

Probing GPDs through the exclusive photoproduction of a  $\gamma\rho$  pair  
with a large invariant mass

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Nucleon and nuclear structure through di-lepton production

Trento, 24-28 October 2016

RB, B. Pire, L. Szymanowski, S. Wallon

[arXiv:1609.03830\[hep-ph\]](https://arxiv.org/abs/1609.03830)

# Transversity of the nucleon using hard processes

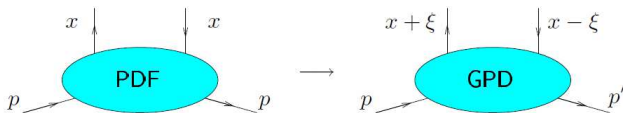
## What is transversity?

- Transverse spin content of the proton:

$$\begin{aligned}
 |\uparrow\rangle(x) &\sim |\rightarrow\rangle + |\leftarrow\rangle \\
 |\downarrow\rangle(x) &\sim |\rightarrow\rangle - |\leftarrow\rangle
 \end{aligned}$$

spin along  $x$                       helicity states

- Observables which are sensitive to helicity flip thus give access to transversity  $\Delta_{Tq}(x)$ . Poorly known.
- Transversity GPDs are experimentally very elusive quantities.

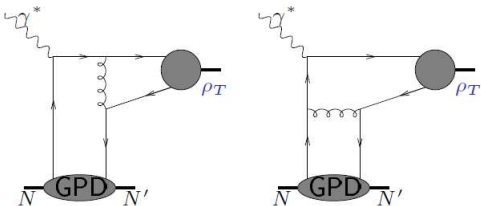


- For massless (anti)particles, chirality = (-)helicity
- **Transversity is thus a chiral-odd quantity**
- Since (in the massless limit) QCD and QED are chiral-even ( $\gamma^\mu, \gamma^\mu\gamma^5$ ), **the chiral-odd quantities ( $1, \gamma^5, [\gamma^\mu, \gamma^\nu]$ ) which one wants to measure should appear in pairs**

# Transversity of the nucleon using hard processes: using a two body final state process?

## How to get access to transversity GPDs?

- the dominant DA of  $\rho_T$  is of twist 2 and chiral-odd ( $[\gamma^\mu, \gamma^\nu]$  coupling)
- unfortunately  $\gamma^* N^\uparrow \rightarrow \rho_T N' = 0$ 
  - This cancellation is true at any order : such a process would require a helicity transfer of 2 from a photon.
  - lowest order diagrammatic argument:



$$\gamma^\alpha [\gamma^\mu, \gamma^\nu] \gamma_\alpha \rightarrow 0$$

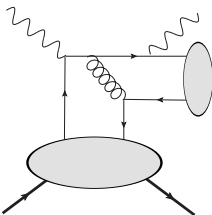
# Transversity of the nucleon using hard processes: using a two body final state process?

## Can one circumvent this vanishing?

- This vanishing only occurs at **twist 2**
- At twist 3 this process does not vanish [Ahmad, Goldstein, Liuti], [Goloskokov, Kroll]
- However processes involving **twist 3 DAs** may face problems with factorization (end-point singularities)  
can be made safe in the high-energy  $k_T$ -factorization approach [Anikin, Ivanov, Pire, Szymanowski, Wallon]
- One can also consider a 3-body final state process [Ivanov, Pire, Szymanowski, Teryaev], [Enberg, Pire, Szymanowski], [El Beiyad, Pire, Segond, Szymanowski, Wallon]

# Probing **chiral-odd** GPDs using $\rho$ meson + photon production

Processes with **3 body final states** can give access to **chiral-odd GPDs**

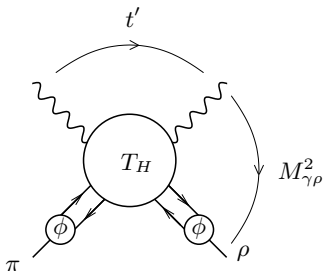


Typical non-zero diagram for a **transverse**  $\rho$  meson

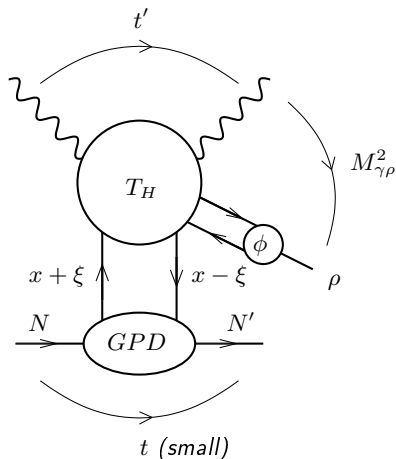
The previous argument does not apply anymore.

