Heavy Quarkonium suppression in a fireball

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Work done in collaboration with N. Brambilla, J. Soto and A. Vairo

Outline

Introduction

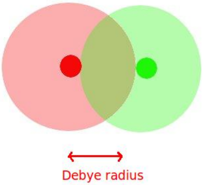
The original idea of Matsui and Satz (1986)

- Quarkonia is quite stable in the vacuum.
- Phenomena of colour screening, quantities measurable in Lattice QCD at finite temperature (static) support this. For example Polyakov loop.
- Dissociation of heavy quarkonium in heavy-ion collisions due to colour screening signals the creation of a quark-gluon plasma.

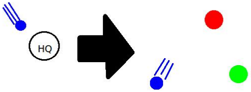
Colour screening

$$V(r) = -\alpha_s \frac{e^{-m_D r}}{r}$$

At finite temperature



Another mechanism, the decay width



- This effect makes the peak in the spectral function broader. It can arrive to a point where it is so broad that it does not make sense to speak of a bound state anymore.

Laine et al. perturbative potential (2007)

$$V(r) = -\alpha_s C_F \left[m_D + \frac{e^{-m_D r}}{r} \right] - i\alpha_s T C_F \phi(m_D r)$$

with

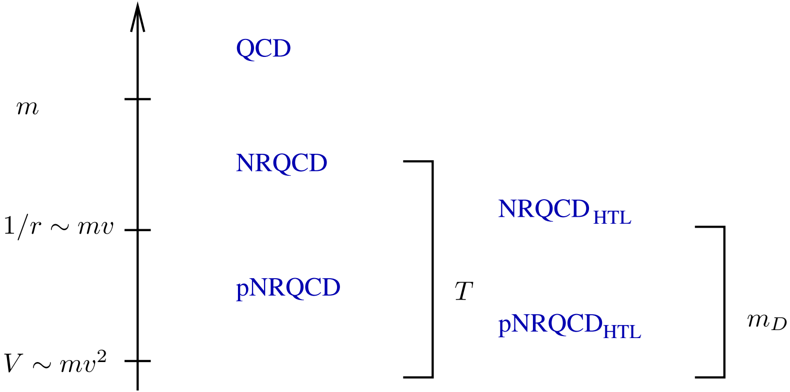
$$\phi(x) = 2\int_0^\infty \frac{dzz}{(z^2+1)^2}\left(1 - \frac{\sin(zx)}{zx}\right)$$

- This potential was obtained through the Wilson loop in Minkownski space at finite temperature.
- It has an imaginary part that has to be related with a decay width.

Question

- We talked in a generic way of a potential.
- It is historically assumed that HQ in a medium follows a Schrödinger eq. How do we show this from first principles? What is the potential that one has to put in?
- Use modern EFT techniques that have been succesful in $T = 0$ computations.

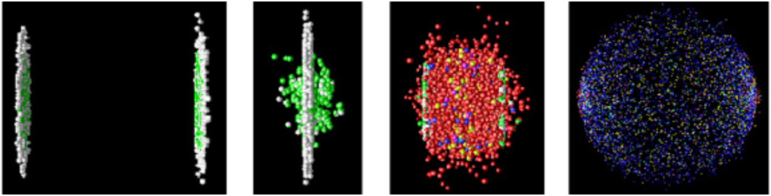
Effective field theories



(Brambilla, Ghiglieri, Petreczky And Vairo, M. A. E and Soto)
Talks of P. Petreczky and S. Biondini in this workshop.

Out of equilibrium

We have a explosion



Assumption 1:

- Medium homogeneous in space and isotropic with an effective temperature T_{eff}.

What is measured?

- HQ is detected by its decay into leptons.
- Electromagnetic process, much slower than the physics that happens inside the fireball.

Assumption 2:

- HQ is measured throught the decay into leptons that happens after freeze-out.

