Quantum diffusion approach to sub-Coulomb fusion of light nuclei

V. V. Sargsyan*

*Joint Institute For Nuclear Research

ECT*, Trento, Italy
October 7 - 11, 2013

Outline

- Quantum Diffusion Approach: Formalism
- Reactions with heavy spherical nuclei
- Effects of deformation and neutron transfer in fusion reactions with heavy nuclei
- Reactions with light nuclei
- Summary

The assumptions of the QD approach

The quantum diffusion approach based on the following assumptions:

1. **The capture (fusion) can be treated on the one collective variable: the relative distance between the colliding nuclei.**
2. **The internal excitations (for example, low-lying collective modes such as dynamical quadropole and octupole excitations of the target and projectile, etc.) can be presented as an environment which is coupled with the collective variable** ***PRE71,016121(2005).***

Our approach takes into account the fluctuation and dissipation effects in the collisions of heavy ions which model the coupling with various channels.

The capture cross section

The capture cross-section is a sum of partial capture cross-sections

$$\sigma(E_{\text{c.m.}}) = \sum_J \sigma_c(E_{\text{c.m.}}, J) = \pi \lambda\!\!\!^{-2} \sum_J (2J+1) P_{cap}(E_{\text{c.m.}}, J)$$

$\lambda\!\!\!^{-2} = \hbar^2 / (2\mu E_{\text{c.m.}})$ **--- the reduced de Broglie wavelength**

$P_{cap}(E_{\text{c.m.}}, J)$ **--- the partial capture probability at fixed energy and angular momenta**

The partial capture probability obtained by integrating the propagator G from the initial state *(R_0,P_0)* at time <u>*t=0*</u> to the finale state *(R, P)* at time *t*:

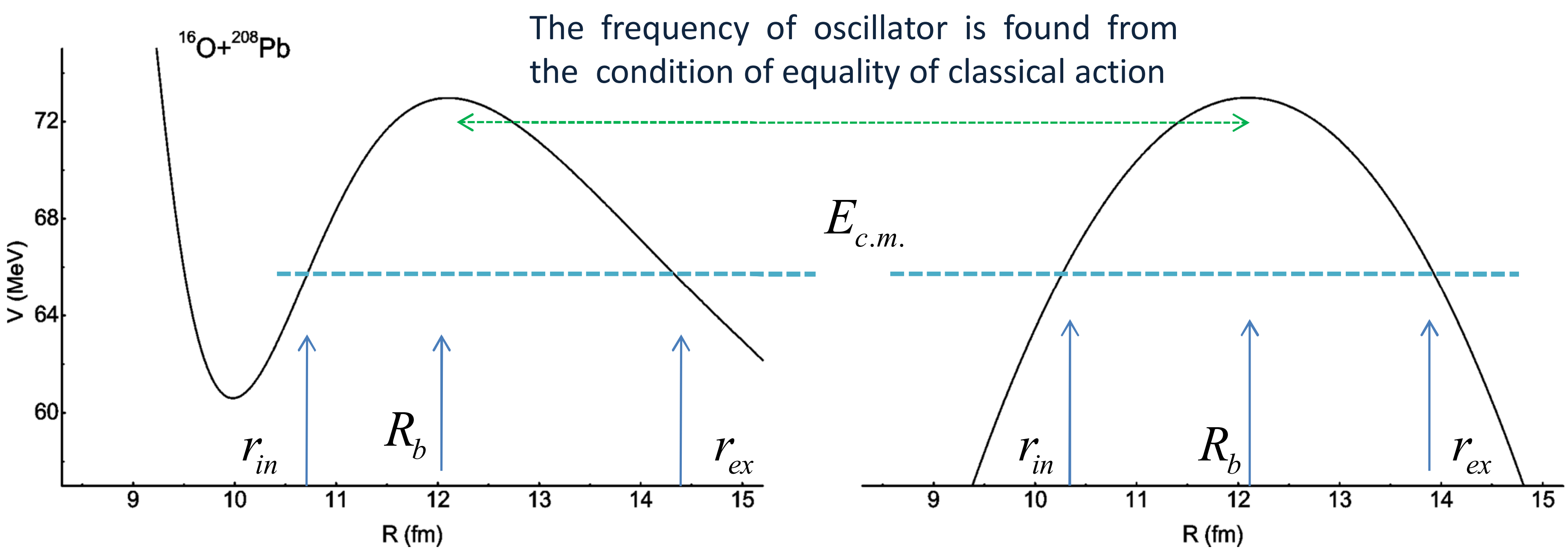
$$P_{cap} = \lim_{t\to\infty} \int_{-\infty}^{r_{in}} dR \int_{-\infty}^{\infty} dP\, G(R, P, t | R_0, P_0, 0)$$

r_{in} **--- the internal turning point**

Approximation:

realistic nucleus-nucleus potential → inverted oscillator

- **The real interaction between nuclei can be approximated by the inverted oscillator.**



Nucleus-nucleus potential:

1. **Density - dependent Migdal forces**
2. **Woods-Saxon parameterization for nucleus density**

Adamian et al., Int. J. Mod. Phys E 5, 191 (1996).

Propagator for the inverted oscillator

- For the inverted oscillator the propagator has the form:

$$G\left(P,R,t\,\middle|\,P_0,R_0,t=0\right)=\frac{1}{2\pi\sqrt{\Sigma_{RR}(t)\Sigma_{PP}(t)-\Sigma_{PR}^2(t)}}\times$$

$$\exp\left[-\frac{1}{2[\Sigma_{RR}(t)\Sigma_{PP}(t)-\Sigma_{PR}^2(t)]}\left(\Sigma_{PP}(t)\left(R-\overline{R(t)}\right)^2+\Sigma_{RR}(t)\left(P-\overline{P(t)}\right)^2-2\Sigma_{PR}(t)\left(P-\overline{P(t)}\right)\left(R-\overline{R(t)}\right)\right)\right]$$

$\overline{R(t)}$, $\overline{P(t)}$ --- the mean value of the collective coordinate and momentum

$\frac{1}{2}\Sigma_{ij}(t)$ --- the variances and $\Sigma_{ij}(t=0)=0, \quad (i,j=R,P)$

Dadonov, Man'ko, Tr. Fiz. Inst. Akad. Nauk SSSR 167, 7 (1986).

- The expression for the capture probability

$$P_{cap}=\lim_{t\to\infty}\frac{1}{2}\operatorname{erfc}\left[\frac{\overline{R(t)}}{\sqrt{\Sigma_{RR}(t)}}\right]$$

The formalism of quantum-diffusion approach

The starting point is the full Hamiltonian of the system:

$$H_{tot} = H_{coll} + H_{inter} + V_{coupling}$$

$$H_{coll} = \frac{1}{2\mu} P^2 - \frac{\mu\omega^2}{2} R^2$$

- **The Hamiltonian of the collective subsystem**

$$H_{inter} = \sum_{\nu} \hbar\omega_\nu b_\nu^+ b_\nu$$

- **The Hamiltonian of the internal subsystem**

$$V_{coupling} = \sum_{\nu} V_\nu (R) \left(b_\nu^+ + b_\nu\right)$$

- **The Hamiltonian of the coupling between the collective and internal subsystems**

The system of Heisenberg operators related to the collective and internal motions

The equations for the collective and internal coordinates are:

$$\begin{cases} \dfrac{dR}{dt} = \dfrac{1}{\mu} P \\ \dfrac{dP}{dt} = -\mu\omega^2 R - \sum_{\nu} \dfrac{\partial V_\nu(R)}{\partial R}\left(b_\nu^+ + b_\nu\right) \end{cases}$$

$$\begin{cases} \dfrac{d}{dt} b_\nu^+ = i\omega_\nu b_\nu^+ + \dfrac{i}{\hbar} V_\nu(R) \\ \dfrac{d}{dt} b_\nu = -i\omega_\nu b_\nu - \dfrac{i}{\hbar} V_\nu(R) \end{cases}$$

The solution for the internal coordinates:

$$b_\nu^+(t) + b_\nu(t) = f_\nu^+(t) + f_\nu(t) - \frac{2V_\nu(R)}{\hbar\omega_\nu} - \frac{1}{\hbar\omega_\nu}\int_0^t d\tau \, \frac{\partial V_\nu(R(\tau))}{\partial R}\left[e^{i\omega_\nu(t-\tau)} - e^{-i\omega_\nu(t-\tau)}\right]$$

Here: $f_\nu(t) = \left[b_\nu(0) + \dfrac{1}{\hbar\omega_\nu} V_\nu(R(0)) \right] \exp(-i\omega_\nu t)$

- **The equations for the collective coordinates depends on the initial distribution of the internal states!**

$$\frac{dP}{dt} = -\mu\omega^2 R(t) - \int_0^t K((t,\tau), R) R'(\tau) d\tau + F(t, R)$$

The random force and dissipative kern

➢ The equation for the collective momentum contains dissipative kern and random force:

Dissipative Kern — Random force

$$\frac{dP}{dt} = -\mu\omega^2 R(t) - \int_0^t K\left((t,\tau), R\right) R'(\tau)\, d\tau + F\left(R, t\right)$$

