ECT*
EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

2nd Workshop on The Proton mass:
At the Heart of Most Visible Matter
3-7 April 2017

Longitudinal and transverse spin-orbit correlations

Based on : [Bhoonah, C.L. (2017)]
[C.L. (2014)]

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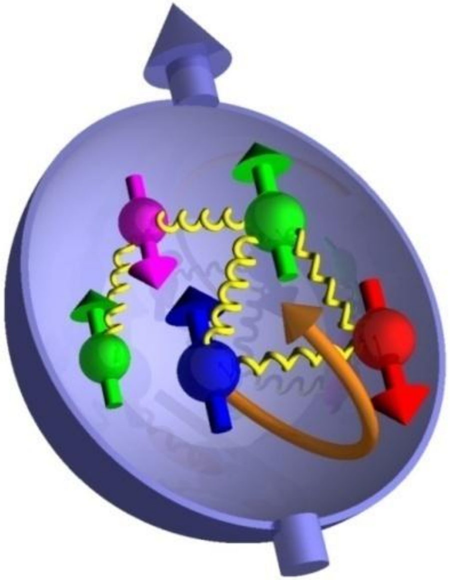
April 4, ECT*, Trento, Italy

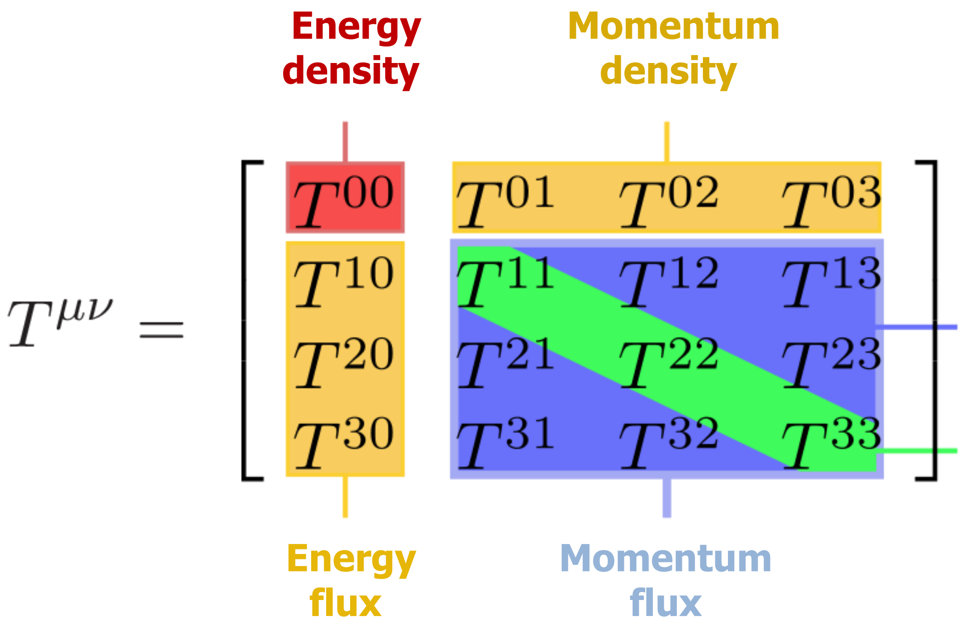
Outline



- Energy-momentum tensor
- Spin-dependent decomposition
- Link with GPDs
- Lattice and quark model estimates
- Conclusions

Energy-momentum tensor

A lot of interesting physics is contained in the EM tensor !



