Trento. February 6 - 10, 2017
Unraveling the complexity of nuclear systems:
single-particle and collective aspects through the looking glass

Structure properties of ^{10,11}Li from (p, pn) and (p, d) reactions

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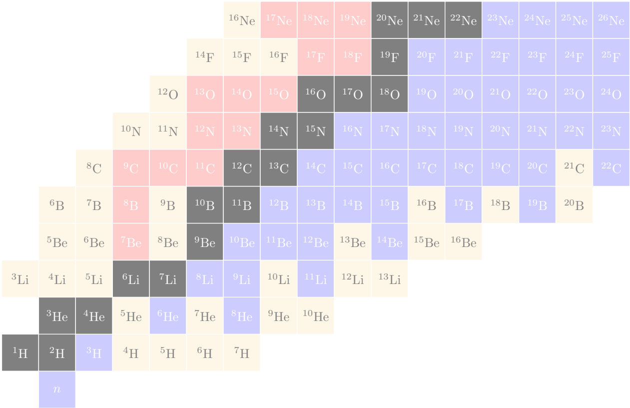
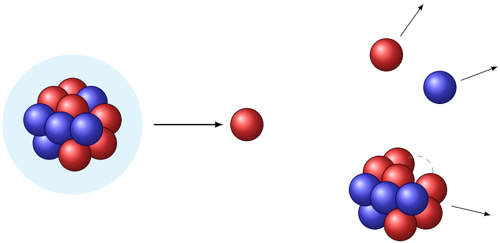

^{16}Ne ^{17}Ne ^{18}Ne ^{19}Ne ^{20}Ne ^{21}Ne ^{22}Ne ^{23}Ne ^{24}Ne ^{25}Ne ^{26}Ne
^{14}F ^{15}F ^{16}F ^{17}F ^{18}F ^{19}F ^{20}F ^{21}F ^{22}F ^{23}F ^{24}F ^{25}F
^{12}O ^{13}O ^{14}O ^{15}O ^{16}O ^{17}O ^{18}O ^{19}O ^{20}O ^{21}O ^{22}O ^{23}O ^{24}O
^{10}N ^{11}N ^{12}N ^{13}N ^{14}N ^{15}N ^{16}N ^{17}N ^{18}N ^{19}N ^{20}N ^{21}N ^{22}N ^{23}N
^{8}C ^{9}C ^{10}C ^{11}C ^{12}C ^{13}C ^{14}C ^{15}C ^{16}C ^{17}C ^{18}C ^{19}C ^{20}C ^{21}C ^{22}C
^{6}B ^{7}B ^{8}B ^{9}B ^{10}B ^{11}B ^{12}B ^{13}B ^{14}B ^{15}B ^{16}B ^{17}B ^{18}B ^{19}B ^{20}B
^{5}Be ^{6}Be ^{7}Be ^{8}Be ^{9}Be ^{10}Be ^{11}Be ^{12}Be ^{13}Be ^{14}Be ^{15}Be ^{16}Be
^{3}Li ^{4}Li ^{5}Li ^{6}Li ^{7}Li ^{8}Li ^{9}Li ^{10}Li ^{11}Li ^{12}Li ^{13}Li
^{3}He ^{4}He ^{5}He ^{6}He ^{7}He ^{8}He ^{9}He ^{10}He
^{1}H ^{2}H ^{3}H ^{4}H ^{5}H ^{6}H ^{7}H
n

(p, pN) reactions

- A nucleus and a proton collide

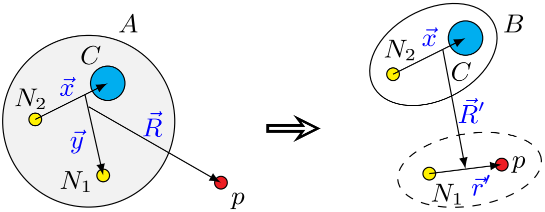
  One nucleon is removed leaving a residual fragment



- High energies to increase mean free path of nucleon inside nucleus
- Proton-target knockout (p, pn) or $(p, 2p)$

  The interaction happens between the proton and a single nucleon
- Used to extract spectroscopic information of nuclei

Three-body structure ## (p,pN) reaction framework



$$\underbrace{(C+N_1+N_2)}_{A}+p \quad \Rightarrow \quad \underbrace{(C+N_2)}_{B}+N_1+p$$

