High-energy QCD evolution from Slavnov-Taylor identities

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Outlook

- ▷ The Color Glass Condensate (CGC)
- ▷ Effective Field Theory of the CGC
- ▷ CGC and QCD Symmetries
- ▷ BRST Symmetry, Layers ang Gauge-fixing
- ▷ Quantum effects and Slavnov-Taylor identities
- ▷ Deformed gauge field backgrounds and quantum Yang-Mills Equations of motion
- ▷ Evolution Equations
- ▷ Conclusions

The Color Glass Condensate (CGC)

At small *x* gluon distribution functions *saturate*.
For momenta around the **saturation scale**

$$Q_s \gg \Lambda_{QCD}$$

gauge fields become strong

$$A_\mu \sim \frac{1}{g}$$

while occupation numbers are of the order $\sim \frac{1}{\alpha_s}$

α_s is small, but the theory is non-perturbative.

Effective Field Theory of the CGC

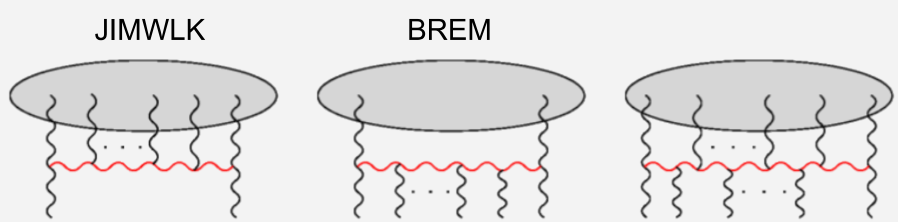
Small x-gluons are described as classical colour fields radiated by colour sources ρ at higher rapidity.

The fast colour sources are in turn described by a **probability distribution function** $W_\tau[\rho]$.

Since α_s is small, we can use reliably **perturbation theory** to compute radiative corrections induced by the exchange of quantum gluons.

CGC and QCD Symmetries

The Effective Field Theory (EFT) for the CGC can be rigorously derived from **QCD** and its ***quantum symmetries*** (Slavnov-Taylor identities) in the presence of the classical colour fields (treated as a background)



From scale Λ ...

One-step quantum evolution with gluon number changing vertices
(Y.Hatta et al., Nuclear Physics A764 (2006) 423)

... to scale $b\Lambda$

Probability distribution at scale Λ

Evolution Equations
(in different flavours: BFKL, JIMWLK, ...)

Probability distribution at scale $b\Lambda$

Layers

Separate the gluon field as

$$A_\mu^a = \widehat{A}_\mu^a + a_\mu^a + \delta A_\mu^a$$

$\widehat{A}_\mu^a$ **classical gluon fields** solution of the YM equation of motion in the presence of the sources ρ.
They have support (in the infinite momentum frame) only for $|p^+| \geq \Lambda$

a_μ^a **semi-fast gluon modes** with support $b\Lambda \leq |p^+| \leq \Lambda$ (with $b \ll 1$ but such that $\alpha_s \log 1/b \ll 1$).
These are the **quantum fluctuations** to be integrated out during the one-step quantum evolution

δA_μ^a **ultra-soft modes** with support $|p^+| \leq b\Lambda$

Effective Field Theory at Tree Level

The action one starts from can be written as

$$S = S_{YM} + S_{MV} + S_W$$

where

- S_{YM} is the Yang-Mills action,
- S_{MV} is the McLerran-Venugopalan functional (its precise form is not relevant in what follows) and
- S_W is a gauge-invariant interaction term involving the source ρ (color-rotated by Wilson lines built with the help of the soft modes δA, in order to preserve gauge-invariance of the EFT).

BRST Symmetry

The BRST symmetry is obtained by replacing the gauge parameters with the ghost fields.

A special treatment is reserved to background fields, that are shifted to a classical anticommuting external source Ω.

BRST is the symmetry holding at the quantum level for the full effective action

(in the functional form of the Slavnov-Taylor identities).

BRST Symmetry and Layers

The full gluon field must transform as a gauge field,
δA must also transform as a gauge field.

