Coulomb phase stability and quark confinement

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Analytic Flashes of QCD

- Perturbation theory: Asymptotic freedom
- Strong coupling: Confinement
- Intermediate regime: Quarkonia (NRQCD)

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Pictures Confinement

- Lattice simulations
- Dual Superconductor
- AdS/CFT dualities

Gribov's Quark Confinement

One Electron Atomic Spectrum

$$E_{n,j} = mc^2 \left[1 + \left(\frac{Z\alpha}{n - (j + \frac{1}{2}) + \sqrt{(j + \frac{1}{2})^2 - (Z\alpha)^2}} \right)^2 \right]^{-\frac{1}{2}}$$

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Vacuum instability for

Z > 137

The running coupling of α_s in the IR limit is large !!.

Gribov's Quark Confinement

Quark Green function Dyson-Schwinger equation

$$D^2 G^{-1}(k) = \frac{(N^2-1)\alpha_s}{2N\pi}\partial_\mu G^{-1}(k)\; G(k)\; \partial^\mu G^{-1}(k) + \mathcal{O}(\frac{\alpha^2}{2})$$

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Instability for large strong coupling α_s constant

$$\alpha_s > \alpha_{crit} = \frac{2N\pi}{(N^2-1)}\left(1 - \sqrt{\frac{2}{3}}\right)$$

$$\alpha_{crit} = 0.43 \qquad N = 3$$

Gribov's Quark Confinement

Heavy quark potential:

$$S_E^{YM}(A) = -\frac{1}{2g_s^2} \int d^4x \, Tr(F^{\mu\nu} F_{\mu\nu}) + Q \int dx^0 \, A_0^3$$

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Solution of motion equations (Coulomb)

$$(A_0)^3(\overrightarrow{x}) = i\frac{g_s^2 Q}{4\pi|\overrightarrow{x}|} = i\frac{\alpha}{|\overrightarrow{x}|}, \quad \alpha = \frac{g_s^2 Q}{4\pi},$$

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Instability of Euclidean functional integral

$$\delta^{(2)}S = -\int d^4x\, Tr\left(\tau^\mu(-\delta_{\mu\nu}D^2 + D_\mu D_\nu - 2[F_{\mu\nu}.\cdot])\,\tau^\nu\right),$$

Gribov's Unstability

Eigenvalue problem

$$(-\delta_{\mu\nu}D^2 + D_\mu D_\nu - 2[F_{\mu\nu}, \cdot])\tau^\nu = \lambda^2 \tau_\mu$$

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Unstable magnetic modes

$$\vec{\tau}(\vec{x}) = \frac{\vec{x} \times \vec{\mathrm{n}}}{|\vec{x}|}\,\phi(|\vec{x}|)\,\mathsf{T}_{12}, \qquad \tau_0 = 0 \qquad (l = 1),$$

$$\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{2-\alpha^2}{r^2}\right)\phi(r) = \lambda^2 \phi(r)$$

Singular Hamiltonians

One dimensional problem

$$H = -\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{2}\frac{g^2}{x^2}$$

in the half line $x \in (0, \infty)$

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Applications

- 2-body problems. Centrifugal potential
- 3-body problem (Efimov effect)
- Relativistic Coulomb problem

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$$\lim_{x=0} 2x\psi'(x) = \lim_{x=0} \left(1 + 2\sqrt{\tfrac{1}{4} + g^2}\,\tanh[\sqrt{\tfrac{1}{4} + g^2}\,\log(\Lambda\, x)] \right) \psi(x),$$

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$$\lim_{x=0} \left(1 + 2\sqrt{-g^2-\tfrac{1}{4}}\cot[\sqrt{-g^2-\tfrac{1}{4}}\log(\Lambda x)]\right)\psi(x) = \lim_{x=0} 2x\psi'(x)$$

with periodicity in Λ

$$\Lambda \Leftrightarrow \Lambda\, e^{2\pi/\sqrt{-g^2-1/4}}$$

Efimov effect

Singular Hamiltonians

Infinite sequence of negative energy levels

$$E_n = 2\Lambda^2 \exp\left(\frac{-2\pi n}{2\sqrt{-g^2-1/4}} + \frac{i}{2\sqrt{-g^2-1/4}} \log \frac{\Gamma(1-i\sqrt{-g^2-1/4})}{\Gamma(1+i\sqrt{-g^2-1/4})} \right)$$

$$\frac{E_{n+1}}{E_n} = \exp[-2\pi/(\sqrt{g-1/4})].$$

Partial breaking of conformal symmetry (Conformal Anomaly)

$$\begin{aligned} G &= \frac{1}{4}(xp + px) \\ D &= tH - G \\ K &= -t^2 + 2Dt + \frac{1}{2}x^2 \end{aligned}$$