Energy density

Momentum density

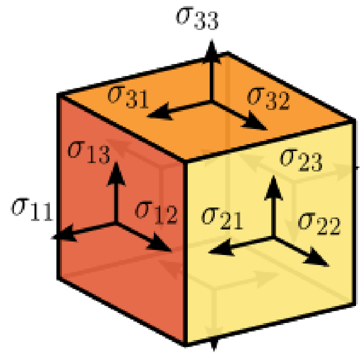
$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

Energy flux

Momentum flux

Shear stress

Normal stress **(pressure)**



Rest frame

$$M = \int \mathrm{d}^3 r \, T^{00}(\vec{r})$$

$$L^i = \int \mathrm{d}^3 r \, \epsilon^{ijk} r^j T^{0k}(\vec{r})$$

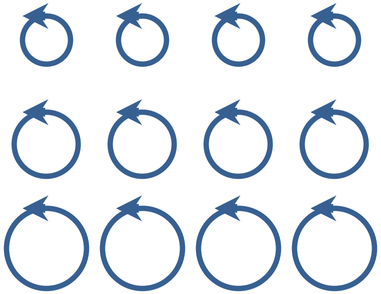
Energy-momentum tensor

In presence of spin density $T^{0i} \neq T^{i0}$

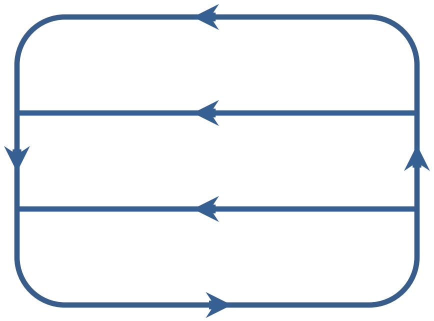
$$\begin{cases} J^{\mu\alpha\beta} = x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha} + S^{\mu\alpha\beta} \\ \partial_\mu T^{\mu\nu} = \partial_\mu J^{\mu\alpha\beta} = 0 \quad \Rightarrow \quad T^{[\alpha\beta]} = -\partial_\mu S^{\mu\alpha\beta} \end{cases}$$

Belinfante
« improvement »

$$\begin{aligned} T_B^{\mu\nu} &\equiv T^{\mu\nu} + \tfrac{1}{2}\partial_\lambda [S^{\lambda\mu\nu} + S^{\mu\nu\lambda} + S^{\nu\mu\lambda}] \\ &= T_B^{\nu\mu} \end{aligned}$$



Spin density gradient



Four-momentum circulation

Rest frame

$$M = \int \mathrm{d}^3 r\, T_B^{00}(\vec{r})$$

$$J^i = \int \mathrm{d}^3 r\, \epsilon^{ijk} r^j T_B^{0k}(\vec{r})$$

No « spin » contribution !

Energy-momentum tensor

Quark energy-momentum tensor

$$\hat{T}^{\mu\nu}_q = \overline{\psi}\gamma^\mu \tfrac{i}{2}\overleftrightarrow{D}^\nu\psi$$

$$\overline{\psi}\gamma^{[\mu}\tfrac{i}{2}\overleftrightarrow{D}^{\nu]}\psi = -\varepsilon^{\mu\nu\alpha\beta}\partial_\alpha(\overline{\psi}\gamma_\beta\gamma_5\psi)$$

based on QCD EOM

General parametrization

[Bakker, Leader, Trueman (2004)]

$$\langle p'|\hat{T}^{\mu\nu}|p\rangle = \overline{u}(p')\left[\frac{P^{\{\mu}\gamma^{\nu\}}}{2}A(t) + \frac{P^{\{\mu}i\sigma^{\nu\}\Delta}}{4M}B(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M}C(t)\right.$$

$$\left. + \underbrace{\underset{\text{Non-conservation}}{M g^{\mu\nu}\bar{C}(t)} + \underset{\text{Asymmetry}}{\frac{P^{[\mu}\gamma^{\nu]}}{2}D(t)}}_{\text{Higher twist}}\right] u(p)$$

$$\boxed{\begin{aligned} A_q + A_G &= 1 \\ B_q + B_G &= 0 \\ \bar{C}_q + \bar{C}_G &= 0 \end{aligned}}$$

Sum rules

$$J_z = \tfrac{1}{2}\left[A(0) + B(0)\right]$$

[Ji (1997)]