This fixes the BRST transformation of all the modes of the gluon gauge field:

$$s\delta A_\mu^a = \mathcal{D}_\mu^{ab}[\delta A]c^b$$

$$s\widehat{A}_\mu^a = \Omega_\mu^a; \qquad s\Omega_\mu^a = 0.$$

$$sa_\mu^a = gf^{abc}\left(\widehat{A}_\mu^b + a_\mu^a\right)c^c - \Omega_\mu^a$$

Gauge-fixing

The **intermediate gluon modes** are quantized in the Light Cone (LC) gauge (so that the ghost decouple).

The **soft modes** δA in the Coulomb gauge (so that the relation between the classical background fields and the color sources is simple)

$$\widehat{\mathcal{F}}^a(p) = \theta(|p^+| - b\Lambda)\theta(\Lambda - |p^+|)n^\mu a^a_\mu(p) - \theta(b\Lambda - |p^+|)ip^i\delta A^a_i(p)$$

Color sources fluctuations

We call $\alpha = B^+$ the background field in the Coulomb gauge
and split the colour source ρ into a background and a quantum fluctuation.

$$\hat{\rho}(x) = -U(x)\ \nabla_T^2 \alpha(x)\ U^\dagger(x)$$

The **classical** relation between the background field and the color sources.

$$U(x)^\dagger = P \exp\left\{ ig \int_{-\infty}^{x^-} \mathrm{d}z^- \alpha(x^+, z^-, \mathbf{x}_T) \right\}$$

Correlators of quantum fluctuations $\delta\rho$ in the EFT will be identified with the correlators of the new classical statistical distribution W at the scale $b\Lambda$

$$\rho \to \widehat{\rho} + \delta\rho.$$

The Effective Action for the CGC

Now one can integrate over the quantum fluctuations a_μ^a .

The effective action $\widetilde{\Gamma}$ is 1-PR (one particle reducible) w.r.t. a_μ^a, the semifast ghost, antighost and Nakanishi-Lautrup multiplier modes.
It is 1-PI w.r.t. all the other fields.

It obeys the Slavnov-Taylor identities (STI) generated by BRST invariance.

The correlators of the fluctuations $\delta\rho$ are to be identified with the correlators of the updated classical statistical distribution after one-step quantum evolution.

The Effective Action for the CGC

One starts from

$$\Gamma = \Gamma[\delta A, a, c, \bar{c}, b, \delta\rho; \widehat{A}, \widehat{\rho}; \delta A^*, a^*, c^*, \delta\rho^*; \Omega]$$

The integration happens on the semi-fast modes
of the gauge field, of the ghosts and antighosts and of the
Nakanishi-Lautrup multiplier.

Define the connected generating functional according to

$$\mathcal{W} = \Gamma + \int \mathrm{d}^4x \, J_\Phi \Phi; \qquad J_\Phi = -(-1)^{\epsilon(\Phi)} \frac{\delta\Gamma}{\delta\Phi}; \qquad \Phi = \frac{\delta\mathcal{W}}{\delta J_\Phi}; \qquad \frac{\delta\mathcal{W}}{\delta\Phi^*} = \frac{\delta\Gamma}{\delta\Phi^*}$$

The Effective Action for the CGC

The ST identity for the connected generating functional is

$$\int \mathrm{d}^4 z\, \Omega^a_\mu(z) \frac{\delta \mathcal{W}}{\delta \widehat{A}^a_\mu(z)} = \int \mathrm{d}^4 z\, \frac{\delta \mathcal{W}}{\delta(\delta A^{*a}_\mu(z))} J_{\delta A^\mu_a}(z) + \int \mathrm{d}^4 z\, \frac{\delta \mathcal{W}}{\delta a^{*a}_\mu(z)} J_{a^\mu_a}(z)$$
$$- \int \mathrm{d}^4 z\, \frac{\delta \mathcal{W}}{\delta(\delta c^{*a}(z))} J_{\delta c^a}(z) - \int \mathrm{d}^4 z\, \frac{\delta \mathcal{W}}{\delta c^{*a}(z)} J_{c^a}(z)$$
$$- \int \mathrm{d}^4 z\, \frac{\delta \mathcal{W}}{\delta J_{\delta b^a}(z)} J_{\delta \bar{c}^a}(z) - \int \mathrm{d}^4 z\, \frac{\delta \mathcal{W}}{\delta J_{b^a}(z)} J_{\bar{c}^a}(z)$$
$$+ \int \mathrm{d}^4 z\, \frac{\delta \mathcal{W}}{\delta(\delta \rho^{*a}(z))} J_{\delta \rho^a}(z).$$

