A fit to the Compton-scatteering data on proton

Alexander Mushkarenkov

University of Massachusetts Amherst, USA

ECT* workshop, 2013

Problem statement

Purpose:

- Extract electrical, magnetic and spin polarizabilities for proton by a fit of theoretically calculated $d\sigma/d\Omega$ and Σ_{2x} for the Compton-scattering to experimental data.
- Investigate model dependency of the results.

Models:

- Lvov, DR, A. L'vov, V. Petrun'kin, M. Schumacher, Phys. Rev. C 55 (1997) 359–377
  Varying α, β, γ_π, SAID inputs
- Pasquini, DR, B. Pasquini, D. Drechsel, M. Vanderhaeghen, Phys. Rev. C 76 (2007) 015203
  Varying α, β, γ_{E1E1}, γ_{E1M2}, γ_{M1M1}, γ_{M1E2}
- Pascalutsa, ChPT, V. Lensky, V. Pascalutsa, Eur. Phys. J. C (2010) 65: 195–209
  Varying α, β, γ_{E1E1}, γ_{E1M2}, γ_{M1M1}, γ_{M1E2}

Experimental data:

- TAPS@MAMI, 65 points for $d\sigma/d\Omega$ at $E_\gamma = 60 - 160$ MeV, EPJ A 10, p.207 (2001)
- LARA@MAMI, 340 points for $d\sigma/d\Omega$ at $E_\gamma = 265 - 400$ MeV, EPJ A 12, p.231 (2001)
- CB@MAMI, 4 points for Σ_{2x} at $E_\gamma = 285$ MeV, Phil Martel's PhD thesis

Some of the previous works and model predictions

	α/α-β	β/α+β	$\gamma_{E1E1}/\gamma_{13}$	$\gamma_{M1M1}/\gamma_{14}$	γ_0	γ_π/γ_π^*
MPG	10.7±0.3	3.1∓0.3	-	-	-	-
LP	11.2±0.7	3.9±0.7	-3.3	3.0	-0.9	7.3
PDV	12.1	1.6	-4.3	2.9	-1.0	-38.7
TAPS (1)	11.9±0.5	1.2±0.7	-	-	-1.12	-37.1
TAPS (2)	12.2±0.8	0.8±0.9	-	-	-1.12	-35.9±2.3
LARA	10.0±1.5	-	-	-	-	-37.1±0.6
LEGS	10.4±1.8	13.3±0.9	3.9±0.5	-2.2±0.3	-1.55±0.15	-27.2±2.3
DAPHNE	-	-	-	-	-1.01±0.08	-

Systematical errors are not shown. Units are 10^{-4} fm^{-3} for α and β, 10^{-4} fm^{-4} for γ's.

- MPG: McGovern, Phillips, Grießhammer, EPJ A (2013) 49, p.12, χEFT fit to $d\sigma/d\Omega$
- LP: Lensky, Pascalutsa, EPJ C (2010) 65, p.195, ChPT prediction
- PDV: Pasquini, Drechsel, Vanderhaeghen, PR C 76 (2007) 015203, DR with SAID-SP02K as input
- TAPS: EPJ A 10, p.207 (2001), fit with Lvov's model with SAID-SM99K as input to measured $d\sigma/d\Omega$ at E_γ=55–165 MeV. (1): γ_π is fixed, (2): γ_π is free.
- LARA: EPJ A 12, p.231 (2001), fit with Lvov's model with SAID-SM99K and $(\alpha - \beta)$ from the TAPS work as an input to measured $d\sigma/d\Omega$ at E_γ=250–800 MeV.
- LEGS: PR C 64, (2001) 025203, $d\sigma/d\Omega$ and Σ_3 at $E_\gamma = 213-333$ MeV, combined analysis π-production and compton scattering data.
- DAPHNE: PRL 87, 022033 (2001), GDH experiment.