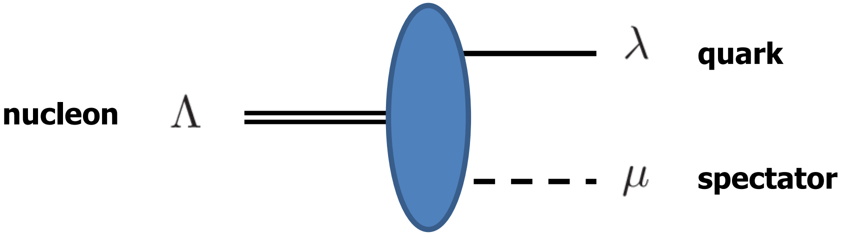
$$L_z = \tfrac{1}{2}\left[A(0) + B(0) + \underset{-2S_z}{D(0)}\right]$$

[Shore, White (2000)]

t **dependence** ⟷ **spatial distribution**

Cf. Luca Mantovani's talk

Proton spin structure

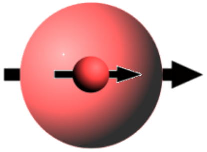


$$l_z = \Lambda - \lambda - \mu$$

Quark spin

$$\langle\langle S_z^q\rangle\rangle \sim \frac{1}{2(2\pi)^3}\sum_{\Lambda,\lambda,\mu} \Lambda\,\lambda\,|\psi^{\Lambda}_{\lambda,\mu}|^2 \sim \langle S_z^N S_z^q\rangle$$

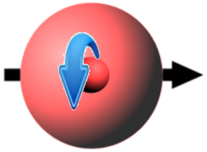
$\overline{\psi}\gamma^+\gamma_5\psi$



Quark OAM

$$\langle\langle L_z^q\rangle\rangle \sim \frac{1}{2(2\pi)^3}\sum_{\Lambda,\lambda,\mu} \Lambda\,l_z\,|\psi^{\Lambda}_{\lambda,\mu}|^2 \sim \langle S_z^N L_z^q\rangle$$

$\overline{\psi}\gamma^+(\vec{r}_\perp \times i\vec{D}_\perp)_z\psi$



Quark spin-orbit correlation

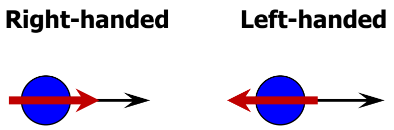
$$\langle\langle C_z^q\rangle\rangle \sim \frac{1}{2(2\pi)^3}\sum_{\Lambda,\lambda,\mu} \lambda\,l_z\,|\psi^{\Lambda}_{\lambda,\mu}|^2 \sim \langle S_z^q L_z^q\rangle$$

$\overline{\psi}\gamma^+\gamma_5(\vec{r}_\perp \times i\vec{D}_\perp)_z\psi$

[C.L., Pasquini (2011)]
[C.L. (2014)]

Parity-odd energy-momentum tensor

Chiral decomposition



$$\hat{T}^{\mu\nu} = \overline{\psi}\gamma^\mu \tfrac{i}{2}\overset{\leftrightarrow}{D}^\nu \psi \qquad \hat{T}^{+\nu} = \hat{T}^{+\nu}_R + \hat{T}^{+\nu}_L$$

$$\hat{T}^{\mu\nu}_5 = \overline{\psi}\gamma^\mu\gamma_5 \tfrac{i}{2}\overset{\leftrightarrow}{D}^\nu \psi \qquad \hat{T}^{+\nu}_5 = \hat{T}^{+\nu}_R - \hat{T}^{+\nu}_L$$

$$\hat{T}^{+\nu}_a = \overline{\psi}_a\gamma^+ \tfrac{i}{2}\overset{\leftrightarrow}{D}^\nu \psi_a \qquad a = R, L$$

$$\psi_{R,L} = \tfrac{\mathbb{1}\pm\gamma_5}{2}\,\psi$$

General parametrization

[C.L. (2014)]