The Effective Action for the CGC

We integrate over the semifast modes by defining

$$\widetilde{\Gamma} = \mathcal{W}|_{J_a=J_c=J_{\bar{c}}=J_b=0} + \int \mathrm{d}^4 z\, J_{\delta A_a^\mu}(z)\delta A_a^\mu(z) + \int \mathrm{d}^4 z\, J_{\delta\rho^a}(z)\delta\rho^a(z)$$
$$+ \int \mathrm{d}^4 z\, J_{\delta c_a}(z)\delta c_a(z) + \int \mathrm{d}^4 z\, J_{\delta \bar{c}_a}(z)\delta \bar{c}_a(z) + \int \mathrm{d}^4 z\, J_{\delta b_a}(z)\delta b_a(z)$$

By setting

$$J_{a_a^\mu} = J_{c^a} = J_{\bar{c}^a} = J_{b^a} = 0$$

we obtain finally the ST identity for the effective action

$$\mathcal{S}\widetilde{\Gamma} \equiv \int \mathrm{d}^4 z \left[\Omega_\mu^a(z) \frac{\delta\widetilde{\Gamma}}{\delta \widehat{A}_\mu^a(z)} + \frac{\delta\widetilde{\Gamma}}{\delta(\delta A_\mu^{*a}(z))} \frac{\delta\widetilde{\Gamma}}{\delta(\delta A_\mu^a(z))} + \frac{\delta\widetilde{\Gamma}}{\delta(\delta c^{*a}(z))} \frac{\delta\widetilde{\Gamma}}{\delta(\delta c^a(z))} \right.$$
$$\left. + \frac{\delta\widetilde{\Gamma}}{\delta(\delta\rho^{*a}(z))} \frac{\delta\widetilde{\Gamma}}{\delta(\delta\rho^a(z))} + \delta b^a(z) \frac{\delta\widetilde{\Gamma}}{\delta(\delta\bar{c}^a(z))} \right] = 0.$$

Quantum Effects

After quantum fluctuations are integrated out

- the splitting $A^a_\mu = \widehat{A}^a_\mu + a^a_\mu + \delta A^a_\mu$ is modified, since the background gets deformed.

- the classical YM equation of motion

$$\hat{\rho}(x) = -U(x)\,\boldsymbol{\nabla}^2_{\mathrm{T}}\alpha(x)\,U^\dagger(x)$$

is in general also deformed.

The STI take care of all the magic!

Deformed Background

Classical Approximation

$$A_\mu = \widehat{A}_\mu + \delta A_\mu + a_\mu$$

Full Quantum Theory

$$\widehat{A}_\mu \to \widehat{A}_\mu + \widetilde{\Gamma}_{\Omega_\mu \delta A^*_\nu} \widehat{A}_\nu + \ldots$$

This relation follow from the STI.