# Probing GPDs using $\rho$ meson + photon production

- We consider the process  $\gamma N \rightarrow \gamma \rho N'$
- Collinear factorization of the amplitude for  $\gamma + N \rightarrow \gamma + \rho + N'$  at large  $M_{\gamma\rho}^2$

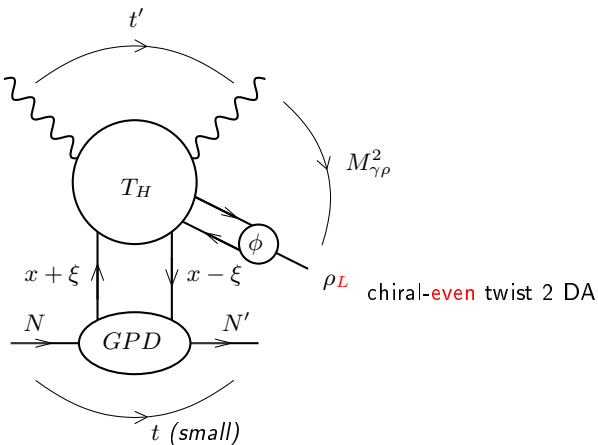


large angle factorization  
à la Brodsky Lepage



# Probing chiral-even GPDs using $\rho$ meson + photon production

Processes with 3 body final states can give access to chiral-even GPDs



chiral-even twist 2 GPD

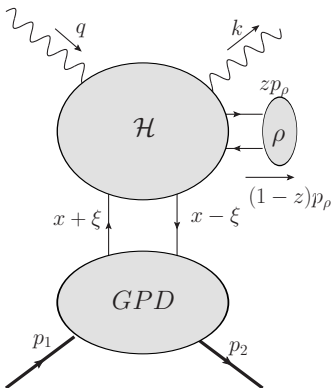




# Master formula based on leading twist 2 factorization

$$\mathcal{A} \propto \int_{-1}^1 dx \int_0^1 dz T(x, \xi, z) \times H(x, \xi, t) \Phi_\rho(z) + \dots$$

- Both the DA and the GPD can be either **chiral-even** or **chiral-odd**.
- At twist 2 the **longitudinal  $\rho$  DA** is **chiral-even** and the **transverse  $\rho$  DA** is **chiral-odd**.
- Hence we will need both **chiral-even** and **chiral-odd** non-perturbative building blocks and hard parts.



## Kinematics

## Kinematics to handle GPD in a 3-body final state process

- use a **Sudakov** basis :  
light-cone vectors  $p$ ,  $n$  with  $2p \cdot n = s$
- assume the following kinematics:
  - $\Delta_{\perp} \ll p_{\perp}$
  - $M^2, m_{\rho}^2 \ll M_{\gamma\rho}^2$

- initial state particle momenta:

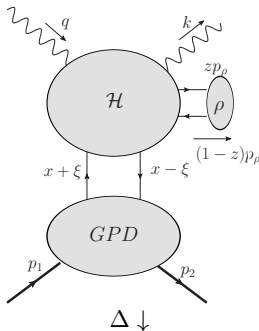
$$q^{\mu} = n^{\mu}, \quad p_1^{\mu} = (1 + \xi) p^{\mu} + \frac{M^2}{s(1+\xi)} n^{\mu}$$

- final state particle momenta:

$$p_2^{\mu} = (1 - \xi) p^{\mu} + \frac{M^2 + \vec{p}_t^2}{s(1 - \xi)} n^{\mu}$$

$$k^{\mu} = \alpha n^{\mu} + \frac{(\vec{p}_t - \vec{\Delta}_t/2)^2}{\alpha s} p^{\mu} + p_{\perp}^{\mu} - \frac{\Delta_{\perp}^{\mu}}{2},$$

$$p_{\rho}^{\mu} = \alpha_{\rho} n^{\mu} + \frac{(\vec{p}_t + \vec{\Delta}_t/2)^2 + m_{\rho}^2}{\alpha_{\rho} s} p^{\mu} - p_{\perp}^{\mu} - \frac{\Delta_{\perp}^{\mu}}{2},$$



# Non perturbative **chiral-even** building blocks

- Helicity conserving GPDs at twist 2 :

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left( -\frac{1}{2}z^- \right) \gamma^+ \psi \left( \frac{1}{2}z^- \right) | p_1, \lambda_1 \rangle$$

$$= \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[ H^q(x, \xi, t) \gamma^+ + E^q(x, \xi, t) \frac{i\sigma^{\alpha+} \Delta_\alpha}{2m} \right] u(p_1, \lambda_1)$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left( -\frac{1}{2}z^- \right) \gamma^+ \gamma^5 \psi \left( \frac{1}{2}z^- \right) | p_1, \lambda_1 \rangle$$

$$= \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[ \tilde{H}^q(x, \xi, t) \gamma^+ \gamma^5 + \tilde{E}^q(x, \xi, t) \frac{\gamma^5 \Delta^+}{2m} \right] u(p_1, \lambda_1)$$

- We will consider the simplest case when  $\Delta_\perp = 0$ .
- In that case and in the forward limit  $\xi \rightarrow 0$  only the  $H^q$  and  $\tilde{H}^q$  terms survive.