If A is Borromean, the unbound fragment B will eventually decay

Spectroscopic information of the projectile

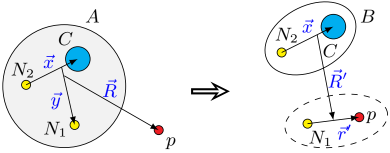
probe the continuum wave function of the unbound fragment

Transfer to Continuum

(TC)

Transfer to Continuum (TC)

- No IA assumed
- No factorization approximation
- Participant/spectator approach



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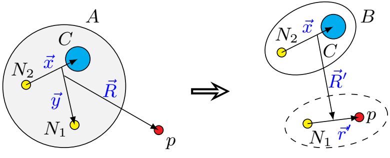


➢ Prior representaton of the T-matrix

$$\mathcal{T}_{if} = \left\langle \varphi_{\vec{q}}^{(-)}(\vec{x}) \Psi_f^{(-)}(\vec{r}', \vec{R}') \middle| V_{pN_1} + U_{pB} - U_{pA} \middle| \Phi_A(\vec{x}, \vec{y}) \chi_{pA}^{(+)}(\vec{R}) \right\rangle ,$$

where

$\varphi_{\vec{q}} \equiv$ continuum wave function of the binary fragment B

$\Psi_f \equiv$ final (pN)-B wave function

$\Phi_A \equiv$ g.s. wave function of the initial 3b composite A

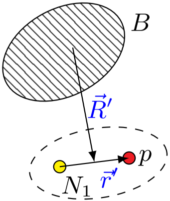
$\chi_{pA} \equiv$ distorted p-A wave

Extension of [*PRC 92 (2015) 044605*]

Final wave function

Expanded in proton-nucleon states (CDCC)

$$\Psi_f(\vec{r}',\vec{R}') \simeq \sum_{n\mathcal{J}\Pi} \phi_n^{\mathcal{J}\Pi}(k_n,\vec{r}')\chi_n^{\mathcal{J}\Pi}(\vec{K}',\vec{R}')$$



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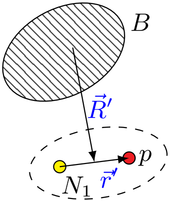
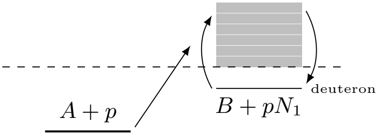


$$\Psi_f(\vec{r}', \vec{R}') \simeq \sum_{n\mathcal{J}\Pi} \phi_n^{\mathcal{J}\Pi}(k_n, \vec{r}')\chi_n^{\mathcal{J}\Pi}(\vec{K}', \vec{R}')$$

Basis of N discretized bins

$$\phi_n^{\mathcal{J}\Pi}(k_n, \vec{r}') = \sqrt{\frac{2}{\pi N}} \int_{k_n-1}^{k_n} \phi_{pN_1}^{\mathcal{J}\Pi}(k, \vec{r}')dk.$$



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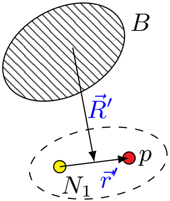

$$\Psi_f(\vec{r}',\vec{R}') \simeq \sum_{n\mathcal{J}\Pi} \phi_n^{\mathcal{J}\Pi}(k_n,\vec{r}')\chi_n^{\mathcal{J}\Pi}(\vec{K}',\vec{R}')$$

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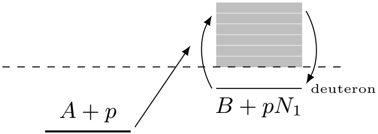
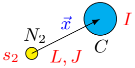


If we select the (p,d) channel TC reduces to DWBA

$$\Psi_f(\vec{r}',\vec{R}') \simeq \phi_d(\vec{r}')\chi_{d\text{-}B}(\vec{R}')$$

2b continuum state of fragment B



$$\varphi^{(+)}_{\vec{q},\sigma_2,\iota}(\vec{x}) = \frac{4\pi}{qx} \sum_{LJJ_TM_T} i^L Y^*_{LM}(\widehat{q}) \langle LMs_2\sigma_2|JM_J\rangle \times \langle JM_J I\iota|J_TM_T\rangle f^{J_T}_{LJ}(qx) \left[\mathcal{Y}_{Ls_2J}(\widehat{x}) \otimes \psi_I\right]_{J_TM_T}$$

