On a theory of γd scattering: Towards model independence

(jointly with M.I. Levchuk)

I. NN-potential-based model of γd scattering:
   - overview of its components (2000)
   - recent improvements/corrections (2008)
   - merits, content comparison with EFT
   - results vs data
   - problems (wishes)

II. Short-range physics → effective theory via minimal substitution, some examples

III. Short-range uncertainties (estimates)

IV. Prospects

A.I. L'vov (LPI, Moscow) **@ Workshop on Compton scattering off Protons and Light Nuclei. ECT*, Trento, Italy. July 31, 2013**

Potential-based model of elastic γd scattering (M. Levchuk, A. L. (2000) and updates)

Nonrelativistic description

$$H = kin + H_{N,\,em} + V_{NN} + H_{NN,em} + \Delta_{NN,em} + \text{retardation}$$

1) $H_{N,\,em}$ = charge, magnetic moment, leading relativistic corrections (SO, polarizability-like effects)

polarizabilities $\alpha_{E1}, \beta_{M1}, \alpha_{E2}, \beta_{M2}, \alpha_{E\nu}, \beta_{M\nu}, \gamma_{E1}, \gamma_{M1}, \gamma_{E2}, \gamma_{M2}$

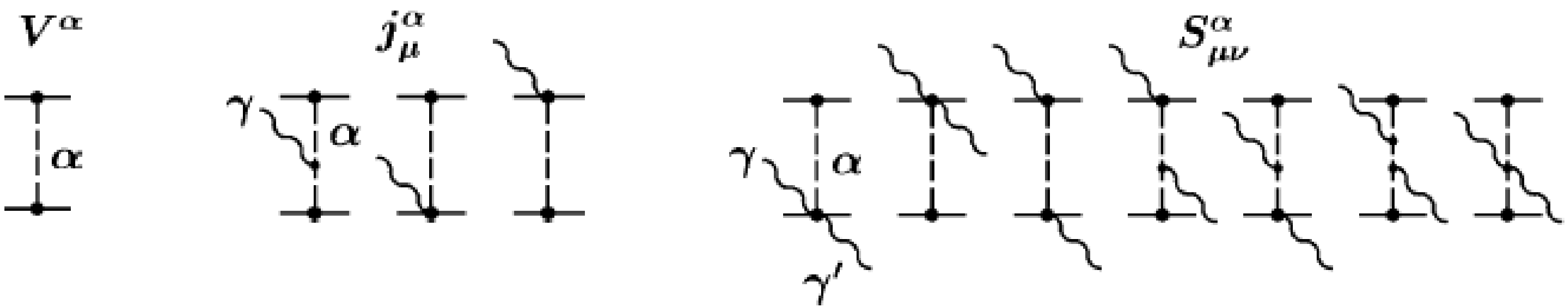
(with α_{E1}, β_{M1} free; others = DR based)

(very accurate description of the γN scattering amplitude)

2) V_{NN} = nonrelativistic Bonn OBE potential (OBEPR)

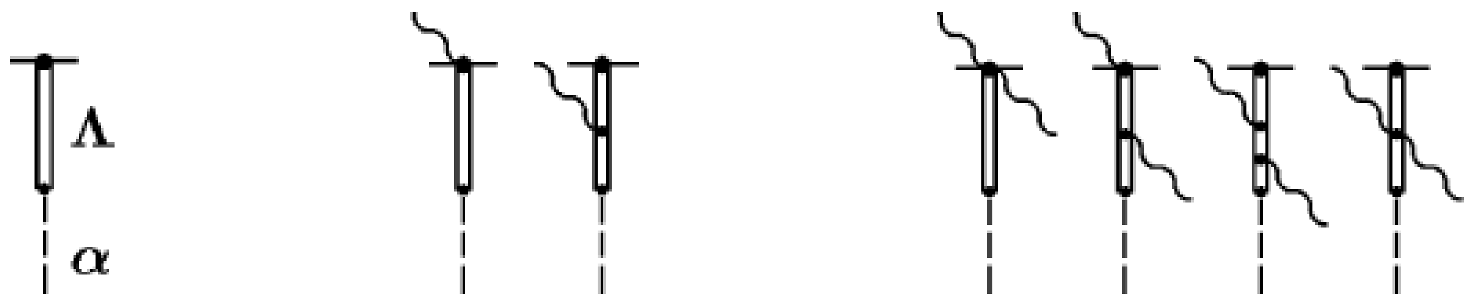
(π, ρ, ω, σ, …; formfactors)

3) $H_{NN,\,em}$ associated with these meson exchanges (currents $O(e)$ and seagulls $O(e^2)$)