Goodness of fit to the LP and PDV models

$$\chi^2 = \frac{1}{N}\sum_{i=0}^{N}\left(\frac{y_i^{exp}(E_\gamma,\theta) - y^{th}(E_\gamma,\theta,\alpha,\beta,\{\gamma\})}{\sigma_i^{exp}(E_\gamma,\theta)}\right)^2$$

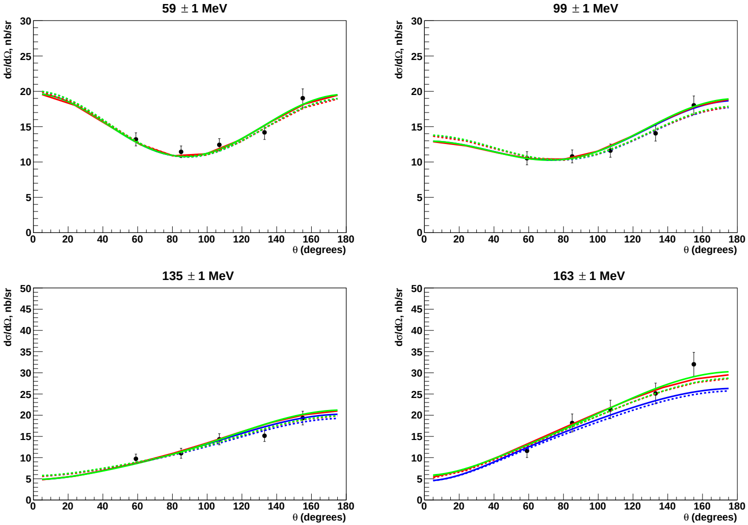
TAPS: $d\sigma/d\Omega$, 65 points, $E_\gamma = 55 - 165$ MeV

LARA: $d\sigma/d\Omega$, 340 points, $E_\gamma = 250 - 400$ MeV

CB: Σ_{2x}, 4 points, $E_\gamma = 285$ MeV

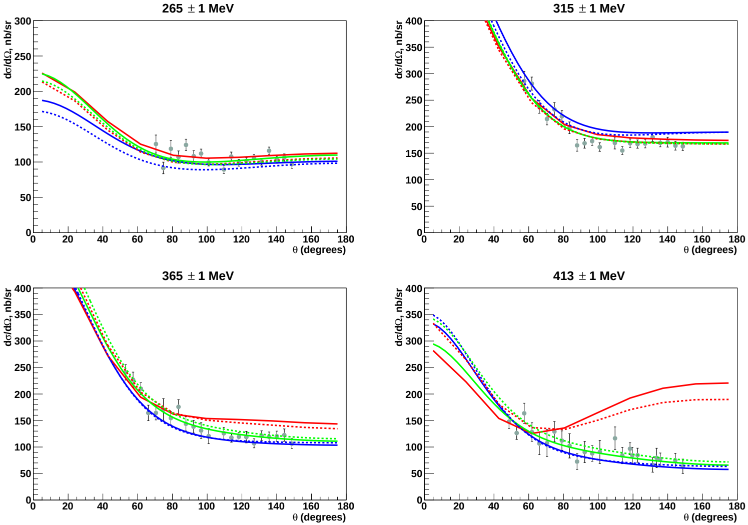
		TAPS	LARA	TAPS+LARA	TAPS+LARA+CB	CB
LP	Lvov	1.48	1.5	1.49	-	-
	Pasquini	1.48	10.8	9.26	9.2	0.64
	Pascalutsa	1.47	3.01	2.77	2.75	0.66
PDV	Lvov	1.25	1.80	1.71	-	-
	Pasquini	1.25	7.04	$\sim$6	$\sim$6	0.39
	Pascalutsa	1.42	2.61	2.42	2.40	0.72