$$\langle p'|\hat{T}^{\mu\nu}_5|p\rangle = \overline{u}(p')\left[\frac{P^{\{\mu}\gamma^{\nu\}}\gamma_5}{2}\,\tilde{A}(t) + \frac{P^{\{\mu}\Delta^{\nu\}}\gamma_5}{4M}\,\tilde{B}(t)\right.$$

$$\left. + \frac{P^{[\mu}\gamma^{\nu]}\gamma_5}{2}\,\tilde{C}(t) + \frac{P^{[\mu}\Delta^{\nu]}\gamma_5}{4M}\,\tilde{D}(t) + Mi\sigma^{\mu\nu}\gamma_5\tilde{F}(t)\right] u(p)$$

Higher twist

Longitudinal spin-orbit correlations

$$C_z = \tfrac{1}{2}\,[\tilde{A}(0) + \tilde{C}(0)] \qquad \textbf{Asymmetric}$$

$$\mathbb{C}_z = \tfrac{1}{2}\,\tilde{A}(0) \qquad \textbf{Belinfante}$$

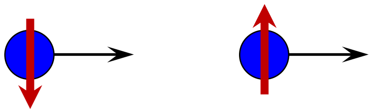
Chiral-odd energy-momentum tensor

Transversity decomposition

$$\hat{T}^{\mu\nu} = \overline{\psi}\gamma^\mu \tfrac{i}{2}\overleftrightarrow{D}^\nu\psi \qquad \hat{T}^{+\nu} = \hat{T}^{+\nu}_\uparrow + \hat{T}^{+\nu}_\downarrow$$

$$\hat{T}_5^{\lambda\mu\nu} = \overline{\psi} i\sigma^{\lambda\mu}\gamma_5 \tfrac{i}{2}\overleftrightarrow{D}^\nu\psi \qquad \hat{T}_5^{j+\nu} = \hat{T}^{+\nu}_\uparrow - \hat{T}^{+\nu}_\downarrow$$

Transversity basis



$$\hat{T}^{+\nu}_a = \overline{\psi}_a\gamma^+ \tfrac{i}{2}\overleftrightarrow{D}^\nu\psi_a \qquad a = \uparrow, \downarrow$$

$$\psi_{\uparrow,\downarrow} = \frac{\mathbb{1} \pm \gamma^j\gamma_5}{2}\,\psi$$

General parametrization

[Bhoonah, C.L. (2017)]

$$\begin{aligned}\langle p'|\hat{T}_5^{\lambda\mu\nu}|p\rangle = \overline{u}(p')\Big[&\frac{P^\nu P^{[\lambda}\Delta^{\mu]}\gamma_5}{2M^2}\,A_T(t) + \frac{P^\nu P^{[\lambda}\gamma^{\mu]}\gamma_5}{M}\,B_T(t)\\ &+ \frac{\Delta^\nu\Delta^{[\lambda}\gamma^{\mu]}\gamma_5}{4M}\,C_T(t) + P^\nu i\sigma^{\lambda\mu}\gamma_5\,D_T(t)\\ &+ \frac{g^{\nu[\lambda}\Delta^{\mu]}\gamma_5}{2}\,\tilde{A}_T(t) + M\,g^{\nu[\lambda}\gamma^{\mu]}\gamma_5\,\tilde{B}_T(t) + \frac{P^{[\lambda}i\sigma^{\mu\nu]}\gamma_5}{2}\,\tilde{D}_T(t)\Big]\,u(p)\end{aligned}$$

Higher twist

Transverse spin-orbit correlations

$$C_j = -\frac{M}{2\sqrt{2}P^+}\left[B_T(0) + 2\tilde{B}_T(0) + 4\tilde{D}_T(0)\right]$$ **Asymmetric**

$$\mathbb{C}_j = -\frac{M}{2\sqrt{2}P^+}\left[B_T(0) + 2\tilde{B}_T(0) - 2D_T(0)\right]$$ **Belinfante**

