Thermal correlation functions in gauge theories at general frequencies

Felix Ziegler
in collaboration with Jan M. Pawlowski and Alexander Rothkopf
Heidelberg University

Pawlowski, Rothkopf, arXiv:1610.09531 [hep-lat]
Pawlowski, Rothkopf, Ziegler, in preparation

Phase diagram of strongly interacting matter:
From Lattice QCD to Heavy-Ion Collisions -
Trento (Italy) - November 30, 2017

Overview

- Introduction
  - Physics motivation of real-time dynamics from lattice QCD
  - Challenges in the reconstruction of spectral functions
- Novel simulation approach for thermal fields on the lattice with non-compact Euclidean time
  - Setup
  - Scalar theories in 0+1 and 3+1 dimensions
  - $SU(2)$ gauge theory in 3+1 dimensions
- Summary and outlook

Introduction

Physics motivation

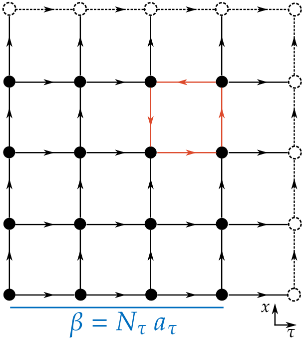
- Thermal physics of hot strongly interacting matter produced in heavy ion collisions
  - Transport phenomena
  - In-medium modification of heavy bound states
- Transport coefficients are real-time quantities related to the energy-momentum tensor (EMT) correlation function
- Example: shear viscosity

$$\eta = \lim_{\omega \to 0} \frac{1}{20} \frac{\rho(\omega, 0)}{\omega}, \quad \rho(\omega, \vec{p}) = \int \frac{d^4x}{(2\pi)^4} e^{-i\omega x_0 + i\vec{p}\cdot\vec{x}} \langle [T_{12}(x), T_{12}(0)] \rangle .$$

$\Rightarrow$ need spectral function $\rho(\omega, \vec{p})$

Lattice QCD at finite temperature

- Gauge fields on links
  $U_\mu(x) = \exp(i g a_\mu A_\mu^a(x)\, T^a)$
- Dynamical fermions with realistic masses
- finite extent in imaginary time
  $1/T = \beta = N_\tau a_\tau$



$$\langle O(U)\rangle = \frac{1}{Z}\int \mathcal{D}U\, O(U)\, \exp(-S_E^{\mathrm{QCD}}[U])$$

$$P(U_k) = e^{-S_E^{\mathrm{QCD}}[U_k]} \Rightarrow \langle O(U)\rangle \approx \frac{1}{N_{\mathrm{cf}}}\sum_{k=1}^{N_{\mathrm{cf}}} O(U_k)$$

Reconstruction of spectral functions and its challenges

- Back to real-time EMT-correlator:

$$\rho(\omega,\vec{p}) = \int \frac{d^4x}{(2\pi)^4} e^{-i\omega x_0 + i\vec{p}\cdot\vec{x}} \langle [T_{12}(x), T_{12}(0)] \rangle$$

- Spectral function connects physical real-time observable with Euclidean time simulation

$$D(\tau) \propto \int \mathrm{d}^3x \, \langle T_{12}(\tau,\vec{x}) T_{12}(0,0) \rangle = \int \mathrm{d}\mu \, \frac{\cosh[\mu(\tau-\beta/2)]}{\sinh[\mu\,\beta/2]} \rho(\mu)$$

For the reconstruction technique used in the following see

Y.Burnier, Alexander Rothkopf, Phys.Rev.Lett. 111 (2013) 182003

Reconstruction of spectral functions and its challenges

Two main conceptual problems of standard spectral reconstructions

- Problem 1:

$$D(\tau) = \int_0^\infty \mathrm{d}\mu \, \frac{\cosh[\mu(\tau - \beta/2)]}{\sinh[\mu\,\beta/2]} \rho(\mu)$$

