

# ORDER PARAMETER FOR CONFINEMENT

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# PLAN OF THE TALK

- ▶ A FEW REMARKS ON COLOUR CONFINEMENT.
- ▶ ORDER PARAMETERS AND GAUGE INVARIANCE.
- ▶ ORDER PARAMETER FOR DUAL SUPERCONDUCTIVITY OF QCD VACUUM.

# REMARKS ON CONFINEMENT- 1

QUARKS STRICTLY CONFINED IN NATURE

▶  $\frac{n_q}{n_p} \leq 10^{-27}$                       EXPECT  $\approx 10^{-12}$

▶  $\frac{\sigma_q}{\sigma_{TOT}} \leq 10^{-15}$                       EXPECT  $O(1)$

NATURAL EXPLANATION : A SYMMETRY.

DECONFINING TRANSITION A CHANGE OF SYMMETRY.

## REMARKS ON CONFINEMENT-2

- ▶ GAUGE INVARIANCE UNAFFECTED BY DECONFINING TRANSITION.
- ▶ D.O.F. INVOLVED LIVE ON THE SURFACE AT SPATIAL INFINITY (TOPOLOGY).
- ▶ TOPOLOGICAL EXCITATIONS IN  $(3+1)d$  MONOPOLES [ $S_2 \rightarrow SU(2)$ ], IN  $(2+1)d$  VORTICES [ $S_1 \rightarrow U(1)$ ]
- ▶ CONFINEMENT BY DUAL SUPERCONDUCTIVITY OF THE VACUUM [ 'tHooft , Mandelstam 1975]

# ORDER PARAMETER AND GAUGE INVARIANCE-1

## ▶ HIGGS MODEL

$$L = -\frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + D_\mu \phi^\dagger D_\mu \phi - V(\phi^\dagger \phi)$$

$$D_\mu \phi \equiv (\partial_\mu + igA_\mu) \phi \quad V(\rho^2) = \rho^2(m^2 + \lambda\rho^2), \quad A_\mu \equiv T^a A_\mu^a$$

CHANGE VARIABLES :  $\phi, \phi^\dagger \longrightarrow \rho, \hat{\phi}$

$$\phi = \rho \hat{\phi} \quad \rho = \rho^\dagger \geq 0 \quad \hat{\phi}^\dagger \hat{\phi} = 1, \quad \phi^\dagger \phi = \rho^2$$

$$L = -\frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \partial_\mu \rho \partial_\mu \rho + g^2 \rho^2 \hat{\phi}^\dagger \tilde{A}_\mu \tilde{A}_\mu \hat{\phi} - V(\rho^2)$$

$$\tilde{A}_\mu \equiv A_\mu - \frac{i}{g} [\partial_\mu \hat{\phi}] \hat{\phi}^\dagger \quad \text{GAUGE COVARIANT}$$

## ▶ $SU(2) \times U(1)$ : BOSONIC SECTOR STANDARD MODEL.

## ▶ $U(1)$ : $\hat{\phi} = \exp(-ig\Theta)$

$$\tilde{A}_\mu = A_\mu - \partial_\mu \Theta \quad \text{GAUGE INVARIANT.}$$

## ▶ $SU(2)$ , $\Phi$ IN THE FUNDAMENTAL REPRESENTATION :

$$\hat{\phi}^\dagger \tilde{A}_\mu \tilde{A}_\mu \hat{\phi} = \tilde{A}_\mu^a \tilde{A}_\mu^a$$

# ORDER PARAMETER AND GAUGE INVARIANCE-2

## ▶ EQUATIONS OF MOTION

$U(1)$

$$\partial_\mu F_{\mu\nu} + 2g^2 \rho^2 \tilde{A}_\nu = 0$$

$$\partial_\mu [\rho^2 \tilde{A}_\mu] = 0$$

$$\square \rho - g^2 \rho \tilde{A}_\mu \tilde{A}_\mu + m^2 \rho \left(1 + \frac{\lambda \rho^2}{m^2}\right) = 0$$