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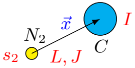

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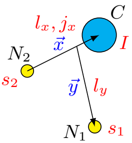
3b g.s. wave function of A

Analytical THO method
[*PRC 88 (2013) 014327*]



$$\Phi^{j\mu}_A(\vec{x},\vec{y}) = \sum_\beta w^j_\beta(x,y)\left\{\left[\mathcal{Y}_{l_x s_2 j_x}(\widehat{x})\otimes\psi_I\right]_{j_1}\otimes\left[Y_{l_y}(\widehat{y})\otimes\psi_{s_1}\right]_{j_2}\right\}_{j\mu}$$

Consistent with 2b wave function: same potential and couplings

- Assume $(V_{pN_1} + U_{pB} - U_{pA})$ does not change the state of B
- Define overlaps:

$$\psi_{LJJ_TM_T}(q,\vec{y}) = \int \frac{f_{LJ}^{J_T}(qx)}{x} \left[\mathcal{Y}_{Ls_2J}(\widehat{x}) \otimes \psi_I\right]_{J_TM_T} \Phi_A^{j\mu}(\vec{x},\vec{y})d\vec{x}$$

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- Auxiliary amplitudes

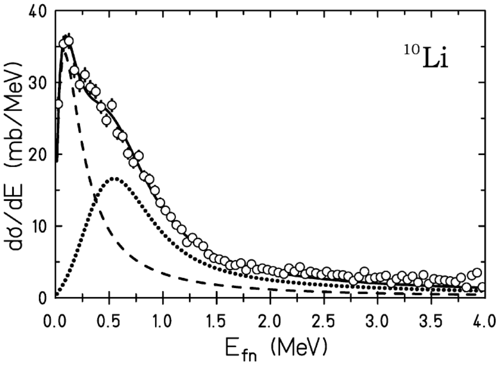
$$\mathcal{T}_{if}^{LJJ_TM_T} \equiv \langle \Psi_f^{(-)}(\vec{r}',\vec{R}')|V_{pN_1}+U_{pB}-U_{pA}|\psi_{LJJ_TM_T}(q,\vec{y})\chi_{pA}^{(+)}\rangle$$

➠ Cross sections

$$\frac{d\sigma_n^2}{d\Omega_B d\varepsilon_x} \propto \sum \left|\mathcal{T}_{if}^{LJJ_TM_T}\right|^2$$

^{11}Li$(p,pn)^{10}$Li

ALADIN-LAND setup at GSI [Aksyutina *et al.*, PLB 666 (2008) 430]



280 AMeV

spectroscopic information extracted through fitting

reaction dynamics not considered

TC calculations [spin of ^{9}Li ignored, $I^\pi = 0^+$]

- ^{10}Li (^{9}Li $+ n$)

  $2s_{1/2}$ virtual state: $a = -20.9$ fm

  $1p_{1/2}$ resonance at $\sim$ 0.5 MeV

  $1d_{5/2}$ state around 4.5 MeV

- ^{11}Li (^{9}Li $+ n + n$)