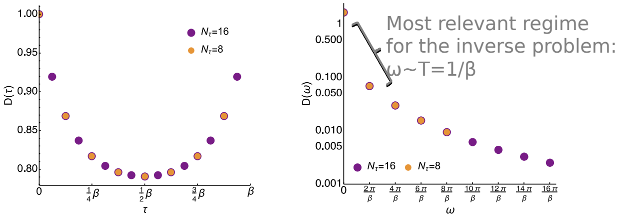
Extraction from imaginary time correlator ill-posed exponentially hard inversion problem.
→ Go to imaginary frequencies and use Källén-Lehman spectral representation

$$D(\omega_n) = \int_0^\infty \mathrm{d}\mu \, \frac{2\,\mu}{\omega_n^2 + \mu^2} \, \rho(\mu)$$

Reconstruction of spectral functions and its challenges

Two main conceptual problems of standard spectral reconstructions

- **Problem 2:** Increasing the number of points along Euclidean time axis does not help!



- Standard lattice simulations only access Matsubara frequencies $\omega_n = 2\pi T n,\ \ n \in \mathbb{Z}$.

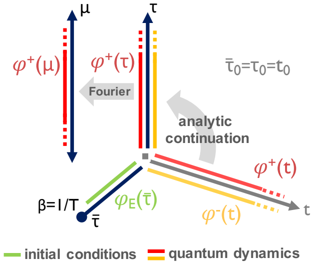
Setup of a novel computational approach

Analytic continuation and general imaginary frequencies

Thermal field theory as real-time initial value problem

$$Z = \int_{\varphi_E(0)=\varphi_E(\beta)} \mathcal{D}\varphi_E \, e^{-S_E[\varphi_E]} \int_{\varphi^+(t_0,\vec{x})=\varphi_E(0)}^{\varphi^-(t_0,\vec{x})=\varphi_E(\beta)} \mathcal{D}\varphi \, e^{i S_M[\varphi^+] - i S_M[\varphi^-]}$$

- Sampling of init. conditions $\varphi^+(t_0)$ on compact Euclidean time lattice, $\bar{\tau} \in [0, \beta]$
- Use: in thermal equilibrium $G^{++} = \langle \varphi^+ \varphi^+ \rangle$ sufficient to describe ρ
- Cut open real-time path at $t_0 = \infty$ and rotate to imaginary time axis $\rightarrow$ simulate $\varphi^+(\tau)$ with $\tau \in [0, \infty)$



Simulating scalar fields

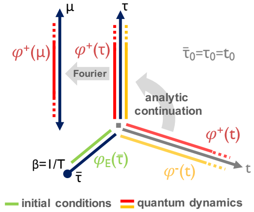
$$S_E = \int d\tau \left[\underbrace{\frac{1}{2}(\partial_\tau \varphi_E)^2 + \frac{1}{2} m^2 \varphi_E^2}_{S_E^0} + \underbrace{\frac{\lambda}{4!}\varphi_E^4}_{S_E^{\text{int}}} \right]$$

$$\partial_{t_5} \varphi_E(\bar{\tau}_k) = -\frac{\delta S_E}{\delta \varphi_E(\bar{\tau}_k)} + \eta(\bar{\tau}_k)$$

$$\partial_{t_5} \varphi^+(\omega_l) = -\frac{\delta S_E^0}{\delta \varphi^+(\omega_l)} - \frac{\delta S_E^{\text{int}}}{\delta \varphi^+(\tau_j)} \frac{\delta \varphi^+(\tau_j)}{\varphi^+(\omega_l)} + \eta(\omega_l)$$

- Use Stochastic Quantization and sample φ_E and φ^+ concurrently from Langevin equations
- Imaginary frequency update in Fourier space
  → kinetic term diagonal and improved convergence
- Thermal initial conditions enter via interaction term in drift

