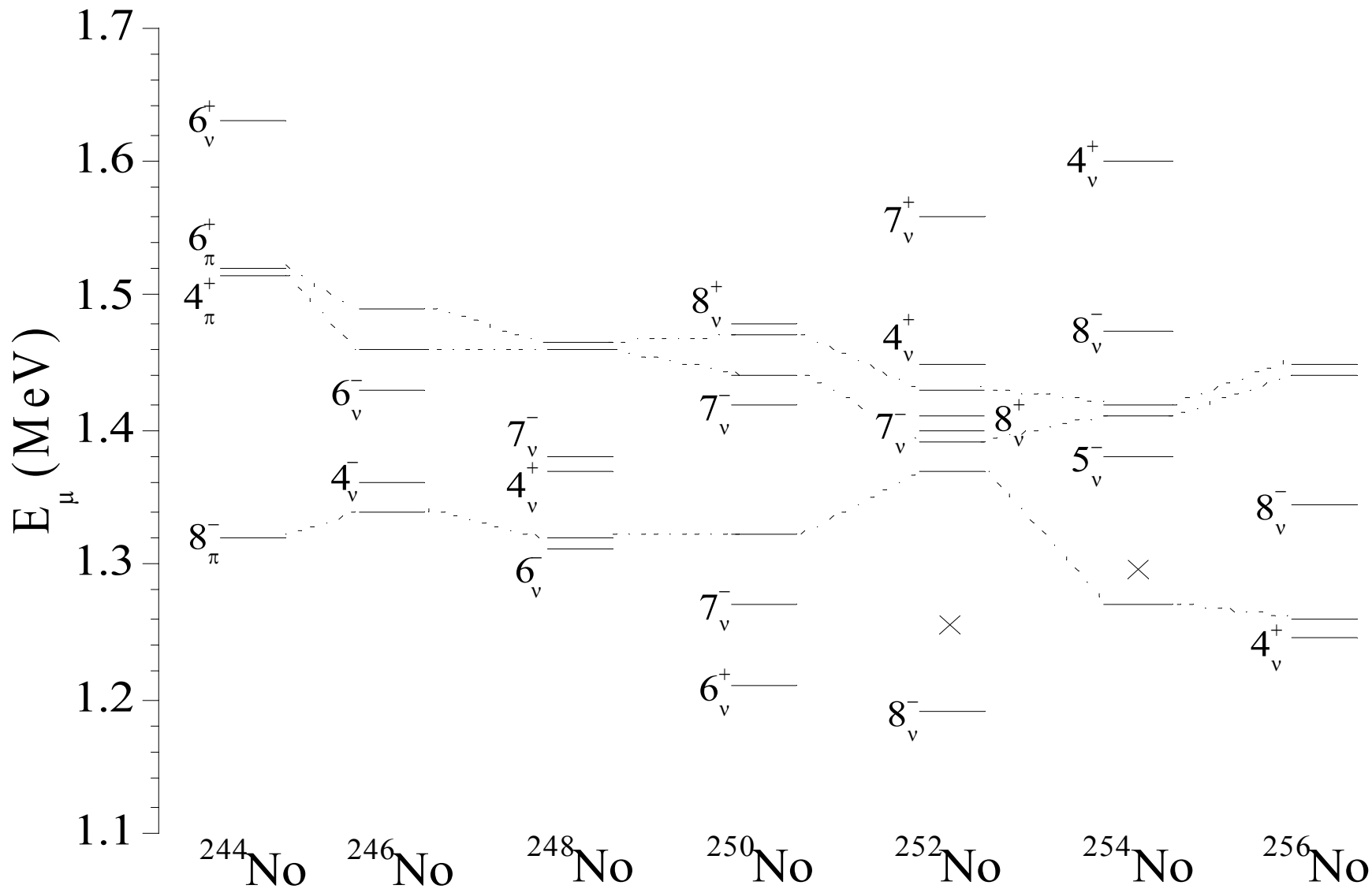
Calculated (th) and experimental (exp) energies  $E_{g-}$  of two-quasiparticle states with  $K^\pi = 8^-_{\nu}$  compared for the isotones from  $^{176}\text{Yb}$  to  $^{184}\text{Pt}$ .

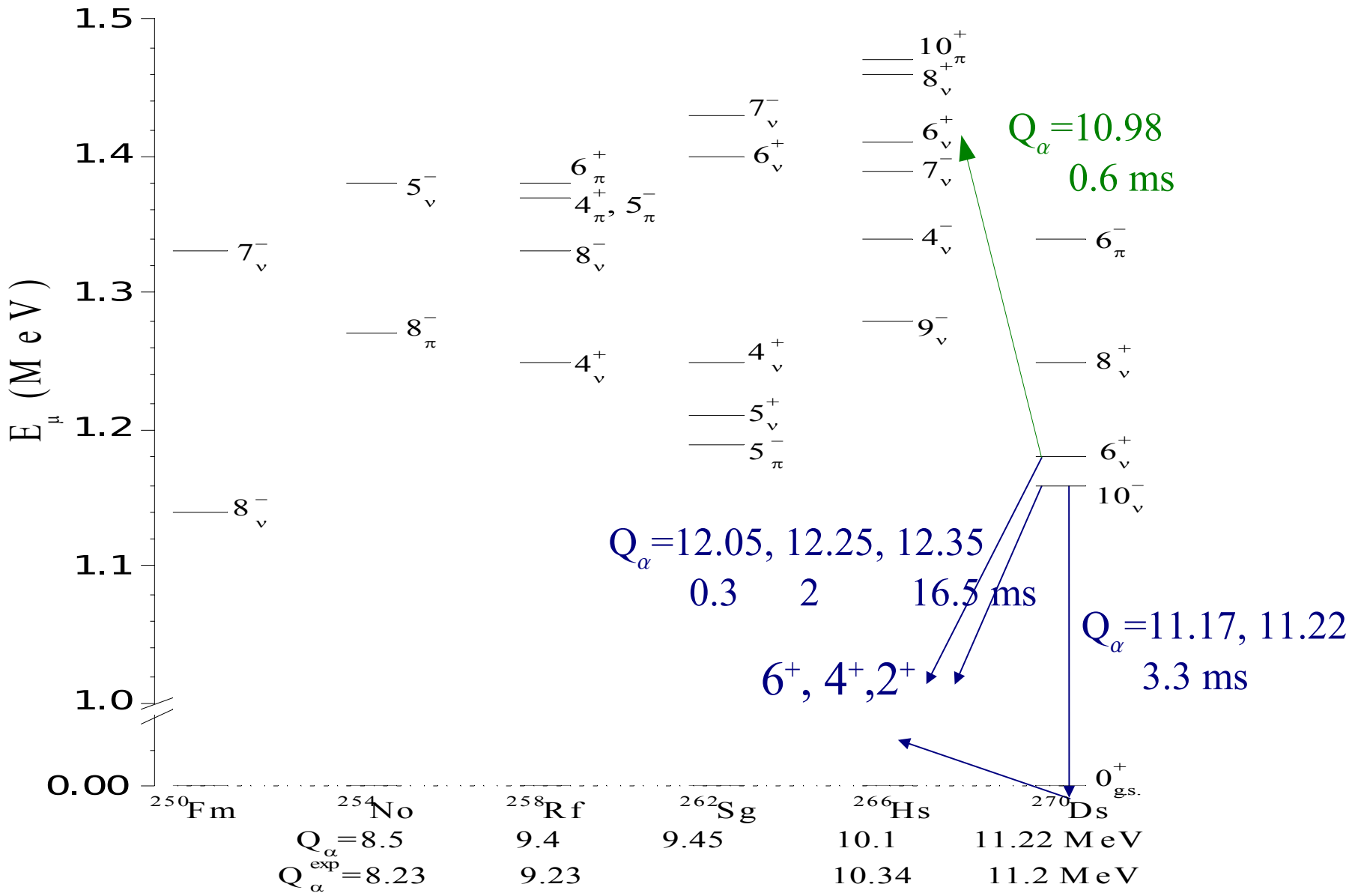




Energies of two-quasiproton states in alpha-decay chains containing indicated nuclei with  $Z=120$