❖ The expressions for the dissipative kern and random force:

$$K\left(t,\tau\right) = \sum_{\nu} \frac{1}{\hbar\omega_\nu} \left\{ \frac{\partial V_\nu(R(t))}{\partial R}, \frac{\partial V_\nu(R(\tau))}{\partial R} \right\}_+ \cos\left(\omega_\nu [t-\tau]\right)$$

$$F\left(R,t\right) = -\sum_{\nu} \frac{\partial V\left(R(t)\right)}{\partial R} \left[f_\nu^+(t) + f_\nu(t) \right]$$

➢ The random force satisfied the following conditions:

$$\left\langle\left\langle f_\nu^+\left(t\right) f_{\nu'}^+\left(t'\right)\right\rangle\right\rangle = \left\langle\left\langle f_\nu\left(t\right) f_{\nu'}\left(t'\right)\right\rangle\right\rangle = 0 \qquad \left\langle\left\langle f_\nu^+\left(t\right) f_{\nu'}\left(t'\right)\right\rangle\right\rangle = \delta_{\nu,\nu'} n_\nu \mathrm{e}^{i\omega_\nu (t-t')}$$

$$\left\langle\left\langle f_\nu\left(t\right) f_{\nu'}^+\left(t'\right)\right\rangle\right\rangle = \delta_{\nu,\nu'}\left(n_\nu + 1\right) \mathrm{e}^{-i\omega_\nu (t-t')}$$

$$n_\nu = \left[e^{\hbar\omega_\nu / T} - 1 \right]^{-1}$$ --- the occupations numbers for phonons

The linear coupling between collective and internal subsystems

The case of linear coupling by coordinate:

$$V_{coupling} = \frac{\lambda^{1/2}}{\hbar} R \sum_{\nu} \Gamma_{\nu} \left(b_{\nu}^{+} + b_{\nu} \right) + \frac{\lambda}{\hbar^2} R^2 \sum_{\nu} \frac{\left|\Gamma_{\nu}\right|^2}{\hbar \omega_{\nu}}$$

Is used to compensate the renormalization of the potential

The random force by the momenta

$$F(t) = \sum_{\nu} F^{\nu}(t) = \frac{\lambda^{1/2}}{\hbar} \sum_{\nu} \Gamma_{\nu} \left(f_{\nu}^{+}(t) + f_{\nu}(t) \right)$$

$$f_{\nu}^{+}(t) = \left[b_{\nu}^{+}(0) + \frac{\lambda^{1/2}}{\hbar^2 \omega_{\nu}} \Gamma_{\nu} R(0) \right] \exp[i \omega_{\nu} t]$$

The dissipative kern

$$K(t-\tau) = \frac{2\lambda}{\hbar^2} \sum_{\nu} \frac{\left|\Gamma_{\nu}\right|^2}{\hbar \omega_{\nu}} \cos\left(\omega_{\nu} (t-\tau)\right)$$

From summation to integral

One can assumes some spectra for the environment and replace the summation over the integral:

$$\sum_{\nu} \frac{\Gamma_{\nu}^{2}}{\hbar\omega_{\nu}} \rightarrow \int_{0}^{\infty} d\Omega\, \rho(\Omega) \frac{g(\Omega)^{2}}{\hbar\Omega} \ldots = \frac{1}{\pi} \int_{0}^{\infty} d\Omega \frac{\gamma^{2}}{\gamma^{2}+\Omega^{2}} \ldots$$

In the markovian limit

$$K(t) = \gamma\, e^{-t\gamma}$$

The analytical expressions for the first and second moments in case of linear coupling

$$\overline{R(t)} = A_t R_0 + B_t P_0$$

$$\Sigma_{RR}(t) = \frac{2\hbar^2 \lambda \gamma^2}{\pi} \int_0^t d\tau' B_{\tau'} \int_0^t d\tau'' B_{\tau''} \int_0^\infty d\Omega \frac{\Omega}{\Omega^2 + \gamma^2} \coth\left[\frac{\hbar\Omega}{2T}\right] \cos[\Omega(\tau' - \tau'')]$$

$$A_t = \sum_{i=1}^{3} \beta_i \left[s_i (s_i + \gamma) + \hbar\lambda\gamma/\mu\right] e^{s_i t}$$

$$B_t = \frac{1}{\mu} \sum_{i=1}^{3} \beta_i (s_i + \gamma) e^{s_i t}$$

Functions determine the dynamic of the first and second moments

$$\beta_1 = [(s_1 - s_2)(s_1 - s_3)]^{-1}$$

$$\beta_2 = [(s_2 - s_1)(s_2 - s_3)]^{-1}$$

$$\beta_3 = [(s_3 - s_1)(s_3 - s_2)]^{-1}$$

s_i --- are the roots of the following equation

$$(s + \gamma)(s^2 - \omega^2) + \hbar\lambda\gamma s / \mu = 0$$

The features of quantum diffusion approach

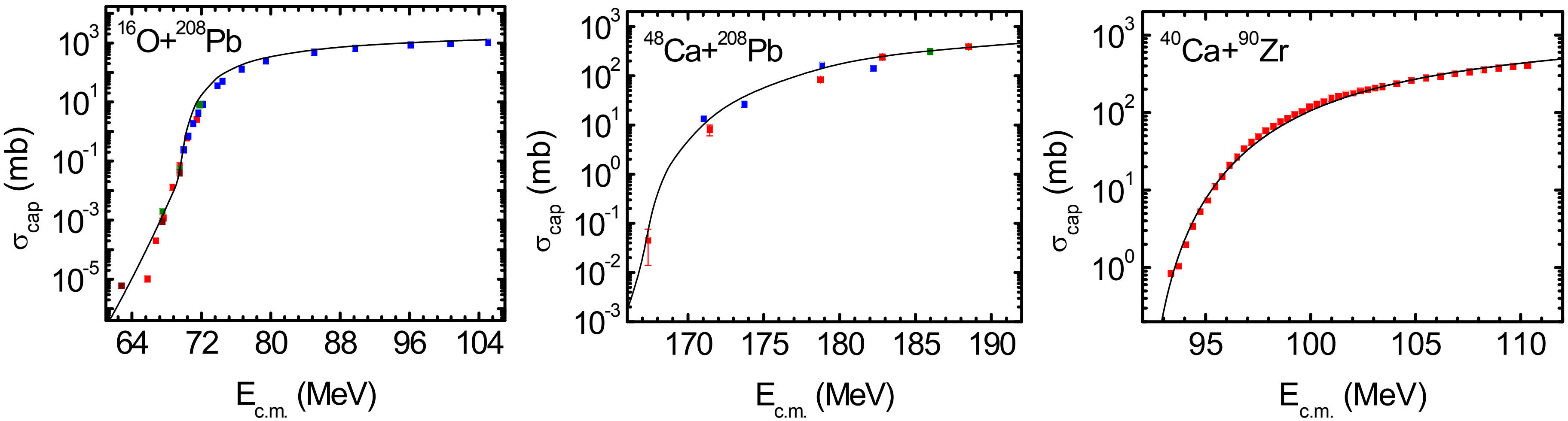
1. **The coupling with respect to the relative coordinate results in a random force and a dissipation kernel.**
2. **The integral term in the equations of motion means that the system is non-Markovian and has a "memory" of the motion over the trajectory preceding the instant *t*.**
3. **The random force and the dissipation kernel are dependent on the dynamical coordinate of the collective subsystem. The temperature and the fluctuations are set using the distribution of the initial coordinates and momenta of the internal subsystem (at t = 0).**
4. **To determine the statistical properties of these fluctuations, we consider an ensemble of initial states in which an initial collective coordinate q(0) is set, and the initial thermostat operators are chosen from the canonical ensemble.**

Sargsyan et. al., EPJ A45, 125 (2010); EPJA47, 38, 2011;
PRA 83, 062117 (2011); PRA 84, 032117 (2011)