- Helicity conserving (vector) DA at twist 2 :

longitudinal polarization

$$\langle 0 | \bar{u}(0) \gamma^\mu u(x) | \rho^0(p, s) \rangle = \frac{p^\mu}{\sqrt{2}} f_\rho \int_0^1 du e^{-iup \cdot x} \phi_{\parallel}(u)$$

# Non perturbative **chiral-odd** building blocks

- Helicity flip GPD at twist 2 :

$$\begin{aligned}
 & \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left( -\frac{1}{2}z^- \right) i\sigma^{+i} \psi \left( \frac{1}{2}z^- \right) | p_1, \lambda_1 \rangle \\
 = & \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[ H_T^q(x, \xi, t) i\sigma^{+i} + \tilde{H}_T^q(x, \xi, t) \frac{P^+ \Delta^i - \Delta^+ P^i}{M_N^2} \right. \\
 + & \left. E_T^q(x, \xi, t) \frac{\gamma^+ \Delta^i - \Delta^+ \gamma^i}{2M_N} + \tilde{E}_T^q(x, \xi, t) \frac{\gamma^+ P^i - P^+ \gamma^i}{M_N} \right] u(p_1, \lambda_1)
 \end{aligned}$$

- We will consider the simplest case when  $\Delta_\perp = 0$ .
- In that case and in the forward limit  $\xi \rightarrow 0$  only the  $H_T^q$  term survives.
- Transverse  $\rho$  DA at twist 2 :

$$\langle 0 | \bar{u}(0) \sigma^{\mu\nu} u(x) | \rho^0(p, s) \rangle = \frac{i}{\sqrt{2}} (\epsilon_\rho^\mu p^\nu - \epsilon_\rho^\nu p^\mu) f_\rho^\perp \int_0^1 du e^{-iup \cdot x} \phi_\perp(u)$$

# Models for DAs

## Asymptotical DAs

We take the asymptotic form of the (normalized) DAs:

conformal symmetry,  $\mu_F^2 \rightarrow \infty$

$$\phi_{\parallel}(z) = 6z(1-z),$$

$$\phi_{\perp}(z) = 6z(1-z).$$

# Model for GPDs: based on the Double Distribution ansatz

## Realistic Parametrization of GPDs

- GPDs can be represented in terms of **Double Distributions** [Radyushkin] based on the **Schwinger** representation of a toy model for GPDs in scalar  $\phi^3$  theory

$$H^q(x, \xi, t=0) = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(\beta + \xi\alpha - x) f^q(\beta, \alpha)$$

- ansatz for these Double Distributions [Radyushkin]:

- chiral-even sector:

$$f^q(\beta, \alpha, t=0) = \Pi(\beta, \alpha) q(\beta) \Theta(\beta) - \Pi(-\beta, \alpha) \bar{q}(-\beta) \Theta(-\beta),$$

$$\tilde{f}^q(\beta, \alpha, t=0) = \Pi(\beta, \alpha) \Delta q(\beta) \Theta(\beta) + \Pi(-\beta, \alpha) \Delta \bar{q}(-\beta) \Theta(-\beta).$$

- chiral-odd sector:

$$f_T^q(\beta, \alpha, t=0) = \Pi(\beta, \alpha) \delta q(\beta) \Theta(\beta) - \Pi(-\beta, \alpha) \delta \bar{q}(-\beta) \Theta(-\beta),$$

- $\Pi(\beta, \alpha) = \frac{3}{4} \frac{(1-\beta)^2 - \alpha^2}{(1-\beta)^3}$  : profile function

- simplistic factorized ansatz for the  $t$ -dependence:

$$H^q(x, \xi, t) = H^q(x, \xi, t=0) \times F_H(t)$$

with  $F_H(t) = \frac{C^2}{(t-C)^2}$  a standard **dipole form factor** ( $C = .71$  GeV)

# Model for GPDs: based on the Double Distribution ansatz

## Sets of PDFs

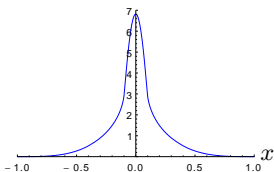
- $q(x)$  : unpolarized PDF [GRV-98]
- $\Delta q(x)$  polarized PDF [GRSV-2000]
- $\delta q(x)$  : transversity PDF [Anselmino *et al.*]