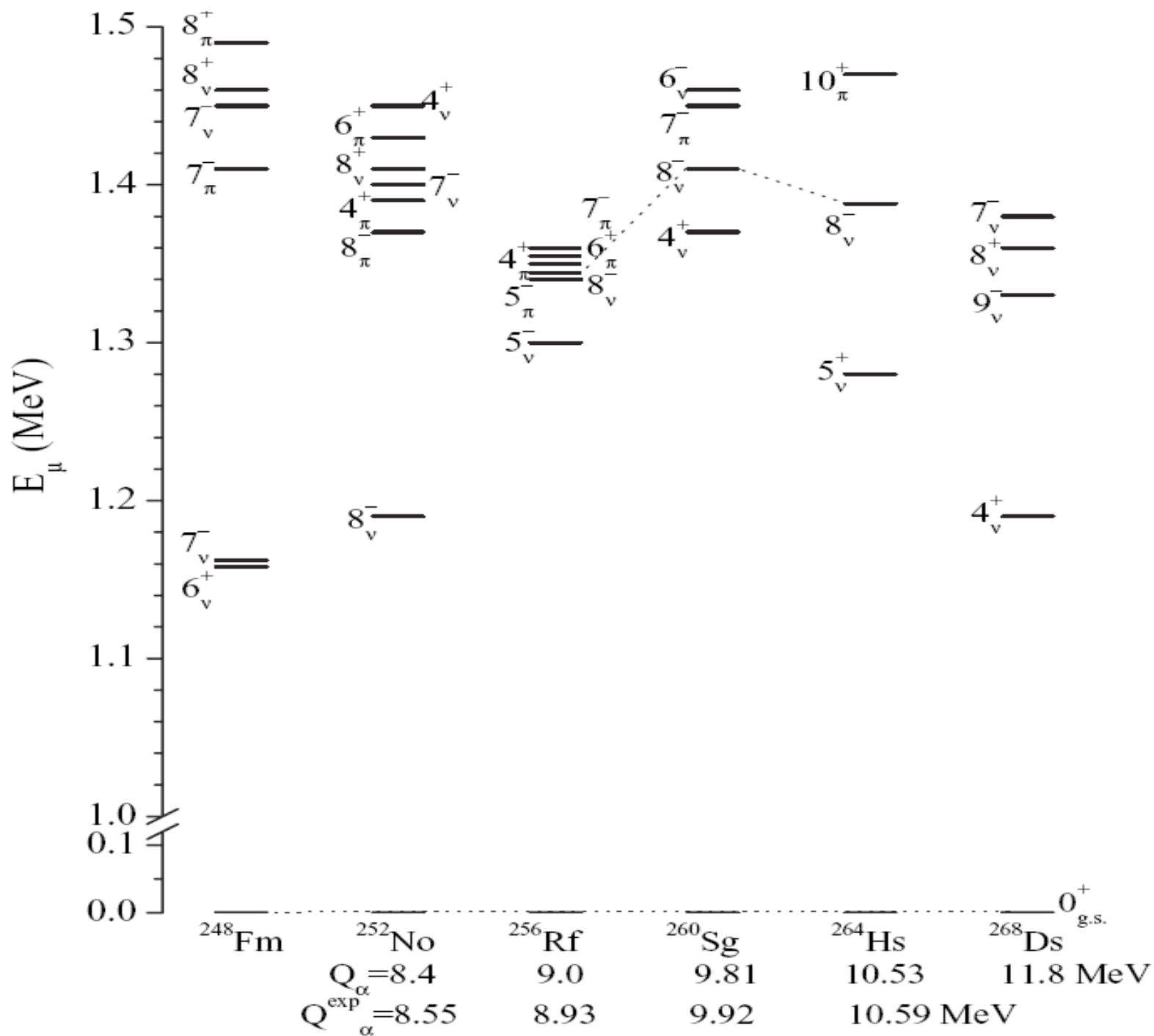






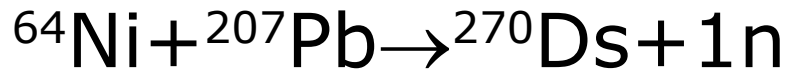
Nucleus	Decay mode	$Q_\alpha$ (MeV)	$T_\alpha$	$T_{1/2}^{\text{exp.}}$
$^{270}\text{Ds}$	$0_{\text{g.s.}}^+ \xrightarrow{\alpha} 0_{\text{g.s.}}^+$	11.22	0.17 ms	0.1 ms, % $\alpha \approx 100$
	$0_{\text{g.s.}}^+ \xrightarrow{\alpha} 2_{\text{g.s.}}^+$	11.17	3.3 ms	
	$10_{\nu}^-, 6_{\nu}^+ \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+ \xrightarrow{\alpha} 0_{\text{g.s.}}^+$	11.22		
	$10_{\nu}^- \xrightarrow{\alpha} 10_{K=1^-}^- \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+$	11.3	10 ms	
	$6_{\nu}^+ \xrightarrow{\alpha} 6_{\text{g.s.}}^+ \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+$	12.05	0.3 ms	
	$6_{\nu}^+ \xrightarrow{\alpha} 4_{\text{g.s.}}^+ \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+$	12.25	2 ms	
	$6_{\nu}^+ \xrightarrow{\alpha} 2_{\text{g.s.}}^+ \xrightarrow{\gamma} 0_{\text{g.s.}}^+$	12.35	16.5 ms	
	$6_{\nu}^+ \xrightarrow{\alpha} 6_{\nu}^+$	10.98	0.6 ms	
$^{266}\text{Hs}$	$0_{\text{g.s.}}^+ \xrightarrow{\alpha} 0_{\text{g.s.}}^+$	10.1	37 ms	2.3 ms, % $\alpha \approx 100$
	$9_{\nu}^-, 4_{\nu}^- \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+ \xrightarrow{\alpha} 0_{\text{g.s.}}^+$	10.1		
	$9_{\nu}^- \xrightarrow{\alpha} 9_{K=1^-}^- \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+$	10.33	530 ms	
	$4_{\nu}^- \xrightarrow{\alpha} 4_{K=1^-}^- \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+$	10.7	67 ms	
$^{262}\text{Sg}$	$0_{\text{g.s.}}^+ \xrightarrow{\alpha} 0_{\text{g.s.}}^+$	9.45	250 ms	6.9 ms, %SF $\geq 78$
	$5_{\pi}^-, 5_{\nu}^+ \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+ \xrightarrow{\text{SF}}$ $5_{\pi}^-, 5_{\nu}^+ \xrightarrow{\text{SF}}$			
$^{258}\text{Rf}$	$0_{\text{g.s.}}^+ \xrightarrow{\alpha} 0_{\text{g.s.}}^+$	9.4	72 ms	14.7 ms, % $\alpha \approx 31 \pm 11$ , $T_\alpha^{\text{exp.}} = 47_{-12}^{+24}$ ms
	$4_{\nu}^+, 8_{\nu}^- \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+ \xrightarrow{\text{SF}, \alpha}$ $4_{\nu}^+, 8_{\nu}^- \xrightarrow{\text{SF}}$			
	$4_{\nu}^+ \xrightarrow{\alpha} 4_{\text{g.s.}}^+ \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+$	10.51	11 ms	
$^{254}\text{No}$	$0_{\text{g.s.}}^+ \xrightarrow{\alpha} 0_{\text{g.s.}}^+$	8.45	9.4 s	51 s, % $\alpha \approx 90$
	$8_{\pi}^- \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+ \xrightarrow{\alpha} 0_{\text{g.s.}}^+$			





Nucleus	Decay mode	$Q_\alpha$ (MeV)	$T_\alpha$	$T_{1/2}^{\text{exp.}}$
$^{268}\text{Ds}$	$0_{\text{g.s.}}^+ \xrightarrow{\alpha} 0_{\text{g.s.}}^+$	11.8	$9.4 \mu\text{s}$	
	$4_{\text{v}}^+ \xrightarrow{\alpha} 4_{\text{g.s.}}^+$	12.86	$8 \mu\text{s}$	
	$9_{\text{v}}^- \xrightarrow{\alpha} 9_{\text{g.s.}}^- \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+$	12.07	0.26 ms	
	$8_{\text{v}}^+ \xrightarrow{\alpha} 8_{\text{g.s.}}^+ \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+$	12.71	$15 \mu\text{s}$	
$^{264}\text{Hs}$	$0_{\text{g.s.}}^+ \xrightarrow{\alpha} 0_{\text{g.s.}}^+$	10.53	1.7 ms	$\approx 0.8 \text{ ms}, \% \alpha \approx 50, \% \text{SF} \approx 50$
	$8_{\text{v}}^- \xrightarrow{\alpha} 8_{\text{v}}^-$	10.508	1.9 ms	
	$5_{\text{v}}^+ \xrightarrow{\alpha} 4_{K=1}^- \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+$	11.04	55 ms	
	$8_{\text{v}}^- \xrightarrow{\alpha} 8_{K=1}^- \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+$	10.87	27 ms	
$^{260}\text{Sg}$	$0_{\text{g.s.}}^+ \xrightarrow{\alpha} 0_{\text{g.s.}}^+$	9.81	26 ms	$3.6 \text{ ms}, \% \alpha \approx 50, \% \text{SF} \approx 50$
	$8_{\text{v}}^- \xrightarrow{\alpha} 8_{\text{v}}^-$	9.92	13.4 ms	
	$4_{\text{v}}^+ \xrightarrow{\alpha} 4_{\text{g.s.}}^+ \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+$	11.04	2.7 ms	
$^{256}\text{Rf}$	$0_{\text{g.s.}}^+ \xrightarrow{\alpha} 0_{\text{g.s.}}^+$	9	1 s	$6.4 \text{ ms}, \% \alpha \approx 0.32, \% \text{SF} \approx 99.68$
	$8_{\text{v}}^- \xrightarrow{\alpha} 8_{\text{v}}^-$	9.12	0.45 s	
$^{252}\text{No}$	$0_{\text{g.s.}}^+ \xrightarrow{\alpha} 0_{\text{g.s.}}^+$	8.4	13.6	$2.4 \text{ s}, \% \alpha > 66.7, \% \text{SF} = 32.2$
	$8_{\pi}^- \xrightarrow{\gamma'/s} 0_{\text{g.s.}}^+ \xrightarrow{\alpha}$	8.4		

The nuclei  $^{248,252}\text{Fm}$ ,  $^{256}\text{No}$ , and  $^{262}\text{Sg}$  produced in the reactions  $^{20}\text{Ne} + ^{232}\text{Th}$ ,  $^{12}\text{C} + ^{248}\text{Cm}$ ,  $^{22}\text{Ne} + ^{238}\text{U}$ , and  $^{18}\text{O} + ^{249}\text{Cf}$  seem to be good candidates for studying the low-lying  $K$  isomers.



ground state:  $E_{\text{CN}}^* \approx 14 \text{ MeV}$

2qp isomer state:  $E_{\text{CN}}^* \approx 14 - E_{\mu} \approx 12.8 \text{ MeV}$

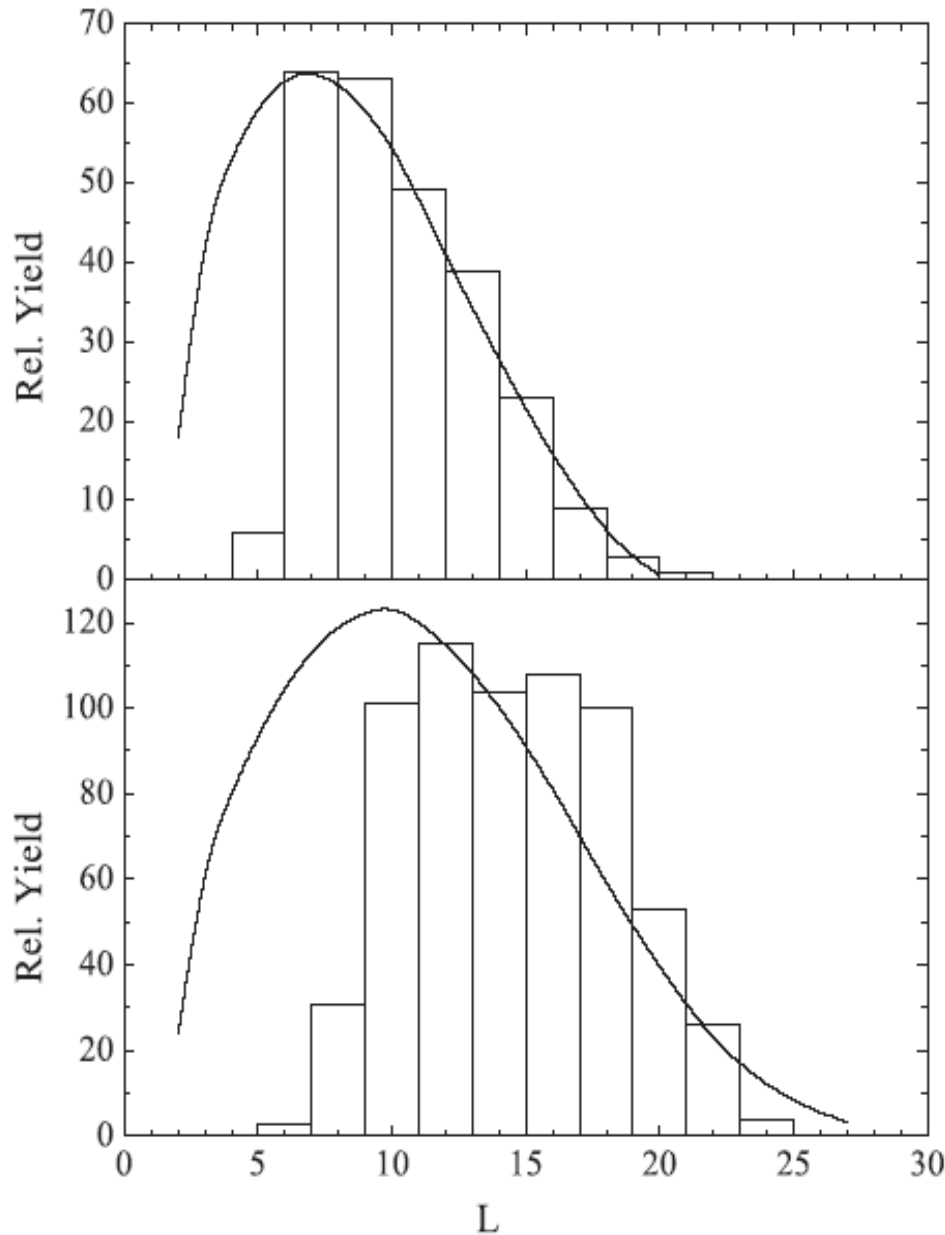
$W_{\text{sur}}(\text{isomer})/W_{\text{sur}}(\text{gs}) \approx 2$

For  $E_{\mu} \approx 1.2 \text{ MeV}$ , the population of isomer state is  $\approx \exp(-(E_{\mu} - E_{\text{rot}})/T) \approx 0.32$ .