What is measured?

$$\int d^4x\, d^4y e^{i(k_1+k_2)(x_1-x_2)} Tr(\rho J^\mu(x) J_\mu(y))$$

(in thermal eq. McLerran and Toimela (1985))
where

- J_μ is the electromagnetic current and we focuss on the component given by HQs.
- k_1 and k_2 is the momentum of the out-going leptons.
- x_0, $y_0 \gg t_{FO}$, where t_{FO} is the time in which we arrive to freeze-out.

Assumption 2:(in a more precise way)
In computing the lepton emission rate the integration from $x_0 = 0$ to $x_o = t_{FO}$ is negligible. The same happens for y_0.

What is measured?

$$\int d^4x\, d^4y e^{i(k_1+k_2)(x_1-x_2)} Tr(\rho J^\mu(x) J_\mu(y))$$

where

- J_μ is the electromagnetic current and we focuss on the component given by HQs.
- k_1 and k_2 is the momentum of the out-going leptons.
- $x_0,\ y_0 \gg t_{FO}$, where t_{FO} is the time in which we arrive to freeze-out.

Setting of the problem
Compute $Tr(\rho J^\mu(t_{FO},\mathbf{x}) J_\mu(t_{FO},\mathbf{y}))$ assuming that we know $Tr(\rho J^\mu(t_0,\mathbf{x}) J_\mu(t_0,\mathbf{y}))$ at some previous time. Assume that after t_{FO} evolution is like in the vacuum.

The case $\frac{1}{r} \gg T_{eff}$

Electromagnetic current in NRQCD

Because $M \gg \frac{1}{r} \gg T_{eff}$ we can start with the NRQCD Lagrangian at $T = 0$.
We also need to know what is J_μ in NRQCD.

$$J^0(x) = Q(\psi^\dagger(x)\psi(x) - \chi^\dagger(x)\chi(x))$$

The Fourier transform of this part does not contribute to lepton emission. Emission of soft photons.

Electromagnetic current in NRQCD

Because $M \gg \frac{1}{r} \gg T_{eff}$ we can start with the NRQCD Lagrangian at $T = 0$.

We also need to know what is J_μ in NRQCD.

$$J^i(x) = Q(e^{i2m_Q t}\psi^\dagger(x)\sigma^i\chi(x) - e^{-i2m_Q t}\chi^\dagger(x)\sigma^i\psi(x))$$

Color singlet with a spin $S = 1$.

$$Tr(\rho\psi^\dagger(x)\sigma^i\chi(x)\chi^\dagger(y)\sigma^i\psi(y))$$

Electromagnetic current in pNRQCD

Because $\frac{1}{r} \gg T_{eff}$ we can start with pNRQCD Lagrangian at $T = 0$.
Now we need the electromagnetic current in pNRQCD

$$\chi_i^\dagger(x)\sigma^i\psi_j(x) \rightarrow S_{ij}(x)$$

i and j are spinorial index that we normally do not write. Isotropy assumes that all directions of polarization are equally probable.

$$Tr(\rho S^\dagger(x)S(y))$$

- S has to be understood as the projection to the spin 1 state.
- It is the number of particles operator, which makes sense.

Ordering matters

In previous works we computed the time-ordered correlator

$$\langle \mathcal{T} S(y) S^\dagger(x) \rangle \,,$$

gives information about the binding energy and the decay width. Compare with lattice and check if you are describing well thermal equilibrium.
Now we compute

$$Tr(\rho S^\dagger(x) S(y)) \,,$$

information about the distribution of particles. Relevant to compare with experiments.
pNRQCD is a QFT, so is able to deal with the two types of correlators.