Burkardt's correlation

Instant-form correlation with Belinfante tensor

$$\langle \delta^x J^x \rangle = \mathbb{C}_j|_{\mathrm{IF}}$$

Transversity asymmetry of total AM

Rest frame $\langle \delta^x J^x \rangle = \frac{1}{2} D_T(0)$ [Burkardt (2005)] [Burkardt (2006)]

Moving frame $\langle \delta^x J^x \rangle = \frac{1}{2} \left[\frac{E-M}{M} B_T(0) + D_T(0)\right]$ [Bhoonah, C.L. (2017)]

Transverse total AM

Rest frame $\langle J^x \rangle = \frac{1}{2} [A(0) + B(0)]$ [Ji (1997)] [Burkardt (2005)]

Moving frame $\langle J^x \rangle = \frac{1}{2} \left\{\frac{E-M}{M} B(0) + [A(0) + B(0)]\right\}$ [Leader (2012)]

Link with GPDs

GPD correlators

$$F^{[\Gamma]} = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\, \langle p'|\overline{\psi}(-\tfrac{z^-}{2})\Gamma\underset{\uparrow}{\mathcal{W}}\psi(\tfrac{z^-}{2})|p\rangle$$

Straight LF Wilson line

Mellin moment of GPDs

$$\frac{1}{2(P^+)^2}\,\langle p'|\hat{T}^{\mu+}|p\rangle = \int \mathrm{d}x\, x\, F^{[\gamma^\mu]}$$

Vector

$$\frac{1}{2(P^+)^2}\,\langle p'|\hat{T}_5^{\mu+}|p\rangle = \int \mathrm{d}x\, x\, F^{[\gamma^\mu\gamma_5]}$$

Axial-vector

$$\frac{1}{2(P^+)^2}\,\langle p'|\hat{T}_5^{\lambda\mu+}|p\rangle = \int \mathrm{d}x\, x\, F^{[i\sigma^{\lambda\mu}\gamma_5]}$$

Tensor

Leading twist ($\mu=+$)

Vector

$$\int \mathrm{d}x\, x\, H(x,\xi,t) = A(t) + 4\xi^2 C(t)$$

$$\int \mathrm{d}x\, x\, E(x,\xi,t) = B(t) - 4\xi^2 C(t)$$

[Ji (1997)]

Axial-vector

$$\int \mathrm{d}x\, x\, \tilde{H}(x,\xi,t) = \tilde{A}(t)$$

$$\int \mathrm{d}x\, x\, \tilde{E}(x,\xi,t) = \tilde{B}(t)$$

[Hägler (2004)]
[Diehl, Hägler (2005)]

Tensor

$$\int \mathrm{d}x\, x\, H_T(x,\xi,t) = -\tfrac{t}{4M^2}\, A_T(t) - B_T(t) + D_T(t)$$

$$\int \mathrm{d}x\, x\, E_T(x,\xi,t) = A_T(t) + B_T(t)$$

$$\int \mathrm{d}x\, x\, \tilde{H}_T(x,\xi,t) = -\tfrac{1}{2}\, A_T(t)$$

$$\int \mathrm{d}x\, x\, \tilde{E}_T(x,\xi,t) = -\xi C_T(t)$$

[Hägler (2004)]
[Diehl, Hägler (2005)]

Sub-leading twist

Parametrization **[Meissner, Metz, Schlegel (2009)]**

Vector

$$\int \mathrm{d}x\, x\, H_{2T}(x,\xi,t) = 0$$

$$\int \mathrm{d}x\, x\, E_{2T}(x,\xi,t) = 0$$

$$\int \mathrm{d}x\, x\, \tilde{H}_{2T}(x,\xi,t) = -2\xi C(t)$$

$$\int \mathrm{d}x\, x\, \tilde{E}_{2T}(x,\xi,t) = -\tfrac{1}{2}\,[A(t) + B(t) - D(t)]$$

[Penttinen *et al.* (2000)]
[Kiptily, Polyakov (2004)]