3a) formfactors

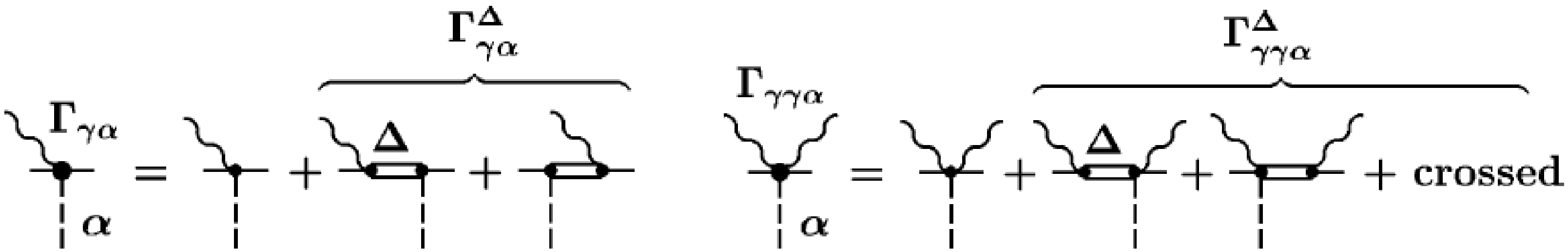
(treated as if the effective particles invented to generate the formfactors are real heavy mesons coupled to photons)



Consistency of V_{NN} and $H_{NN,em}$ ensures gauge invariance

Non-potential corrections:

Δ-excitation

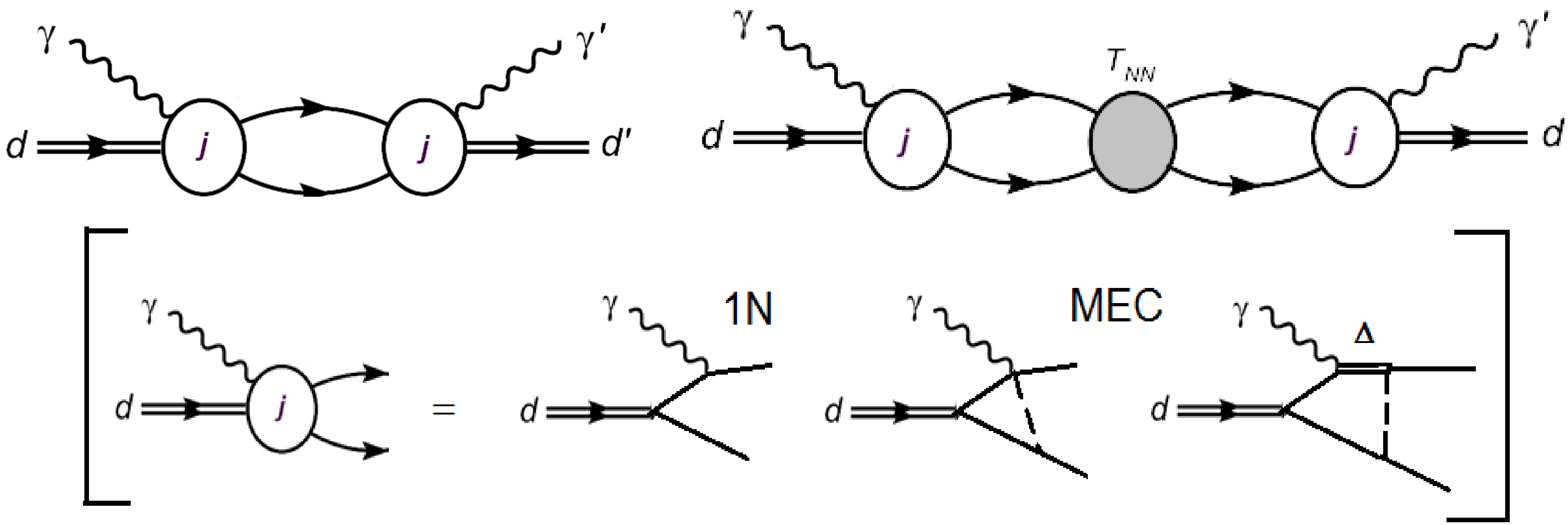


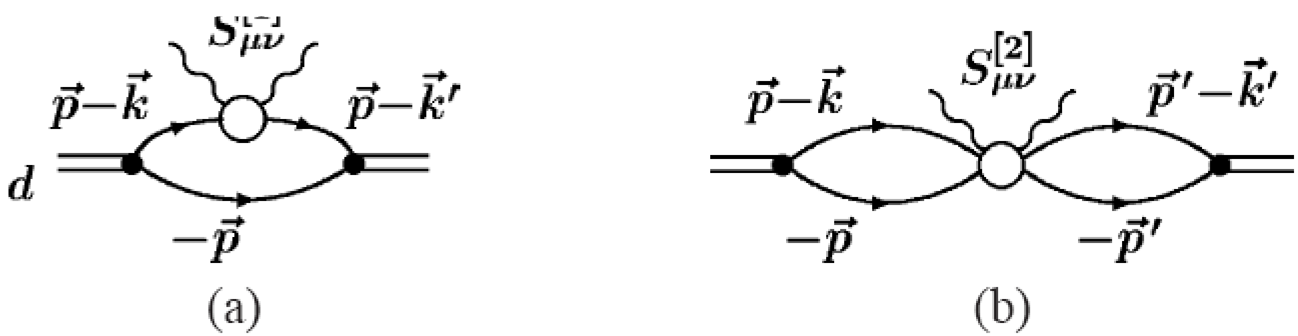
(a) (b)

Retardation in the pion propagator (in the seagull)

$$\frac{1}{\mathbf{K}^2 + m_\pi^2} \rightarrow \frac{1}{\mathbf{K}^2 + m_\pi^2 - \omega^2} \simeq \frac{1}{\mathbf{K}^2 + m_\pi^2} + \frac{\omega^2}{(\mathbf{K}^2 + m_\pi^2)^2}$$

Structure of the amplitude (includes up to 4 loops!)