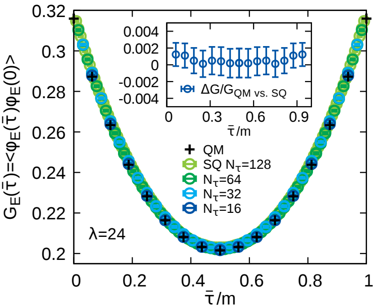
- Temperature in φ_E via compact temporal path
- Temperature in φ^+ via initial condition $\varphi^+(t_0)$

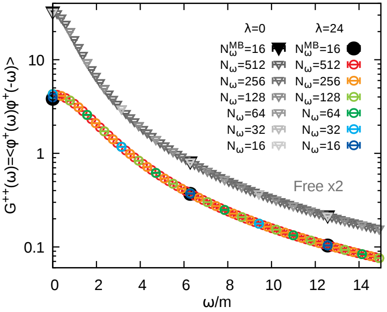


Numerical results for scalar field theories

0+1 dimensional real scalar field

Two-point correlation function





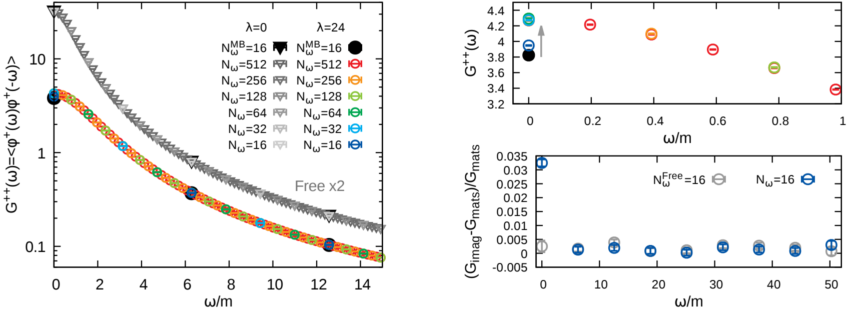
QM (an-)harmonic oscillator vs. stoch. quantization result on the compact Euclidean time lattice

Free and interacting theory from general frequency simulations

Pawlowski, Rothkopf, arXiv:1610.09531 [hep-lat]