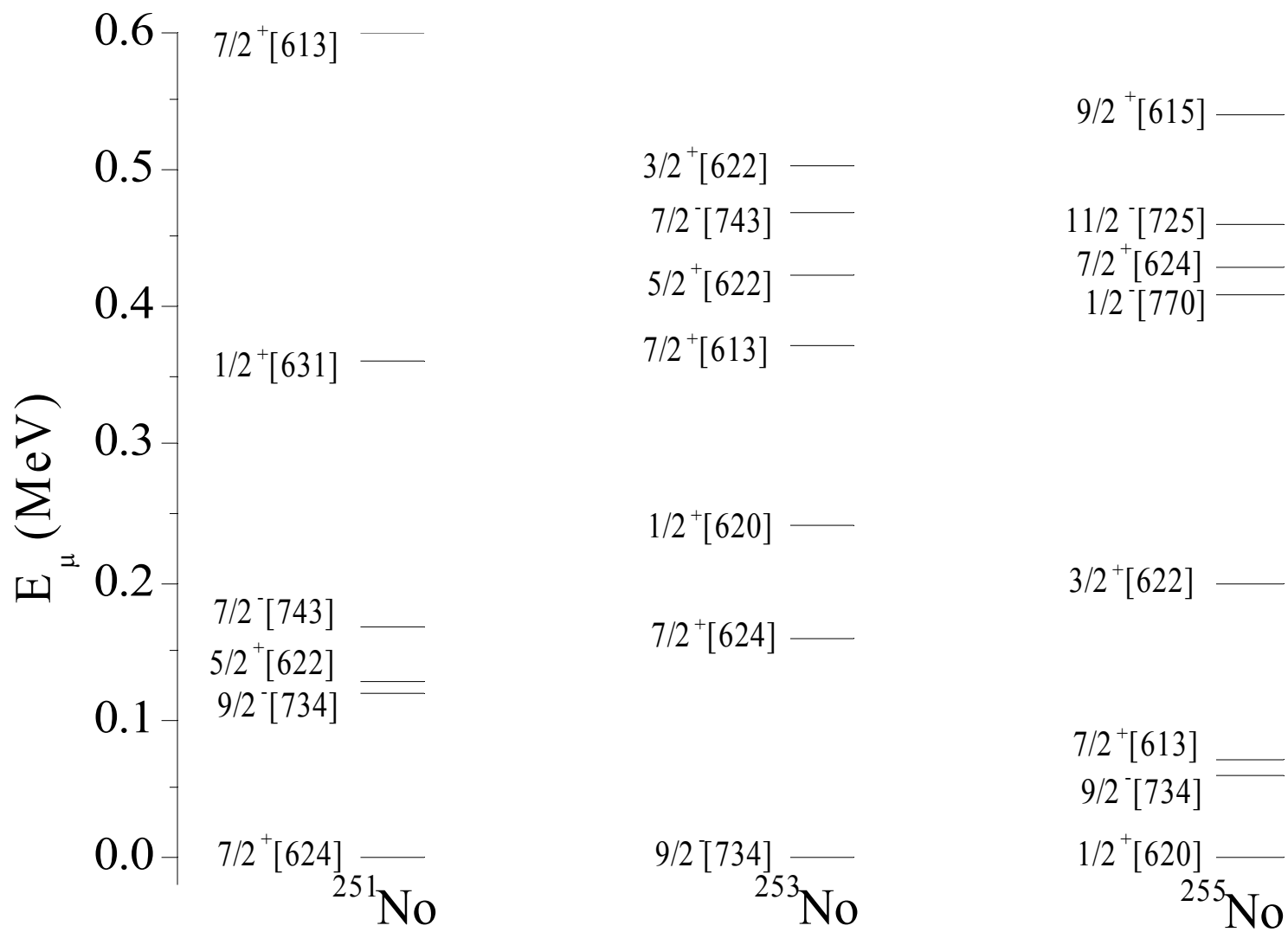
$\sigma_{\text{ER}}(\text{gs})/\sigma_{\text{ER}}(\text{isomer}) \approx 0.68:0.64$

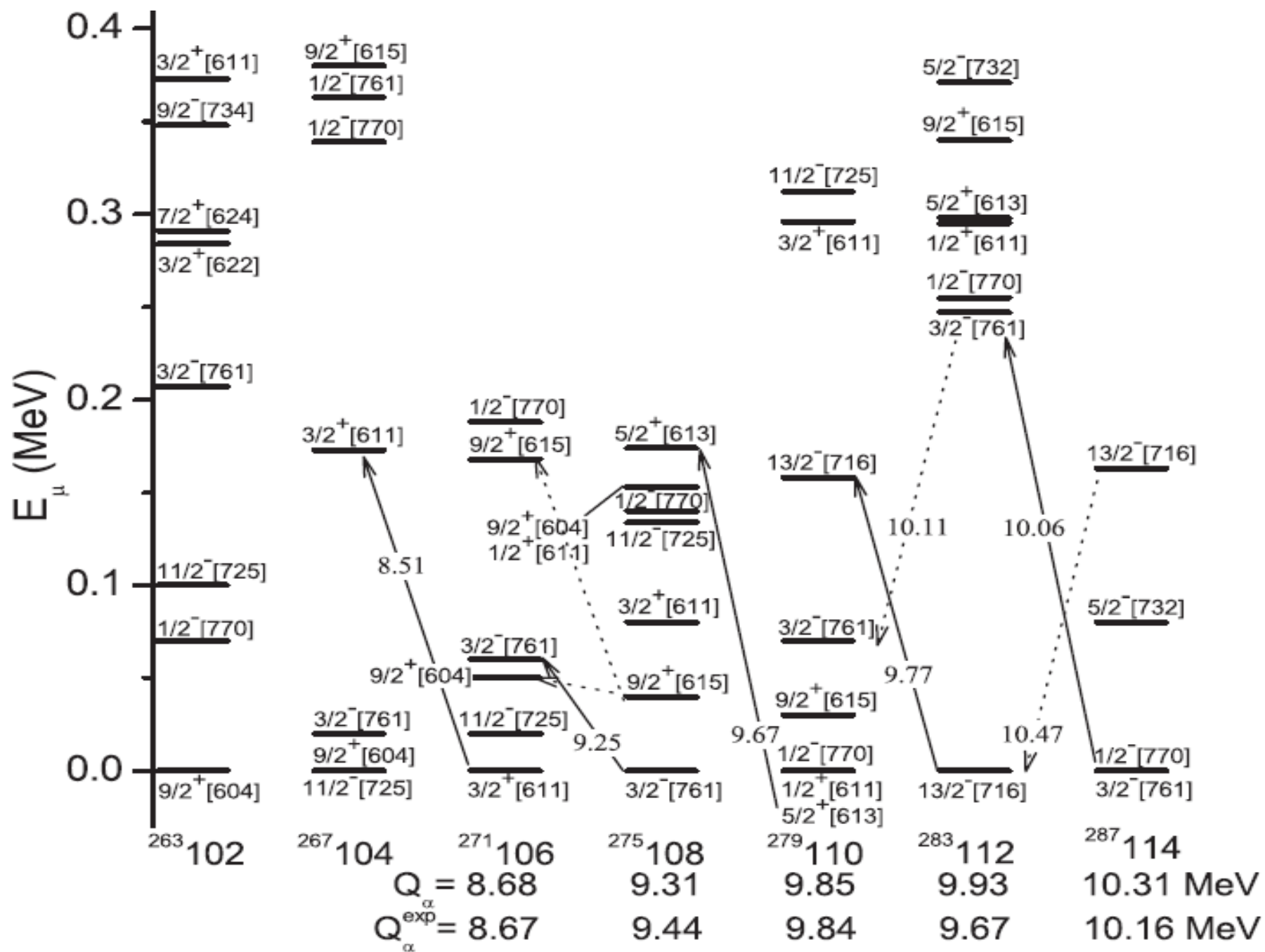
exp.:  $\approx 1:1$

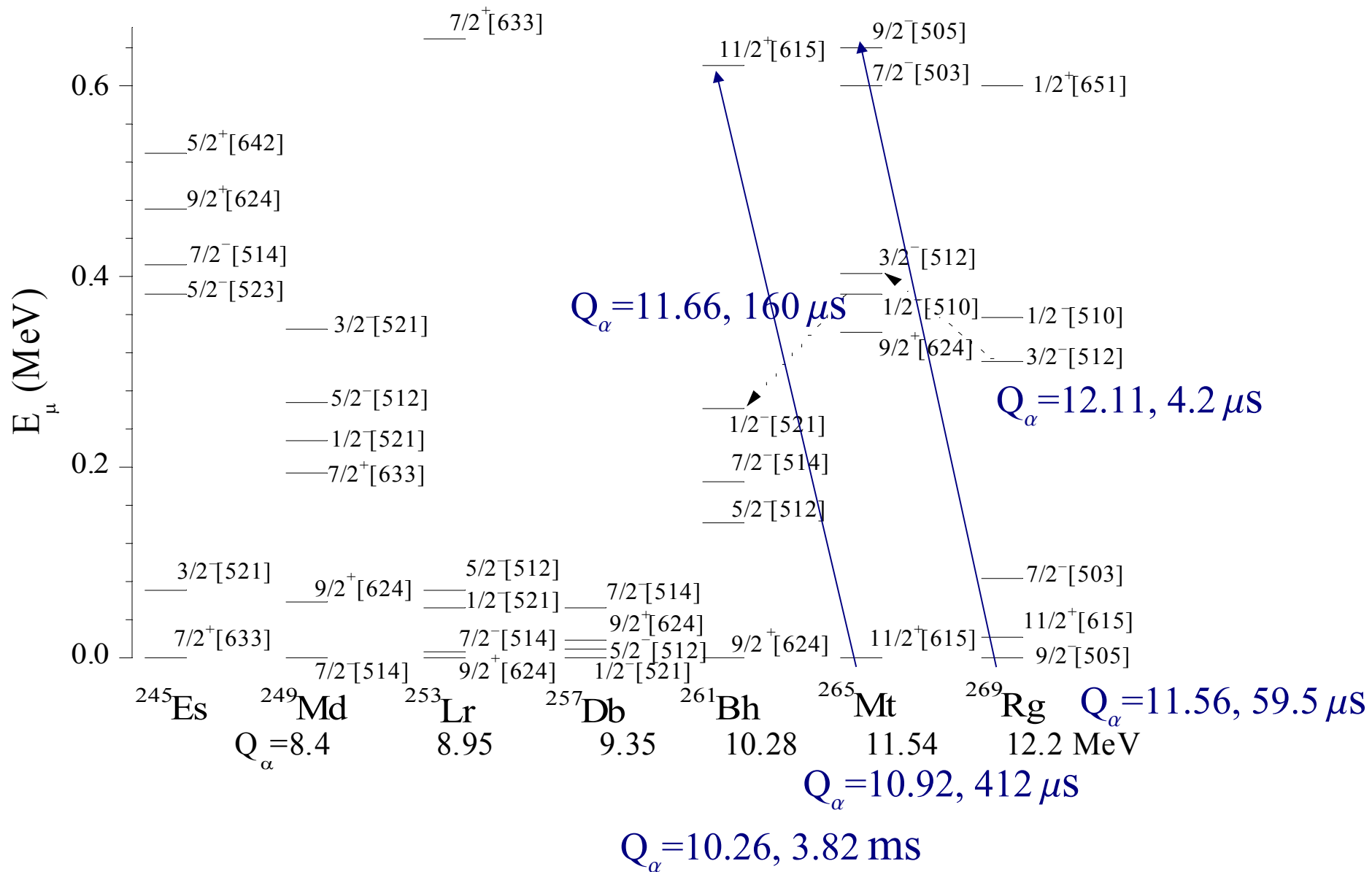
PRC 81, 024320 (2010)



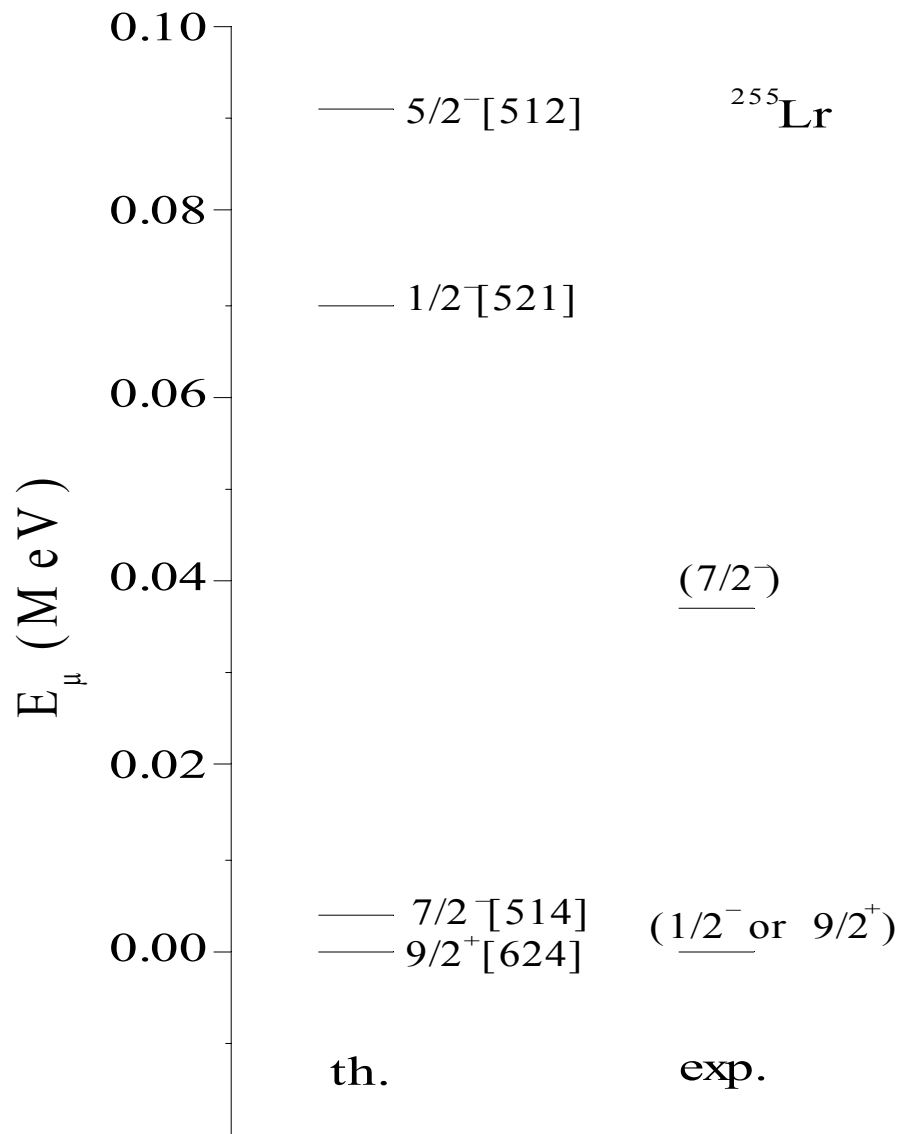
The entry spin distribution of  $^{254}\text{No}$ , produced in the reaction  $^{208}\text{Pb}(^{48}\text{Ca}, 2n)^{254}\text{No}$  at beam energies 215 MeV (upper part) and 219 MeV (lower part). The experimental data from Phys. Rev. Lett. 84, 3542 (2000) are presented by squares.