Correspondence with EFT: We have all contributions included into EFT calculations (perhaps, not the same numerically). Plus full consistency of the NN-potential, wave function, NN-rescattering off-shell T-matrix, MEC, mesonic seagulls.

Check point: low-energy limit of the scattering amplitude (in e^2/M)

+0.842 Res (1- & 2-body currents), no intermediate NN rescattering
-0.100 Res, NN rescattering term (2008 year)
-1.000 1-body seagull
-0.239 2-body seagull

-0.497 (instead of the correct -0.500) [as of 2008]

Computer power! In 2000 we were unable to calculate, without further numerical approximations, 4 loops
(in 2000 the sum was -0.47 owing to mismatch between Paris (PEST) & OBEPR; Paris (separable PEST) potential was used to evaluate NN rescattering in the previous version of the code)

Another improvement of 2008. Bug in the old code:

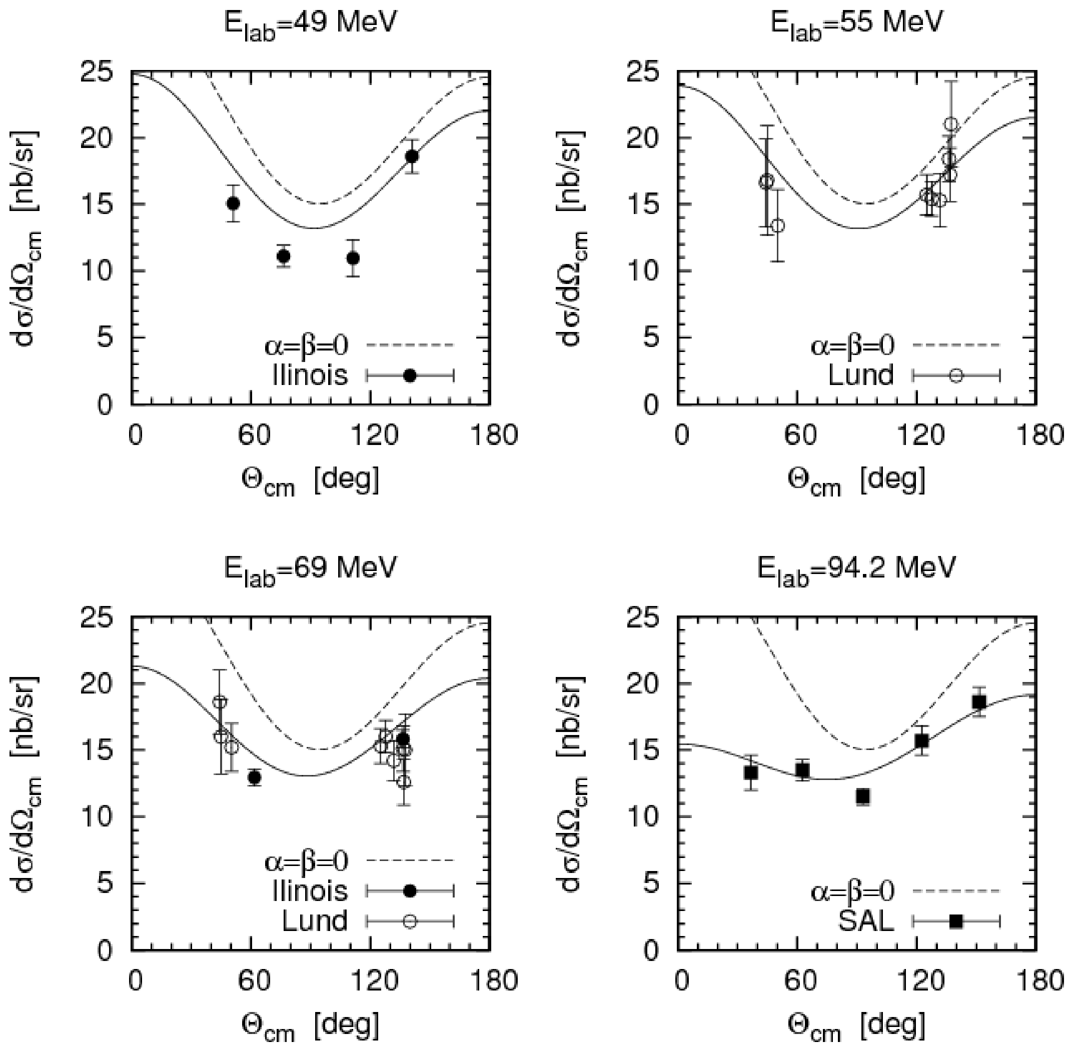
propagator of the Δ-isobar in the two-body current j was taken with wrong sign in the crossed term

(when internally $k \rightarrow -k'$, $k' \rightarrow -k$,
the old energy of the Δ was kept in another internal variable and erroneously was not changed properly)

This lead to a strong decrease of the differential cross section, especially at backward angles, and to dramatic (erroneous) decrease in the fitted $\alpha-\beta$.