0+1 dimensional real scalar field

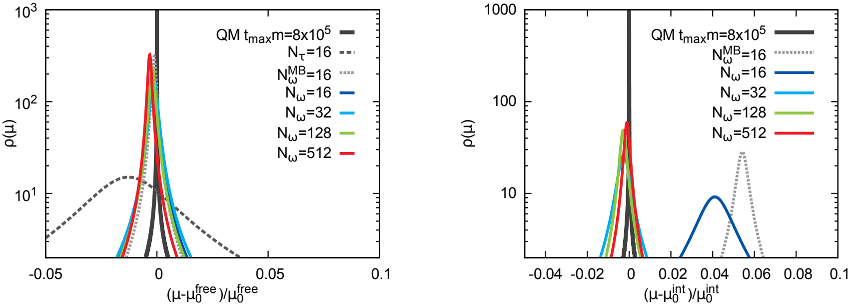
Two-point correlation function



Convergence properties of the correlator

0+1 dimensional real scalar field

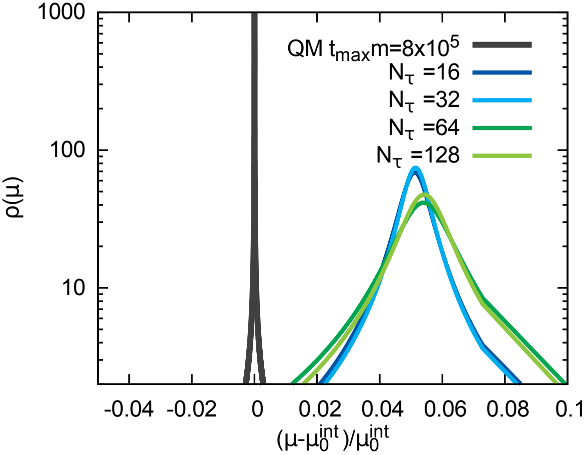
Spectral functions



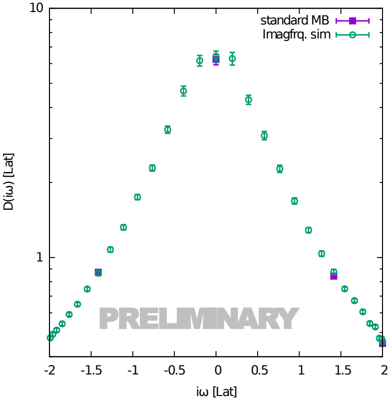
- General imaginary frequencies capture physical properties correctly.
- Information from standard compact Euclidean simulation insufficient.

0+1 dimensional real scalar field

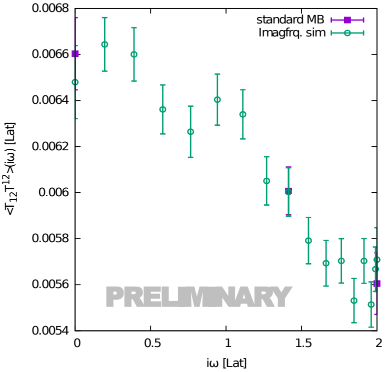
Spectral reconstruction from a **standard** compact Euclidean time correlator $G_E(\bar{\tau})$ does not improve by simply increasing the number of temporal lattice points.



3+1 dimensional complex scalar field



Field correlator



EMT correlator

Pawlowski, Rothkopf, arXiv:1710.02672 [hep-lat]

$SU(2)$ GAUGE THEORY

Simulating gauge fields

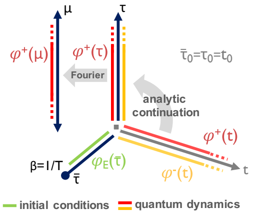
- Wilson plaquette action

$$S_E[U] = \frac{2}{g^2} \sum_x \sum_{\mu<\nu} \mathrm{Re}[\mathbb{1} - U_{\mu\nu}(x)] = \frac{a^4}{2\,g^2} \sum_x \sum_{\mu,\nu} \mathrm{tr}[F_{\mu\nu}(x)^2] + O(a^2)$$

- Langevin update

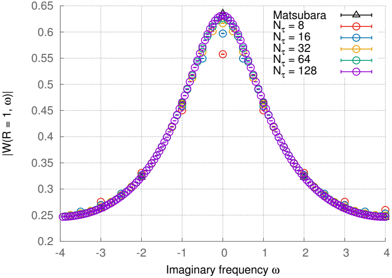
$$U^+_{x,\mu}(t_5) \to U^+_{x,\mu}(t_5+\varepsilon) = \exp\left(i\,X_\varepsilon\right)\,U^+_{x,\mu}(t_5)$$
$$X_\varepsilon = (-\varepsilon D_{x\mu a} S_E[U^+] + \sqrt{\varepsilon}\eta_{x\mu a})\,T^a$$

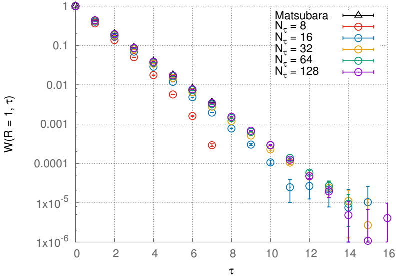
- Simulate in coordinate space only → preserve gauge invariance
- Thermal initial conditions enter via staple term in the drift



Pawlowski, Rothkopf, Ziegler, work in progress

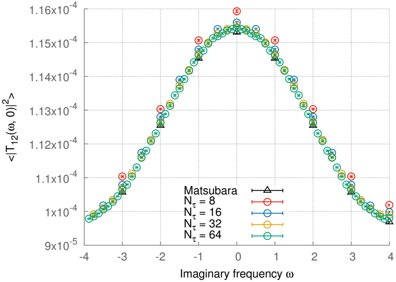
Wilson loop (confined phase)

