
Why **local** gauge invariance is necessary

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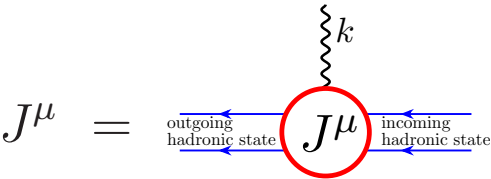


Outline

- Preliminaries
- Gauge invariance basics
- Implementation of **local** gauge invariance
- A practical example
- Summary



Setup: Currents



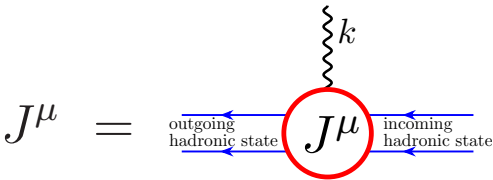
Photon may be real or virtual,
incoming or outgoing

All hadrons on-shell:

$$k_\mu J^\mu = 0 \quad (\text{necessary condition}) \quad \Rightarrow \quad \text{gauge invariance}$$



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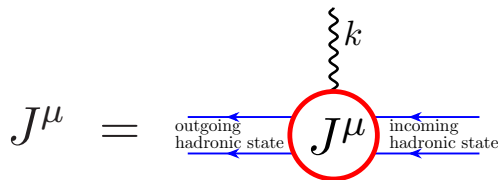
Transition currents:



B: Baryon
R: Resonance



Setup: Currents

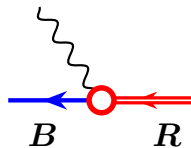
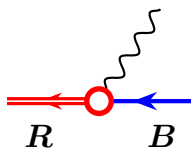


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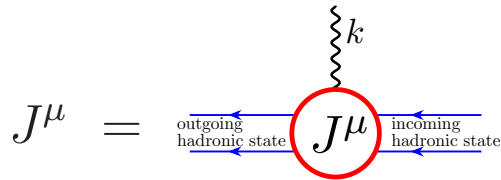


B: Baryon
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Current operators for transition currents must necessarily be **manifestly transverse**. Hence, **gauge-invariance condition is trivially satisfied for on- and off-shell baryons**.



Setup: Currents

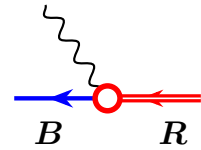
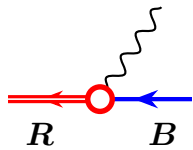


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Transition currents:



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Current operators for transition currents must necessarily be **manifestly transverse**. Hence, **gauge-invariance condition is trivially satisfied for on- and off-shell baryons**.

Why bother?



Production Current

← time →

$$\begin{aligned}
 M^\mu &= \underbrace{\text{[s-channel diagram]}}_{\text{s-channel}} + \underbrace{\text{[u-channel diagram]}}_{\text{u-channel}} + \underbrace{\text{[t-channel diagram]}}_{\text{t-channel}} + \underbrace{\text{[interaction current diagram]}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

Generic expressions; summations over all possible intermediate states implied



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 M^\mu &= \underbrace{\text{Diagram 1}}_{s\text{-channel}} + \underbrace{\text{Diagram 2}}_{u\text{-channel}} + \underbrace{\text{Diagram 3}}_{t\text{-channel}} + \underbrace{\text{Diagram 4}}_{\text{interaction current}} \\
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Generic expressions; summations over all possible intermediate states implied

- s -channel term contains transition-current contributions

Diagrams apply to spacelike process; for timelike process, read diagrams in time-reversed order

- Entire production current must be gauge invariant

Without it, wrong background contribution for extraction of form factors



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Diagrams apply to spacelike process; for timelike process, read diagrams in time-reversed order

- Entire production current must be gauge invariant

Without it, wrong background contribution for extraction of form factors

- Any approximation will likely destroy gauge invariance

- For a microscopic theory, it is **not** sufficient to fix $k_\mu M^\mu = 0$ on shell



Next...

- Gauge invariance basics
- Implementation of **local** gauge invariance
- A practical example
- Summary



Gauge Invariance

$$\begin{aligned}
 M^\mu &= \underbrace{\text{Diagram 1}}_{\text{s-channel}} + \underbrace{\text{Diagram 2}}_{\text{u-channel}} + \underbrace{\text{Diagram 3}}_{\text{t-channel}} + \underbrace{\text{Diagram 4}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

The diagrams show four Feynman-like diagrams for the transition matrix element M^μ . Each diagram has two incoming blue lines (representing hadrons) and one outgoing wavy line (representing a photon).
 - s-channel: A red dashed line connects the two incoming hadrons, with a blue dot at the vertex. A red arrow points from the vertex to the photon line.
 - u-channel: A red dashed line connects the incoming hadrons, with a blue dot at the vertex. A red arrow points from the vertex to the photon line.
 - t-channel: A red dashed line connects the incoming hadrons, with a red dot at the vertex. A red arrow points from the vertex to the photon line.
 - interaction current: A green circle labeled 'C' is connected to the photon line. A red arrow points from the vertex to the photon line.