After the correction made all available data are well described with “natural” $\alpha-\beta$ expected from DR.

L-L 2008:



Fit of the isoscalar dipole polarizabilities:

$\alpha+\beta = 14.6$ (Baldin)

OBEPR (A)

$\alpha-\beta = 9.1 \pm 1.6$

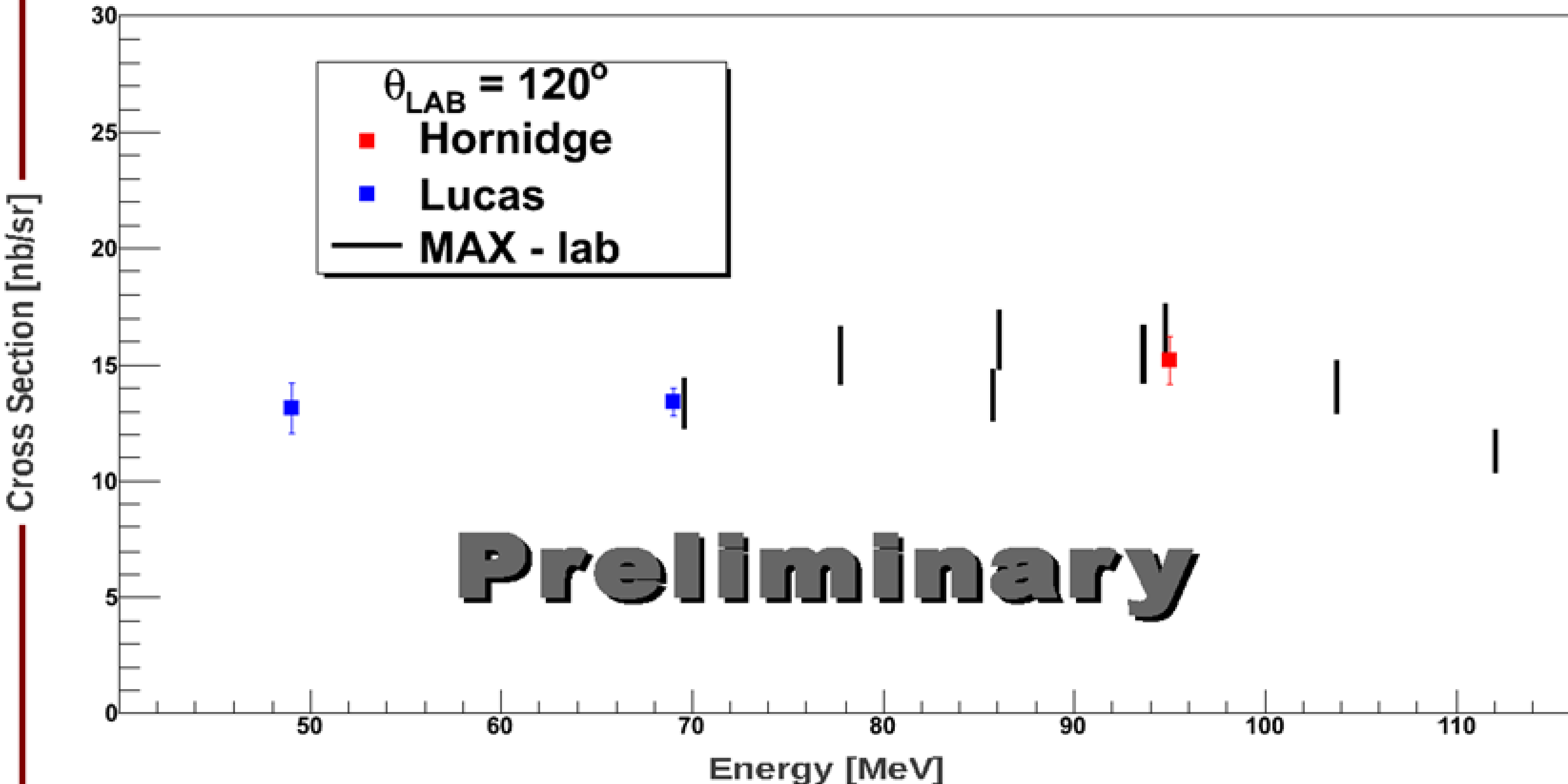
$\chi^2 = 31 / (29 - 1)$

$[\ \alpha+\beta = 15.2 \pm 1.5\ ]$

$[\ \alpha-\beta = 9.3 \pm 1.7\ ]$

In this framework γd-scattering in the whole range of energies (from 0 to about 100 MeV) and angles is successfully described. Moreover…

Preliminary Results from d(γ,γ)