Linear coupling limit

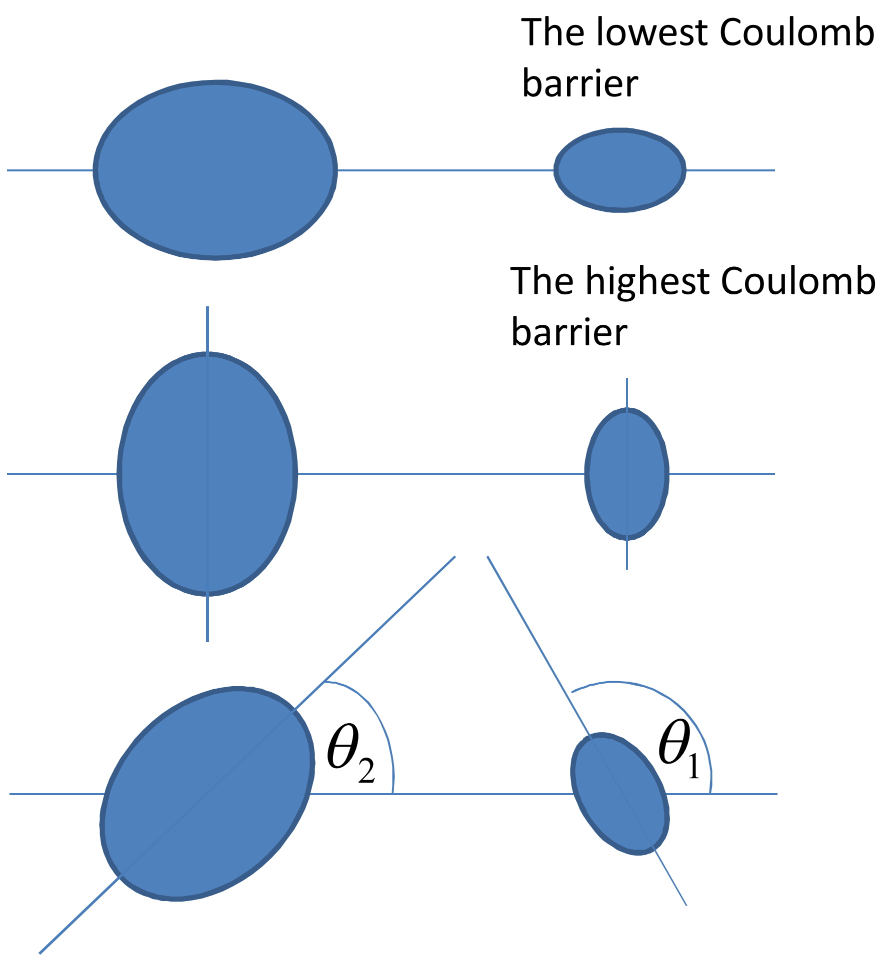
Reactions with heavy spherical nuclei

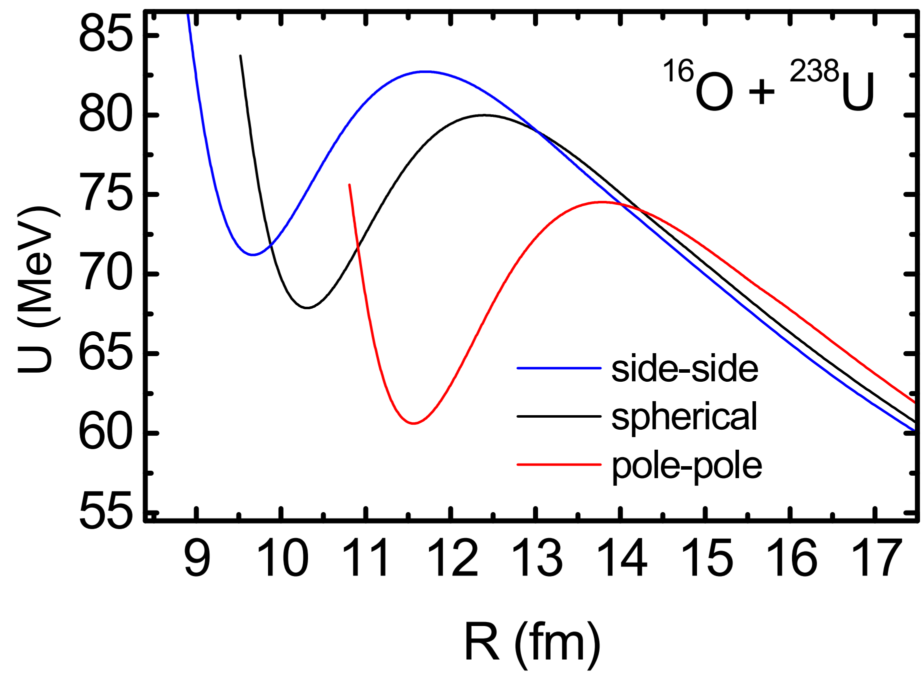
- ✓ Reactions with spherical nuclear are good test for the verification of the approach.
- ✓ Good agreement between the theory and experimental data is an argument in favor of the correct approximation of the potential.
- ✓ Using these reactions we fixed the parameters used in calculation.
- ✓ The linear coupling approximation (constant friction) seems to be suitable for the considered reactions.



- ➢ Reactions with the spherical nuclei more clearly shows the behavior of the excitation function.
- ➢ The change of the slope of the excitation function means, that at sub-barrier energies the diffusion becomes a dominant component.

Role of deformation of nuclei in capture process

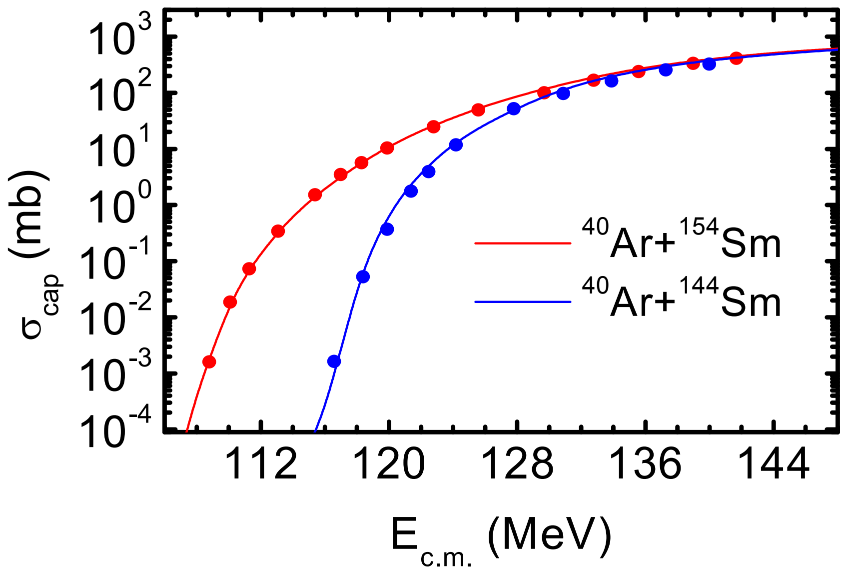


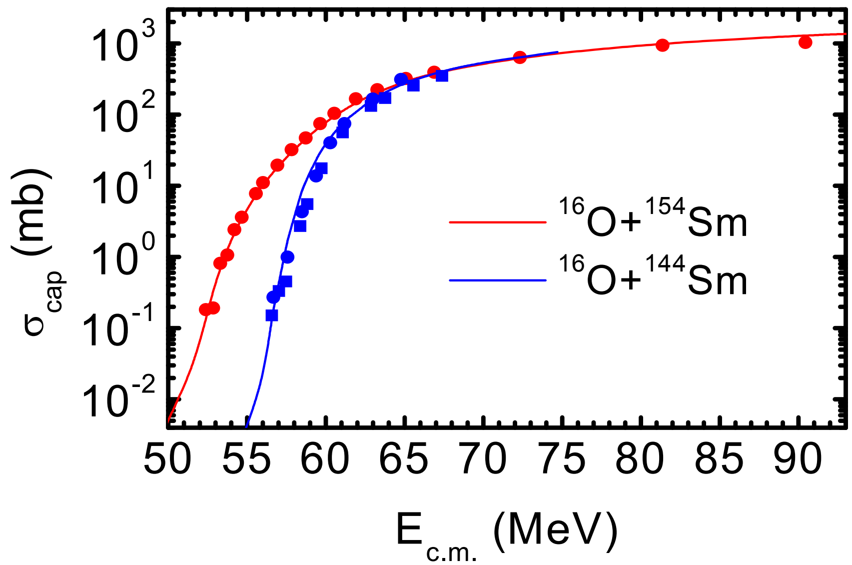


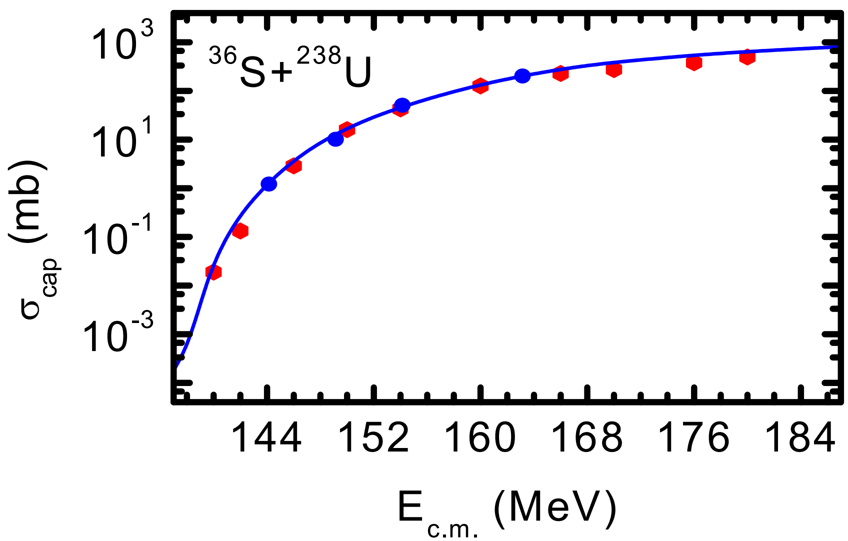
- **At fixed bombarding energy the capture occurs above or below the Coulomb barrier depending on mutual orientations of colliding nuclei !**

$$\sigma_{cap}(E_{c.m.}) = \int_0^{\pi/2} d\theta_1 \sin\theta_1 \int_0^{\pi/2} d\theta_2 \sin\theta_2 \sigma_{cap}(E_{c.m.}, \theta_1, \theta_2)$$