Global gauge invariance

$$k_\mu M^\mu = 0$$

all external hadrons on-shell

$$\Phi \rightarrow \Phi e^{-i\Lambda}$$



Gauge Invariance

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 M^\mu &= \underbrace{\text{[s-channel diagram]}}_{\text{s-channel}} + \underbrace{\text{[u-channel diagram]}}_{\text{u-channel}} + \underbrace{\text{[t-channel diagram]}}_{\text{t-channel}} + \underbrace{\text{[interaction current diagram]}}_{\text{interaction current}} \\
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Fixing global gauge invariance does **not** mean internal damage is fixed as well



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The diagrams show four Feynman-like diagrams for the amplitude M^μ . Each diagram has an incoming blue arrow from the bottom left and an outgoing blue arrow from the bottom right. A wavy line representing a gauge boson is attached to the top of the diagram.

- s-channel:** A red dashed line connects the two external lines through a vertex labeled 's' (pink circle). A blue dot is on the internal line.
- u-channel:** A red dashed line connects the two external lines through a vertex labeled 'u' (pink circle). A blue dot is on the internal line.
- t-channel:** A red dashed line connects the two external lines through a vertex labeled 't' (pink circle). A red dot is on the internal line.
- interaction current:** A green circle labeled 'C' is connected to the two external lines. A red dot is on the internal line.

Local gauge invariance

$$\Phi \rightarrow \Phi e^{-i\lambda(x)}$$



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$$\begin{aligned}
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off-shell relations



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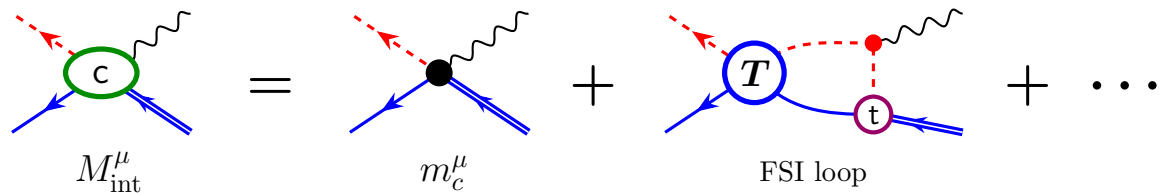
off-shell relations

local gauge invariance \Rightarrow implies existence of e.m. field

Without gWTI underlying e.m. field is damaged



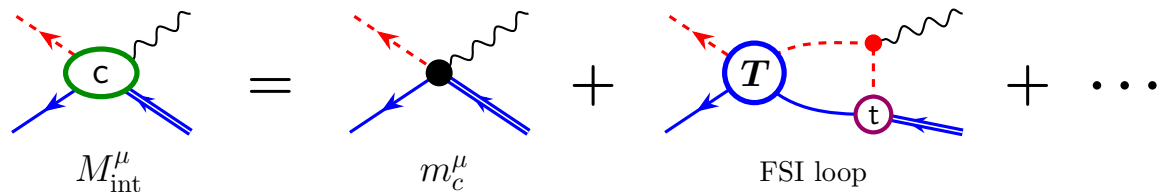
Implementation: Reorganize interaction current



■ Interaction current contains **all** contributions from final-state interactions



Implementation: Reorganize interaction current



■ Interaction current contains **all** contributions from final-state interactions

■ Decompose FSI loops into longitudinal and transverse contributions

For details, see HH, Nakayama, Krewald, PRC74, 045202 (2006); HH, Huang, Nakayama, PRC83, 065502 (2011); HH, Wang, He, PRC92, 055503 (2015)

■ Impose generalized Ward-Takahashi identities to longitudinal part



Generalized Ward-Takahashi Identities

$$(1) \quad k_\mu M^\mu = (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$$

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Only two relations are independent \Rightarrow Use (2) & (3)

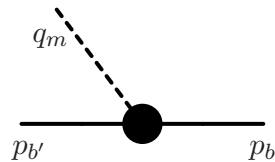


Generalized Ward-Takahashi Identities

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Hadronic vertex



$$F(p_{b'}, p_b) = \mathbf{G}(q_m) \boldsymbol{\tau} f(q_m^2, p_{b'}^2, p_b^2)$$

$$\begin{cases} f_s(s) = f(M_m^2, M_{b'}^2, s) \\ f_u(u) = f(M_m^2, u, M_b^2) \\ f_t(t) = f(t, M_{b'}^2, M_b^2) \end{cases}$$