Luke Myers, 30.07.2013. This workshop

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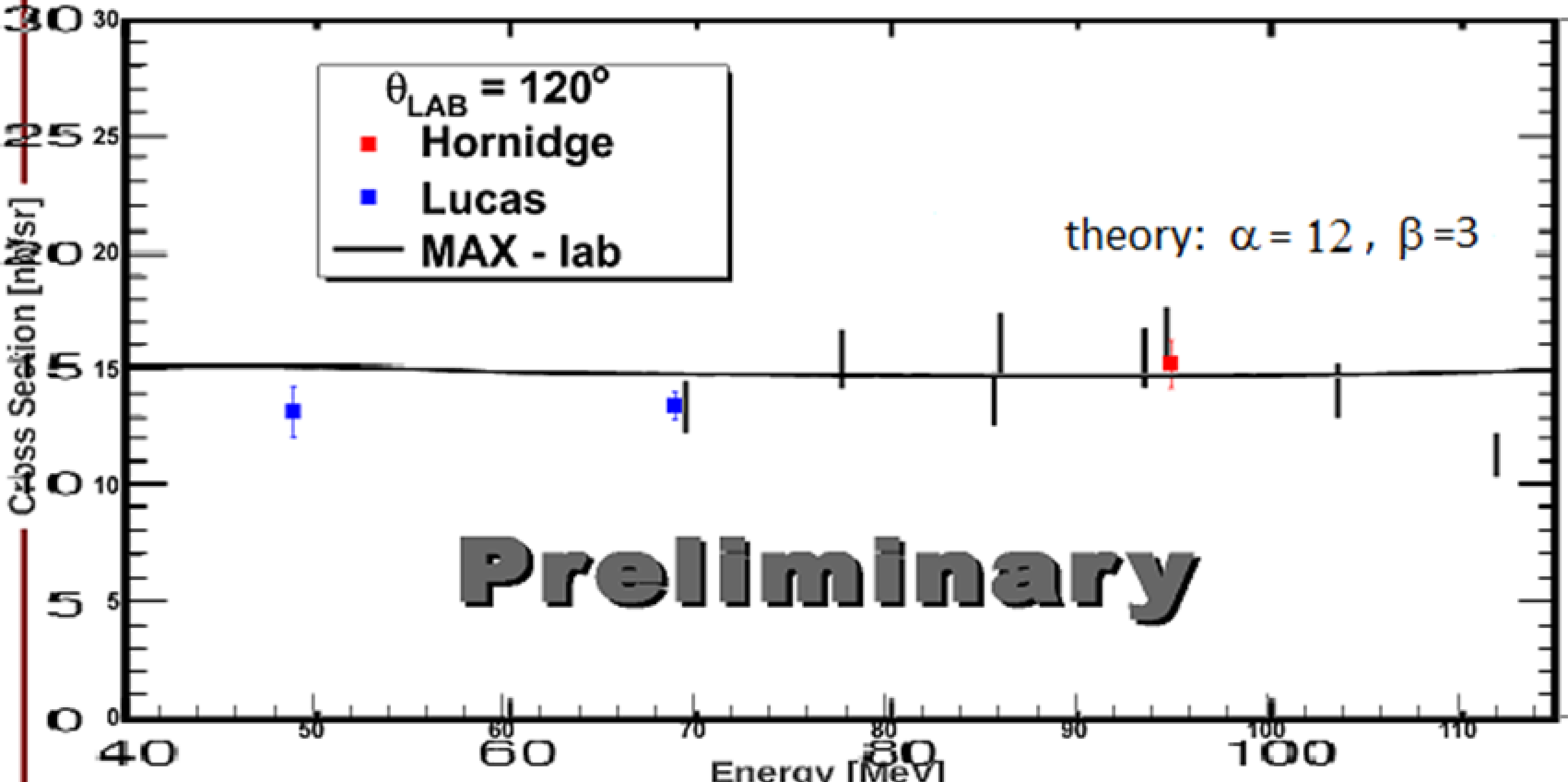

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What else?

Model dependence:
To which extent results are sensitive to the NN potential used?

Problem:
How to build E.M. currents consistent with complicated potentials (non necessarily OBE)?

In future we plan to use the minimal substitution.
Below is some theoretical background.

Minimal substitution for highly nonlinear (nonlocal) interactions

Prescription is to replace momenta of charged particles participating in the interaction (effective Lagrangian) with $p_\mu \rightarrow p_\mu - e A_\mu$

When the effective Lagrangian has p^2, we replace it with $(p - eA)^2$ and extract e.m. coupling as

$$(-eA)p + p(-eA)$$

Or, in the momentum representation,

$$(-e\varepsilon)\, p + (p+k)\, (-e\varepsilon),$$

where p and k are incoming momenta of the particle and the photon.

If we have, in the effective Lagrangian, a polynomial momentum dependence like

$$L = C_{\mu\nu\ldots\alpha\beta} \quad p_\mu \, p_\nu \, p_\lambda \ldots p_\alpha \, p_\beta$$

(with p-independent coefficients C with n indices), the minimal e.m. coupling becomes

$$C_{\mu\nu\ldots\alpha\beta} \quad [$$
$$(-e\varepsilon)_\mu \, p_\nu \, p_\lambda \ \ldots p_\alpha \, p_\beta$$
$$+ \ (p+k)_\mu \, (-e\varepsilon)_\nu \, p_\lambda \ldots p_\alpha \, p_\beta$$
$$+ \ (p+k)_\mu \, (p+k)_\nu (-e\varepsilon)_\lambda \ \ldots p_\alpha \, p_\beta \ \ldots$$
$$+ \ (p+k)_\mu \, (p+k)_\nu \, (p+k)_\lambda \ \ldots (-e\varepsilon)_\alpha \, p_\beta$$
$$+ \ (p+k)_\mu \, (p+k)_\nu \, (p+k)_\lambda \ldots (p+k)_\alpha (-e\varepsilon)_\beta \ ]$$

This sum can be found in a closed form in the case of the totally symmetric (over indices) coefficients C.