Comparison with experimental data







- **The effect depends on the charges and deformations of the colliding nuclei.**
- **The used averaging procedure seems to work correct.**

Sargsyan et. al., PRC 85, 037602 (2012)

Role of neutron transfer

- **Neutrons are insensitive to the Coulomb barrier and, therefore, their transfer starts at larger separations before the projectile is captured by the target nucleus.**

- **It is generally thought that the sub-barrier capture (fusion) cross section increases because of the neutron transfer.**

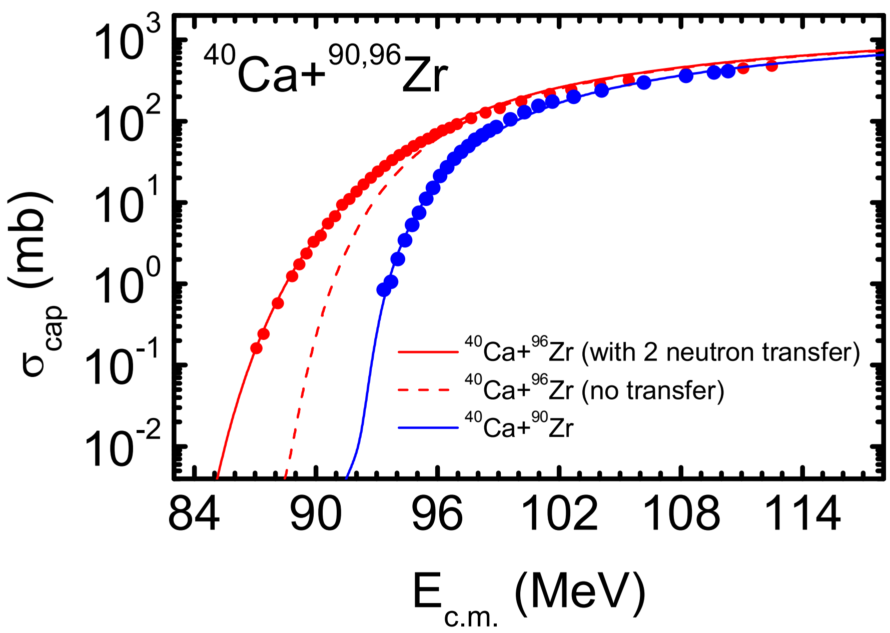
- **The present experimental data (for example ^{60}Ni + ^{100}Mo system,** *F. Scarlassara, EPJ Web Of Conf. 17, 05002 (2011)*) **specify in complexity of the role of neutron transfer in the capture (fusion) process and provide a useful benchmark for theoretical models.**

Why the influence of the neutron transfer is strong in some reactions, but is weak in others ?

The model assumptions

- Sub-barrier capture depends on two-neutron transfer with positive Q-value.

- Before the crossing of Coulomb barrier, 2-neutron transfer occurs and lead to population of first 2+ state in recipient nucleus (donor nucleus remains in ground state).

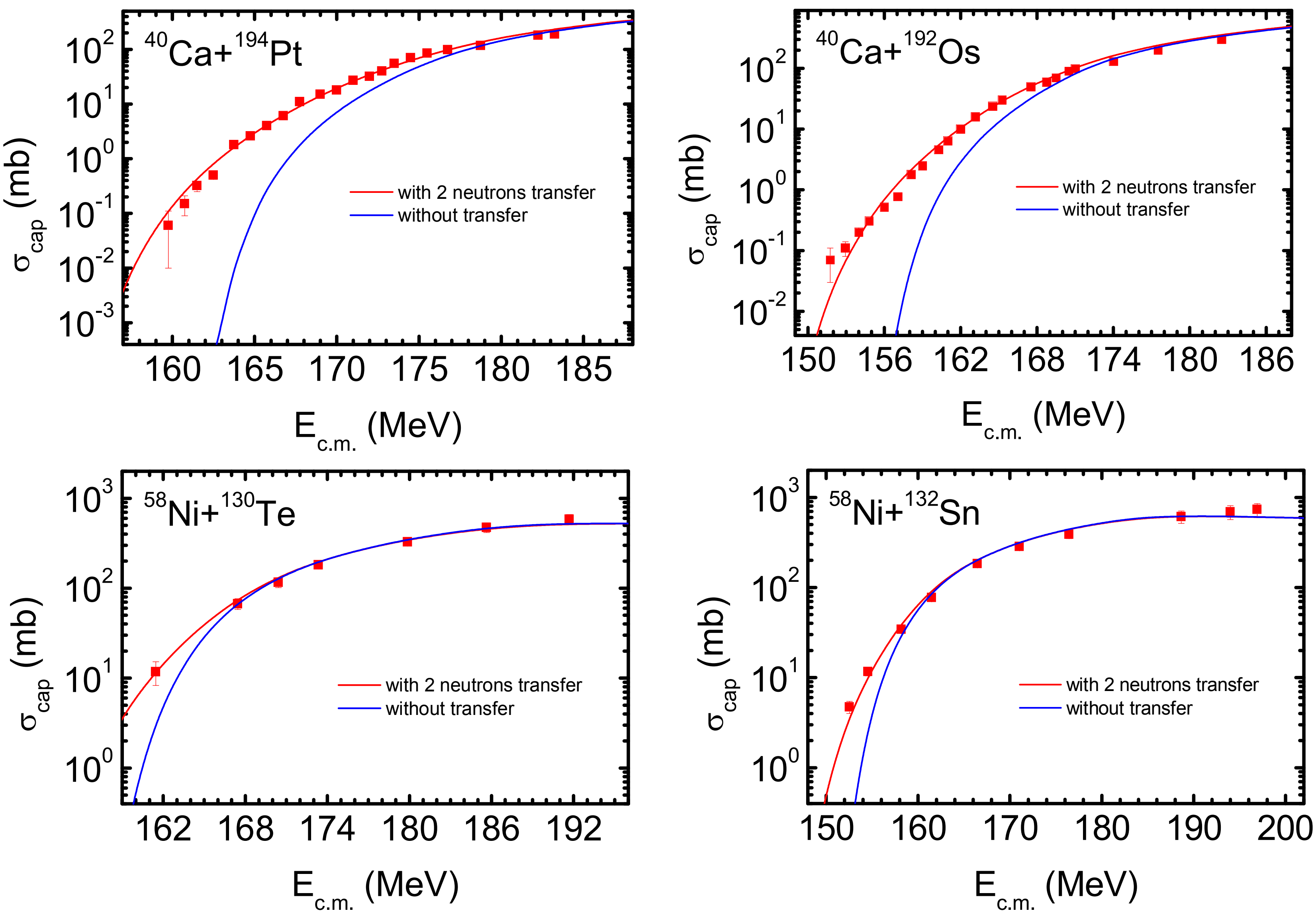


- Large enhancement of the excitation function for the $^{40}Ca+^{96}Zr$ reaction with respect to the $^{40}Ca+^{96}Zr$ reaction!

- Taking into account the transfer of two neutrons we reproduce the experimental data!

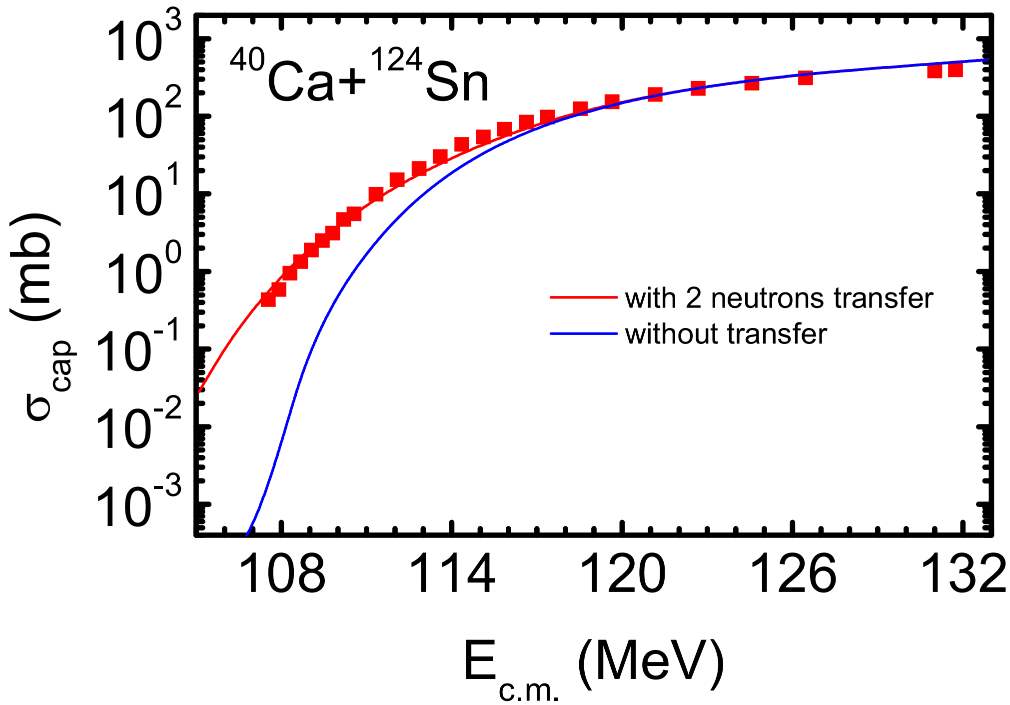
Sargsyan et. al., PRC 84, 064614 (2011)
PRC 85, 024616 (2012)

Reactions with two neutron transfer



Pair transfer ?