Lvov, Pasquini, Pascalutsa and $\frac{d\sigma}{d\Omega}$ from TAPS



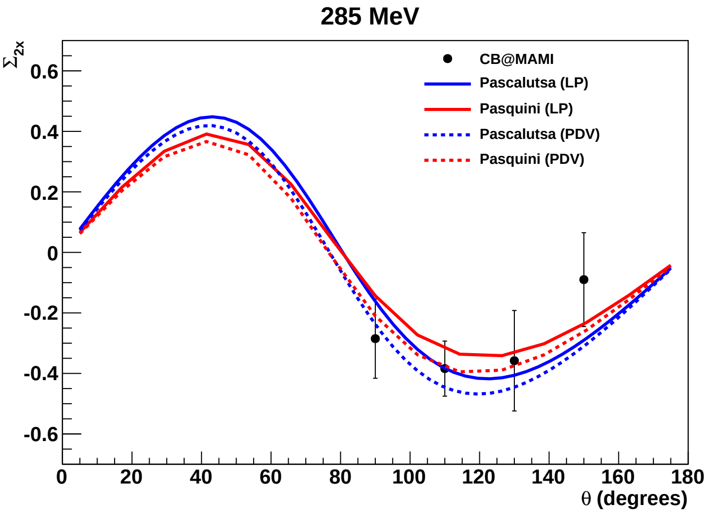
Solid and dashed curves correspond to the LP and PDV predictions for the polarizabilities

Lvov, Pasquini, Pascalutsa and $\frac{d\sigma}{d\Omega}$ from LARA



Solid and dashed curves correspond to the LP and PDV predictions for the polarizabilities

Pasquini, Pascalutsa and Σ_{2x} from CB



The fitting technique

$$\chi^2 = \sum_{exp} \left(\chi^2_{stat.} + \chi^2_{syst.} \right) + \chi^2_{constr.} \rightarrow minimize$$

For a given experiment:

$$\chi^2_{stat.} + \chi^2_{syst.} = \sum_i \left(\frac{f \cdot X_i^{exp} - X^{th}(\alpha, \beta, \gamma)}{f \cdot \sigma_i^{exp}} \right)^2 + \left(\frac{f-1}{f \cdot \delta} \right)^2$$

Optional constraints:

$$\chi^2_{constr.} = [(\alpha + \beta - 13.8)/0.4]^2 + [(\gamma_0 + 1.01)/0.18]^2 + \ldots$$

Linear approximation:

$$X^{th}(\alpha, \beta, \gamma) = X^{th}(\alpha', \beta', \gamma') + \frac{\partial X^{th}}{\partial \alpha}(\alpha - \alpha') + \frac{\partial X^{th}}{\partial \beta}(\beta - \beta') + \sum_i \frac{\partial X^{th}}{\partial \gamma_i}(\gamma - \gamma_i')$$

$$\frac{\partial X^{th}}{\partial \alpha} = const, \frac{\partial X^{th}}{\partial \beta} = const, \frac{\partial X^{th}}{\partial \gamma_i} = const$$

Algorithm:

(1) Choose $\alpha', \beta', \gamma' \rightarrow$ (2) calculate $X^{th}(\alpha', \beta', \gamma')$ and $\frac{\partial X^{th}}{\partial \alpha}$, $\frac{\partial X^{th}}{\partial \beta}$, $\frac{\partial X^{th}}{\partial \gamma_i}$ for each experimental point $\rightarrow$ (3) minimize χ^2 to find new $\alpha', \beta', \gamma' \rightarrow$ go to (2). Repeat (2) $\rightarrow$ (3) $\rightarrow$ (2) until

$$\Delta\alpha' < 0.01 \ \& \ \Delta\beta' < 0.01 \ \& \ \Delta\gamma' < 0.01$$

.