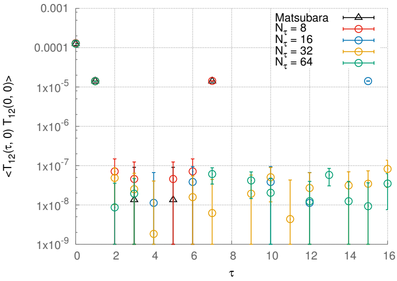




- Lattice sizes: $4^3 \times 8$ (Matsubara)
  $4^3 \times N_\tau$ (General frequencies)
- $\beta = 1.8$
- $N_{cf} = 8 \times 10^5$ configurations

EMT correlator (confined phase)





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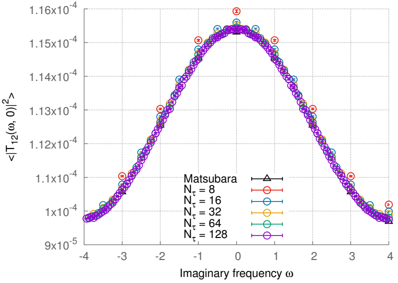

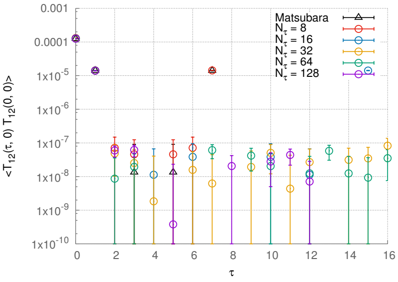

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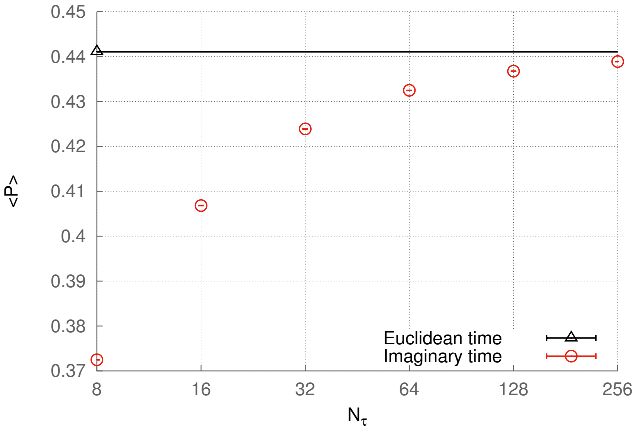
Convergence check (confined phase)



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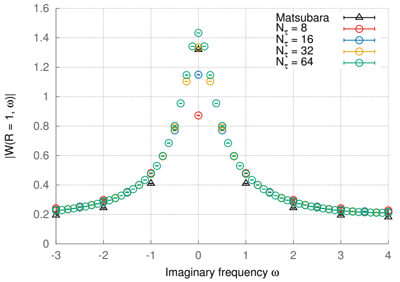
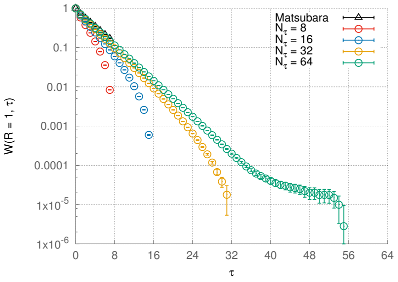
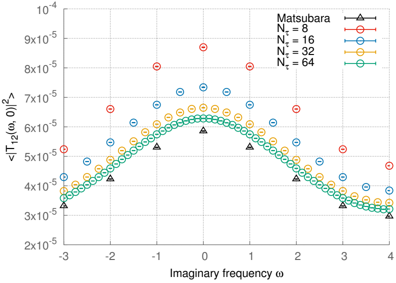
Wilson loop (deconfined phase)