# Model for GPDs: based on the Double Distribution ansatz

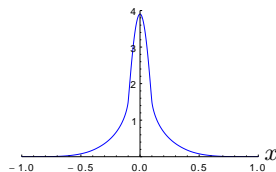
## Typical sets of chiral-even GPDs

$$\xi = .1 \leftrightarrow S_{\gamma N} = 20 \text{ GeV}^2 \text{ and } M_{\gamma\rho}^2 = 3.5 \text{ GeV}^2$$

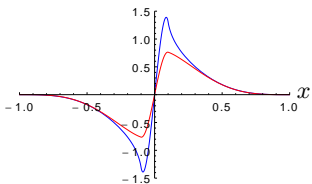
$$\frac{1}{2} H^{u(-)}(x, \xi)$$



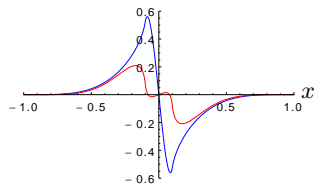
$$\frac{1}{2} H^{d(-)}(x, \xi)$$



$$\frac{1}{2} \tilde{H}^{u(-)}(x, \xi)$$



$$\frac{1}{2} \tilde{H}^{d(-)}(x, \xi)$$



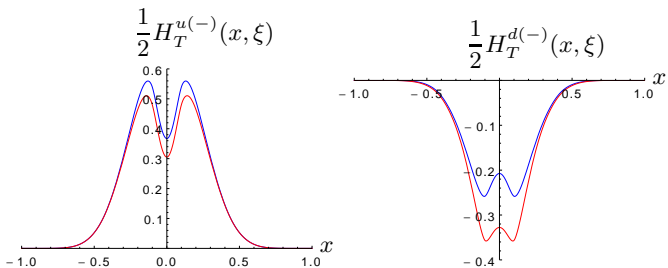
“valence” and “standard”: two GRSV Ansätze for  $\Delta q(x)$



# Model for GPDs: based on the Double Distribution ansatz

## Typical sets of chiral-odd GPDs

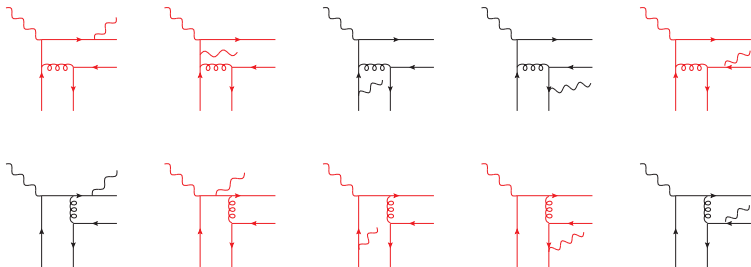
$$\xi = .1 \leftrightarrow S_{\gamma N} = 20 \text{ GeV}^2 \text{ and } M_{\gamma\rho}^2 = 3.5 \text{ GeV}^2$$



“valence” and “standard”: two GRSV Ansätze for  $\Delta q(x)$   
 $\Rightarrow$  two Ansätze for  $\delta q(x)$

# Computation of the hard part

20 diagrams to compute



The other half can be deduced by  $q \leftrightarrow \bar{q}$  (anti)symmetry

Red diagrams cancel in the chiral-odd case

## Final computation

## Final computation

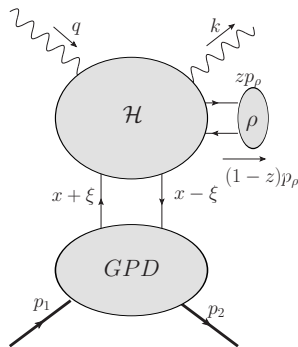
$$\mathcal{A} \propto \int_{-1}^1 dx \int_0^1 dz T(x, \xi, z) H(x, \xi, t) \Phi_\rho(z)$$

- One performs the  $z$  integration **analytically** using an asymptotic DA  $\propto z(1-z)$
- One then plugs our GPD models into the formula and performs the integral w.r.t.  $x$  numerically.
- Differential cross section:

$$\left. \frac{d\sigma}{dt du' dM_{\gamma\rho}^2} \right|_{-t=(-t)_{min}} = \frac{|\overline{\mathcal{M}}|^2}{32S_{\gamma N}^2 M_{\gamma\rho}^2 (2\pi)^3}.$$