- Reactions with $Q_{1n} < 0$ and $Q_{2n} > 0$
- Good agreement between calculations and experimental data is an argument of pair transfer



- By describing sub-barrier capture, we demonstrate indirectly strong spatial 2-neutron correlation and nuclear surface enhancement of neutron pairing
- Indication for Surface character of pairing interaction ?

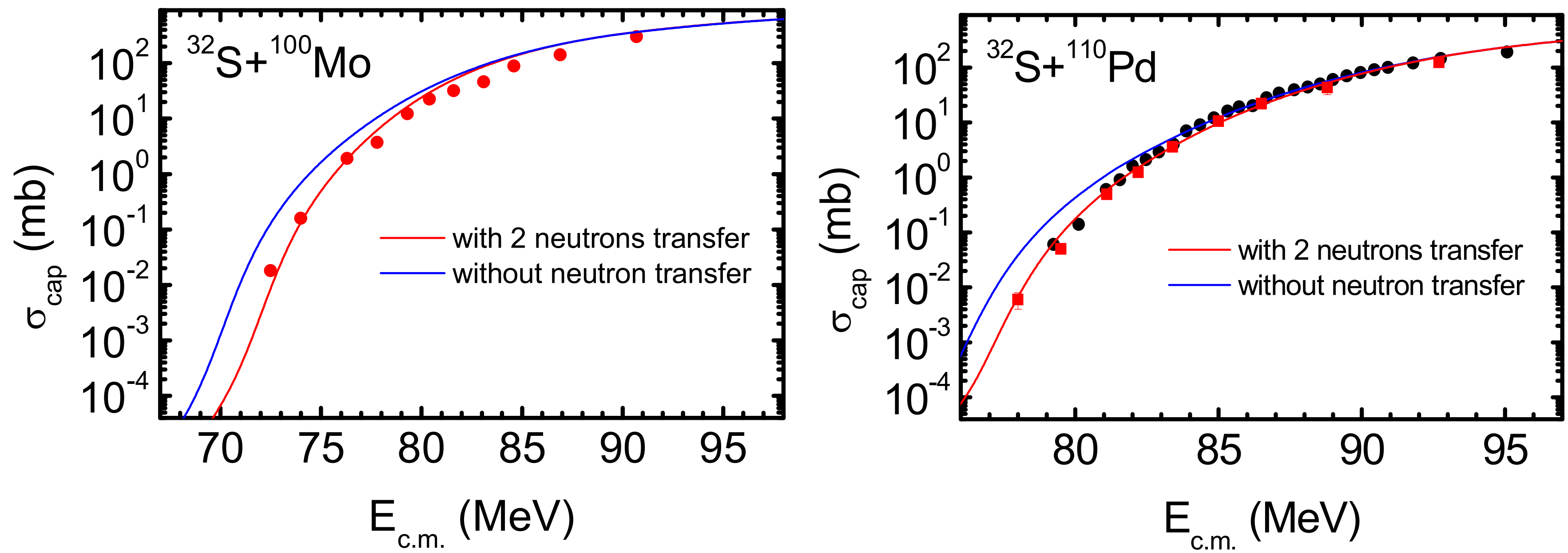
Enhancement or suppression ?

- **2n-transfer can also suppress capture**

- **If deformation of the system decreases due to neutron transfer, capture cross section becomes smaller**

$$^{32}\mathrm{S}(\beta_2=0.31)+^{110}\mathrm{Pd}(\beta_2=0.26)\rightarrow^{34}\mathrm{S}(\beta_2=0.25)+^{108}\mathrm{Pd}(\beta_2=0.24)$$

$$^{32}\mathrm{S}(\beta_2=0.31)+^{100}\mathrm{Mo}(\beta_2=0.26)\rightarrow^{34}\mathrm{S}(\beta_2=0.25)+^{98}\mathrm{Mo}(\beta_2=0.17)$$



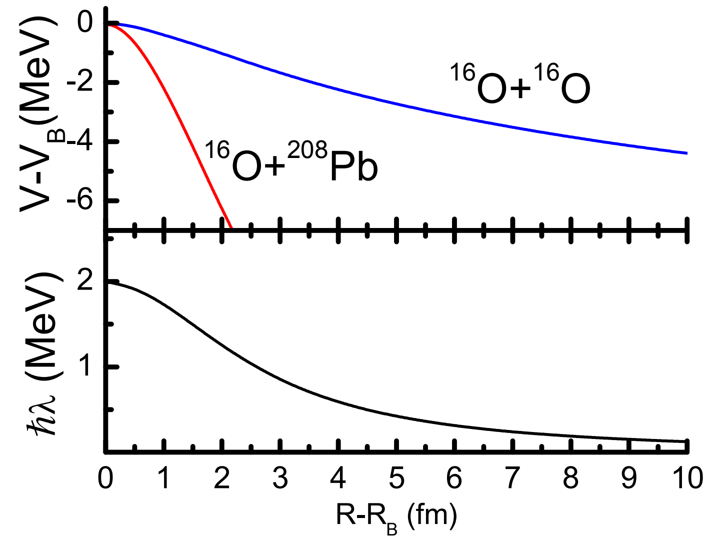
Non-Linear coupling limit

Friction depending on the relative distance of colliding nuclei

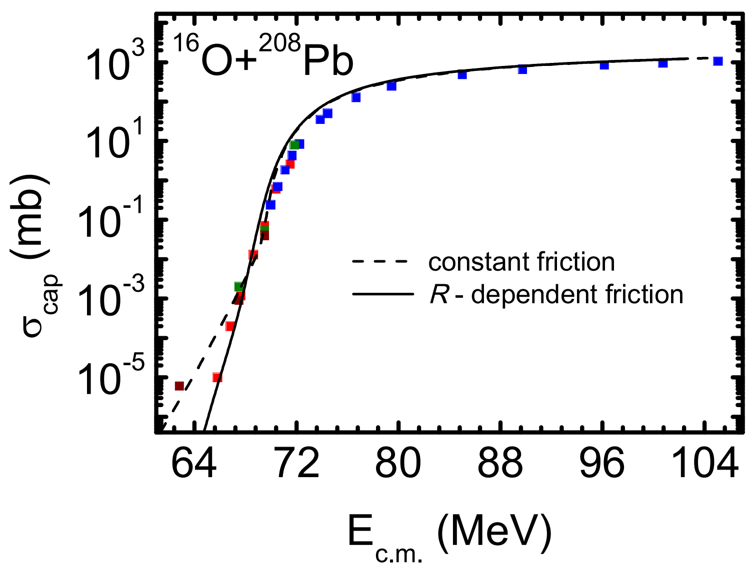
- Frictions is a result of the overlapping of the nuclear densities.
- For the light systems, the coupling parameter should depend on the relative distance between the colliding nuclei and, as a result the friction becomes coordinate-dependent.

$$\lambda = 2 \quad \text{if } R < R_B$$

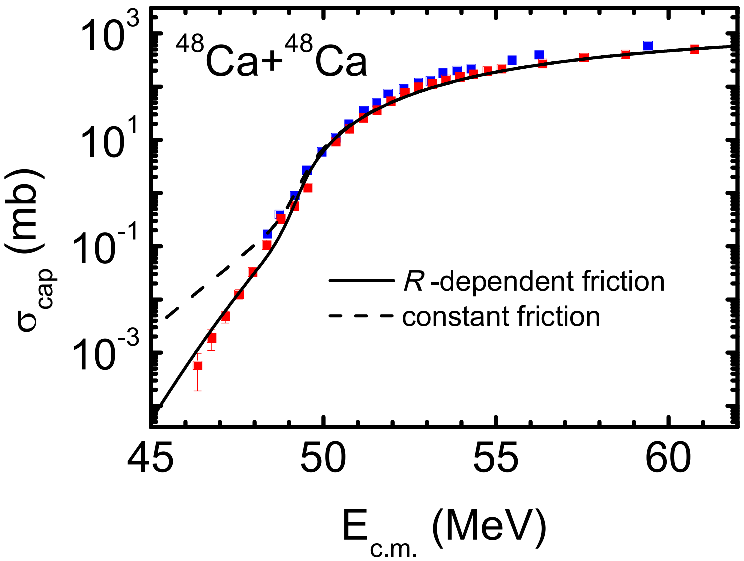
$$\lambda(R - R_B) = \frac{2}{1 + 0.15(R - R_B)^2}$$