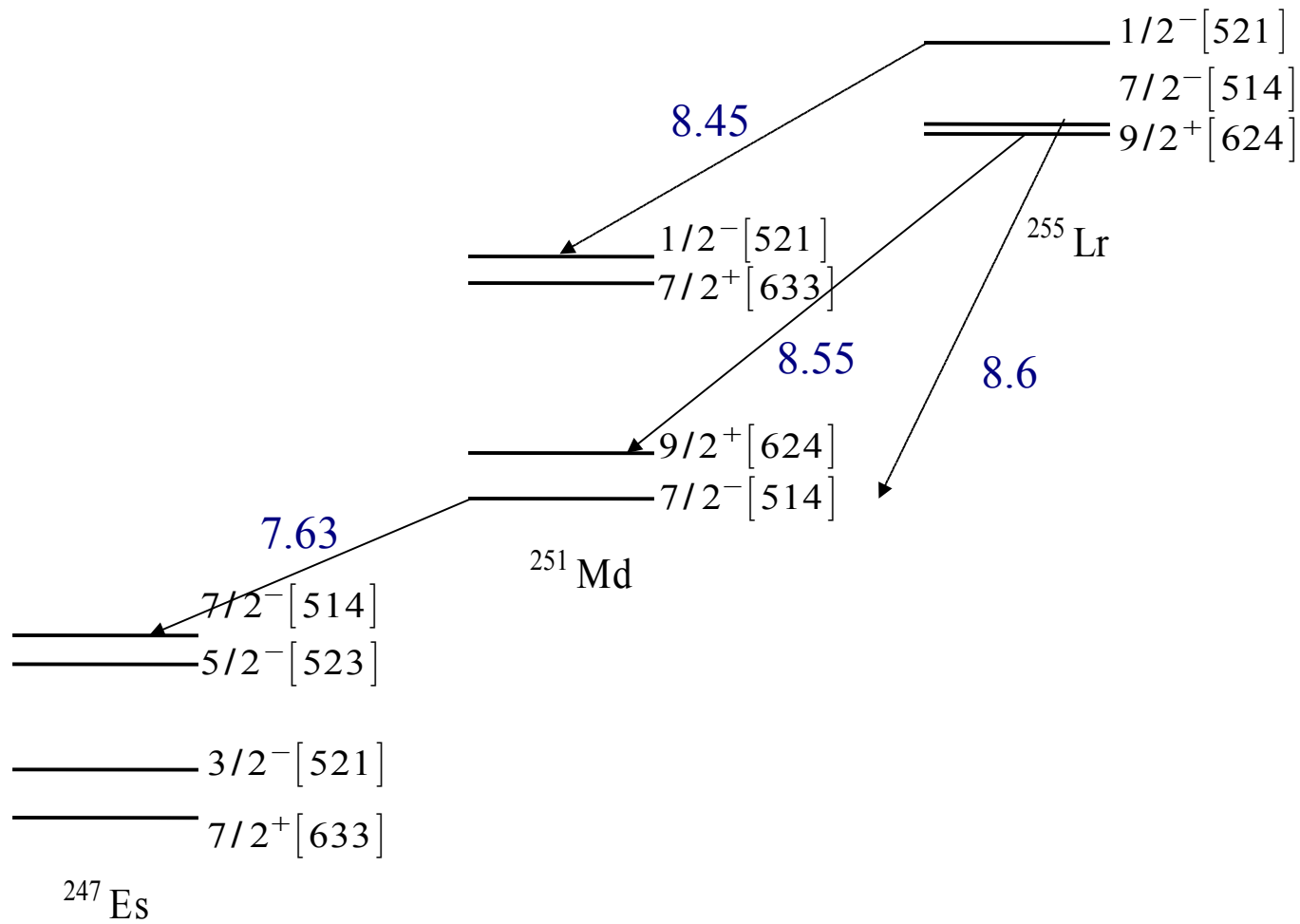






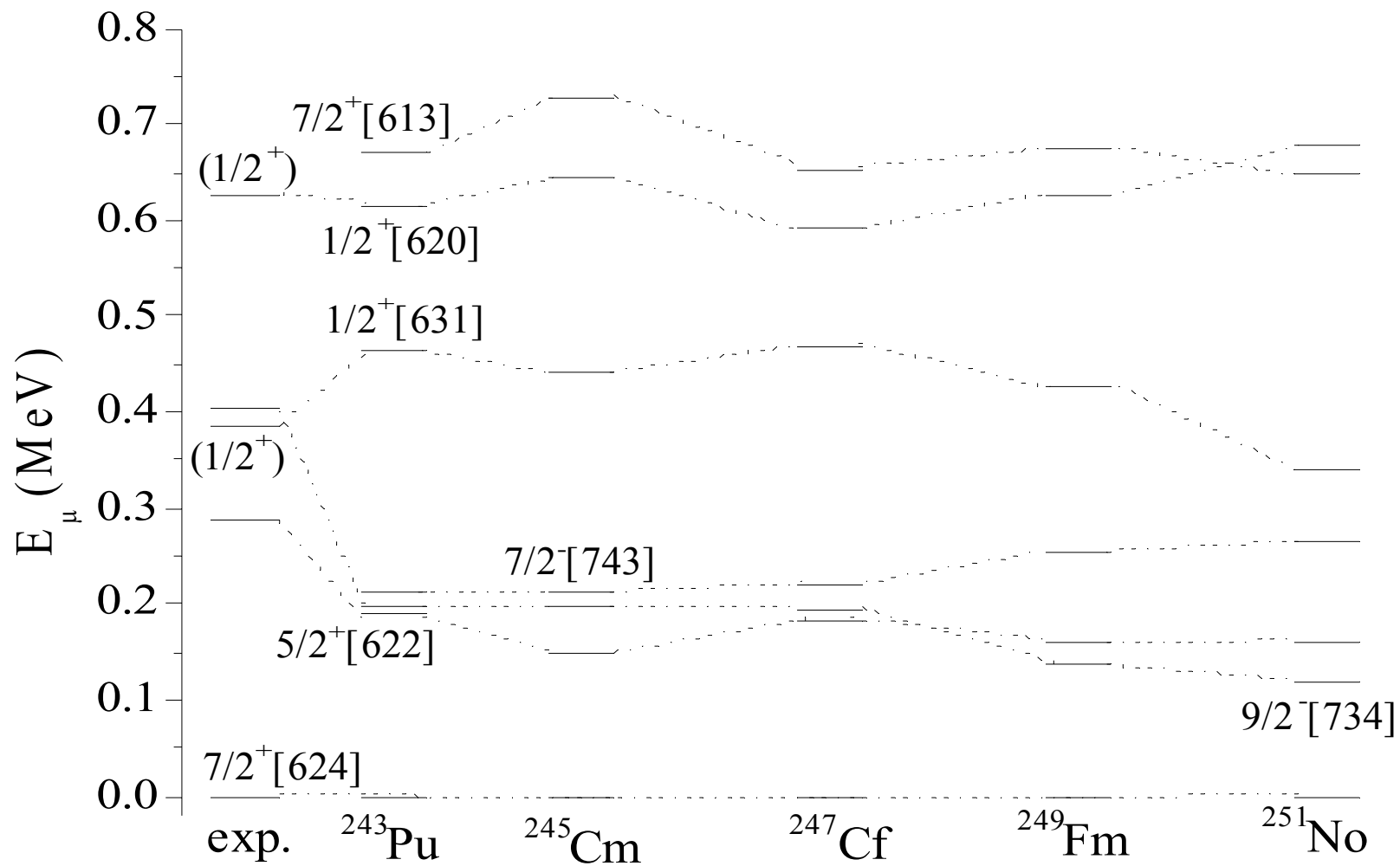






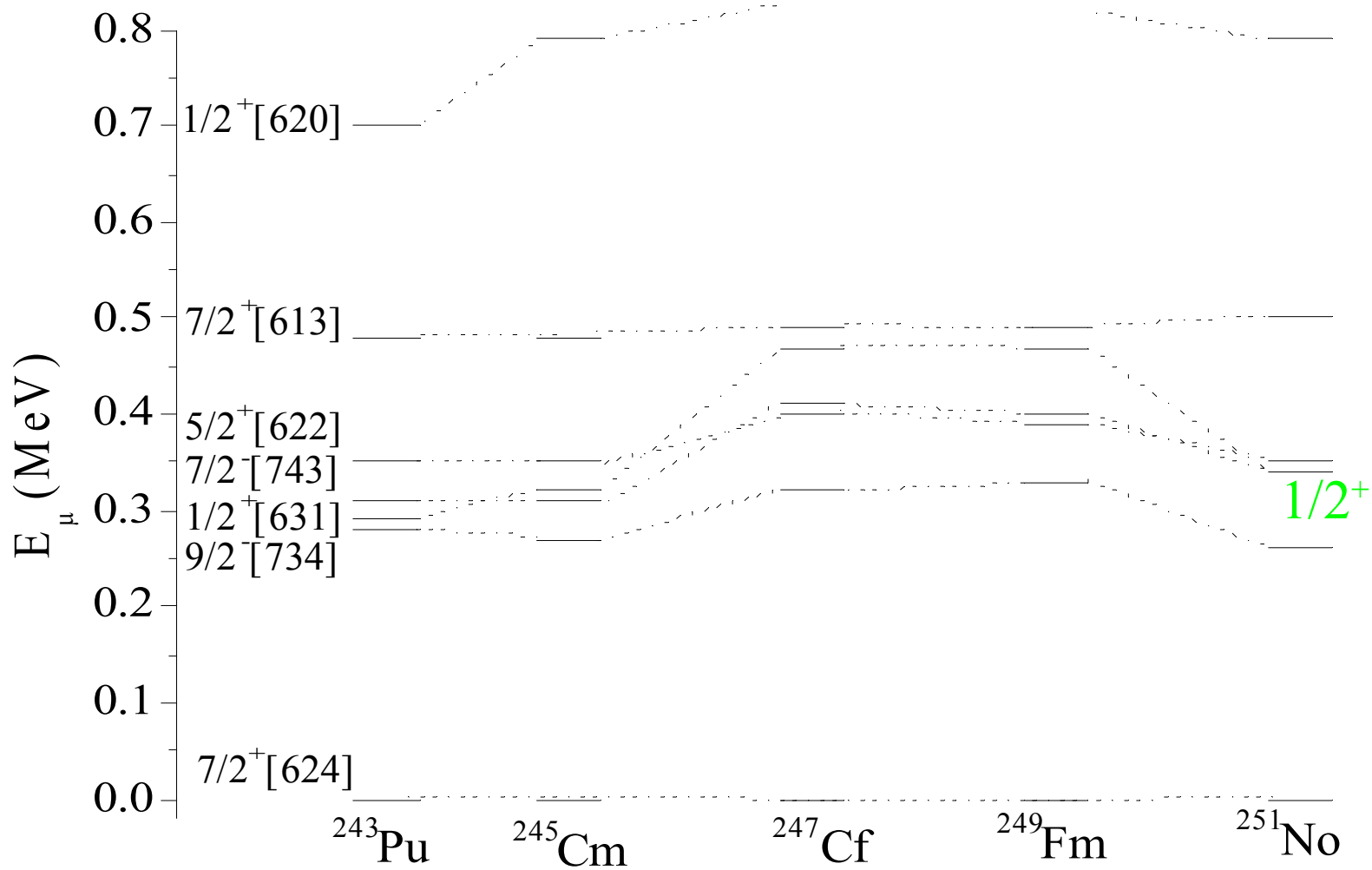
N=149 isotones

TCSM



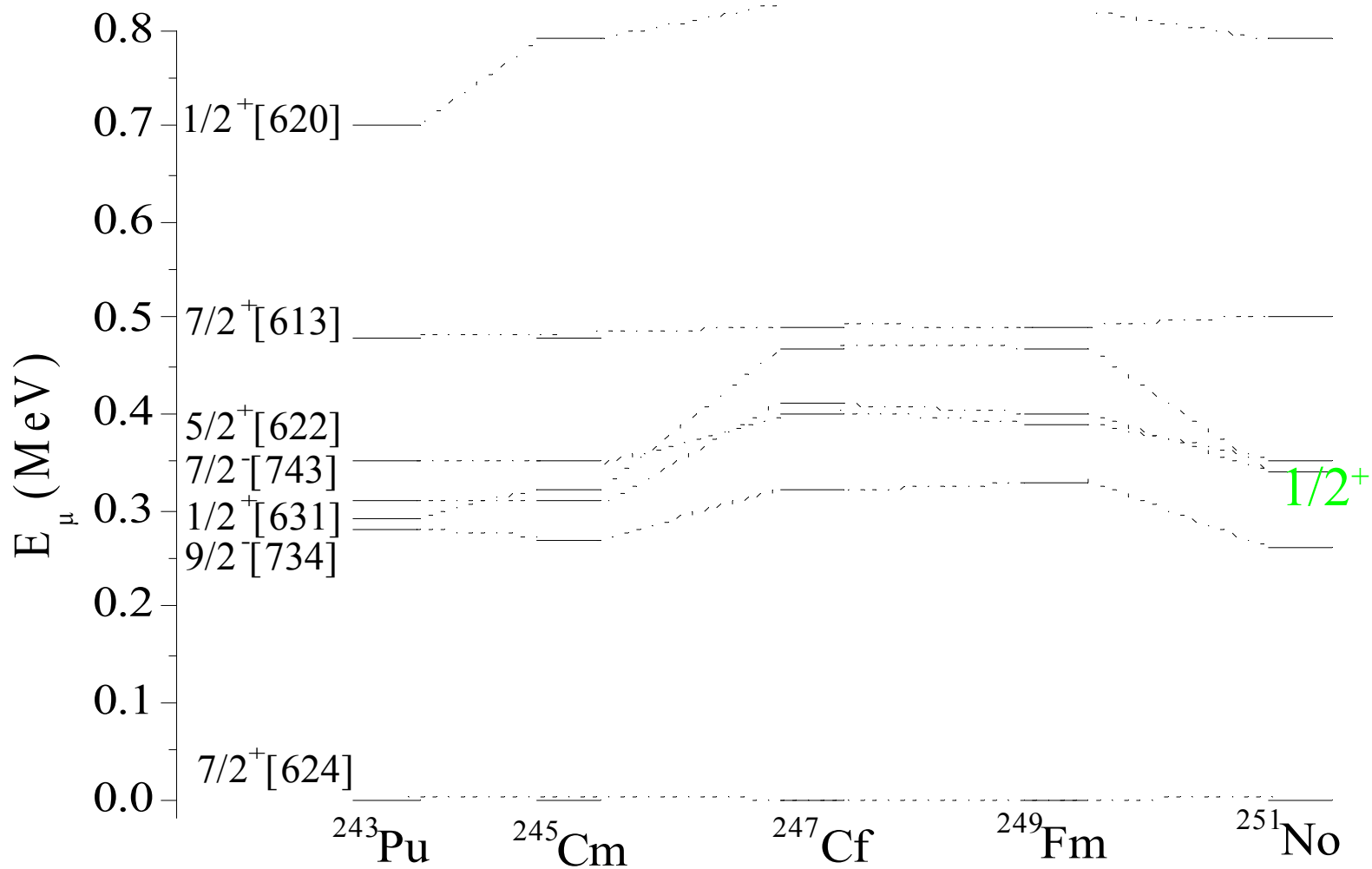
N=149 isotones

QPM



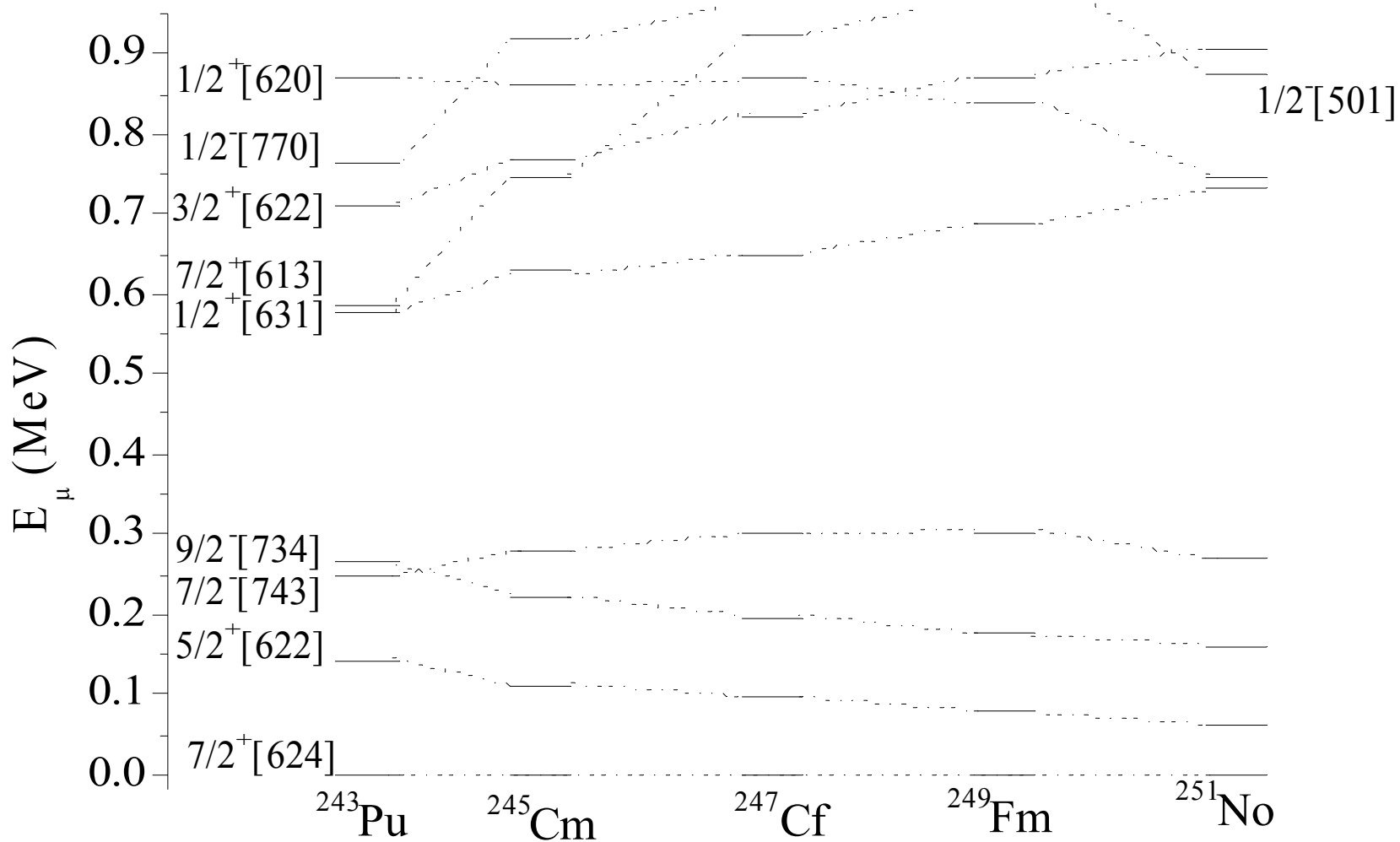
N=149 isotones

QPM



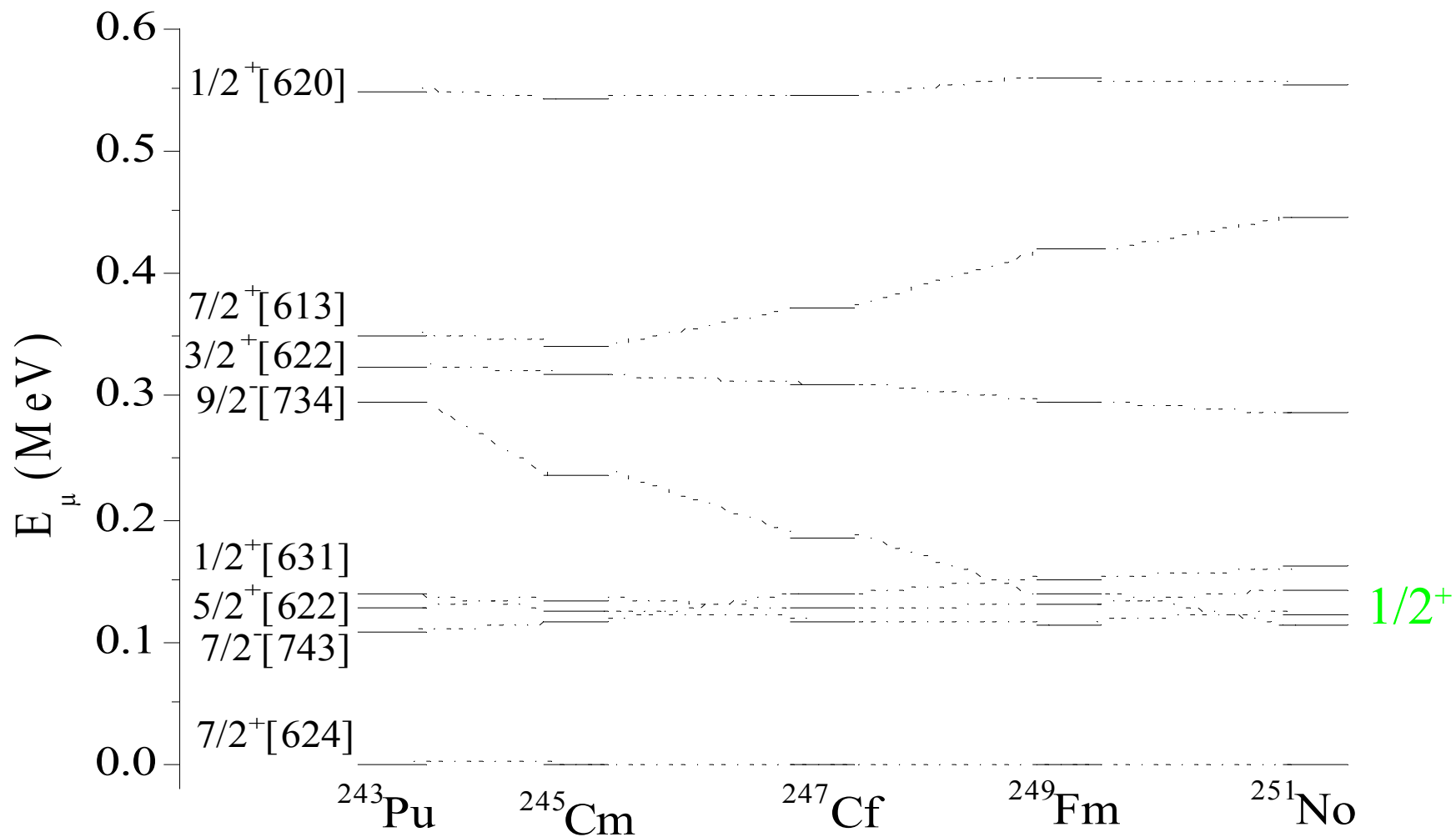
N=149 isotones

SLy4



N=149 isotones

SkP

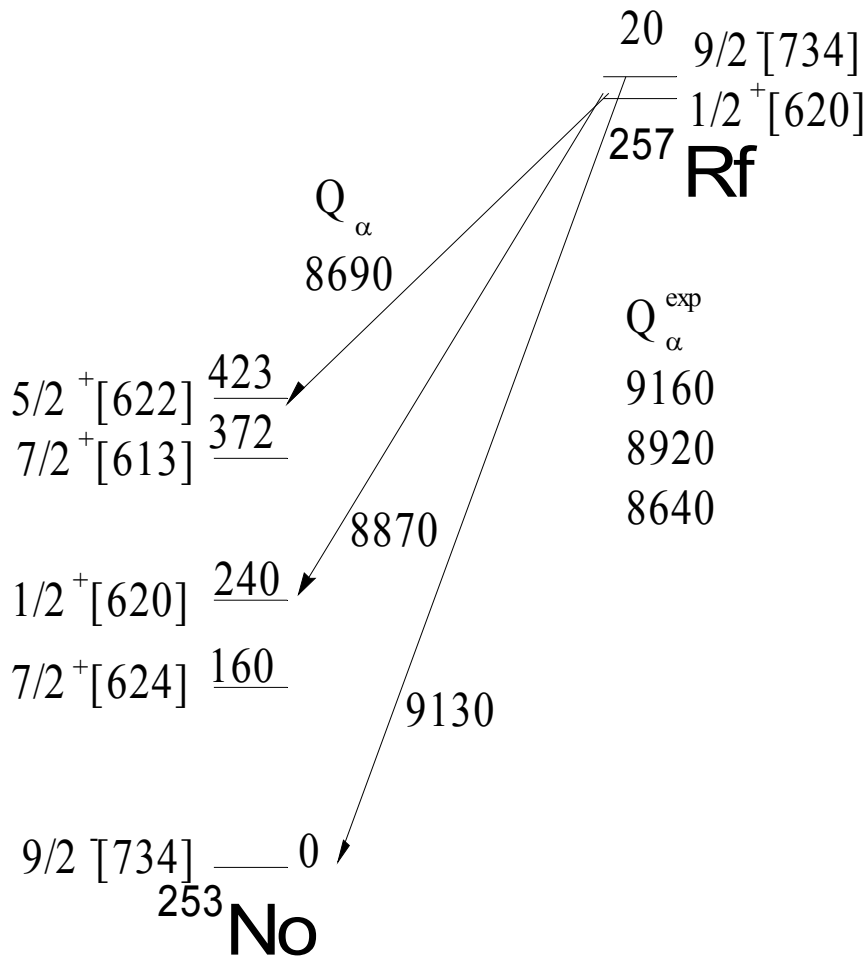


The smooth change of energies of almost all one-quasiparticle states in the isotone chain.