$|\overline{\mathcal{M}}|^2 =$  averaged amplitude squared

- Kinematical parameters:  $S_{\gamma N}^2$ ,  $M_{\gamma\rho}^2$  and  $-u'$

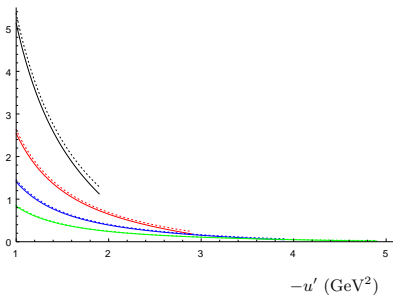


## Fully differential cross section

## Chiral even cross section

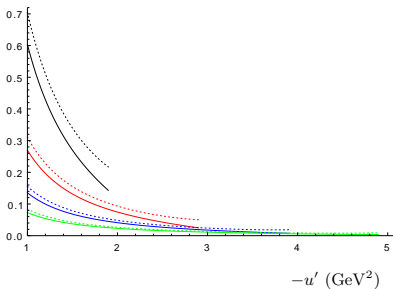
at  $-t = (-t)_{\min}$ 

$$\frac{d\sigma_{\text{even}}}{dM_{\gamma\rho}^2 d(-u') d(-t)} \quad (\text{nb} \cdot \text{GeV}^{-6})$$



proton

$$\frac{d\sigma_{\text{even}}}{dM_{\gamma\rho}^2 d(-u') d(-t)} \quad (\text{nb} \cdot \text{GeV}^{-6})$$



neutron

$$S_{\gamma N} = 20 \text{ GeV}^2$$

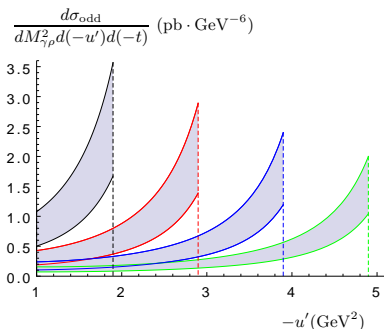
$$M_{\gamma\rho}^2 = 3, 4, 5, 6 \text{ GeV}^2$$

solid: "valence" model

dotted: "standard" model

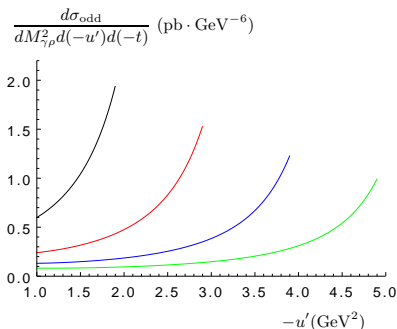
## Fully differential cross section

## Chiral odd cross section

at  $-t = (-t)_{\min}$ 

proton

“valence” and “standard” models,  
each of them with  $\pm 2\sigma$  [S. Melis]



neutron

“valence” model only

$$S_{\gamma N} = 20 \text{ GeV}^2$$

$$M_{\gamma\rho}^2 = 3, 4, 5, 6 \text{ GeV}^2$$

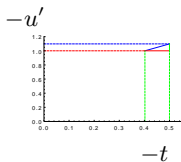
# Phase space integration

Evolution of the phase space in  $(-t, -u')$  plane

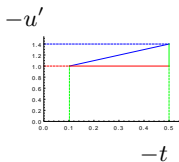
large angle scattering:  $M_{\gamma\rho}^2 \sim -u' \sim -t'$

in practice:  $-u' > 1 \text{ GeV}^2$  and  $-t' > 1 \text{ GeV}^2$  and  $(-t)_{\min} \leq -t \leq .5 \text{ GeV}^2$   
 this ensures large  $M_{\gamma\rho}^2$

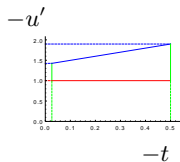
example:  $S_{\gamma N} = 20 \text{ GeV}^2$