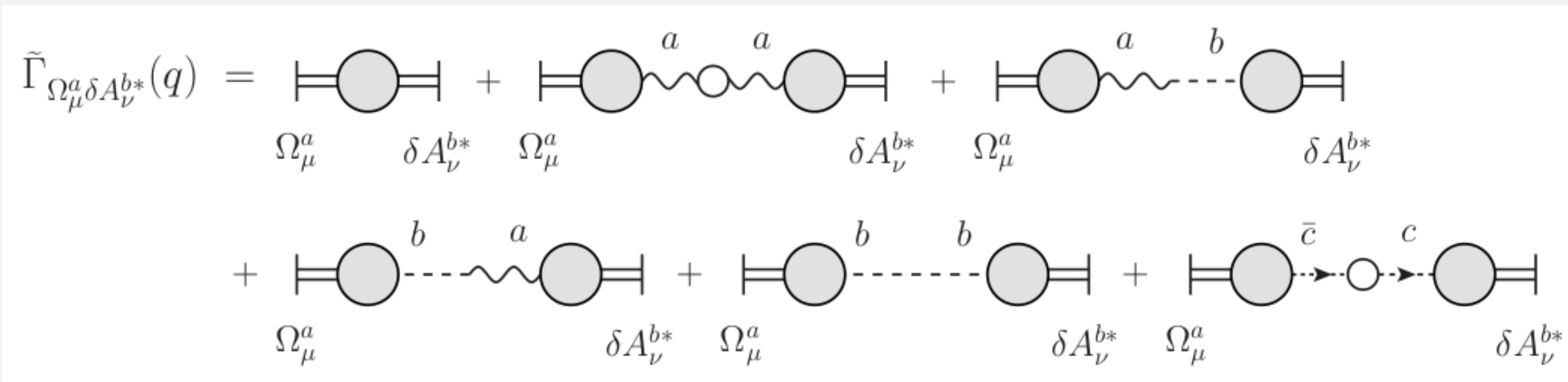
In the **LC gauge** and even at non-zero background the deformation function

$$\widetilde{\Gamma}_{\Omega_\mu \delta A^*_\nu} \equiv \frac{\delta^2 \widetilde{\Gamma}}{\delta \Omega_\mu \delta(\delta A^*_\nu)}$$

is **zero** since there are no interactions between the source Ω and the semifast modes.
Therefore in the LC gauge the background is not deformed.
Incidentally this justifies the expansion around the configuration of the classical background plus δA (at fixed δA) although this is not a minimum of the action and therefore this is not a saddle point approximation.

D.Binosi, A.Q., Phys.Rev. D85 (2012) 121702, arXiv:1203.6637 **[hep-th]** ; Phys.Rev. D85 (2012) 085020, arXiv:1201.1807 **[hep-th]**; Phys.Rev. D84 (2011) 065017, arXiv:1106.3240 **[hep-th]**

Arbitrary Gauge



Since the antifield for the soft gluon modes does not couple to the semifast gluons and one integrates only on the semifast ghost, the above diagrams are zero in any R_ξ- gauge.

Quantum YM Equations of Motion

From the STI one finds

$$\frac{\delta\widetilde{\Gamma}}{\delta\widehat{A}^a_\mu(x)} = -\int \mathrm{d}^4z \, \frac{\delta\widetilde{\Gamma}}{\delta\Omega^a_\mu(x)\delta(\delta\rho^{*b}(z))} \frac{\delta\widetilde{\Gamma}}{\delta(\delta\rho^b(z))}$$

But in the **LC gauge** the Ω-$\delta\rho^*$ deformation function remains classical:

$$\widetilde{\Gamma}_{\Omega^a_\mu \delta\rho^{*b}}(p) = \widetilde{\Gamma}^{(0)}_{\Omega^a_\mu \delta\rho^{*b}}(p)$$

and therefore also after the one-step quantum evolution one can still relate the background field and the color source by the same equation of motion holding at tree-level.

Gauge Invariance of the CGC Effective Action

Since one does not integrate over the soft ghosts, the soft gluons and the quantum fluctuations $\delta\rho$ (as a prescription of the CGC effective approach), the amplitudes involving one δc remain at tree-level.

Therefore by differentiating the ST identity w.r.t. δc_a one gets (at $\delta c = \Omega = 0$) a Ward identity for the CGC effective action:

$$-\partial^\mu \frac{\delta\tilde{\Gamma}}{\delta(\delta A_a^\mu)} + f^{abc}\delta A_c^\mu \frac{\delta\tilde{\Gamma}}{\delta(\delta A_b^\mu)} + f_{abc}\delta\rho_c \frac{\delta\tilde{\Gamma}}{\delta(\delta\rho_b)} = 0$$

Gauge invariance of the CGC effective action follows from the STI.

Evolution Equations

So all one has to do is to work out the evolution equation obeyed by the effective action and eventually translate it into an evolution equation for the classical probability distribution.