  $\varepsilon_{g.s.} = -0.37$ MeV

  $r_{mat} = 3.55$ fm, $r_{ch} = 2.48$ fm

  64% $s_{1/2}$, 30% $p_{1/2}$, 3% $d_{5/2}$

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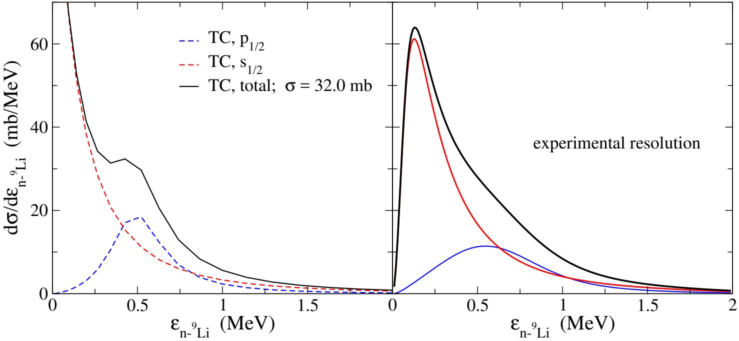
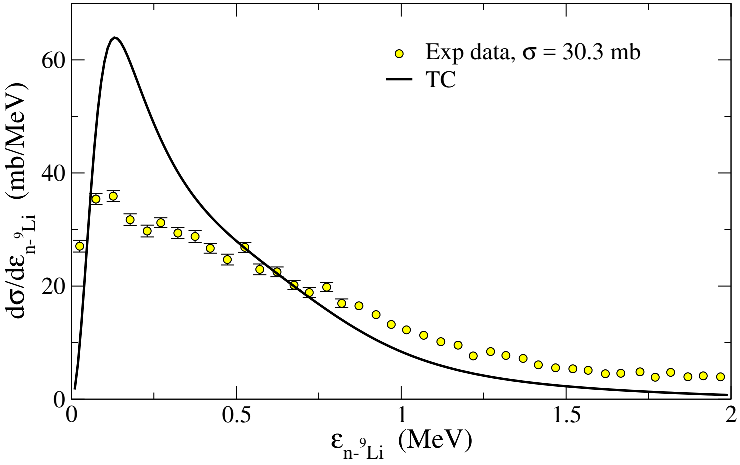
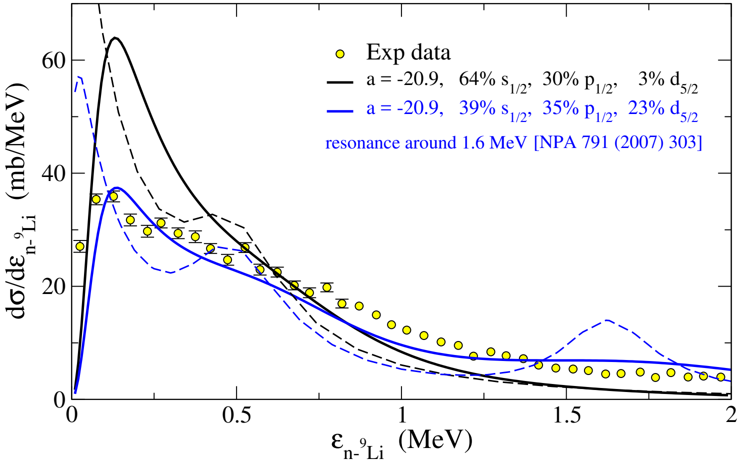




Data from Aksyutina *et al.* [PLB 666 (2008) 430]



Not sufficient to confirm $d_{5/2}$ resonance

Data from Aksyutina *et al.* [PLB 666 (2008) 430]

Include spin of ^{9}Li; $I^{\pi} = 3/2^{-}$

spin-spin splitting:

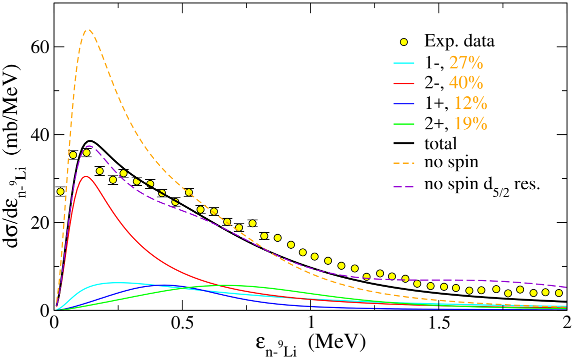
$s_{1/2} \Rightarrow 1^{-}, 2^{-}$

$p_{1/2} \Rightarrow 1^{+}, 2^{+}$

Model:

- ^{10}Li:
  $a = -37.9$ fm (2^{-})
  res. at 0.4, 0.6 MeV
- ^{11}Li:
  $\varepsilon_{g.s.} = -0.37$ MeV
  $r_{mat} = 3.2$ fm
  $r_{ch} = 2.41$ fm

67% s, 31% p

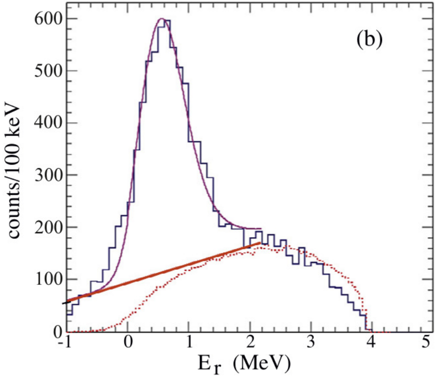


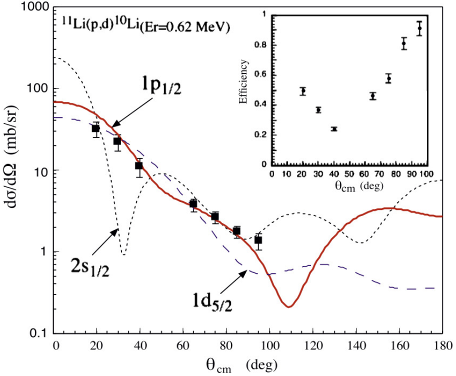
$d_{5/2}$ resonance not required to explain the data