The deformation parameters of the  $N=149$  nuclei treated are almost the same. Although different methods of calculations give various deformations of the ground state. For example, in the case of  $^{251}\text{No}$   $\beta_2=0.234$  and  $\beta_4=0.057$  are resulted from the TCSM,  $\beta_2=0.296$  and  $\beta_4=0.01$  from the HB with Sly4 parameterization.

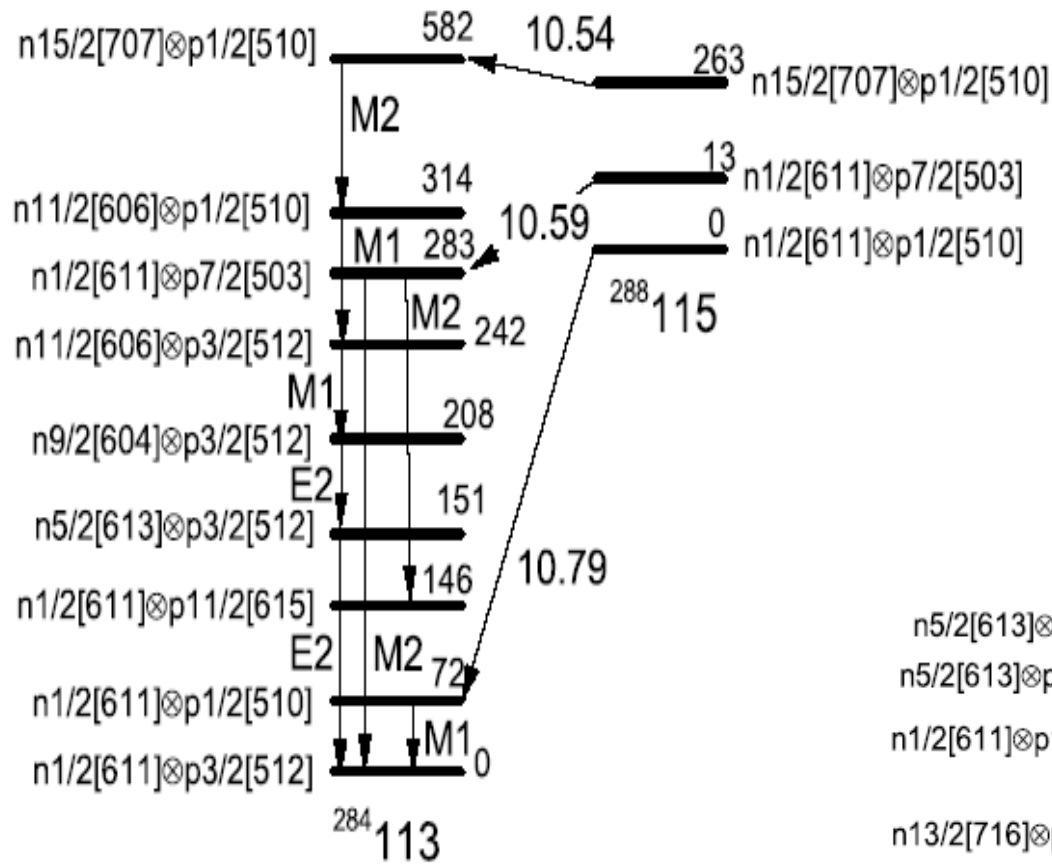