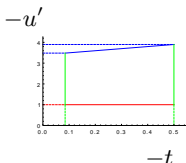
$M_{\gamma\rho} = 2.2 \text{ GeV}^2$



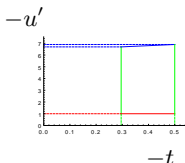
$M_{\gamma\rho} = 2.5 \text{ GeV}^2$



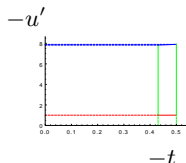
$M_{\gamma\rho} = 3 \text{ GeV}^2$



$M_{\gamma\rho} = 5 \text{ GeV}^2$



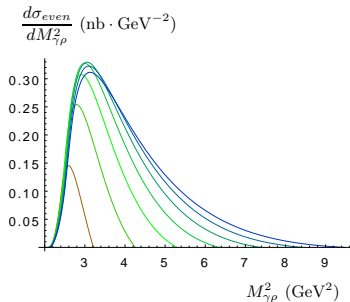
$M_{\gamma\rho} = 8 \text{ GeV}^2$



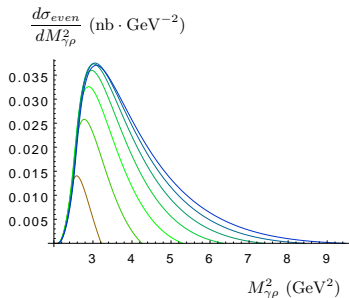
$M_{\gamma\rho} = 9 \text{ GeV}^2$

# Single differential cross section

## Chiral even cross section



proton



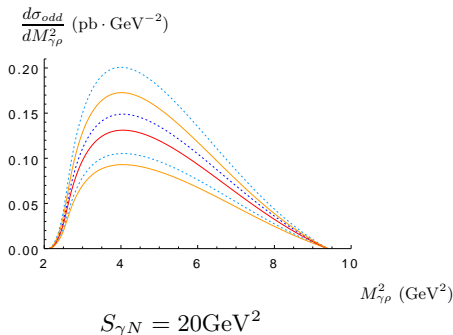
neutron

“valence” scenario

$S_{\gamma N}$  vary in the set 8, 10, 12, 14, 16, 18, 20 GeV<sup>2</sup> (from left to right)

# Single differential cross section

## Chiral odd cross section



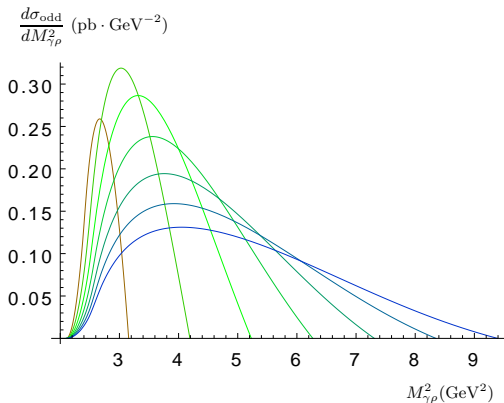
Various ansätze for the PDFs  $\Delta q$  used to build the GPD  $H_T$ :

- *dotted curves*: “standard” scenario
- *solid curves*: “valence” scenario
- *deep-blue* and *red* curves: central values
- *light-blue* and *orange*: results with  $\pm 2\sigma$ .



# Single differential cross section

## Chiral odd cross section

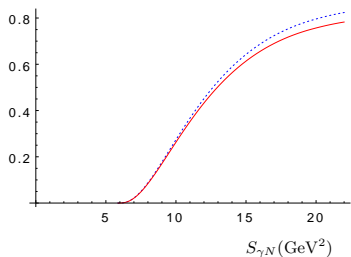


proton, "valence" scenario

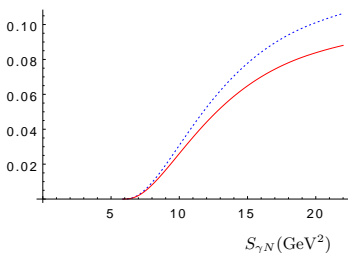
$S_{\gamma N}$  vary in the set 8, 10, 12, 14, 16, 18, 20 GeV<sup>2</sup> (from left to right)

## Integrated cross-section

## Chiral even cross section

 $\sigma_{even}$  (nb)

proton

 $\sigma_{even}$  (nb)

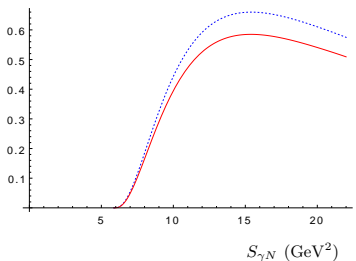
neutron

solid red: "valence" scenario

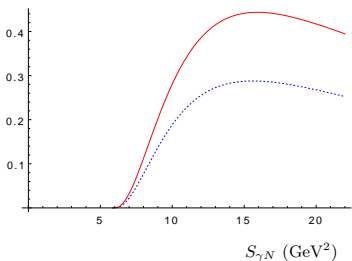
dashed blue: "standard" one

## Integrated cross-section

## Chiral odd cross section

 $\sigma_{odd}$  (pb)

proton

 $\sigma_{odd}$  (pb)

neutron

solid red: "valence" scenario

dashed blue: "standard" one

# Counting rates for 100 days

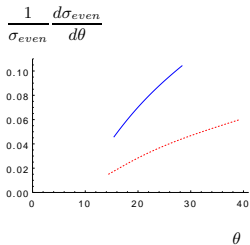
example: JLab Hall B

- untagged incoming  $\gamma \Rightarrow$  Weizsäcker-Williams distribution
- With an expected luminosity of  $\mathcal{L} = 100 \text{ nb}^{-1} \text{ s}^{-1}$ , for 100 days of run:
  - Chiral even case :  $\simeq 6.8 \cdot 10^6 \rho_L$ .
  - Chiral odd case :  $\simeq 7.5 \cdot 10^3 \rho_T$