The Lagrangian of pNRQCD, transformation

$$\mathcal{L}_{pNRQCD} = \int d^3\mathbf{r}\, Tr\left[S^\dagger\left(i\partial_0 - h_s\right)S \right.$$
$$\left. + O^\dagger\left(iD_0 - h_o\right)O\right] + V_A(r)Tr(O^\dagger \mathbf{r} g\mathbf{E} S + S^\dagger \mathbf{r} g \mathbf{E} O)$$
$$+ \frac{V_B(r)}{2} Tr(O^\dagger \mathbf{r} g \mathbf{E} O + O^\dagger O \mathbf{r} g \mathbf{E}) + \mathcal{L}_g + \mathcal{L}_q$$

We can consider a transformation

$$O(t) \to \Omega(t) O(t) \Omega^\dagger(t)$$
$$E^i(t) \to \Omega(t) E^i \Omega^\dagger(t)$$

such that $i\partial_t \Omega(t) = gA_0(t)\Omega(t)$.

The Lagrangian of pNRQCD, transformation

Ω can be a Wilson line connecting some time t' with t.

$$\mathcal{L}_{pNRQCD} = \int d^3\mathbf{r}\, Tr\left[S^\dagger\left(i\partial_0 - h_s\right)S\right.$$
$$\left.+O^\dagger\left(i\partial_0 - h_o\right)O\right] + V_A(r)\,Tr(O^\dagger\mathbf{r}g\mathbf{E}S + S^\dagger\mathbf{r}g\mathbf{E}O)$$
$$+\frac{V_B(r)}{2}Tr(O^\dagger\mathbf{r}g\mathbf{E}O + O^\dagger O\mathbf{r}g\mathbf{E}) + \mathcal{L}_g + \mathcal{L}_q$$

Equivalent to doing the computation in the temporal gauge.

- Advantage: It simplifies the current computation a lot.
- Disadvange: It sweeps the difficulties into the determination of the initial conditions.

Evolution of the number of singlets

$$f_s(x,y) = Tr(\rho S^\dagger(x) S(y))$$

We can use perturbation theory but expanding in r instead of α_s. In the interaction picture

$$i\partial_t S = [S, H_0]$$
$$i\partial_t \rho = [H_I, \rho]$$

Assumption 3:
We assume that HQ is comoving with the medium and that the center of mass momentum is not changed.

Evolution of the number of singlets

$$\partial_t f_S = -i(H_{eff} f_s - f_s H^\dagger_{eff}) + \mathcal{F}(f_o)$$

- $H_{eff} = h_s + \Sigma$ where Σ corresponds with the self-energy that can be obtained in pNRQCD by computing the time-ordered correlator.
- $\mathcal{F}(f_o)$ is a new term that takes into account the process $O \to g + S$. In ensures that the total number of heavy quarks is conserved.
- $\mathcal{F}(f_o)$ is a complicated function of $Tr(\rho O^\dagger O)$ and $\langle E^i E^j \rangle$. The information about the medium enters only in the chromoelectric field correlator. It will not be a Markovian process for any correlator.

Evolution of the number of singlets

Define

$$H = \frac{H_{eff} + H_{eff}^\dagger}{2}$$
$$\Gamma = i(H_{eff} - H_{eff}^\dagger)$$

$$\partial_t f_s = -i[H, f_s] - \frac{1}{2}\{\Gamma, f_s\} + \mathcal{F}(f_o)$$

- Screening.
- Decay.
- Creation.

Evolution of the number of singlets

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- Screening.
- Decay.
- Creation.

Evolution of the octet

Very similar reasoning.

$$f_o^{ab}(x,y) = Tr(\rho O^{\dagger,a}(x) O^b(y))$$

$$\partial_t f_o = -i[H_o, f_o] - \frac{1}{2}\{\Gamma, f_o\} + \mathcal{F}_1(f_s) + \mathcal{F}_2(f_o)$$

Remark:
We have this simple form because $\frac{1}{f_o}\partial_t f_o \ll E$ and we have this result because of the field redefinition we made.

Conservation of number of heavy quarks

$$\partial_t Tr(f_s) + \partial_t Tr(f_o) = 0$$

Ensured by

- Because we did not consider contact interactions in NRQCD that represent anhilation.
- Optical theorem that relates decay width and cross-section.