$SU(2)$  ,  $\phi \in \text{FUND.REPR.}$

$$D_\mu G_{\mu\nu} + 2g^2 \rho^2 \tilde{A}_\nu = 0$$

$$D_\mu [\rho^2 \tilde{A}_{\mu\nu}] = 0$$

- ▶  $m^2 < 0$  STABLE VACUUM :  $\rho^2 = \bar{\rho}^2 = -\frac{m^2}{\lambda}$

VECTOR BOSONS BECOME MASSIVE SPIN 1 PARTICLES

$\mu^2 = 2g^2 \bar{\rho}^2$  : MEISSNER EFFECT AND CONSERVED

LONDON CURRENT  $J_\mu = \bar{\rho}^2 \tilde{A}_\mu$  IN  $U(1)$ .

- ▶ WHAT SYMMETRY IS BROKEN AND WHAT IS THE ORDER PARAMETER (INVARIANT UNDER LOCAL GAUGE TRANSFORMATIONS [Elitzur 1975] )?

# ORDER PARAMETER AND GAUGE INVARIANCE-3

- ▶ WEINBERG Phys.Rev.D7 1068 (1973)  
ORDER PARAMETER  $\langle \phi_u \rangle$ ,  $\phi_u \equiv \phi$  IN UNITARY GAUGE.  
 $\langle \phi^\dagger \phi \rangle = \langle \phi^\dagger \rangle \langle \phi \rangle$  ( CLASSICAL ZERO MODES).  
BROKEN SYMMETRY : GLOBAL GAUGE GROUP.  
Elitzur 1975 SATISFIED.  $U(1)$  :  $\langle \phi_u \rangle = \bar{\rho} = \sqrt{\frac{-m^2}{\lambda}}$
- ▶  $U(1)$  GAUGE INVARIANT CHARGED FIELDS [ Dirac 1955]

$$\Phi \equiv \exp\left(ig \int d^4y A_\rho(y) c_\rho(x-y)\right) \phi(x)$$
$$\partial_\rho c_\rho(x-y) = \delta^4(x-y) \quad \Phi^\dagger \Phi = \phi^\dagger \phi = \rho^2$$

$$\Phi_\Lambda \rightarrow \Phi \exp(-ig\Lambda + ig \int d^4y \partial_\rho \Lambda(y) c_\rho(x-y))$$

$$c_\rho^{(4)}(z) = \frac{1}{2\pi^2} \frac{z_\rho}{z^4};$$

$$c_0^{(3)}(z) = 0, \quad \vec{c}^{(3)}(z) = \delta(z_0) \frac{1}{4\pi} \frac{\vec{z}}{z^3};$$

$$c_0^{(2)}(z) = c_3^{(2)}(z) = 0, \quad \vec{c}^{(2)}(z) = \delta(z_0) \delta(z_3) \frac{1}{2\pi} \frac{\vec{\zeta}}{\zeta^2}, \quad \vec{\zeta} = (z_1, z_2);$$

$$\vec{c}^{(1)}(z) = 0, \quad c_0^{(1)}(z) = \delta^3(\vec{z}) \theta(z_0)$$

# ORDER PARAMETER AND GAUGE INVARIANCE-4

$$L = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \bar{D}_\mu\Phi^\dagger\bar{D}_\mu\Phi - V(\Phi^\dagger\Phi)$$

$$\bar{D}_\mu\Phi = \partial_\mu + ig(A_\mu - \partial_\mu \int d^4y A_\rho(y)c_\rho(x-y))$$

$$c_\rho = c_\rho^{(4)} : A_\mu - \partial_\mu \int d^4y A_\rho(y)c_\rho^{(4)}(x-y) = (\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\square})A_\nu$$

$\langle\phi\rangle$  IN THE LANDAU GAUGE IS A GAUGE INVARIANT ORDER PARAMETER.  $\langle\Phi\rangle = \bar{\rho}$  UP TO A PHASE.

$$c_\rho = c_\rho^{(3)} : A_m - \partial_m \int d^4y A_r(y)c_3^{(3)}(x-y) = (\delta_{mn} - \frac{\partial_m\partial_n}{\nabla^2})A_n$$

$\langle\phi\rangle$  IN THE COULOMB GAUGE IS A GAUGE INVARIANT ORDER PARAMETER.  $\langle\Phi\rangle = \bar{\rho}$  UP TO A PHASE.