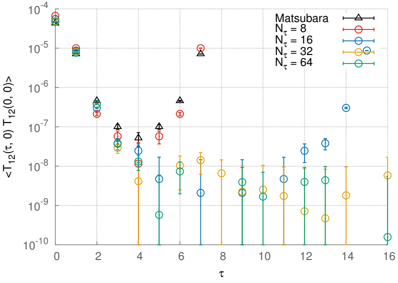




- Lattice sizes: $4^3 \times 8$ (Matsubara)
  $4^3 \times N_\tau$ (General frequencies)
- $\beta = 3.0$
- $N_{\mathrm{cf}} = 1.6 \times 10^6$ configurations

EMT correlator (deconfined phase)





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  $4^3 \times N_\tau$ (General frequencies)
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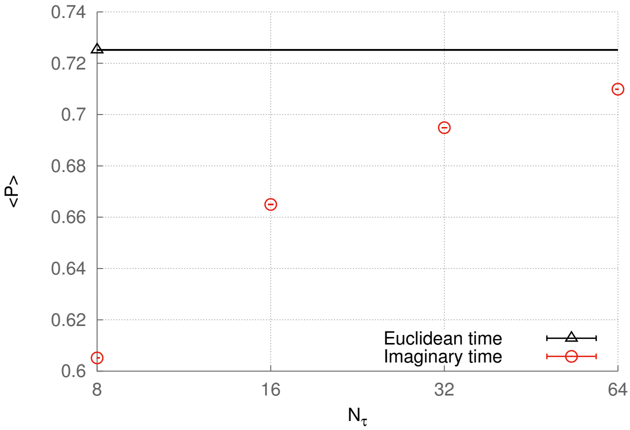
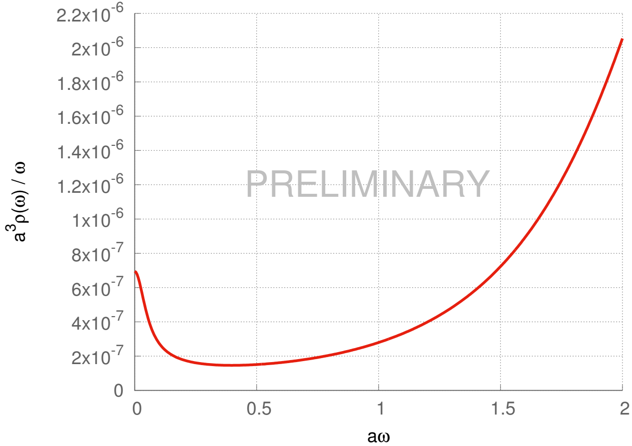
- $N_{\text{cf}} = 1.6 \times 10^6$ configurations

Convergence check (deconfined phase)



- Lattice sizes: $4^3 \times 8$ (Matsubara)
  $4^3 \times N_\tau$ (General frequencies)
- $\beta = 3.0$
- $N_{cf} = 1.6 \times 10^6$ configurations

Spectral function of the EMT correlator



- Lattice sizes: $8^3 \times 8$ (Matsubara)
  $8^3 \times 64$ (General frequencies)
- $\beta = 2.8$
- $N_{\text{cf}} = 5 \times 10^5$ configurations

Pawlowski, Rothkopf, Ziegler, work in progress

Summary

- Thermal fields as initial-value problem formulated in an additional non-compact Euclidean time promising
- Numerical implementation provides significantly improved access to real-time spectral quantities
- Gauge field simulations not restricted to Langevin equation

Outlook

- **Near future:** extract spectral functions and transport properties from the energy-momentum tensor correlator
- Extension to $SU(3)$ gauge theory and full QCD (work in progress)
- Formal developments
- Resolving correlators at small momenta → see talk by Nicolas Wink

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Thank you very much for your attention!