Data from Aksyutina *et al.* [PLB 666 (2008) 430]

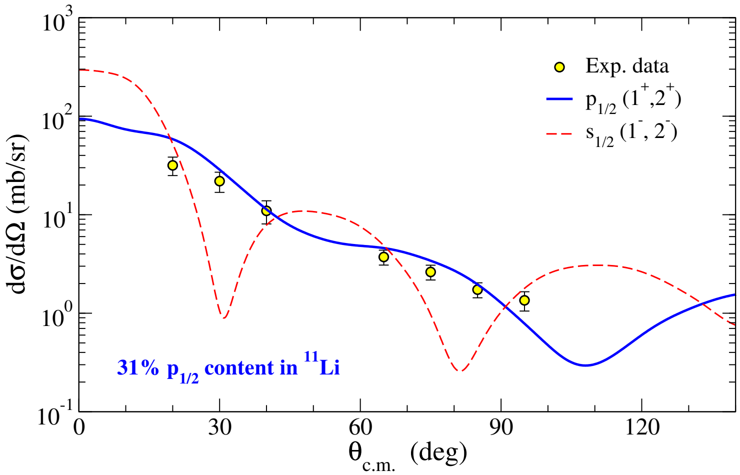
^{11}Li$(p,d)^{10}$Li

IRIS at TRIUMF, 5.7 AMeV [Sanetullaev *et al.*, PLB 755 (2016) 481]





weight $p_{1/2}$: 33%



Same model gives good agreement for (p,pn) and (p,d)

Data from Sanetullaev *et al.* [PLB 755 (2016) 481]

- **Transfer to Continuum (TC)** framework to describe (p, pN) reactions induced by 3b projectiles; reduces to DWBA for (p, d) transfer.
  - Structure information contained in $\langle \Phi_{3b}^{g.s.} | \varphi_{2b}^{q} \rangle$ **overlaps.**
  - Provides absolute cross sections.
- ^{11}Li$(p, pn)^{10}$Li: including spin of ^{9}Li improves agreement.
- ^{11}Li$(p, d)^{10}$Li: angular distribution sensitive to p-wave content only.
  - **Same model** gives good agreement for both reactions.
- Our model is consistent with 31% of p-waves in ^{11}Li, in agreement with recent experimental estimations. The presence of a d resonance in ^{10}Li at low energies and the precise s, d weights are still unclear.

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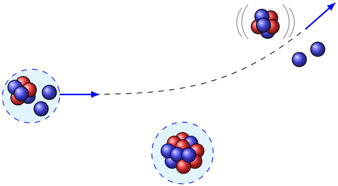
➠ Calculate angular correlations of NN ejectiles.

➠ Include core excitations: ^{14}Be(p,pn) scattering

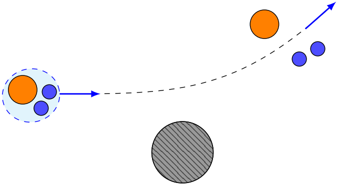
Structure properties ⟺ Nuclear reactions

Microscopic description



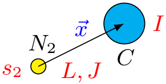
- Many-body problem
- Start from (effective) NN interactions
- Fragment degrees of freedom

Few-body description



- Separation in clusters
- Effective interaction between clusters
- No internal structure or phenomenological description

2b continuum state of fragment B



$$\varphi^{(+)}_{\vec{q},\sigma_2,\iota}(\vec{x}) = \frac{4\pi}{qx} \sum_{LJJ_TM_T} i^L Y^*_{LM}(\widehat{q}) \langle LMs_2\sigma_2|JM_J\rangle \times \langle JM_J I\iota|J_TM_T\rangle f^{J_T}_{LJ}(qx) [\mathcal{Y}_{Ls_2J}(\widehat{x}) \otimes \kappa_I]_{J_TM_T}$$