FOR ANY GAUGE GROUP AND GAUGE  $\langle\phi\rangle$  ORDER PARAMETER UP TO A GLOBAL GAUGE TRANSFORMATION:

$$\int d^4y \partial_\mu^{(y)} [c_\mu^{(4)}(y-x)\phi(y)] = \phi(x) + \int d^4y c_\mu^{(4)}(y-x)\partial_\mu\phi(y)$$

BY TRANSLATION INVARIANCE  $\langle\phi(x)\rangle = \frac{1}{V} \int d^4x \phi(x)$  AND

$$\langle\phi(x)\rangle = \langle\phi(\infty)\rangle - \frac{1}{V} \int d^4z \frac{z_\mu}{z^4} \int d^4y \partial_\mu\phi(y) = \langle\phi(\infty)\rangle$$

GLOBAL SYMMETRY BREAKING.



–  $SU(2)$  ,  $\phi$  FUNDAMENTAL REPRESENTATION  $\rho$  FIXED [ E. Fradkin, S. Shenker Phys. Rev. D19 3682 (1979)]

SYSTEM IN BROKEN PHASE : NO PHASE TRANSITION EXPECTED.

ONLY UNPHYSICAL LINE OF TRANSITIONS OBSERVED ON LATTICE, DUE TO LATTICE ARTEFACTS [ Bonati et al. Nucl.Phys. B828 , 330 (2010)] .

CONSISTENT WITH THE STATEMENTS OF [Fradkin et al].

# ORDER PARAMETER FOR CONFINEMENT -1

– DEFINING A GAUGE INVARIANT ELECTRICALLY OR MAGNETICALLY CHARGED OPERATOR IN ABSENCE OF FUNDAMENTAL HIGGS. [ LATTICE  $U(1)$ ]

$$q, p \quad \exp(-ip\delta)|q\rangle = |q + \delta\rangle \quad \exp(iq\alpha)|p\rangle = |p + \alpha\rangle$$
$$q \leftrightarrow \vec{A}(\vec{x}) \quad p \leftrightarrow \vec{E}(\vec{x})$$

E- CHARGED FIELD  $\epsilon(\vec{x}) = \exp(ig \int d^3y \vec{A}(\vec{y}) \vec{c}^{(3)}(\vec{x} - \vec{y}))$

$$\epsilon(\vec{x})|\vec{E}(\vec{y})\rangle = |\vec{E}(\vec{y}) + g\vec{c}^{(3)}(\vec{x} - \vec{y})\rangle$$

$$\epsilon(\vec{x}) \rightarrow_{\Lambda} \epsilon(\vec{x}) \exp(ig\Lambda(\vec{x})) \quad \text{SINCE} \quad \vec{\nabla} \vec{c}^{(3)}(\vec{x} - \vec{y}) = \delta^3(\vec{x} - \vec{y})$$

$$\vec{c}^{(3)}(\vec{x} - \vec{y}) \rightarrow \vec{c}(\vec{x} - \vec{y}) = \vec{c}^{(3)}(\vec{x} - \vec{y}) - \vec{c}^{(1)}(\vec{x} - \vec{y})$$

$\vec{\nabla} \vec{c}(\vec{x} - \vec{y}) = 0$ , ONLY THE TRANSVERSE COMPONENT OF  $\vec{A}$  CONTRIBUTES, AND  $E(\vec{x}) = \exp(ig \int d^3y \vec{A}(\vec{y}) \vec{c}(\vec{x} - \vec{y}))$  IS THE GAUGE INVARIANT DIRAC VERSION OF  $\epsilon$  LIKE  $\Phi$  WAS OF  $\phi$ .