Application

Lindblad equation

$$\partial_t \rho = -i[H,\rho] + \sum_k (C_k \rho C_k^\dagger - \frac{1}{2}\{C_k^\dagger C_k, \rho\})$$

- Used in open quantum systems and quantum optics. Numerical libraries available to solve it (we used qutip (Johansson, Nation and Nari (2012)).
- Introduced in the world of quarkonium by Akamatsu (2014).
- There is no prescription to find C_k or to tell how many are there.

Further simplification

$$f_s(t, \mathbf{r_x}, \mathbf{r_y})$$

Huge matrix. We can simplify it by making an expansion in spherical armonics and cutting at some point.

- If the initial condition is diagonal in the spherical harmonics space it will remain always so.
- Including up to p-wave gives a good result if you are interested in s-wave. It does not change the results a lot to include also the d-wave.
- Similar arguments apply to f_o in color space. $f_o^{ab} \propto \delta^{ab}$.

Initial conditions

Simple assumption:

- HQ is created at $t = 0$ with a the same probability as in pp collisions.
- Power counting of NRQCD tell us that at LO it will be a Dirac delta function.
- Perturbative processes creating a singlet are $\alpha_s(M_Q)$ suppressed with respect to octets.

We use

$$f_s = \alpha_s(M_Q)\delta^3(\mathbf{r_x} - \mathbf{r_y})$$
$$f_o = \frac{\delta^{ab}\delta}{8}\delta^3(\mathbf{r_x} - \mathbf{r_y})$$

After we normalize. We try different values of δ to see how big is the dependency $\delta = 0.1, 1, 10$.

Approximation to R_{AA}

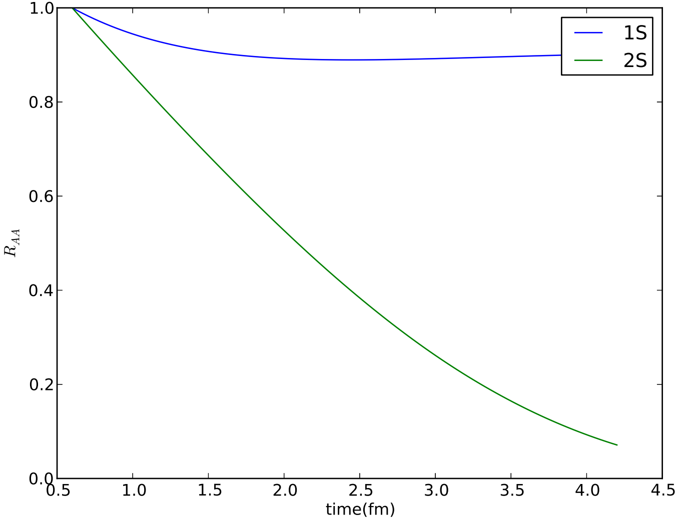
- We assume that the initia probability (up to a factor) is equal in pp and in AA.
- Both evolve in the vacuum up to $t = 0.6fm$. At that moment HQ starts to feel a thermal medium with $T = 475MeV$ that follows Bjorken evolution.
- The ratio of $\langle 1S|f_s|1S\rangle$ in AA and pp so computed will be our approximation to R_{AA}.
- Qualitative idea. We neglect CNM effects, energy loss, asymmetry, viscosity, very naive initial condition...

Toy model for $\frac{1}{r} \gg T_{eff} \gg m_D \gg E$

Nice because at LO $C^i = Ar^i$ where A is some constant (that is proportional to $\sqrt{\kappa}$).
But if you use the perturbative coupling α_s at LHC temperatures you get negative decay widths.
Toy model:do a more or less educated guess about the value of A.

Toy model for $\frac{1}{r} \gg T_{eff} \gg m_D \gg E$

Considering $C^i = 0$. Only screening.



Toy model for $\frac{1}{r} \gg T_{eff} \gg m_D \gg E$

Singlet to octet and viceversa.