Quark Coulomb Instability

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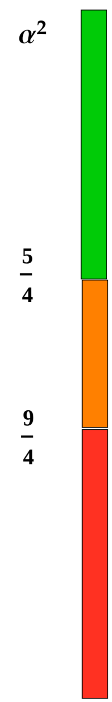
- Three different unstable regimes $j = 1$

  - i)$\alpha^2 < \frac{5}{4} \Rightarrow$ no negative eigenvalues
    (Stability of Coulomb potential)
  - ii) $\frac{5}{4} < \alpha^2 < \frac{9}{4} \Rightarrow$ one negative eigenvalue
    (Instability of Coulomb potential)
  - iii) $\alpha^2 > \frac{9}{4} \Rightarrow \infty$-negative eigenvalues
    (Instability of Coulomb potential)



Broken Conformal Symmetry Λ
M.A. & A. Santagata (2010)

Quark-Antiquark: Meson

Coulomb potential

$$(A_0)^3(\vec{x}) = \frac{i\alpha}{|\vec{x} - L\vec{e}_3|} - \frac{i\alpha}{|\vec{x} + L\vec{e}_3|},$$

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$$\left[\frac{\partial^2}{\partial \rho^2} + \frac{\partial^2}{\partial z^2} - \frac{3}{4\rho^2} + \left(\frac{\alpha}{\sqrt{\rho^2 + (z-L)^2}} - \frac{\alpha}{\sqrt{\rho^2 + (z+L)^2}}\right)^2\right]\phi(\rho, z)$$

$$= \lambda^2 \phi(\rho, z)$$

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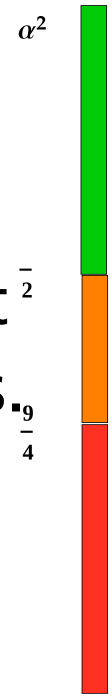
  - i) $\alpha^2 < 2 \Rightarrow$ no negative eigenvalues
    (Stability of Coulomb potential)
  - ii) $2 < \alpha^2 < \frac{9}{4} \Rightarrow$ two negative eigenvalues at large distances and none at short distances.
    (Instability of Coulomb potential)
  - iii) $\alpha^2 > \frac{9}{4} \Rightarrow \infty$-negative eigenvalues
    (Instability of Coulomb potential)