# ORDER PARAMETER FOR CONFINEMENT -2

M-CHARGED FIELD  $\mu(\vec{x}) = \exp(-i\frac{2\pi}{g} \int d^3y \vec{E}(\vec{y}) \vec{A}^{(mon)}(\vec{x} - \vec{y}))$

$$\mu(\vec{x})|\vec{A}(\vec{y})\rangle = |\vec{A}(\vec{y}) + \frac{2\pi}{g} \vec{A}^{(mon)}(\vec{x} - \vec{y})\rangle$$

DUAL VECTOR POTENTIAL  $\vec{Z}$  :  $\vec{E} = \vec{\nabla} \wedge \vec{Z}$

$$\mu(\vec{x}) = \exp(-i\frac{2\pi}{g} \int d^3y (\vec{\nabla} \wedge \vec{Z}(\vec{y})) \vec{A}^{(mon)}(\vec{x} - \vec{y})) =$$

$$\exp(i\frac{2\pi}{g} \int d^3y \vec{Z}(\vec{y}) (\vec{\nabla} \wedge \vec{A}^{(mon)})) = \exp(i\frac{2\pi}{g} \int d^3y \vec{Z} \vec{B}^{(mon)}(\vec{x} - \vec{y}))$$

DUAL GAUGE INVARIANT IF  $\vec{\nabla} \vec{B}^{(mon)} = 0$

—  $\vec{A}^{(mon)}(\vec{z})_m = \vec{A}_{(\pm)} = \epsilon_{3mj} \frac{z_j}{z(z \mp z_3)}$  COULOMBIC + STRING

ALONG 3-AXIS  $\vec{\nabla} \vec{A}_{(\pm)} = 0$  ,  $\vec{\nabla} \vec{B}_{(\pm)} = 0$

—  $\vec{A}^{(mon)}(\vec{z})_m = \vec{A}_{(+)} [z_3 < 0]$  ,  $\vec{A}^{(mon)}(\vec{z})_m = \vec{A}_{(-)} [z_3 > 0]$

WU YANG : COULOMBIC + 2d SINGULARITY AT  $z_3 = 0$ .

# ORDER PARAMETER FOR CONFINEMENT -3

TEST CASE : LATTICE  $U(1)$  GAUGE THEORY, WILSON ACTION. A FIRST ORDER PHASE TRANSITION at  $\beta \equiv \frac{2}{g^2} = \beta_c \approx 1$  FROM A CONFINED PHASE TO A DECONFINED PHASE.

$\langle \mu \rangle \neq 0$  AT  $\beta < \beta_c$  ,  $\langle \mu \rangle = 0$  AT  $\beta \geq \beta_c$ : CONFINING VACUUM IS A DUAL SUPERCONDUCTOR , NORMAL VACUUM IS NOT.

$$\langle \mu \rangle = \frac{Z(S+\Delta S)}{Z(S)} \quad \rho \equiv \partial_\beta \ln(\langle \mu \rangle) = \langle S \rangle_S - \langle (S + \Delta S) \rangle_{(S+\Delta S)}$$
$$\langle \mu \rangle = \exp\left(\int_0^\beta \rho(x) dx\right) \quad \text{SINCE} \quad \langle \mu \rangle(\beta = 0) = 1$$

FINITE SIZE SCALING  $\rightarrow$  CRITICAL INDEXES COMPATIBLE WITH FIRST ORDER .

# ORDER PARAMETER FOR CONFINEMENT -4

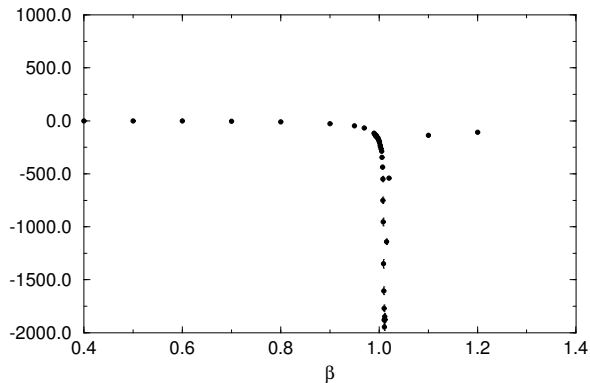


Figure:  $\rho$  versus  $\beta$  A.D.G. ,G.Paffuti P.R.D 56,6816 ,1997

# ORDER PARAMETER FOR CONFINEMENT -5

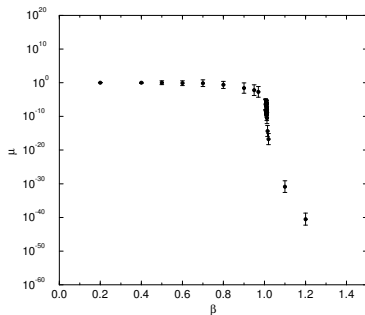


Figure:  $\langle \mu \rangle$  for U(1) lattice gauge theory .