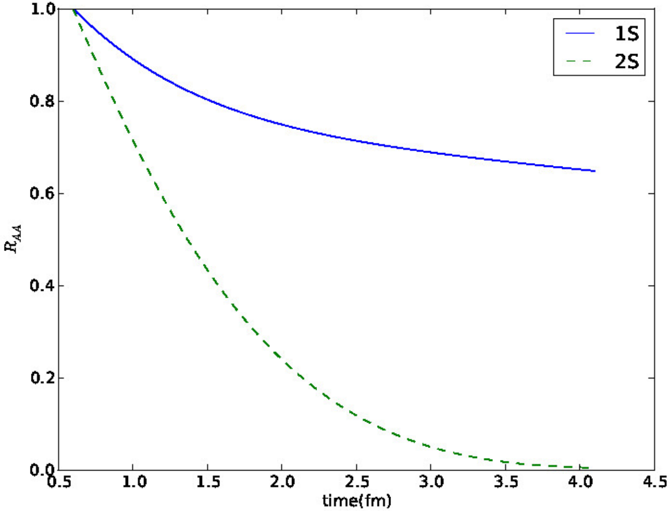
$$C_i^{so} = \sqrt{\frac{2C_F\alpha_s(2\pi T)T}{9}} m_D r_i \log(2) \begin{pmatrix} 0 & \frac{1}{\sqrt{N_c^2-1}} \\ 1 & 0 \end{pmatrix}$$

Octet to octet.

$$C_i^{oo} = \sqrt{\frac{(N_c^2-4)\alpha_s(2\pi T)T}{18N_c}} m_D r_i \log(2) \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

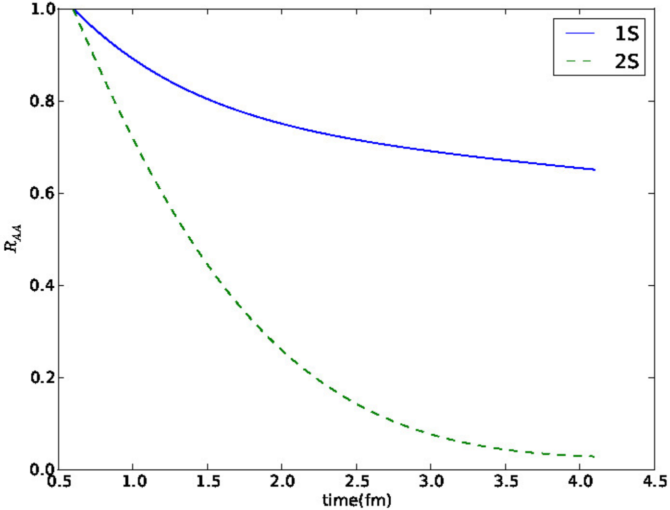
Toy model for $\frac{1}{r} \gg T_{eff} \gg m_D \gg E$