Axial-vector

$$\int \mathrm{d}x\, x\, H'_{2T}(x,\xi,t) = \tfrac{1}{2}\,[\tilde{A}(t) - \tilde{C}(t) + 2\tilde{F}(t)] + \tfrac{t}{4M^2}\,\tfrac{1}{2}\,[\tilde{B}(t) - \tilde{D}(t)]$$

$$\int \mathrm{d}x\, x\, E'_{2T}(x,\xi,t) = -\tfrac{1}{2}\,[\tilde{A}(t) + \tilde{B}(t) - \tilde{C}(t) - \tilde{D}(t)]$$

$$\int \mathrm{d}x\, x\, \tilde{H}'_{2T}(x,\xi,t) = \tfrac{1}{4}\,[\tilde{B}(t) - \tilde{C}(t)]$$

$$\int \mathrm{d}x\, x\, \tilde{E}'_{2T}(x,\xi,t) = 0$$

[Penttinen *et al.* (2000)]
[Kiptily, Polyakov (2004)]

Tensor

$$\int \mathrm{d}x\, x\, H'_{2}(x,\xi,t) = -\xi\,[\tfrac{t}{4M^2}\, C_T(t) + D(t) + \tilde{D}_T(t)]$$

$$\int \mathrm{d}x\, x\, E'_{2}(x,\xi,t) = \xi\,[C_T(t) + D(t) + \tilde{D}_T(t)]$$

$$\int \mathrm{d}x\, x\, \tilde{H}'_{2}(x,\xi,t) = -[(1 - \tfrac{t}{4M^2})\, B_T(t) + \tilde{B}(t) - D_T(t)]$$

$$\int \mathrm{d}x\, x\, \tilde{E}'_{2}(x,\xi,t) = \xi\,[(1 - \tfrac{t}{4M^2})\, A_T(t) + \tilde{A}(t) + D_T(t)]$$

[Bhoonah, C.L. (2017)]

Relations between form factors

Vector

$$\overline{\psi}\gamma^{[\mu}i\overleftrightarrow{D}^{\nu]}\psi = -\varepsilon^{\mu\nu\alpha\beta}\partial_\alpha(\overline{\psi}\gamma_\beta\gamma_5\psi)$$

[C.L., Mantovani, Pasquini (in preparation)]

$$\boxed{D(t) = -G_A(t)}$$

Axial-vector

$$\overline{\psi}\gamma^{[\mu}\gamma_5 i\overleftrightarrow{D}^{\nu]}\psi = 2m\,\overline{\psi}i\sigma^{\mu\nu}\gamma_5\psi - \varepsilon^{\mu\nu\alpha\beta}\partial_\alpha(\overline{\psi}\gamma_\beta\psi)$$

[C.L. (2014)]

$$\boxed{\begin{aligned}\tilde{C}(t) &= \tfrac{m}{2M}\,H_1(t) - F_1(t)\\ \tilde{D}(t) &= \tfrac{m}{2M}\,H_2(t) - F_2(t)\\ \tilde{F}(t) &= \tfrac{m}{2M}\,H_3(t) - \tfrac{1}{2}\left[F_1(t) + \tfrac{t}{4M^2}\,F_2(t)\right]\end{aligned}}$$

Tensor

$$\overline{\psi}i\sigma^{[\lambda\mu}\gamma_5 i\overleftrightarrow{D}^{\nu]}\psi = -2\varepsilon^{\lambda\mu\nu\alpha}\partial_\alpha(\overline{\psi}\psi)$$

$$\overline{\psi}i\sigma^{\lambda\mu}\gamma_5 i\overleftrightarrow{D}_\mu\psi = 2m\,\overline{\psi}\gamma^\lambda\gamma_5\psi + i\partial^\lambda(\overline{\psi}\gamma_5\psi)$$