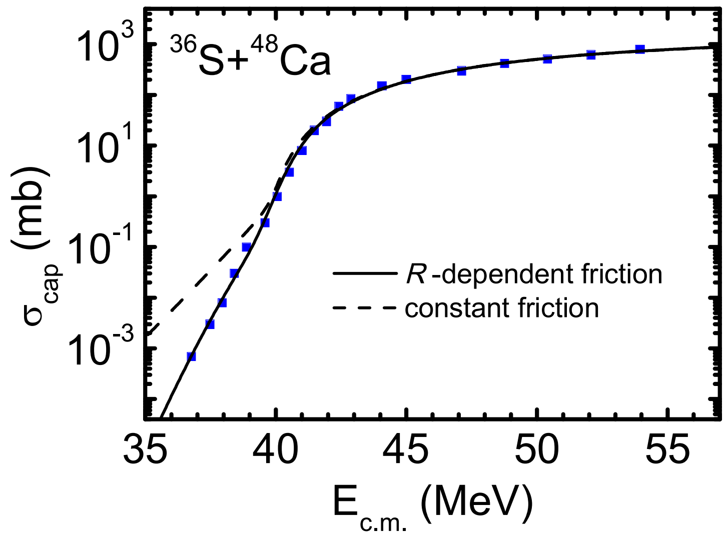


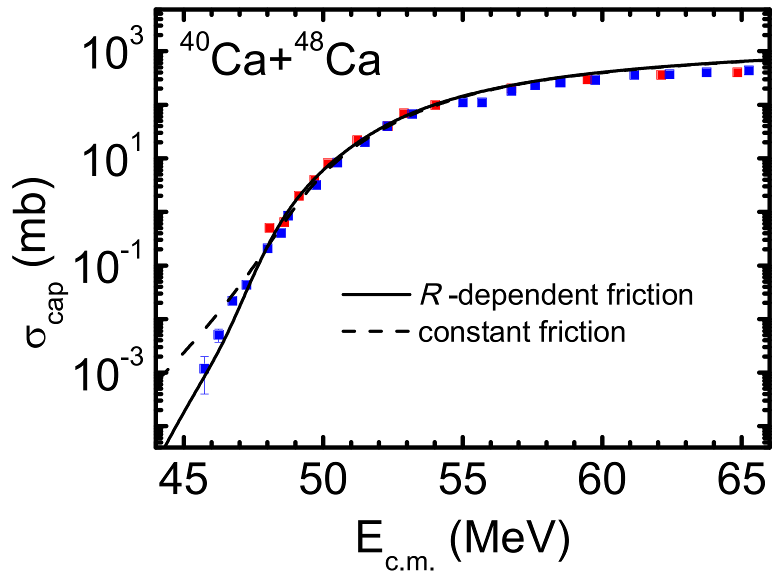
- Comparing the results, obtained with the analytic expressions (<u>constant friction</u>) for the equations of motions with the numerical one (<u>coordinate- dependent</u>), one can assume that the linear coupling limit is suitable for the heavy systems and not very deep sub-barrier energies.