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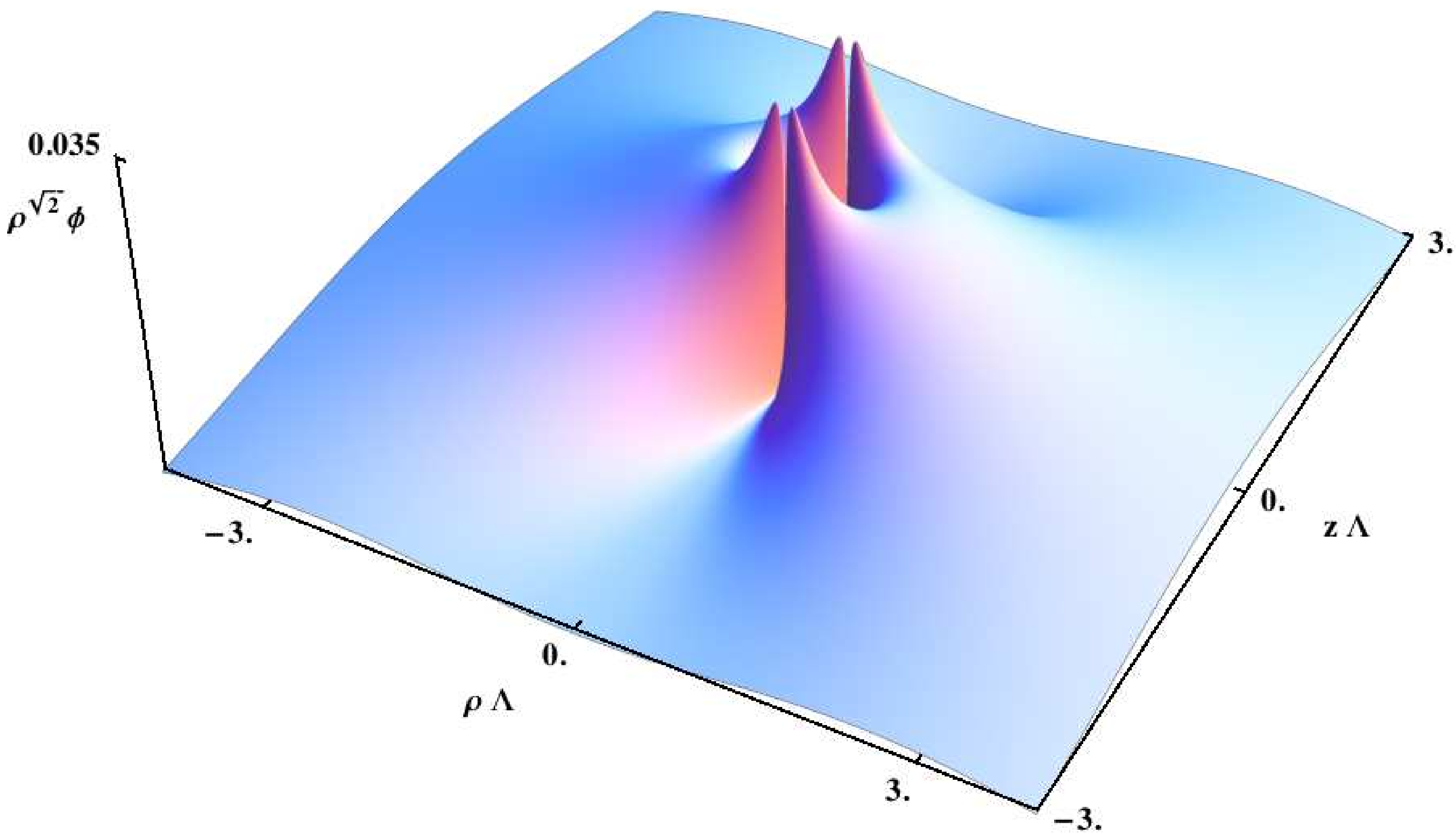
Unstable solutions

$$2 < \alpha^2 < \frac{9}{4}$$

Symmetric solution (Thick string)