Long-living isomer in  $^{251}\text{No}$ :  $1/2^+[631]$ , about 1 s



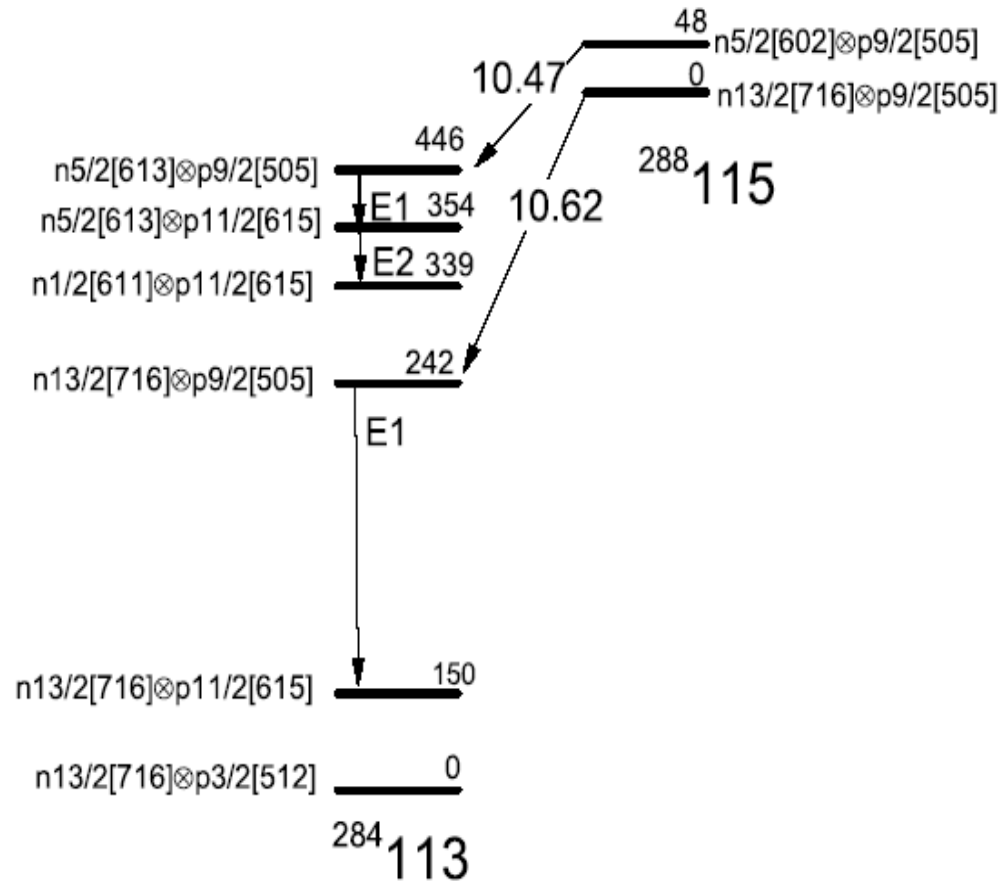


Using the one-quasiparticle spectra calculated with the TCSM for  $^{257}\text{Rf}$  and  $^{253}\text{No}$ , the possible  $\alpha$  decay scheme of  $^{257}\text{Rf}$  is suggested. The calculated  $Q_\alpha$  values are consistent with the experimental values listed.

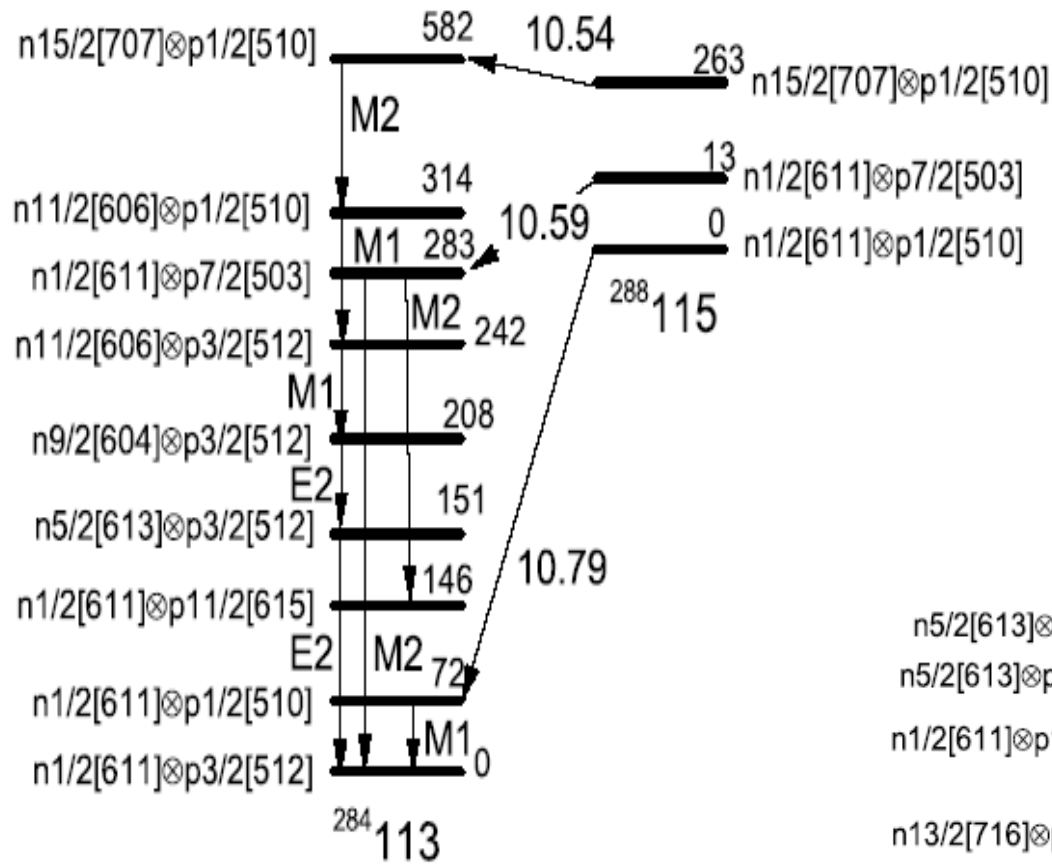




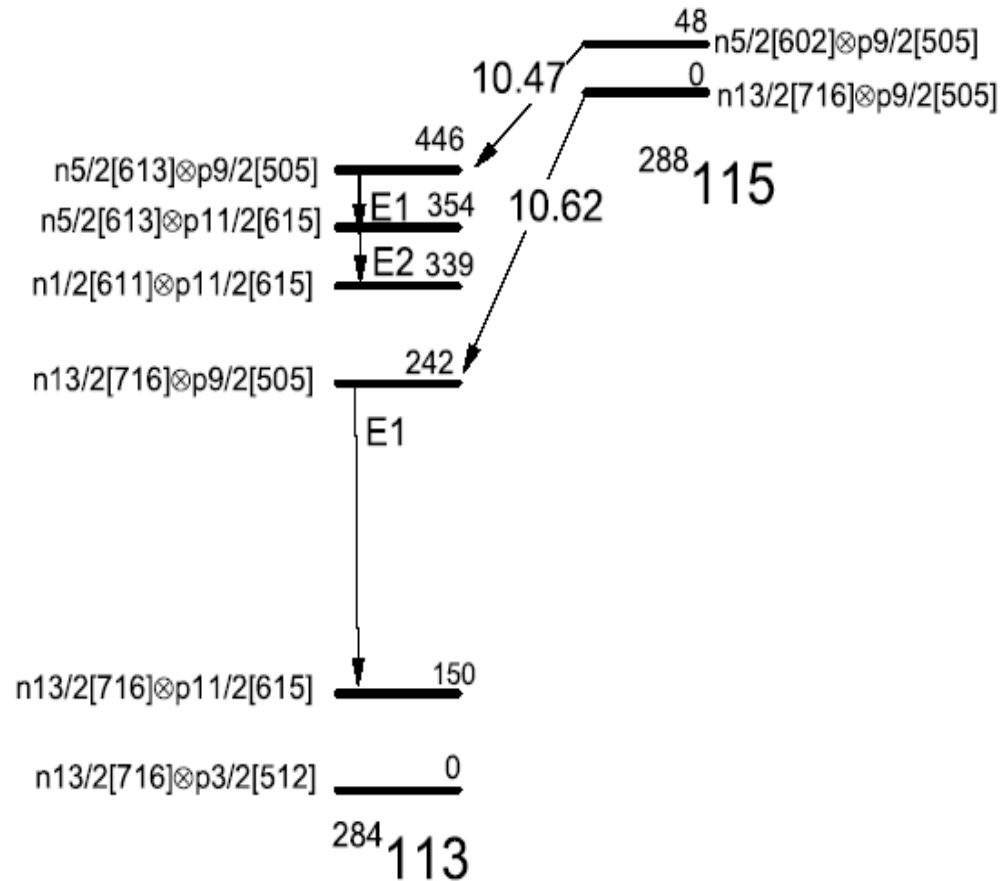
## SHF calculations (SV-bas)