Only S and P-wave



Toy model for $\frac{1}{r} \gg T_{eff} \gg m_D \gg E$

Also D-wave

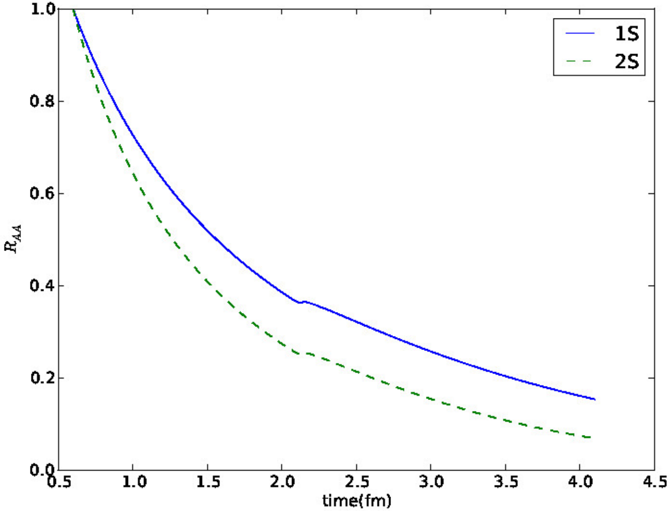


The case $\frac{1}{r} \gg T_{eff} \gg E \gg m_D$

- In this case we do not encounter negative decay width with realistic couplings. We use $\alpha_s(2\pi T)$.
- We computed in JHEP1009(2010)038 (Brambilla, M.A.E, Ghiglieri, Soto and Vairo) the corrections to the singlet binding energy and decay width. Now we also need the octet ones. Straightforward computation.
- In is not trivial to write our equations in Lindblad form with a reasonable number of collapse operators. We need to assume quasistatic limit ($\frac{1}{T}\frac{dT}{d\tau} \ll E$)..

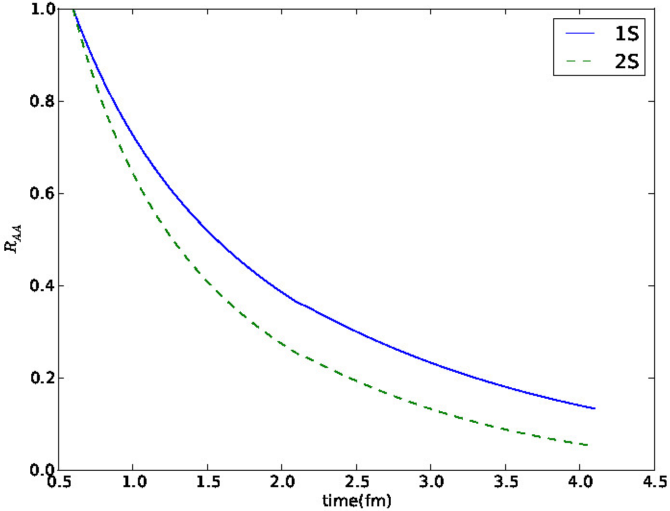
The case $\frac{1}{r} \gg T_{eff} \gg E \gg m_D$

Using $\delta = 1$.



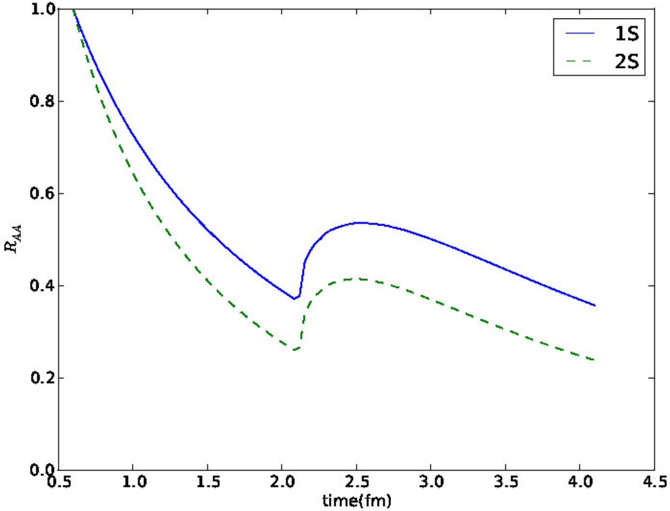
The case $\frac{1}{r} \gg T_{eff} \gg E \gg m_D$

Using $\delta = 0.1$.



The case $\frac{1}{r} \gg T_{eff} \gg E \gg m_D$

Using $\delta = 10$.



Conclusions

Conclusions

- R_{AA} is related with the number of singlets operator in pNRQCD. In the $\frac{1}{r} \gg T$ case a simple set of equations can be found without assuming weak coupling. All information encoded in an Hermitian effective Hamiltonian and in the correlator of chromoelectric fields.
- In some cases these equations can be written in a Lindblad form, as previously seen by Akamatsu in the high temperature regime.
- The octet thermal modifications are very important. We need to include more realistic initial conditions.
- We show results for different temperature regimes.