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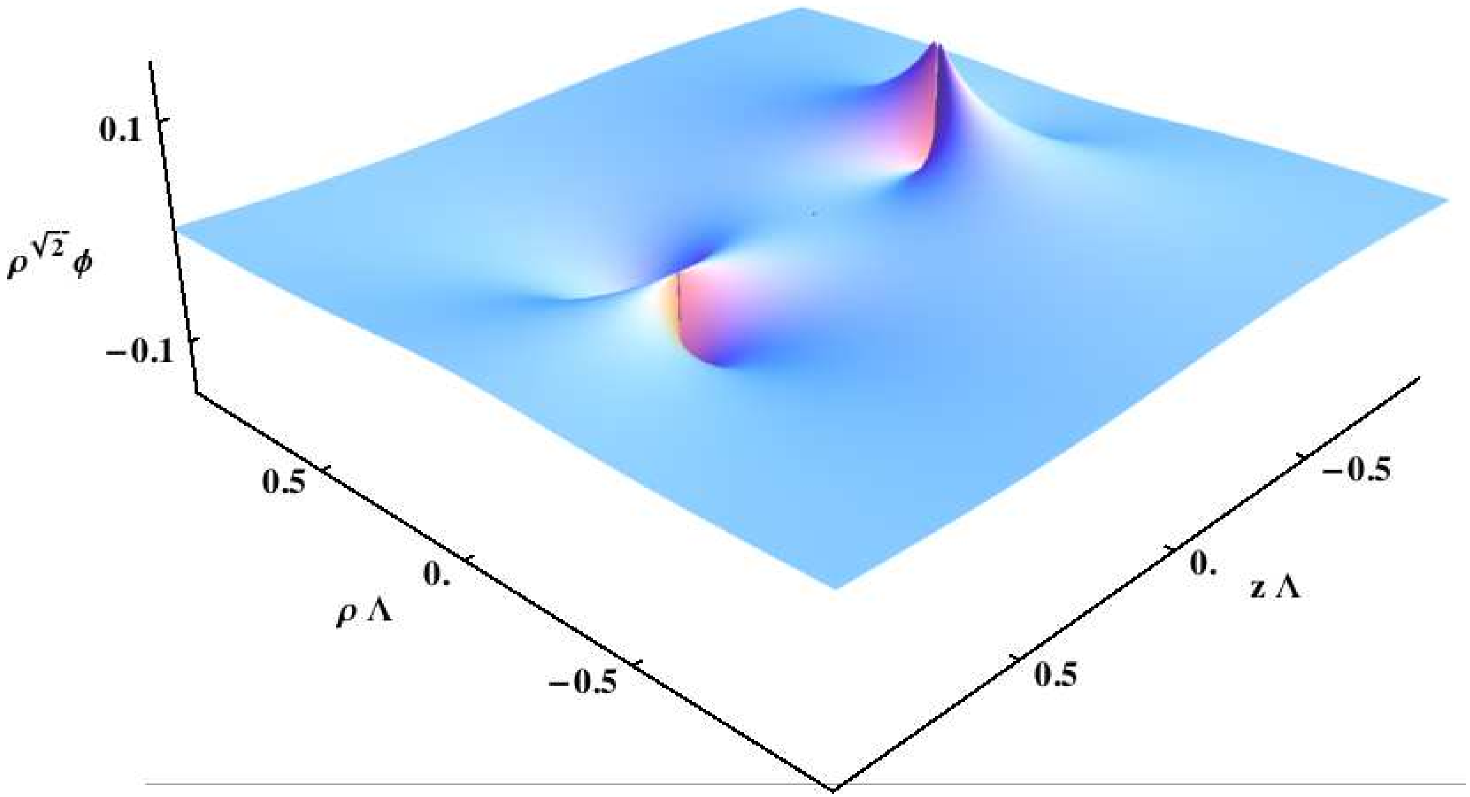
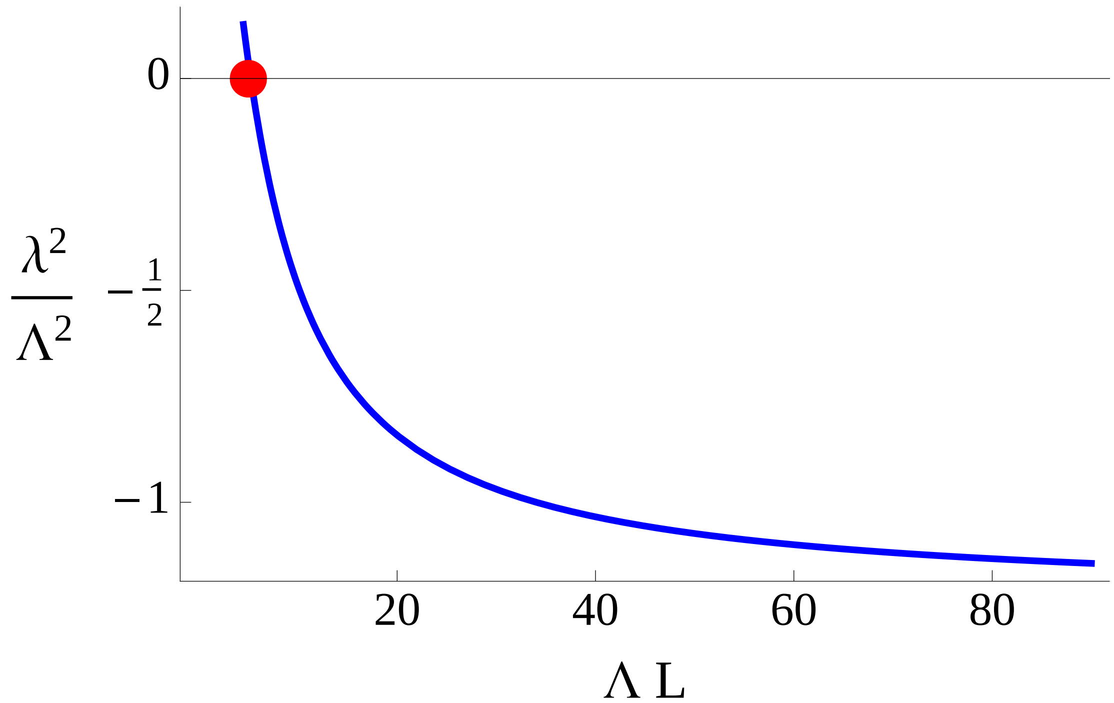
Anti-symmetric solution