[Bhoonah, C.L. (2017)]

$$\boxed{\begin{aligned}D_T(t) + 3\tilde{D}_T(t) &= \Sigma(t)\\ -[(1-\tfrac{t}{4M^2})\,B_T(t) + 3\tilde{B}_T(t) + \tfrac{t}{4M^2}\,C_T(t) - D_T(t)] &= \tfrac{m}{M}\,G_A(t)\\ -[(1-\tfrac{t}{4M^2})\,A_T(t) + 3\tilde{A}_T(t) - C_T(t) + D_T(t)] &= \tfrac{m}{M}\,G_P(t) - \Pi(t)\end{aligned}}$$

Interpretation of leading-twist relations

Longitudinal OAM

$$L_z = \tfrac{1}{2}\int \mathrm{d}x\, x\,[H(x,0,0)+E(x,0,0)] - \tfrac{1}{2}\,G_A(0)$$

$$\rightsquigarrow \quad \langle L_z S^N_z\rangle = \langle J_z S^N_z\rangle - \langle S_z S^N_z\rangle$$

[Ji (1997)]
[Shore, White (2000)]

Longitudinal spin-orbit correlation

$$C_z = \tfrac{1}{2}\int \mathrm{d}x\, x\,\tilde{H}(x,0,0) - \tfrac{1}{2}\,[F_1(0) - \tfrac{m}{2M}\,H_1(0)]$$

$$\rightsquigarrow \quad \langle L_z S_z\rangle = \langle J_z S_z\rangle - \langle S_z S_z\rangle$$

[C.L. (2014)]

Transverse spin-orbit correlation

$$\frac{\sqrt{2}P^+}{M}\,C_j = \tfrac{1}{3}\int \mathrm{d}x\, x\,[H_T(x,0,0)+\tfrac{1}{2}\,\bar{E}_T(x,0,0)] - \tfrac{2}{3}\,[\Sigma(0) - \tfrac{m}{2M}\,G_A(0)]$$

$$\rightsquigarrow \quad \langle L_j T_j\rangle = \langle J_j T_j\rangle - \langle S_j T_j\rangle$$

[Bhoonah, C.L. (2017)]

Some figures

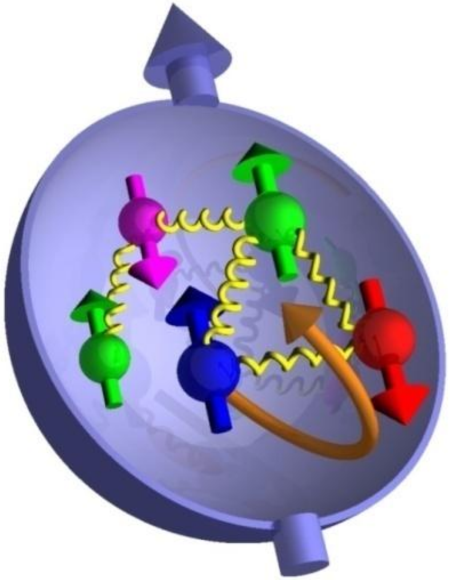
		L_z^u	L_z^d	C_z^u	C_z^d	C_j^u	C_j^d
$\mu^2 \approx 0.26\ \text{GeV}^2$	LFCQM	0.071	0.055	-0.84	-0.54	×	×
	LFχQSM	-0.008	0.077	-0.80	-0.55	×	×
$\mu^2 = 4\ \text{GeV}^2$	Lattice	-0.175	0.205	-0.90	-0.53	-3.6	-2.2

LFCQM, LFχQSM	**[C.L., Pasquini, Vanderhaeghen (2011)]** **[C.L. (2014)]**
Lattice QCD	**[Göckeler *et al.* (2005)]** **[Göckeler *et al.* (2007)]** **[Bratt *et al.* (2010)]** **[Abdel-Rehim *et al.* (2015)]**

☞ **Spin and kinetic OAM of valence quarks are anti-correlated !**

Conclusions

Take home message



- In presence of spin density, EMT is asymmetric
- EMT can be decomposed according to parton polarization
- Spin-orbit correlations complementary to nucleon spin sum rule
- Information encoded in GPDs and standard form factors
- Quark models and Lattice suggest negative spin-orbit correlations