For the totally symmetric C we can arbitrarily interchange vector indices and even omit them (because they can be inserted later in any order). Then the sum in [...] becomes

$$(-e\epsilon)[p^{n-1} + (p+k)p^{n-2} + \ldots + (p+k)^{n-2}p + (p+k)^{n-1}]$$
$$= (-e\epsilon)[(p+k)^n - p^n]/k = (-e\epsilon)\int_0^1 \frac{\partial}{\partial p}(p+xk)^n\, dx$$

Or, restoring indices,

$$j_\mu = e\int_0^1 \frac{\partial}{\partial p^\mu} L(p+xk)\, dx$$

We could expect that the e.m. coupling is related with a derivative of the interaction. Exact answer differs from that guess only by a smearing of the momentum along *k*.

If the interaction depends on several momenta of charged particles, each particle contributes and we get (L., 1986)

$$j_\mu = \sum_a e_a \int_0^1 \frac{\partial}{\partial p_a^\mu} L(p_a \to p_a + xk)\, dx$$

(symmetrized minimal substitution).

The procedure can be repeated to obtain, through a similar next-order derivative plus smearing, a two-photon coupling to the effective interaction L.

The obtained minimal e.m. currents exactly (analytically) satisfy the Ward –Takahashi identities for tree and loop diagrams and lead to exactly gauge invariant amplitudes.

The next funny example shows that in some cases the minimal substitution gives physically very sound results.

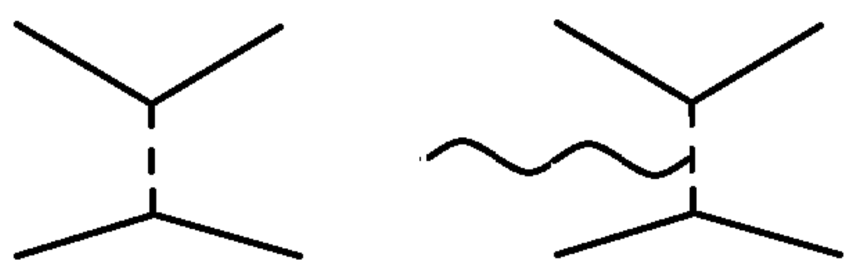
Let us consider an effective interaction caused by an exchange with a charged scalar particle emitted by a proton (its momentum = p_1), which recharges into the neutron (with the momentum = p_2).

$$L(p_1, p_2) = \frac{-g^2}{(p_1 - p_2)^2 - m^2}$$

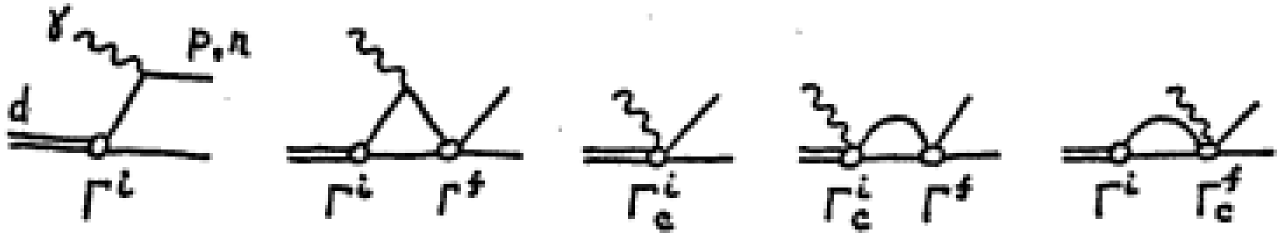
Then the minimal interaction with a real photon ($k^2=0$, $k\varepsilon=0$) will be

$$
\begin{aligned}
j_\mu(p_1,p_2) &= e\int_0^1 \frac{2g^2(p_1-p_2+xk)_\mu}{[(p_1-p_2+xk)^2-m^2]^2}dx \\
&= e\int_0^1 \frac{2g^2(p_1-p_2)_\mu}{[(p_1-p_2)^2+2xk(p_1-p_2)-m^2]^2}dx \\
&= e\frac{-2g^2(p_1-p_2)_\mu}{2k(p_1-p_2)}\left(\frac{1}{(p_1-p_2)^2+2k(p_1-p_2)-m^2}-\frac{1}{(p_1-p_2)^2-m^2}\right) \\
&= \frac{2eg^2(p_1-p_2)_\mu}{[(p_1-p_2+k)^2-m^2][(p_1-p_2)^2-m^2]}
\end{aligned}
$$

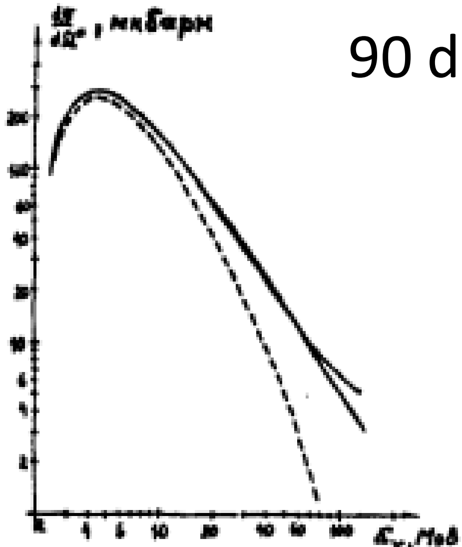
This is exactly the meson exchange current for in-fly photon-"pion" interaction containing 2 pion propagators and 1 e.m. pion vertex.