Fit to the TAPS data

	Conditions	α	β	γ_{E1E1}	γ_{M1M1}	γ_0/γ_{E1M2}	$\gamma_\pi/\gamma_\pi^*/\gamma_{M1E2}$	χ^2
TAPS	SM99K	11.9±0.5	1.2±0.7	-	-	-1.12	-37.1	1.28
Lvov	SM99K	11.9±0.5	1.1±0.7	-	-	-1.12	-37.1	1.28
Lvov	SN11	12.1±0.5	0.9±0.7	-	-	-1.14	-37.1	1.27
Lvov	SM99K, Baldin	12.2±0.4	1.5±0.4	-	-	-1.12	-37.1	1.27
Lvov	SN11, Baldin	12.4±0.4	1.3±0.4	-	-	-1.14	-37.1	1.26
TAPS	SM99K	12.2±0.8	0.8±0.9	-	-	-1.12	-35.9±2.3	1.30
Lvov	SM99K	12.1±0.8	0.9±1.0	-	-	-1.12	-36.3±2.3	1.30
Lvov	SN11	12.1±0.8	0.9±1.0	-	-	-1.14	-37.2±2.4	1.30
Lvov	SM99K, Baldin	12.4±0.7	1.3±0.8	-	-	-1.12	-36.4±2.3	1.29
Lvov	SN11, Baldin	12.3±0.7	1.4±0.8	-	-	-1.14	-37.3±2.4	1.28
Pasquini		11.6±0.5	1.2±0.7	-4.3	2.9	-0.7	9.3	1.28
Pasquini		11.5±0.5	0.7±1.0	-4.3	2.9	-0.7	10.4±1.6	1.29
Pasquini	Baldin	12.0±0.4	1.7±0.4	-4.3	2.9	-0.7	9.3	1.28
Pasquini	Baldin	12.0±0.4	1.7±0.5	-4.3	2.9	-0.7	9.3±1.3	1.30
Pasquini	Baldin	12.4±0.8	1.3±0.8	-3.9±4.0	3.5±1.7	-1.3±4.3	1.8±3.2	1.36
Pascalutsa		11.0±0.5	2.7±0.7	-3.3	3.0	0.2	1.4	1.47
Pascalutsa		11.1±0.5	1.9±0.7	-4.3	2.9	-0.7	9.3	1.40
Pascalutsa	Baldin	12.8±0.8	0.9±0.8	-5.4±3.8	4.3±1.9	-0.4±6.6	14.3±2.27	1.37

$$\gamma_0 = -\gamma_{E1E1} - \gamma_{M1M1} - \gamma_{E1M2} - \gamma_{M1E2}$$

$$\gamma_\pi = -\gamma_{E1E1} + \gamma_{M1M1} - \gamma_{E1M2} + \gamma_{M1E2}$$

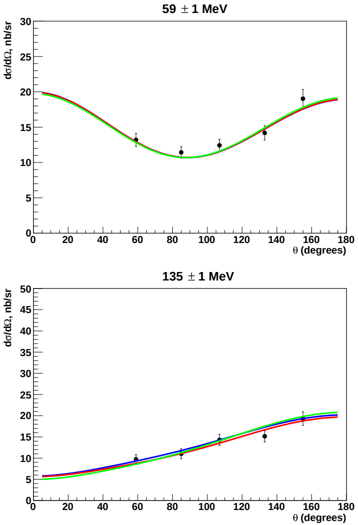
$$\gamma_\pi^* = \gamma_\pi + \gamma^{\pi\, pole}, \; \gamma^{\pi\, pole} = -46.77$$