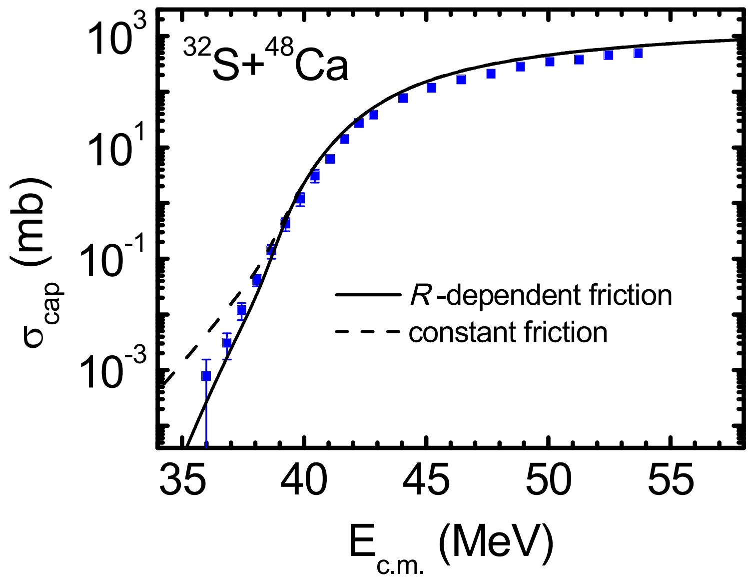


Calculations with constant and *R*-dependent friction

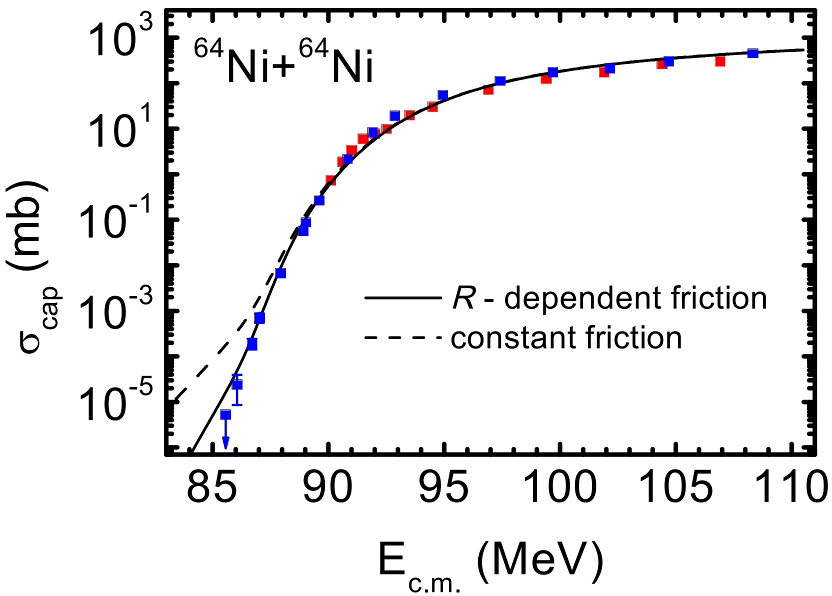


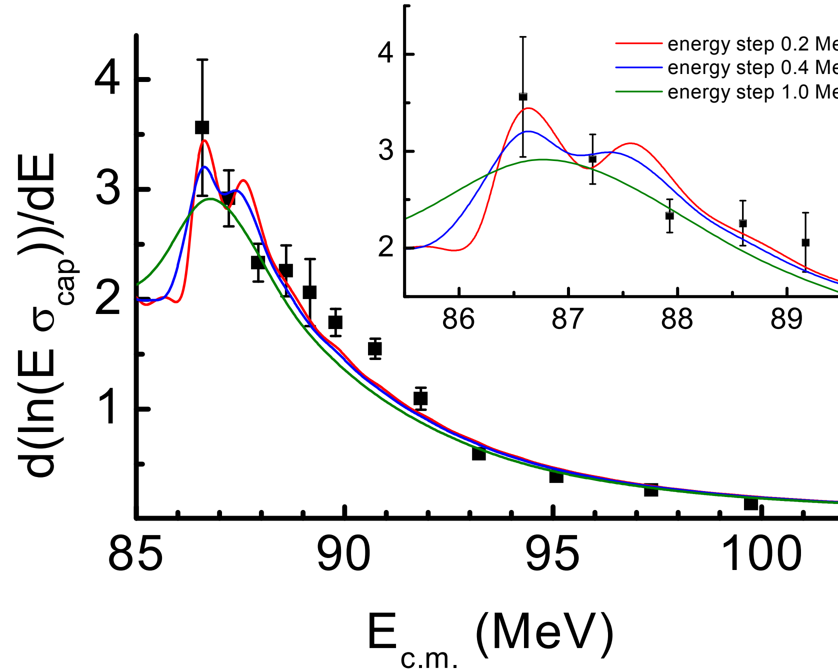


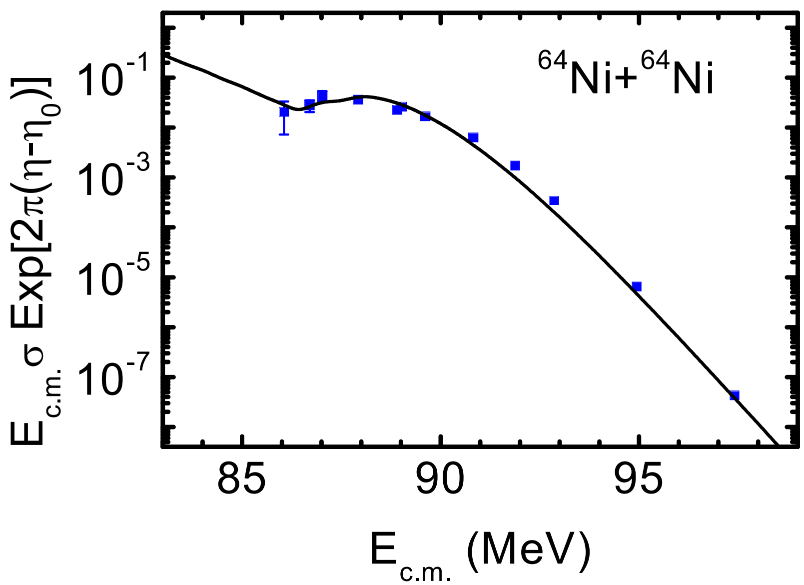




Astrophysical *S* and logarithmic *L* factors

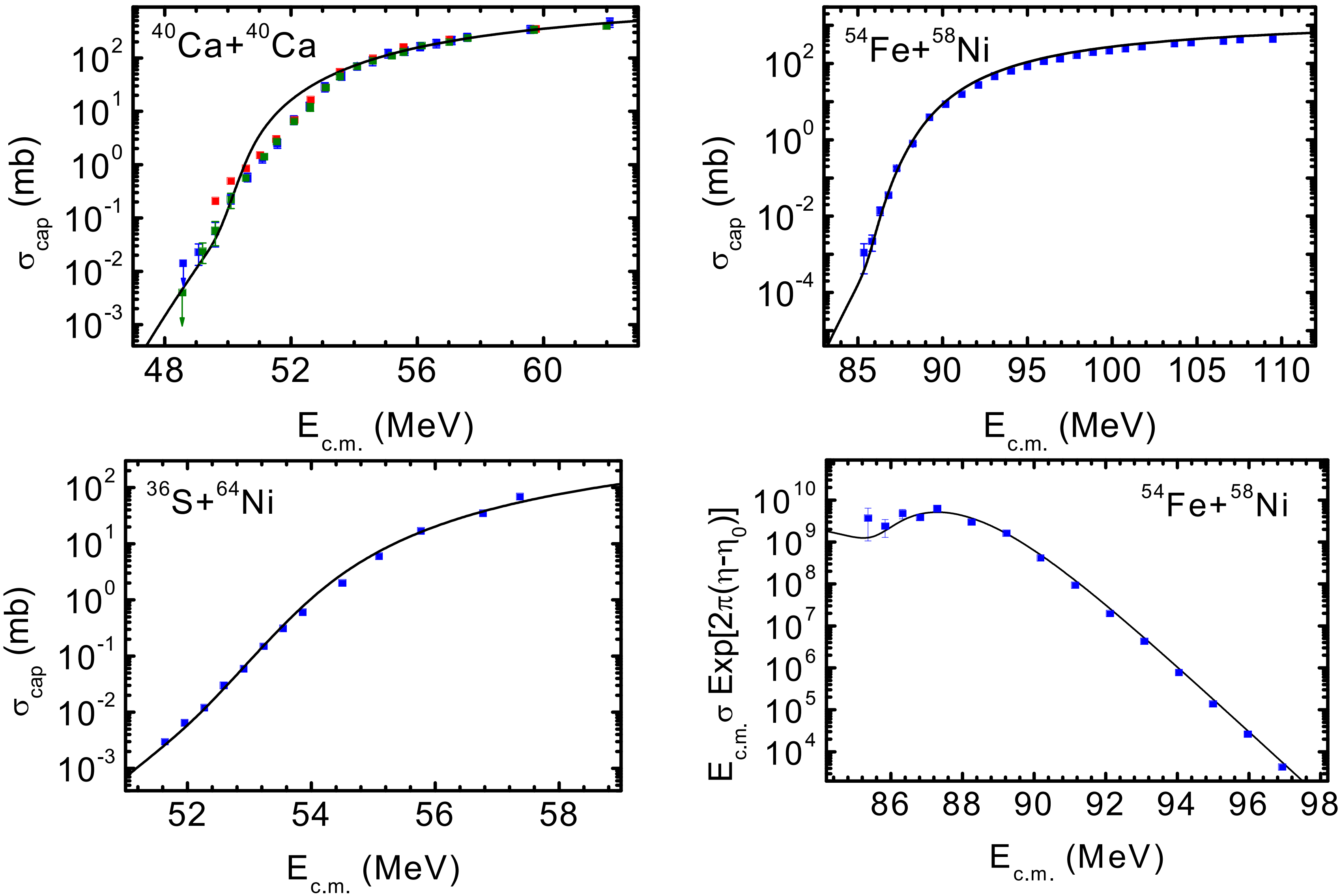




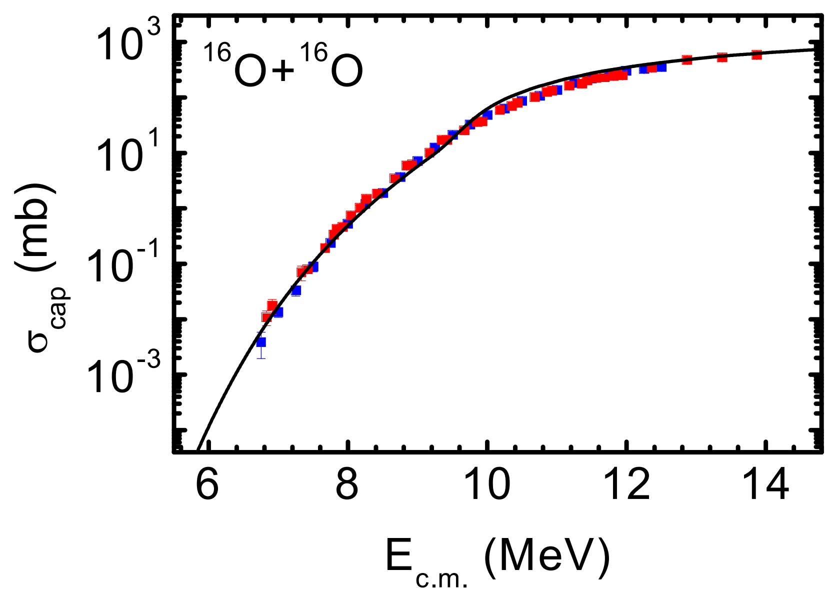


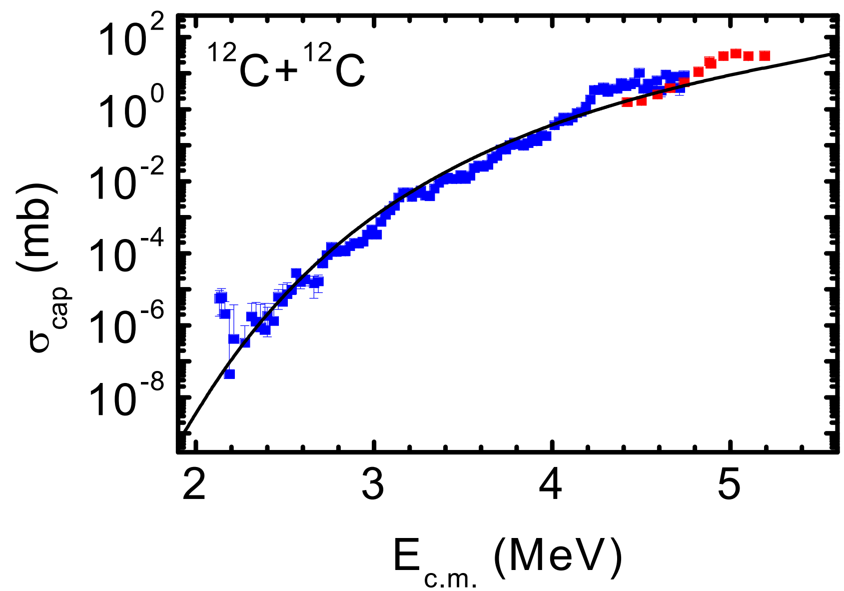
- The maximum in *S*- factor is related to the change of the slope of the excitation function when diffusion becomes dominant factor.
- This changing should be revealed in *L*-factor representation, however the *L*-factor depends on the step of the energy.

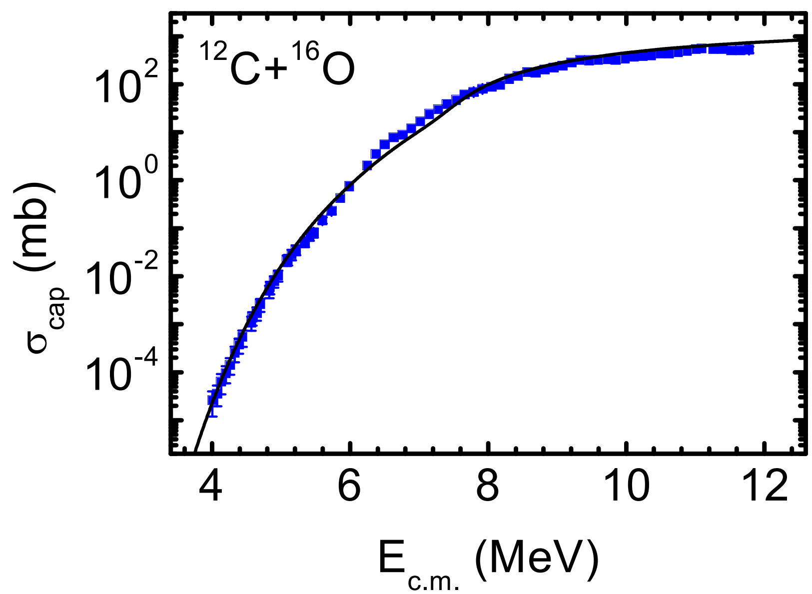
Examples of reactions with medium mass nuclei