Application to $\gamma d \to pn$ (1987)



90 deg



0 deg

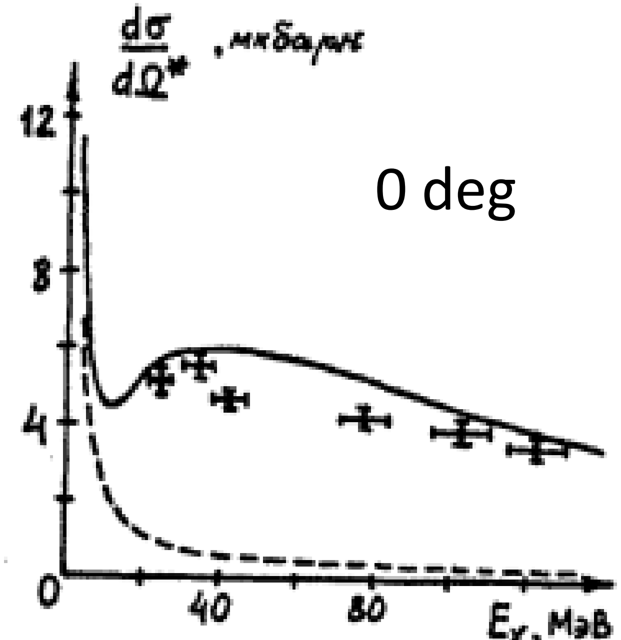


Рис.2. Энергетическая зависимость дифференциального сечения при $\Theta^* = 90°$ (Θ^*-угол в с.ц.и.). Обозначения кривых: ... - усредненные мировые данные [5]; —— - учет всех диаграмм из рис. I; -- — расчет без контактных диаграмм

Рис.3. То же, что на рис. 2 при $\Theta^* = 0$. Экспериментальные данные из [6]

Application to γd scattering (1995)