Fit to $\frac{d\sigma}{d\Omega}$ from TAPS+LARA and Σ_{2x} from CB

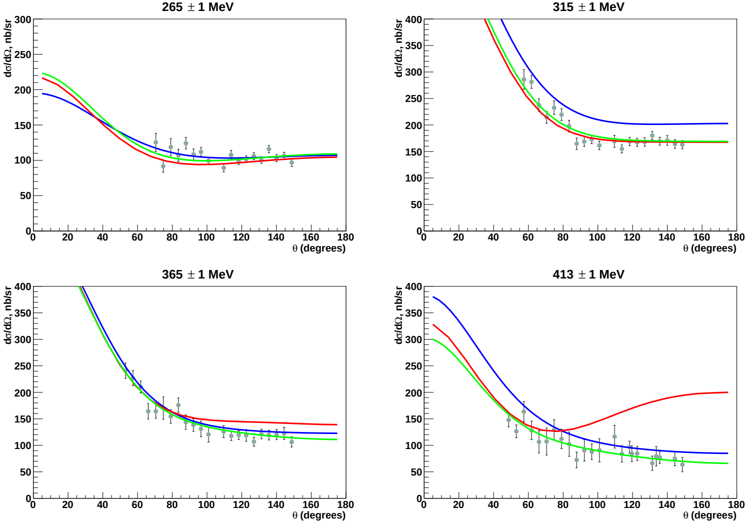
	Conditions	α	β	γ_{E1E1}	γ_{M1M1}	γ_0	$\gamma_\pi / \gamma_\pi^*$	χ^2
LP		11.2±0.7	3.9±0.7	-3.3	3.0	-0.9	7.3	
Pascalutsa	TAPS+LARA	12.6±0.4	1.4±0.4	-3.6±0.3	5.2±0.3	-2.2±0.7	13.4±0.5	1.95
Pascalutsa	TAPS+CB	13.6±1.5	0.0±1.3	-5.4±1.5	3.9±1.4	2.6±8.3	14.6±1.7	1.31
Pascalutsa	All	12.6±0.4	1.4±0.4	-3.6±0.3	5.2±0.3	-2.2±0.7	13.4±0.5	1.95
PDV		12.1	1.6	-4.3	2.9	-1.0	-38.7	
Pasquini	TAPS+CB	12	1.9	-4.5±1.0	2.5±0.6	-1.01	8	1.24
Pasquini	TAPS+CB, *	12.0±0.5	1.7±0.4	-4.5±1.0	2.6±0.7	-1.0±0.2	8.6±1.2	1.23
Lvov	TAPS+LARA	12.3±0.9	2.6±0.8	-	-	-1.14	-36.7±2.3	1.43

* constrained $\gamma_0 = -1.01 \pm 0.18$, $\gamma_\pi = 8.0 \pm 1.8$, $\alpha = 12.0 \pm 0.6$, $\beta = 1.9 \pm 0.5$

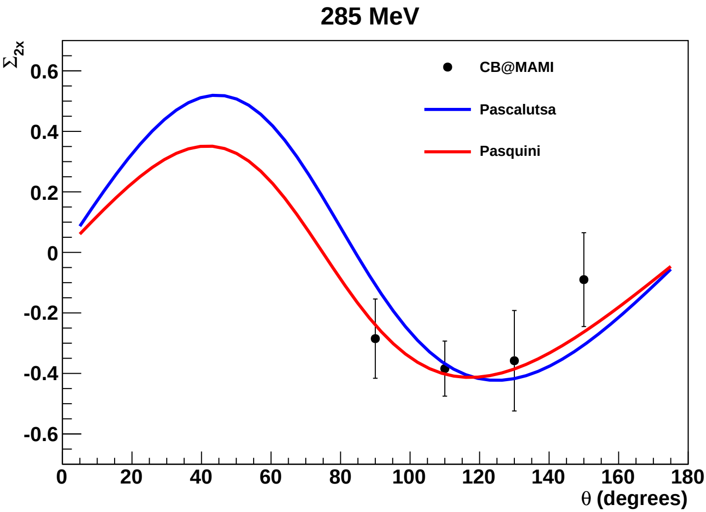
Best fit of Lvov, Pasquini, Pascalutsa and $\frac{d\sigma}{d\Omega}$ from TAPS



Best fit of Lvov, Pasquini, Pascalutsa and $\frac{d\sigma}{d\Omega}$ from LARA



Pasquini, Pascalutsa and Σ_{2x} from CB



Conclusions

- The fit can reproduce the TAPS result. (The global fit was not tested.)
- The result of the Lvov fit do not depend on SAID input.
- Lvov, Pasquini and Pascalutsa give the similar (same) α and β from the fit of the TAPS data.
- It is difficult to make a conclusion on effect of including Σ_{2x} to the fit.
- $\chi^2 = 1.3 - 2.0$
    - Are the published errors are too small?
    - The models cannot reproduce the data?
    - Am I doing something wrong?
    - Is it just a local χ^2 minimum?