TCSM calculations

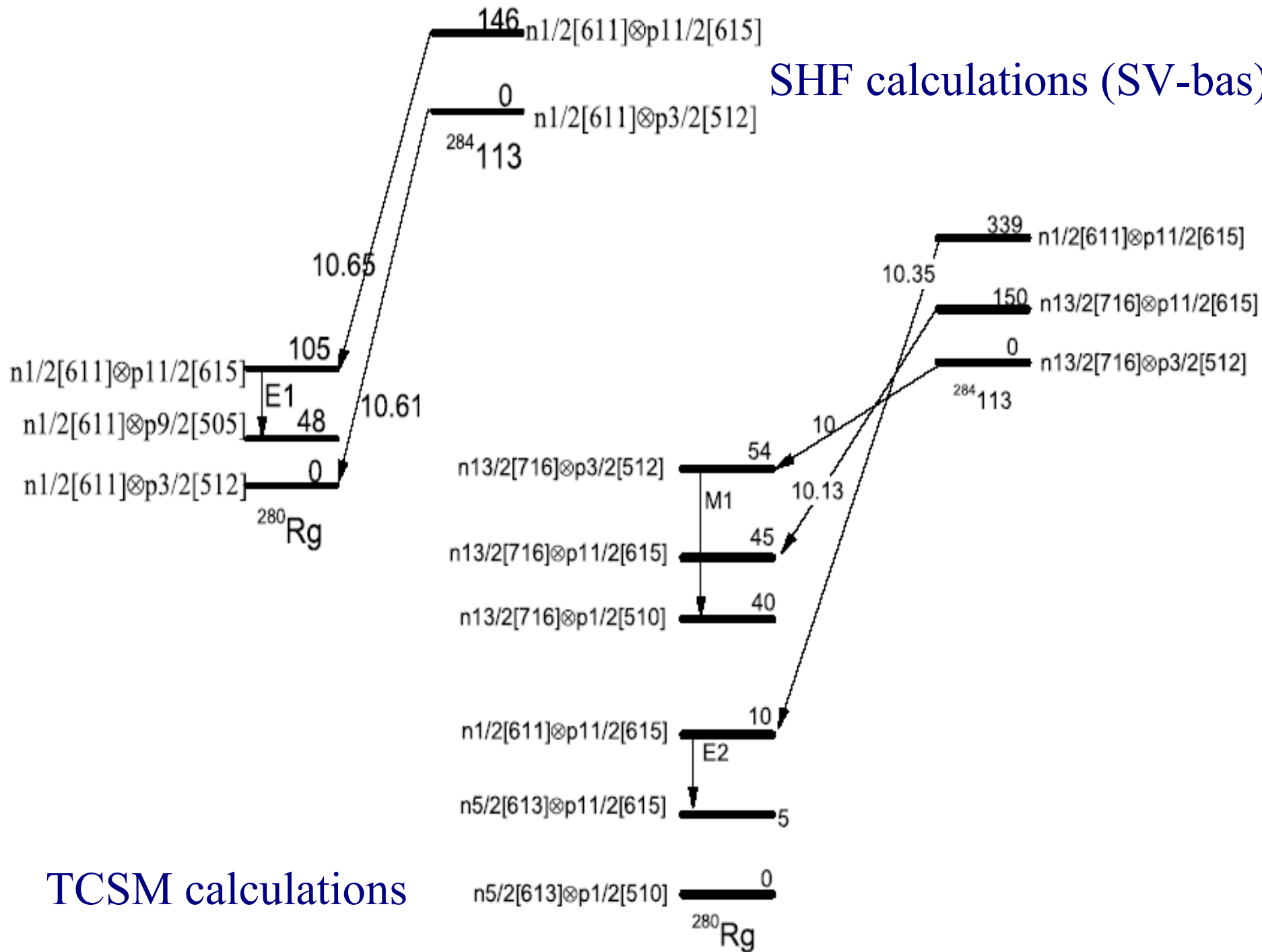


## SHF calculations (SV-bas)



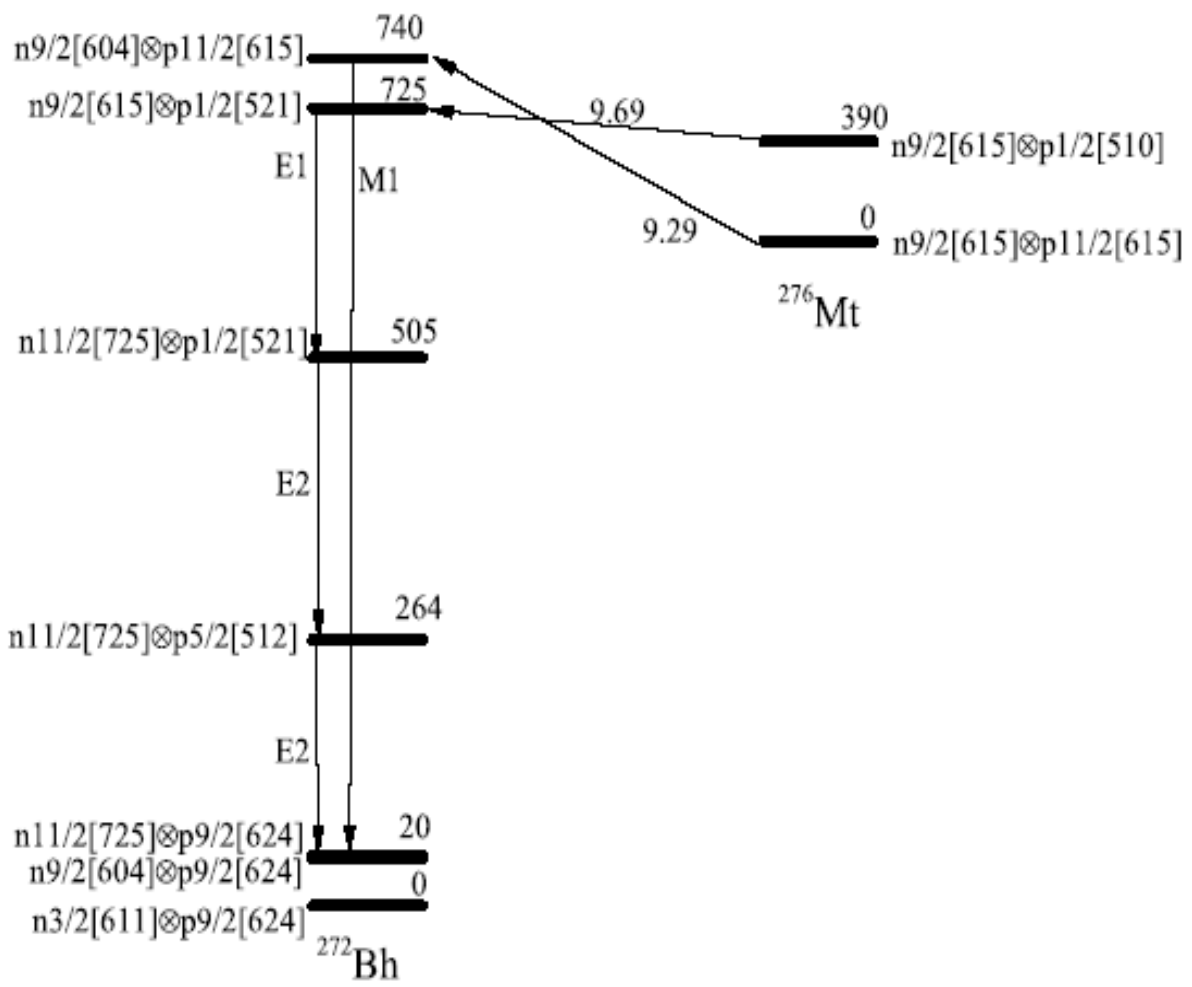
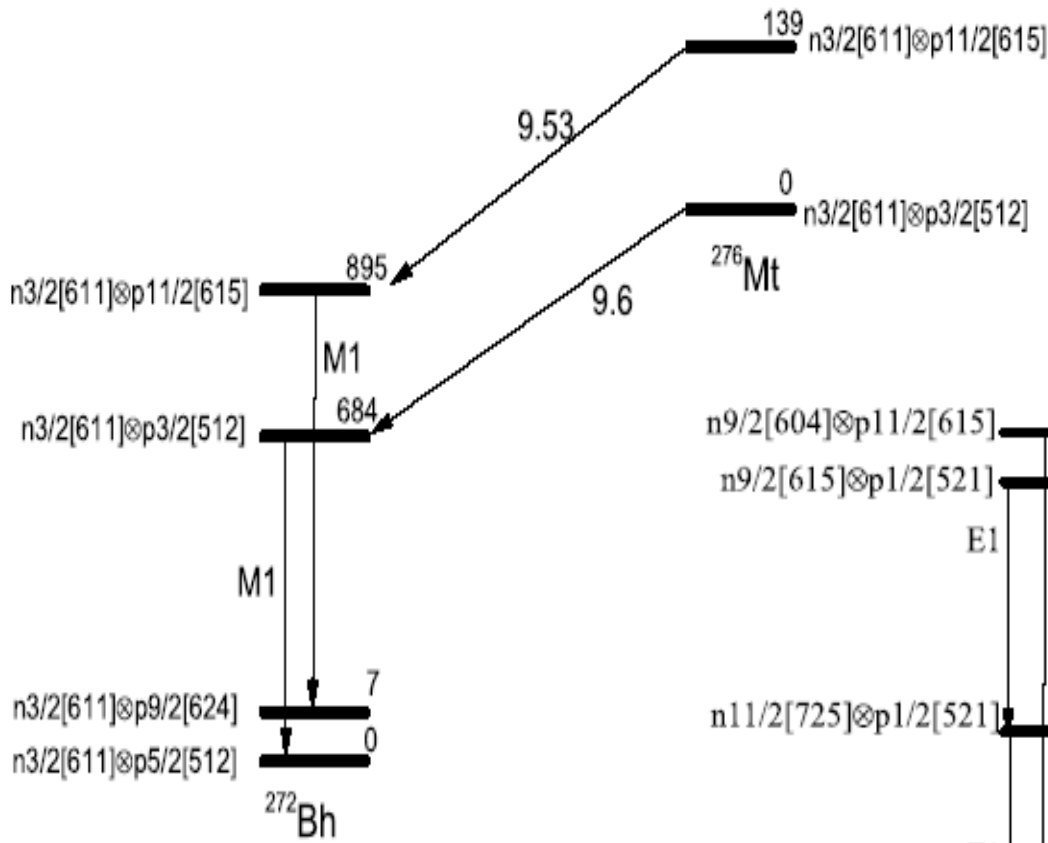
TCSM calculations