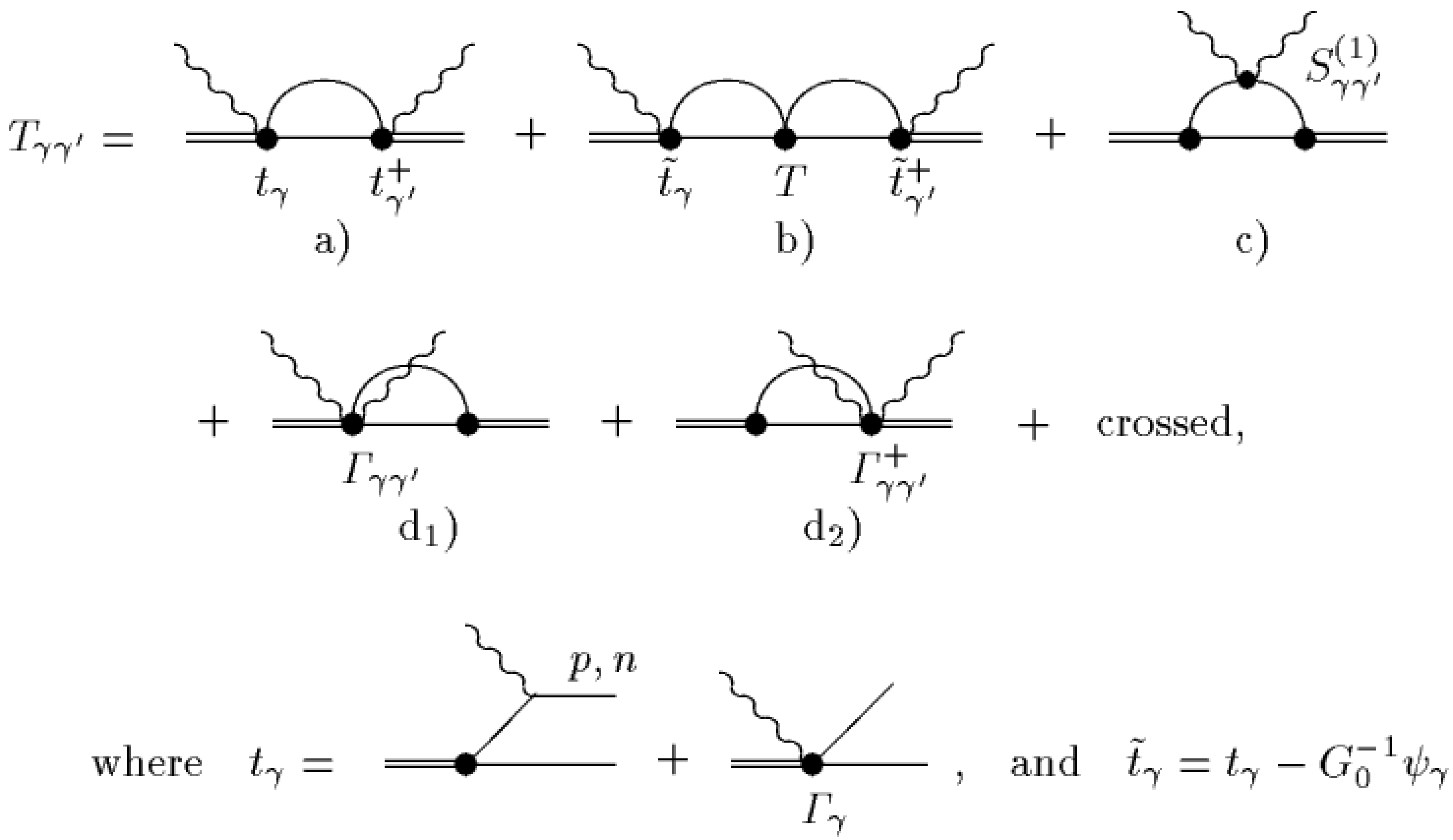


Figure 2. Diagrams of γd-scattering in the effective model of minimal substitution.

Application to γd scattering (1995)

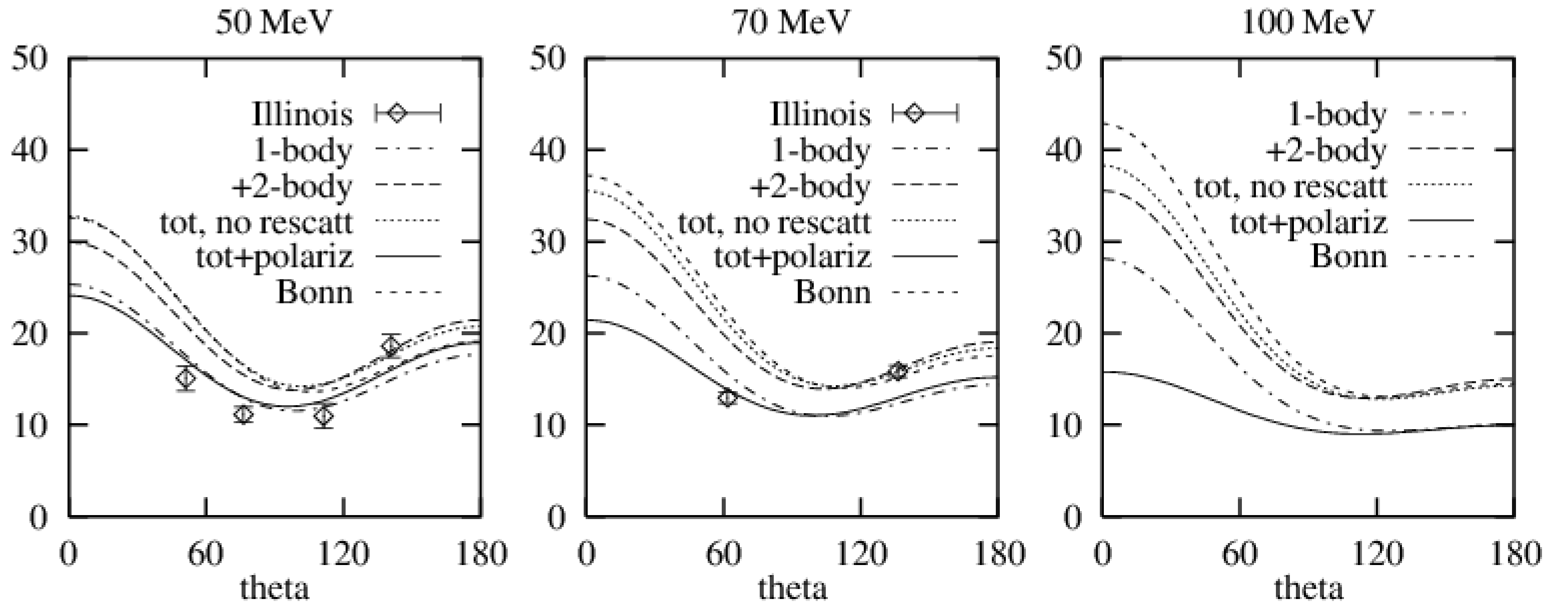


Figure 3. Differential cross section $d\sigma/d\Omega_{\rm cm}$ [nb/sr] in the minimal model (Fig. 1). (1) Dashed-dotted lines: contribution of the one-body current and seagull. (2) Dashed lines: total contribution, i.e. both one- and two-body operators are retained. (3) Dotted lines: total contribution without the rescattering term (Fig. 1b). (4) Solid lines: total contribution with adding the nucleon polarizability. In cases 1–3,5 the polarizability is not included. (5) Short-dashed lines: total contribution without the rescattering (Fig. 2b) in the "Bonn model". Experimental data are from [10].

Later it was widely used to check our results of 2000

Precaution:

In order to have reliable results, one should separately consider contribution of the pion exchange (long range!)

(Minimal substitution = expansion over kR)

So, actually calculations are not so simple.
They are in progress.

Conclusion

The developed potential-based model of γd scattering seems to work very successfully. It provides a sufficiently reliable tool for extracting polarizabilities of nucleons.

The question of a theoretical accuracy (model dependence) is not yet investigated well, however, and further work is desirable here.