# Effects of an experimental angular restriction for the produced $\gamma$ ?

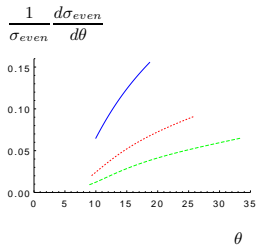
## Angular distribution of the produced $\gamma$ (chiral-even cross section)

after boosting to the lab frame



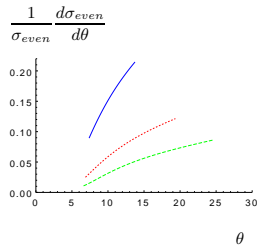
$$S_{\gamma N} = 10 \text{ GeV}^2$$

$$M_{\gamma\rho}^2 = 3, 4 \text{ GeV}^2$$



$$S_{\gamma N} = 15 \text{ GeV}^2$$

$$M_{\gamma\rho}^2 = 3, 4, 5 \text{ GeV}^2$$



$$S_{\gamma N} = 20 \text{ GeV}^2$$

$$M_{\gamma\rho}^2 = 3, 4, 5 \text{ GeV}^2$$

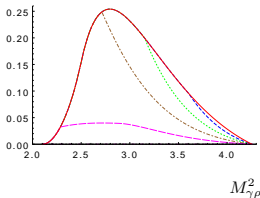
JLab Hall B detector equipped between  $5^\circ$  and  $35^\circ$

# Effects of an experimental angular restriction for the produced $\gamma$ ?

## Angular distribution of the produced $\gamma$ (chiral-even cross section)

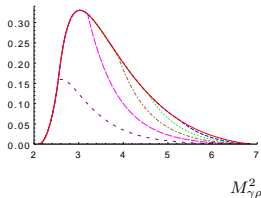
after boosting to the lab frame

$$\frac{d\sigma_{\text{even}}}{dM_{\gamma\rho^2}} (\text{nb. GeV}^{-2})$$



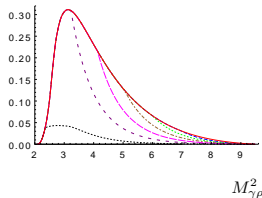
$$S_{\gamma N} = 10 \text{ GeV}^2$$

$$\frac{d\sigma_{\text{even}}}{dM_{\gamma\rho^2}} (\text{nb. GeV}^{-2})$$



$$S_{\gamma N} = 15 \text{ GeV}^2$$

$$\frac{d\sigma_{\text{even}}}{dM_{\gamma\rho^2}} (\text{nb. GeV}^{-2})$$



$$S_{\gamma N} = 20 \text{ GeV}^2$$

$$\theta_{\text{max}} = 35^\circ, 30^\circ, 25^\circ, 20^\circ, 15^\circ, 10^\circ$$

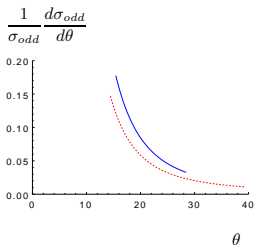
JLab Hall B detector equipped between  $5^\circ$  and  $35^\circ$

$\Rightarrow$  this is safe!

# Effects of an experimental angular restriction for the produced $\gamma$ ?

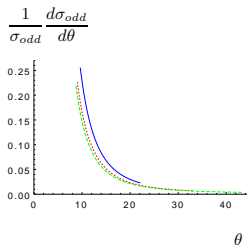
## Angular distribution of the produced $\gamma$ (chiral-odd cross section)

after boosting to the lab frame



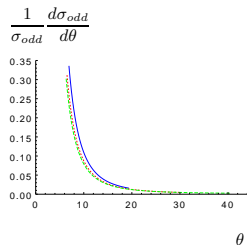
$$S_{\gamma N} = 10 \text{ GeV}^2$$

$$M_{\gamma\rho}^2 = 3, 4 \text{ GeV}^2$$



$$S_{\gamma N} = 15 \text{ GeV}^2$$

$$M_{\gamma\rho}^2 = 3.5, 5, 6.5 \text{ GeV}^2$$



$$S_{\gamma N} = 20 \text{ GeV}^2$$

$$M_{\gamma\rho}^2 = 4, 6, 8 \text{ GeV}^2$$

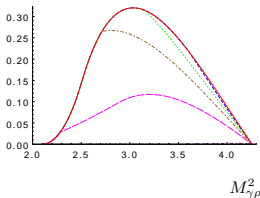
JLab Hall B detector equipped between  $5^\circ$  and  $35^\circ$