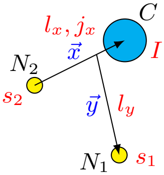
Coupling order: $\vec{L} + \vec{s}_2 = \vec{J}, \qquad \vec{J} + \vec{I} = \vec{J}_T$

Obtain solution for each $(L, J)J_T$:

$$f^{J_T}_{LJ}(qx) \longrightarrow \frac{i}{2} e^{i\sigma_L} \left[H^{(-)}_L(qx) - S^{J_T}_{LJ} H^{(+)}_L(qx) \right]$$

3b g.s. wave function of A

Analytical THO method [*PRC 88 (2013) 014327*]



$$\beta \equiv \{K, l_x, j_x, j_1, l_y, j_2\}$$

$$\vec{l}_x + \vec{s}_2 = \vec{j}_x, \qquad \vec{j}_x + \vec{I} = \vec{j}_1$$

$$\vec{l}_y + \vec{s}_1 = \vec{j}_2, \qquad \vec{j}_1 + \vec{j}_2 = \vec{j}$$

$$\Phi_A^{j\mu}(\vec{x},\vec{y}) = \sum_\beta w_\beta^j(x,y) \left\{ \left[\mathcal{Y}_{l_x s_2 j_x}(\widehat{x}) \otimes \kappa_I \right]_{j_1} \otimes \left[Y_{l_y}(\widehat{y}) \otimes \kappa_{s_1} \right]_{j_2} \right\}_{j\mu}$$

Consistent with 2b wave function: same potential and couplings

- Assume that V_{prior} does not change the state of B
- Define overlaps:

$$\psi_{LJJ_TM_T}(q,\vec{y}) = \int \frac{f_{LJ}^{J_T}(qx)}{x} \left[\mathcal{Y}_{Ls_2J}(\widehat{x}) \otimes \psi_I\right]_{J_TM_T} \; \Phi_A^{j\mu}(\vec{x},\vec{y}) d\vec{x}$$

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So the T-matrix is

$$\mathcal{T}_{if} = \frac{4\pi}{q} \sum_{LJJ_TM_T} (-i)^L Y_{LM}(\widehat{q}) \langle LMs_2\sigma_2|JM_J\rangle \langle JM_JI\iota|J_TM_T\rangle \boldsymbol{\mathcal{T}_{if}^{LJJ_TM_T}}$$

with

$$\boldsymbol{\mathcal{T}_{if}^{LJJ_TM_T}} \equiv \langle \Psi_f^{(-)}(\vec{r}',\vec{R}')|V_{pN_1} + U_{pB} - U_{pA}|\psi_{LJJ_TM_T}(q,\vec{y})\chi_{pA}^{(+)}\rangle$$

Inclusive cross sections

$$\frac{d\sigma_n^2}{d\Omega_B d\varepsilon_x} \propto \sum \left| \mathcal{T}_{if}^{LJJ_TM_T} \right|^2$$

- $\dfrac{d\sigma}{d\varepsilon_x}$, relative energy spectrum of $B(C+N_2)$
- $\dfrac{d\sigma}{d\Omega_B}$, angular distribution of B

➢ Distribution with respect to Ω_q is being developed

^{11}Li$(p,pn)^{10}$Li — **TC** calculations

- Spin of ^{9}Li ignored $(I=0)$
- n-n potential: GPT [*PLB 32 (1970) 591*]
- n-^{9}Li potential: central and spin-orbit terms

➢ p-N potential: Reid93 [*PRC 49 (1994) 2950*]

➢ p-^{11}Li, p-^{10}Li, n-^{10}Li potentials: codes OXBASH and LEA
Folding of HF density and effective NN interactions

$$U(r)=\int d\vec{r}'\rho_X(\vec{r}')t_{NN}^{eff}(|\vec{r}-\vec{r}'|,\rho)$$

➢ Relativistic corrections in FRESCO

Structure model (PNS3)

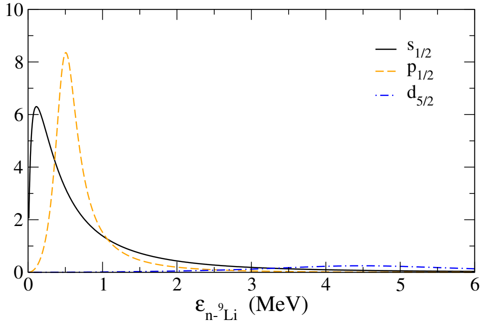
- ^{10}Li (^{9}Li $+ n$)
  $2s_{1/2}$ virtual state: $a = -20.9$ fm
  $1p_{1/2}$ resonance at $\sim$ 0.5 MeV
  $1d_{5/2}$ state around 4.5 MeV