# SHF calculations (SV-bas)



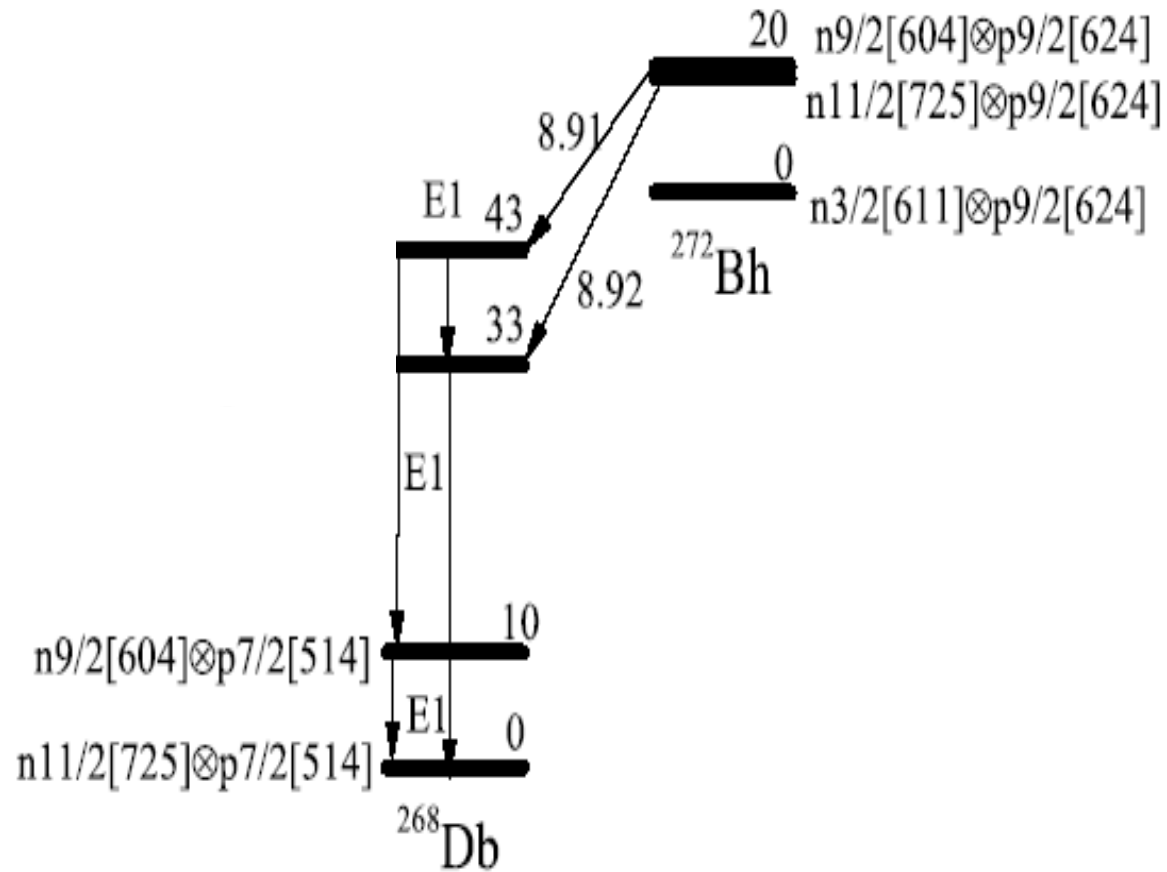
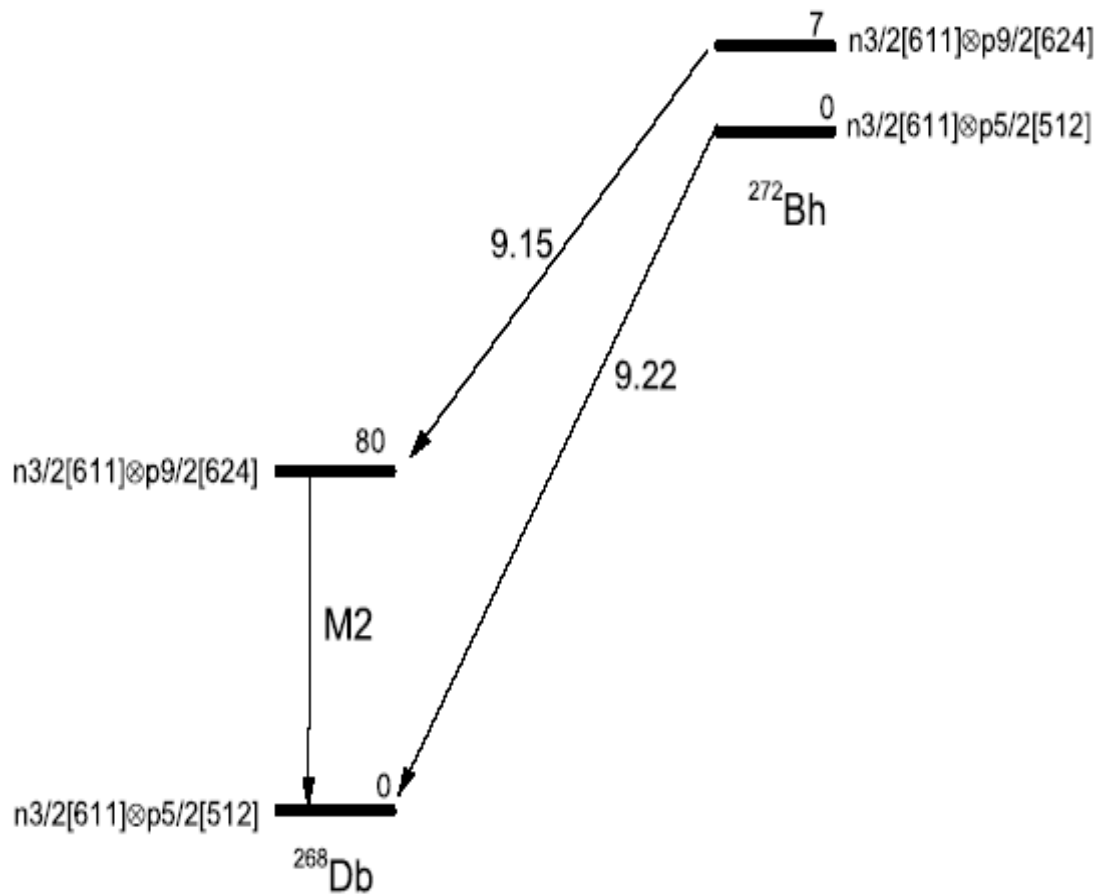
TCSM calculations

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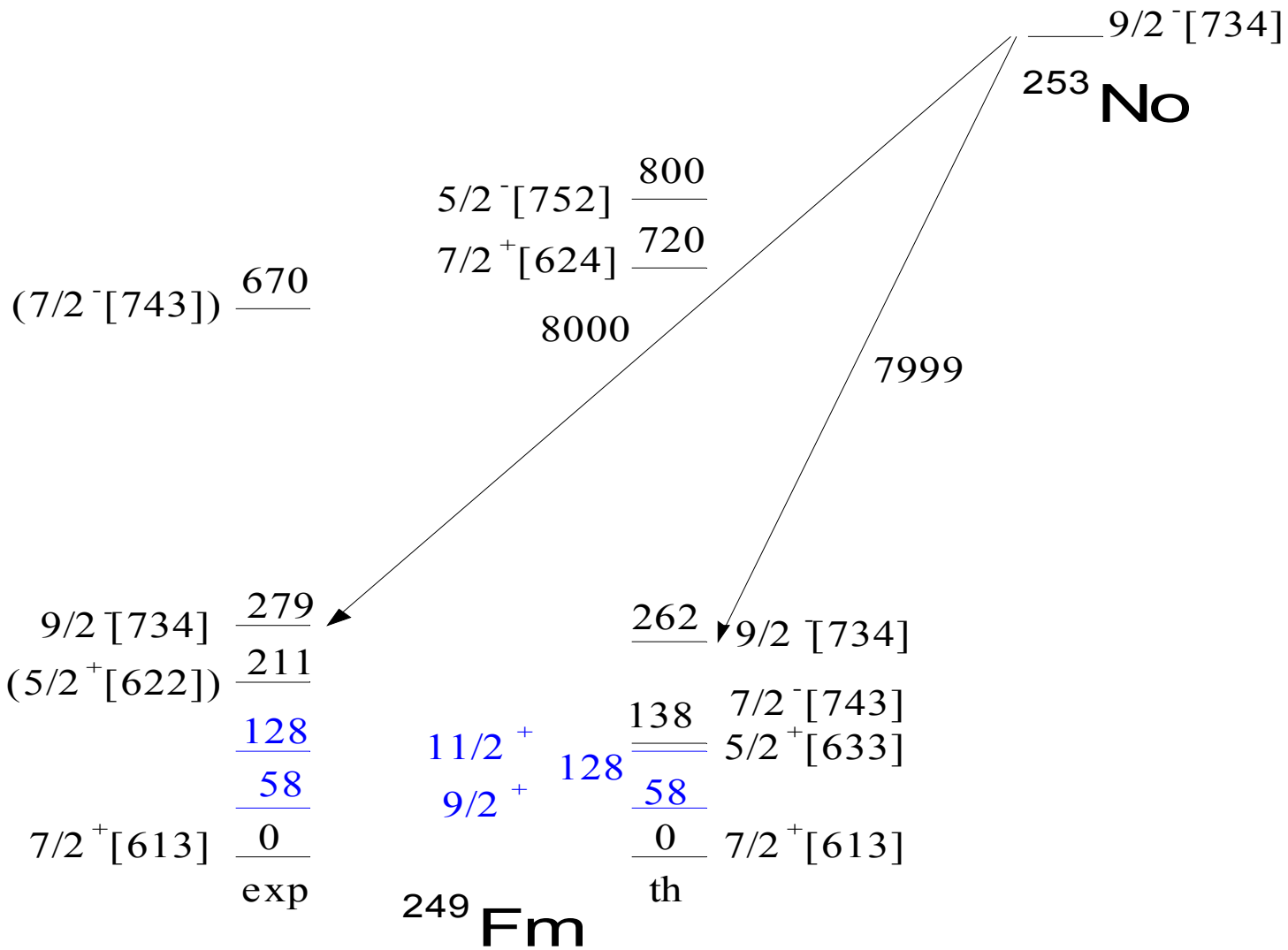
# TCSM calculations





# Summary

- The used simple shape parametrization is suitable to describe some properties of heavy nuclei.
- Description and prediction of one- and two-quasiparticle isomeric states.
- Alpha-decay chain through the isomeric states.
- Isotonic and isotopic trends
- Lifetime of isomeric state
- Fission from isomers
- More experimental data



# Parameters

$$35 \leq N - Z \leq 56$$

*for neutrons*

$$\kappa_n = -0.076 + 0.0058(N - Z) - 6.53 \times 10^{-5}(N - Z)^2 + 0.002 A^{1/3},$$

$$\mu_n = 1.598 - 0.0295(N - Z) + 3.036 \times 10^{-4}(N - Z)^2 - 0.095 A^{1/3},$$

*for protons*

$$\kappa_p = 0.0383 + 0.00137(N - Z) - 1.22 \times 10^{-5}(N - Z)^2 - 0.003 A^{1/3},$$

$$\mu_p = 0.335 + 0.01(N - Z) - 9.367 \times 10^{-5}(N - Z)^2 + 0.003 A^{1/3},$$

The parts in front of the terms with  $A^{1/3}$  vary:

(0.05-0.053) for  $\kappa_n$  , (0.075-0.0768) for  $\kappa_p$  ,

(0.88-0.92) for  $\mu_n$  , (0.58-0.61) for  $\mu_p$

# Strength parameters of pairing interaction

$$G_{\begin{matrix} n \\ p \end{matrix}} = \left( 19.2 \mp 7.4 \frac{N-Z}{A} \right) A^{-1} \text{ MeV}$$

$$A \approx 250 \rightarrow G_n \approx 0.075 \text{ MeV}, G_p \approx 0.085 \text{ MeV}$$

## One-quasiparticle excitations

$$E_{\mu} = \sqrt{(e_{\mu} - e_F)^2 + \Delta^2} - \sqrt{(e_{\mu}' - e_F)^2 + \Delta^2}$$

## Two-quasiparticle excitations

$$E_{\mu} = \sqrt{(e_{\mu} - e_F)^2 + \Delta^2} + \sqrt{(e_{\mu}' - e_F)^2 + \Delta^2}$$

$$\Delta \geq 0.35 \text{ MeV} \rightarrow \text{BCS approximation}$$