The dependence of Γ on b can in principle be two-fold:

a. through the deformation functions

b. through the quantum Green functions for δA and $\delta\rho$

$$
\begin{aligned}
\frac{\partial \Gamma}{\partial b} &= \int \frac{\delta \Gamma}{\delta(\delta A^a_\mu(x))} \frac{\partial}{\partial b}\left(\widehat{A}^a_\mu + \Gamma_{\Omega^a_\mu\, \delta A^{*\nu}_c} \widehat{A}^c_\nu + \cdots\right)(x) \\
&+ \int \frac{\delta \Gamma}{\delta(\delta \rho^a(x))} \frac{\partial}{\partial b}\left(\widehat{\rho}^a + \Gamma_{\Omega^a_\mu\, \delta\rho^*_c} \widehat{\rho}^c + \cdots\right)(x) \\
&+ \int \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{1}{n!m!}\left(\widehat{A}^{a_1}_{\mu_1} + \Gamma_{\Omega^{a_1}_{\mu_1}\, \delta A^{*\nu_1}_{c_1}} \widehat{A}^{c_1}_{\nu_1} + \cdots\right)(x_1) \cdots \left(\widehat{A}^{a_n}_{\mu_n} + \Gamma_{\Omega^{a_n}_{\mu_n}\, \delta A^{*\nu_n}_{c_n}} \widehat{A}^{c_n}_{\nu_n} + \cdots\right)(x_n) \\
&\times \left(\widehat{\rho}^{d_1} + \Gamma_{\Omega^{d_1}_{\mu_1}\, \delta\rho^*_{e_1}} \widehat{\rho}^{e_1} + \cdots\right)(y_1) \cdots \left(\widehat{\rho}^{d_m} + \Gamma_{\Omega^{d_m}_{\mu_m}\, \delta\rho^*_{e_m}} \widehat{\rho}^{e_m}_{\nu_m} + \cdots\right)(y_m) \\
&\times \frac{\partial}{\partial b} \frac{\delta^{m+n}\Gamma}{\delta(\delta A^{a_1}_{\mu_1}(x_1)) \cdots \delta(\delta A^{a_n}_{\mu_n}(x_n)) \delta(\delta\rho^{d_1}(y_1)) \cdots \delta(\delta\rho^{d_m}(y_m))}\Bigg|_{\delta A=\delta\rho=\widehat{A}=\widehat{\rho}=0}
\end{aligned}
$$

This holds to all orders and in any gauge.

Evolution Equations

In the LC gauge there is no b-dependence through the deformation functions.

Moreover the dependence on

$$\tau = \alpha_s \log 1/b$$

is linear in the effective action:

$$\Gamma \sim \tau\, \Delta S_{\text{eff}}$$

$$\Delta S_{\text{eff}} = \sum_{m=0}^{\infty} \frac{1}{m!} \int \underbrace{\frac{\delta^m \Delta S_{\text{eff}}}{\delta(\delta A^-(y_1)) \cdots \delta(\delta A^-(y_m))}}_{\Gamma_m(y_1,\ldots,y_m)} \delta\rho(y_1) \cdots \delta(y_m)$$

Then provided that one operates the identification

$$\langle T[\delta\rho(x_1)\ldots\delta\rho(x_n)]\rangle = \left.\frac{\delta^n \mathrm{W}}{\delta\rho(x_1)\ldots\delta\rho(x_n)}\right|_{\rho=0}$$

one gets the evolution equation

$$\frac{\partial}{\partial\tau}\mathrm{W} = \sum_{n=0}^{\infty} \frac{1}{n!} \Gamma_n(x_1,\ldots,x_n) \frac{\delta^n \mathrm{W}}{\delta\rho(x_1)\ldots\delta\rho(x_n)}$$

Depending on the approximation used to compute ΔS_{eff} one gets the BFKL, the JIMWLK evolution, ...

Conclusions

Evolution equations for the CGC arise from the fundamental QCD symmetries (Slavnov-Taylor identities) in the presence of a background gauge field

A coherent formal framework for deriving the different evolution equations

One can control in a better way gauge-dependence and maybe provide a path towards the inclusion of higher order terms