# Effects of an experimental angular restriction for the produced $\gamma$ ?

## Angular distribution of the produced $\gamma$ (chiral-odd cross section)

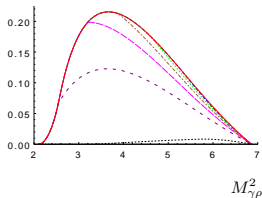
after boosting to the lab frame

$$\frac{d\sigma_{odd}}{dM_{\gamma\rho^2}} \text{ (pb.GeV}^{-2}\text{)}$$



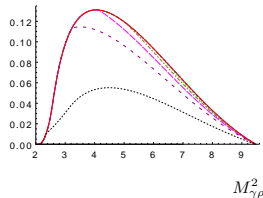
$$S_{\gamma N} = 10 \text{ GeV}^2$$

$$\frac{d\sigma_{odd}}{dM_{\gamma\rho^2}} \text{ (pb.GeV}^{-2}\text{)}$$



$$S_{\gamma N} = 15 \text{ GeV}^2$$

$$\frac{d\sigma_{odd}}{dM_{\gamma\rho^2}} \text{ (pb.GeV}^{-2}\text{)}$$



$$S_{\gamma N} = 20 \text{ GeV}^2$$

$$\theta_{max} = 35^\circ, 30^\circ, 25^\circ, 20^\circ, 15^\circ, 10^\circ$$

JLab Hall B detector equipped between  $5^\circ$  and  $35^\circ$

⇒ this is safe!



## Conclusion

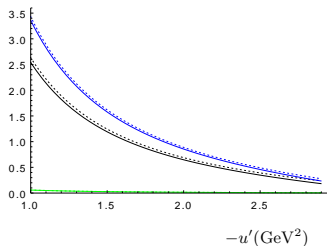
- High statistics for the chiral-even component: enough to extract  $H$  ( $\tilde{H}$ ?) and **test the universality of GPDs**
- In this chiral-even sector: analogy with **Timelike Compton Scattering**, the  $\gamma\rho$  pair playing the role of the  $\gamma^*$ .
- Strong dominance of the chiral-even component w.r.t. the chiral-odd one:
  - In principle the separation  $\rho_L/\rho_T$  can be performed by an angular analysis of its decay products, but this could be very challenging. Cuts in  $\theta_\gamma$  might help
  - Future: **study of polarization observables**  $\Rightarrow$  sensitive to the interference of these two amplitudes
- The **Bethe Heitler** component (outgoing  $\gamma$  emitted from the incoming lepton) is:
  - zero for the chiral-odd case
  - suppressed for the chiral-even case
- Our result can also be applied to **electroproduction** ( $Q^2 \neq 0$ ) after adding **Bethe-Heitler** contributions and interferences.
- Possible measurement at **JLAB** (Hall B, C, D)
- A similar study could be performed at **COMPASS**. **EIC**, **LHC** in UPC?

Backup

## Chiral-even cross section

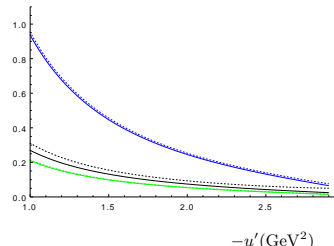
Contribution of  $u$  versus  $d$ 

$$\frac{d\sigma_{\text{even}}}{dM_{\gamma\rho}^2 d(-u')d(-t)} \quad (\text{nb} \cdot \text{GeV}^{-6})$$



proton

$$\frac{d\sigma_{\text{even}}}{dM_{\gamma\rho}^2 d(-u')d(-t)} \quad (\text{nb} \cdot \text{GeV}^{-6})$$



neutron

$M_{\gamma\rho}^2 = 4 \text{ GeV}^2$ . Both vector and axial GPDs are included.

$u + d$  quarks     $u$  quark     $d$  quark

Solid: "valence" model

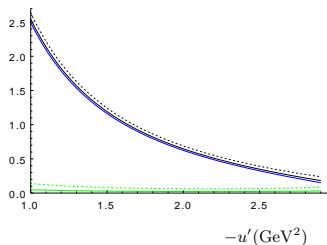
dotted: "standard" model

- $u$ -quark contribution dominates due to the charge effect
- the interference between  $u$  and  $d$  contributions is important and negative.

## Chiral-even cross section

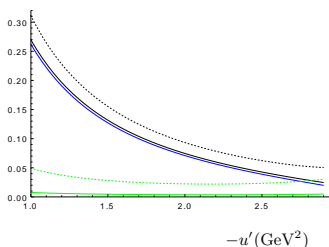
## Contribution of vector versus axial amplitudes

$$\frac{d\sigma_{\text{even}}}{dM_{\gamma\rho}^2 d(-u') d(-t)} \quad (\text{nb} \cdot \text{GeV}^{-6})$$



proton

$$\frac{d\sigma_{\text{even}}}{dM_{\gamma\rho}^2 d(-u') d(-t)} \quad (\text{nb} \cdot \text{GeV}^{-6})$$



neutron

$M_{\gamma\rho}^2 = 4 \text{ GeV}^2$ . Both  $u$  and  $d$  quark contributions are included.

vector + axial amplitudes / vector amplitude / axial amplitude

solid: "valence" model

dotted: "standard" model

- dominance of the vector GPD contributions
- no interference between the vector and axial amplitudes