This phenomenologically motivated ansatz can be improved in a systematic way

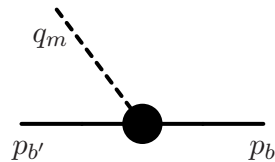


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Interaction-current Ansatz:

$$M_{\text{int}}^\mu = m_c^\mu f_t(t) + \mathbf{G}(q) \mathbf{C}^\mu + T_{\text{int}}^\mu$$

$$k_\mu T_{\text{int}}^\mu \equiv 0$$

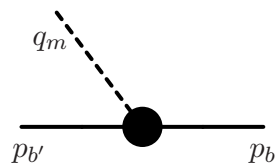


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\Rightarrow Determine C^μ such that (3) is true



Non-singular

$$\begin{aligned}
 C^\mu = & -e_m(2q - k)^\mu \frac{f_t - 1}{t - M_m^2} (\delta_s f_s + \delta_u f_u - \delta_s \delta_u f_s f_u) \\
 & - e_f(2p' - k)^\mu \frac{f_u - 1}{u - M_f^2} (\delta_t f_t + \delta_s f_s - \delta_t \delta_s f_t f_s) \\
 & - e_i(2p + k)^\mu \frac{f_s - 1}{s - M_i^2} (\delta_u f_u + \delta_t f_t - \delta_u \delta_t f_u f_t)
 \end{aligned}$$



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 \end{aligned}$$

where

$$\delta_x = \begin{cases} 1 & \text{channel contributes} \\ 0 & \text{channel does not contribute} \end{cases} \quad x = s, u, t$$



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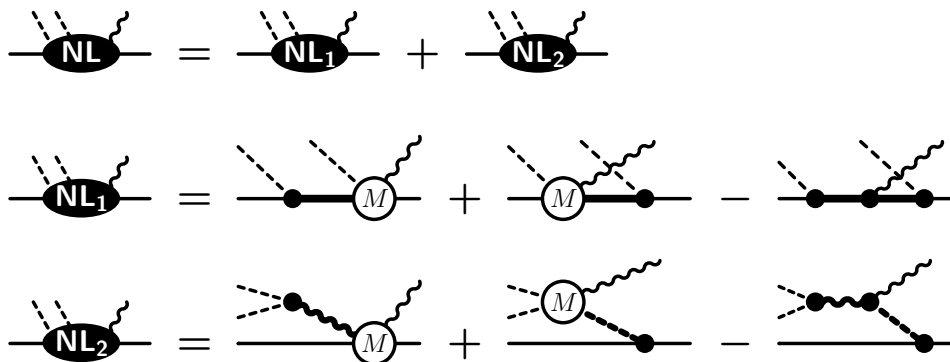
Charge conservation: $Q_m \tau + Q_f \tau - \tau Q_i = e_m + e_f - e_i = 0$

Four-divergence: $k_\mu C^\mu = e_m f_t + e_f f_u - e_i f_s$

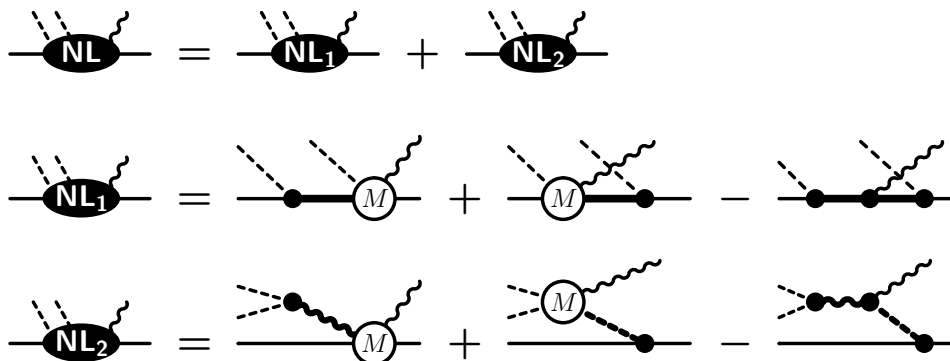
ensures correct four-divergence for M_{int}^μ



Example: Two-pion production at the no-loop level



Example: Two-pion production at the no-loop level:



Without gWTI, this amplitude will not be gauge invariant



Summary

- Global gauge invariance is necessary but not sufficient to provide consistent implementation of the electromagnetic field throughout the interaction volume
- Correct dynamical basis provided by **generalized Ward-Takahashi identities** as they follow from **local** gauge invariance
- Results obtained within microscopic reaction models that do not obey local gauge invariance need to be viewed with caution
- Applications show that contributions from gauge-fixing contact current can be substantial



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Thank you!