^{16}O + ^{16}O, ^{12}C + ^{12}C and ^{12}C + ^{16}O reactions

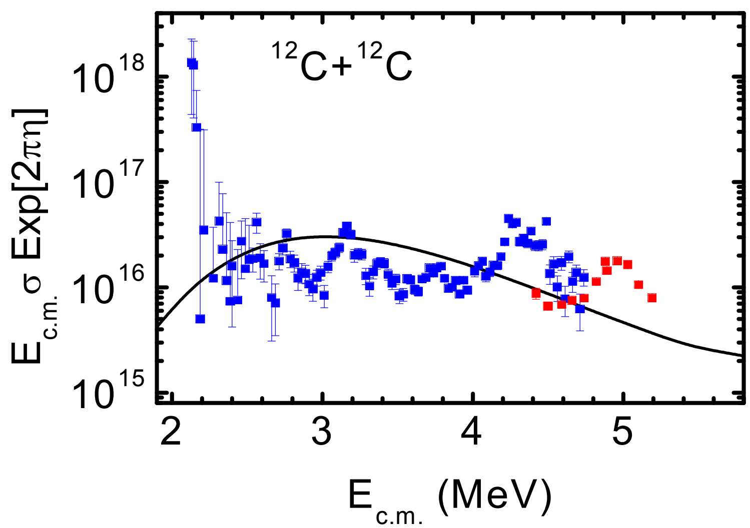


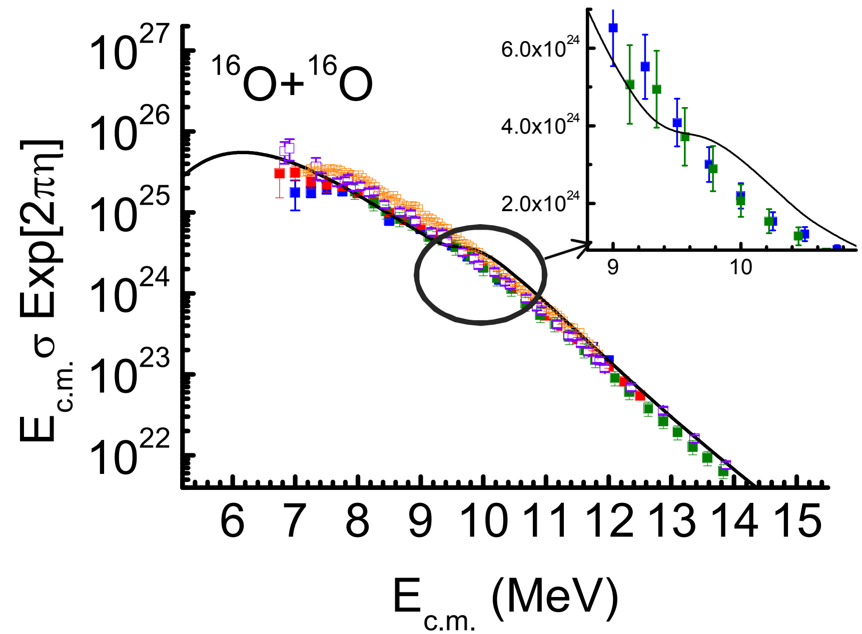


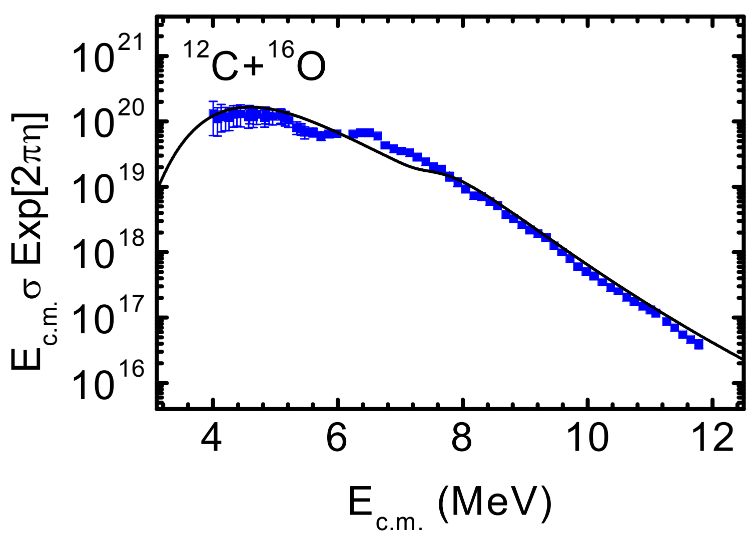


- **The main trends of fusion excitation functions of the considered reactions are well reproduced with QDA approach.**
- **Structure in excitation function --> Alexis Diaz-Torres**

The S-factor for the astrophysical reactions







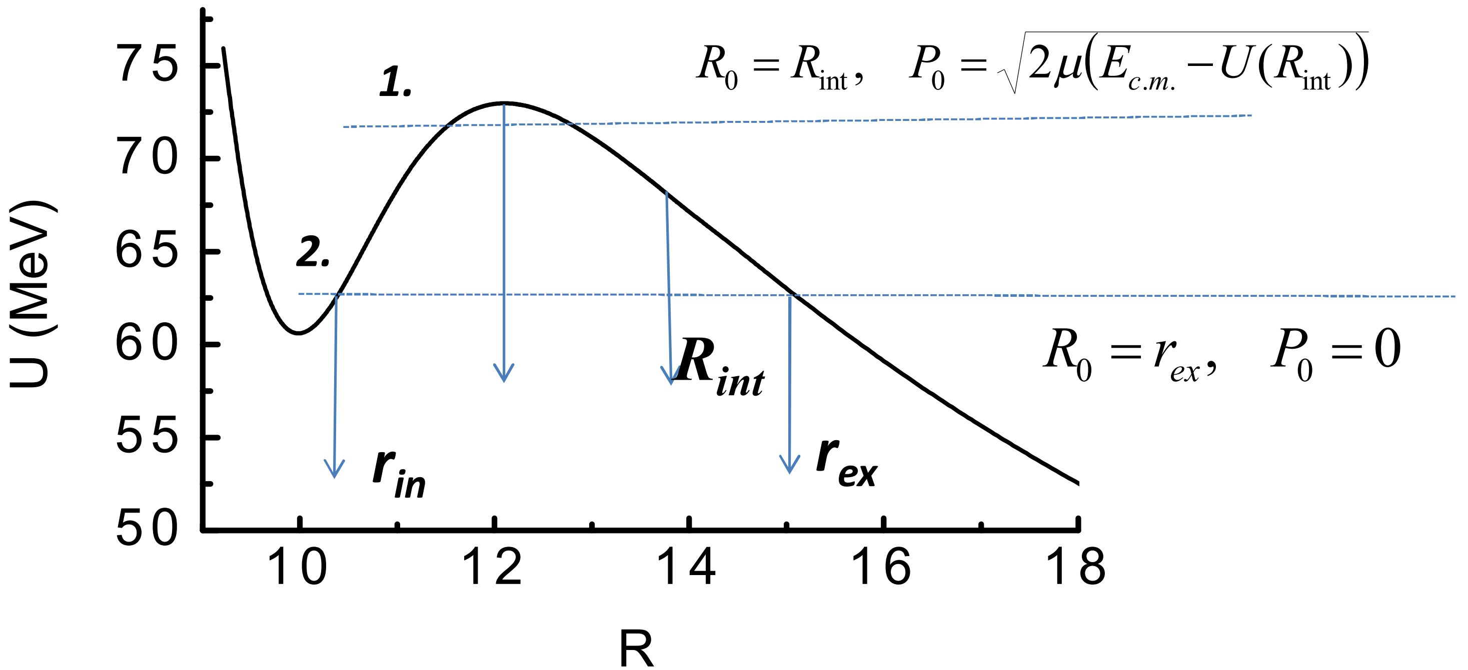
- **The same behavior of the S- factor as in the reactions with heavy and medium nuclei.**

Summary

- The quantum diffusion approach is applied to study the capture process in the reactions with spherical and deformed nuclei at sub-barrier energies. The available experimental data at energies above and below the Coulomb barrier are well described.
- Change of capture cross section after neutron transfer occurs due to change of deformations of nuclei. The neutron transfer is indirect effect of quadrupole deformation.
- Neutron transfer can enhance or suppress or weakly influence the capture cross section.
- The linear coupling limit (constant friction) is suitable for describing the reactions with heavy nuclei. To describe reactions with light nuclei one need to take into account non-linearity of the coupling (R- dependent friction).
- The maximum in S- factor related to the change of the slope of the excitation function and means that diffusion becomes dominant component.
- This changing should be reveal in L-factor representation, however the L-factor depends on the step of the energy.

Initial conditions for two regimes of interaction

The propagator of passing through the barrier: $G(R,P,t;R_0,P_0,t=0)$



Partial capture cross section: $P_{cap} = \lim_{t\to\infty} \int_{-\infty}^{r_{in}} dR \int_{-\infty}^{\infty} dP\, G(R,P,t;R_0,P_0,t=0)$

Total capture cross section: $\sigma_{cap}(E_{c.m.}) = \pi\lambda^2 \sum_J (2J+1) P_{cap}(E_{c.m.},J)$