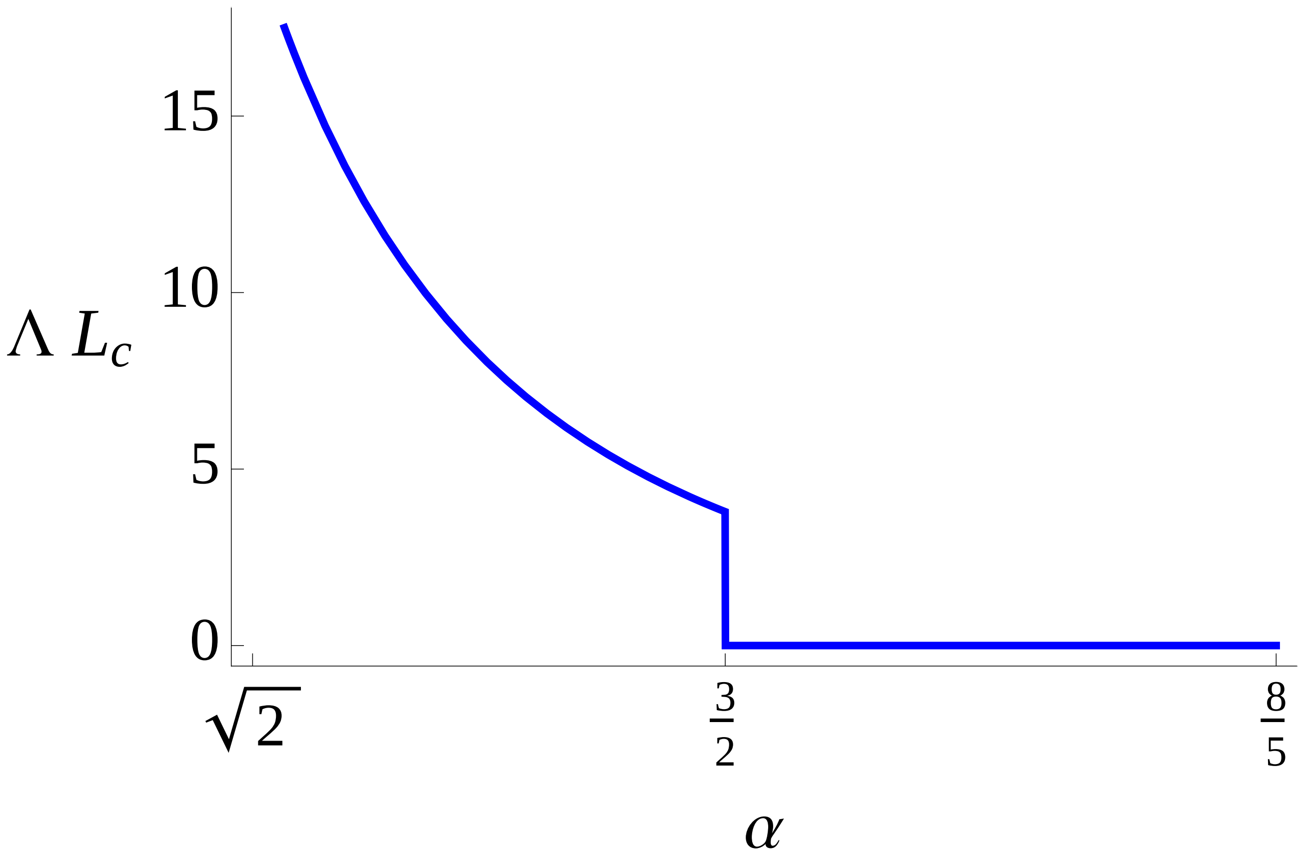
Existence of a critical $q - \bar{q}$ distance

Unstable modes disappear at short $q - \bar{q}$ distances



Existence of a critical $\mathrm{q} - \bar{\mathrm{q}}$ distance

Dependence of critical $\mathrm{q} - \bar{\mathrm{q}}$ distance on α



$\alpha = \sqrt{2}$ $\qquad \alpha_G = 0.43$

COULOMB PHASE INSTABILITIES

- Gribov picture of confinement derived from first principles

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  L$<$L$_c$ Coulomb phase stable (asymt. freedom)

  L$>$L$_c$ Coulomb phase unstable (confinement)

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OUTLOOK

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OUTLOOK

- Finite temperature phase transition?
- Chiral symmetry breaking?

Singular Hamiltonians and AdS/CFT

- Free scalar field in anti-deSitter space-time

$$S(\phi) = \frac{1}{2} \int \frac{dt\, dz}{z^2} \int d^3x \, \left[|\dot{\phi}|^2 + |\partial_z \phi|^2 - |\nabla \phi|^2 - m^2 |\phi|^2 \right]$$

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- Effective Hamiltonian

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