- ▶ MONOPOLES DEFINED AS ABELIAN MONOPOLES IN  $U(1)$  SUBGROUPS OF THE GAUGE GROUP (ABELIAN PROJECTIONS). ABELIAN PROJECTIONS PHYSICALLY EQUIVALENT. [t'Hooft 1981].
- ▶ MONOPOLE CREATION OPERATOR IN A GIVEN ABELIAN PROJECTION (GAUGE CHOICE) [  $SU(2)$  ]:

$$\mu(\vec{x}, x_0) = \exp\left(-i\frac{2\pi}{g} \int d^3y \vec{E}_3(\vec{y}, x_0) T^3 \vec{A}^{(mon)}(\vec{x} - \vec{y})\right) \quad (1)$$

$$\frac{\partial \ln(\langle \mu \rangle)}{\partial \beta} \equiv \rho = \langle S \rangle_S - \langle (S + \Delta S) \rangle_{(S+\Delta S)} \quad (2)$$

- ▶  $\langle \mu \rangle$  INVARIANT UNDER LOCAL GAUGE AND DUAL GAUGE TRANSFORM'S **BY TRANSLATION INVARIANCE.**
- ▶ IF ABELIAN PROJECTIONS ARE EQUIVALENT EQ (1) WITH NO GAUGE FIXING IS THE ORDER PARAMETER.  
 $\langle \mu \rangle \neq 0 \rightarrow$  **CONFINEMENT**,  $\langle \mu \rangle = 0$  **DECONFINEMENT.**

- ▶ CHECK WITH PURE GAUGE  $SU(2)$  ,  $SU(3)$  WHERE CONFINED -DECONFINED ARE WELL DEFINED IN TERMS OF WILSON LOOPS. EXPECT A BEHAVIOR SIMILAR TO  $U(1)$ .
- ▶ SEEMED TO WORK AT THE BEGINNING WHEN VOLUMES WERE RATHER SMALL AND THE GROUPS  $SU(2)$ ,  $SU(3)$  .( PISA GROUP , BARI GROUP 1996-2004).
- ▶ PROBLEMS WITH  $G(2)$  GAUGE GROUP [ Cossu, D'Elia , Di Giacomo, Lucini (2007)] . EXPLORE  $SU(2)$  ,  $SU(3)$  AT LARGER VOLUMES.
- ▶ **DOES NOT WORK!** .  $\rho$  DIVERGES AT LARGE VOLUMES  
→  $\langle \mu \rangle = 0$  ALSO IN THE CONFINED PHASE.  
AN INFRARED PROBLEM.



— OPEN POSSIBILITIES:

1) A TECHNICAL PROBLEM IN THE DEFINITION OF  $\langle \mu \rangle$  .  
A CHARGED OPERATOR HAS INFRARED PROBLEMS :  
BETTER CREATE A MONOPOLE-ANTIMONOPOLE PAIR ,  
SEND THEIR DISTANCE LARGE AND USE CLUSTER  
PROPERTY TO DEFINE  $\langle \mu \rangle$ .

2) NOT ALL OF THE ABELIAN PROJECTIONS ARE  
EQUIVALENT. HISTORICALLY WE WOULD HAVE TRIED THE  
"MAXIMAL ABELIAN GAUGE". VERY DEMANDING  
COMPUTATIONALLY. FIND GOOD ARGUMENTS TO MAKE A  
CHOICE AND REPEAT THE TEST. GOOD CANDIDATE:  
 $\partial_0 A_0 = 0$  GAUGE (POLYAKOV GAUGE) : COMPUTATIONALLY  
AFFORDABLE. THEORETICALLY APPEALING.

UNDERSTANDING INFRARED PROBLEMS :

— COMPUTE  $\rho$  IN STRONG COUPLING EXPANSION [C. Bonati, G. Cossu, M. DElia, A. Di Giacomo Phys Rev D85 065001 (2011)]  $\langle \mu \rangle = \langle \mu | 0 \rangle$  IMPROVE TO  $\langle \mu \rangle_i$

$$\langle \mu \rangle_i = \frac{\langle \mu | 0 \rangle}{\sqrt{\langle 0 | 0 \rangle} \sqrt{\langle \mu | \mu \rangle}} \quad (3)$$

AT ORDER  $\beta^5$   $\rho \approx -\beta^5 V^{\frac{1}{3}}$  ,  $\langle \mu | 0 \rangle \approx \exp(-k\beta^5 V^{\frac{1}{3}})$ , BUT  $\sqrt{\langle \mu | \mu \rangle} \approx \exp(-k\beta^5 V^{\frac{1}{3}})$  AND  $\langle \mu \rangle_i > 0$ .

IN THE POLYAKOV ABELIAN PROJECTION  $k = 0$  . THE FIELD OF THE MONOPOLE COMMUTES WITH THE TIME LINKS.

**INFRARED DIVERGENCE CANCELS IN POLYAKOV ABELIAN PROJECTION.**

– STABLE MONOPOLES NEED HIGGS FIELD, NOT PRESENT IN QCD.  $A_4 = iA_0$  CAN PLAY THE ROLE OF HIGGS FIELD.

[B. Julia, A. Zee Phys.Rev. D11 2227 (1975)]

$$D_j \vec{G}_{ij} = g \vec{\Phi} \wedge D_i \vec{\Phi} - g \vec{A}_0 \wedge D_i \vec{A}_0$$

$$D_i D_i \vec{A}_0 = g \vec{\Phi} \wedge D_0 \vec{\Phi}$$

$$D_\mu D_\mu \vec{\Phi} + \lambda \vec{\Phi} (\Phi^2 + \frac{m^2}{\lambda}) = 0$$

IN THE GAUGE  $\vec{A}_0 = 0$ , [ 'tHooft, Polyakov (1974) ]

$$D_j \vec{G}_{ij} = g \vec{\Phi} \wedge D_i \vec{\Phi}$$

$$D_i D_i \vec{\Phi} + \lambda \vec{\Phi} (\Phi^2 + \frac{m^2}{\lambda}) = 0$$

IN ABSENCE OF HIGGS FIELD ( QCD )

$$D_j \vec{G}_{ij} = g \vec{A}_4 \wedge D_i \vec{A}_4$$

$$D_i D_i \vec{A}_4 = 0$$

SOLUTIONS : B.P.S. MONOPOLES OR DYONS [ D. Diakonov  
(2008)]

POLYAKOV LINE  $L(\vec{x}) = \exp(ig \int_0^{\frac{1}{T}} \vec{A}_4(\vec{x}, t) dt)$

IN THE GAUGE  $\partial_4 A_4 = 0$  :  $L(\vec{x}) = \exp(i \frac{g A_4(\vec{x})}{T})$

THE ABELIAN PROJECTION OF THE MONOPOLES IS THE  
ONE IN WHICH THE POLYAKOV LINE IS DIAGONAL.

# CONCLUSIONS AND OUTLOOK

- ▶ ORDER PARAMETERS WELL DEFINED IN GAUGE THEORIES: TRANSFORM UNDER GLOBAL GAUGE GROUP.
- ▶ LATTICE  $U(1)$  CONFINES ELECTRIC CHARGES BY DUAL SUPERCONDUCTIVITY OF VACUUM. ORDER PARAMETER THE  $v_{ev}$  OF A MAGNETICALLY CHARGED OPERATOR.
- ▶ EXTENDING THE CONSTRUCTION TO NON ABELIAN THEORIES ENCOUNTERS INFRARED PROBLEMS. IMPROVEMENTS OF THE MAGNETICALLY CHARGED OPERATOR  $\mu$  NEEDED.
  - DEFINE  $\mu$  BY CLUSTER PROPERTY
  - STUDY IN DETAIL THE IMPROVED VERSION OF  $\mu$
  - ASSUME THE POLYAKOV GAUGE AS ABELIAN PROJECTION.