- ^{11}Li (^{9}Li $+ n + n$)
  $\varepsilon_{g.s.} = -0.37$ MeV
  $r_{mat} = 3.55$ fm, $r_{ch} = 2.48$ fm
  64% s waves, 31% p waves

Form factors:

$$\eta \equiv \int d\vec{y} |\psi_{LJJ_TM_T}(\vec{y})|^2$$

$$\frac{d\sigma}{d\varepsilon_{n\text{-}^9\mathrm{Li}}} \propto K(\varepsilon_{n\text{-}^9\mathrm{Li}})\,\eta \quad ??$$



Factorization

- p wave follows TC
- s wave distorted

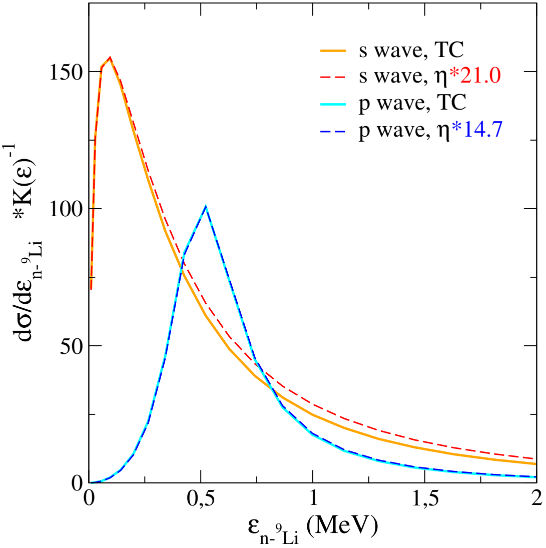
Correct up to certain extent

BUT!!

- **ratio depends on L, J**

  reaction calculation to obtain relative weights

  less ambiguous than fitting



Include spin of ^{9}Li; $I^\pi = 3/2^-$

spin-spin splitting:

$s_{1/2} \Rightarrow 1^-, 2^-$

$p_{1/2} \Rightarrow 1^+, 2^+$

Model (PS6)

- ^{10}Li:

$a = -37.9$ fm (2^-)

res. at 0.4, 0.6 MeV

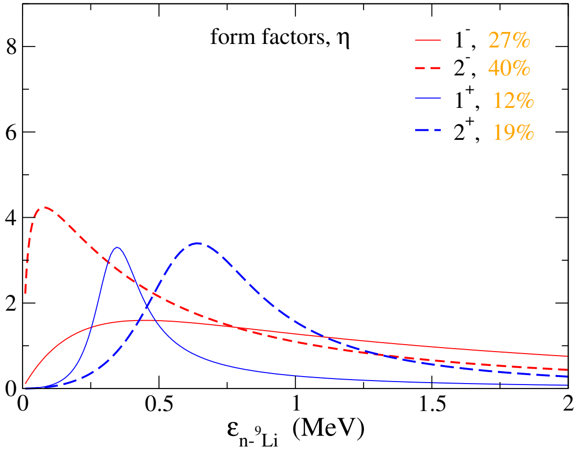
- ^{11}Li:

$\varepsilon_{g.s.} = -0.38$ MeV

$r_{mat} = 3.2$ fm

$r_{ch} = 2.41$ fm

67% s, 31% p



Include spin of ^{9}Li; $I^\pi = 3/2^-$

spin-spin splitting:

$s_{1/2} \Rightarrow 1^-, 2^-$

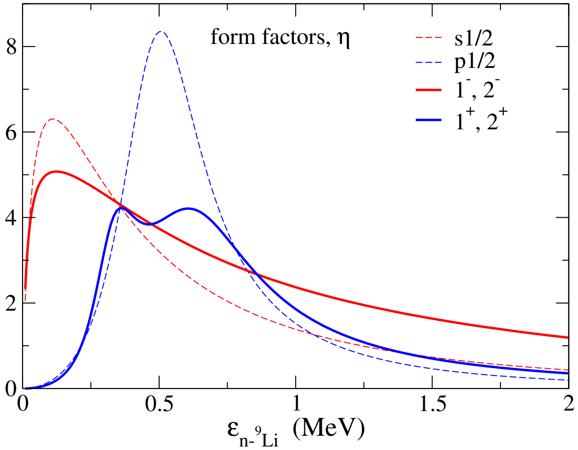
$p_{1/2} \Rightarrow 1^+, 2^+$

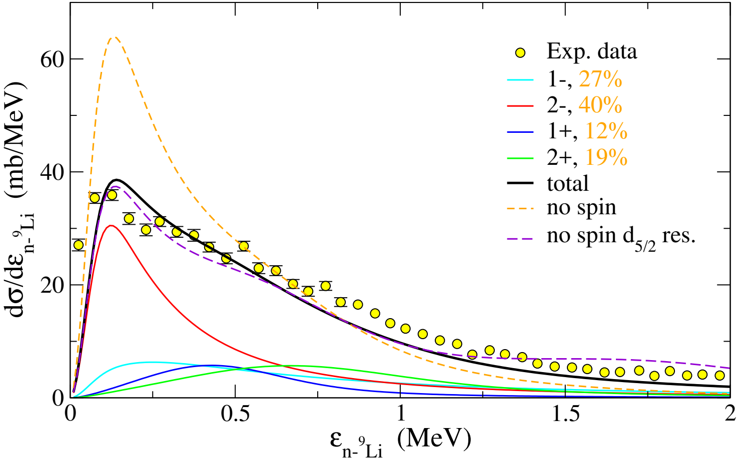
Model (PS6)

- ^{10}Li:
$a = -37.9$ fm (2^-)
res. at 0.4, 0.6 MeV

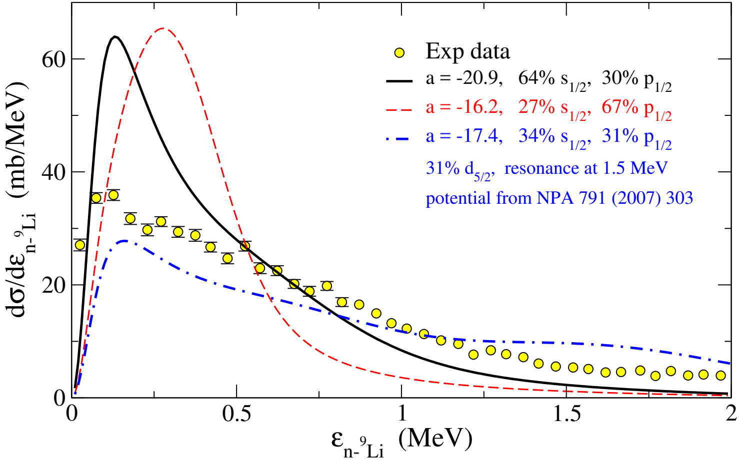
- ^{11}Li:
$\varepsilon_{g.s.} = -0.38$ MeV
$r_{mat} = 3.2$ fm
$r_{ch} = 2.41$ fm

67% s, 31% p

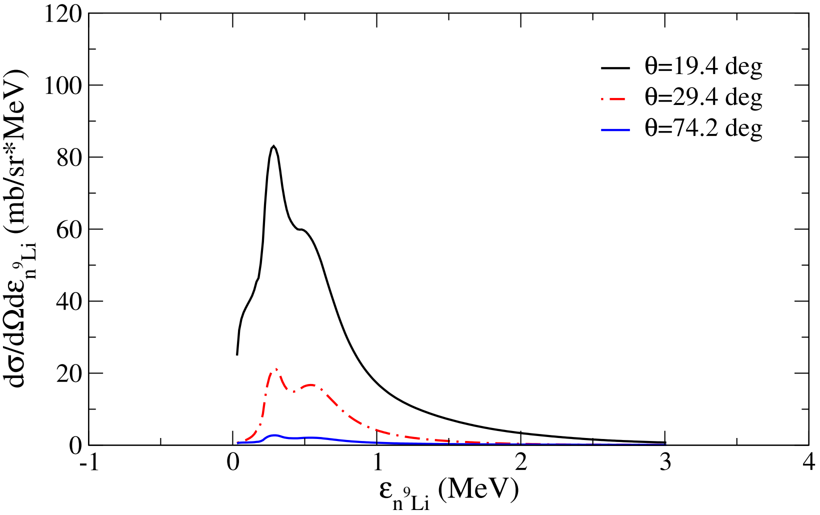




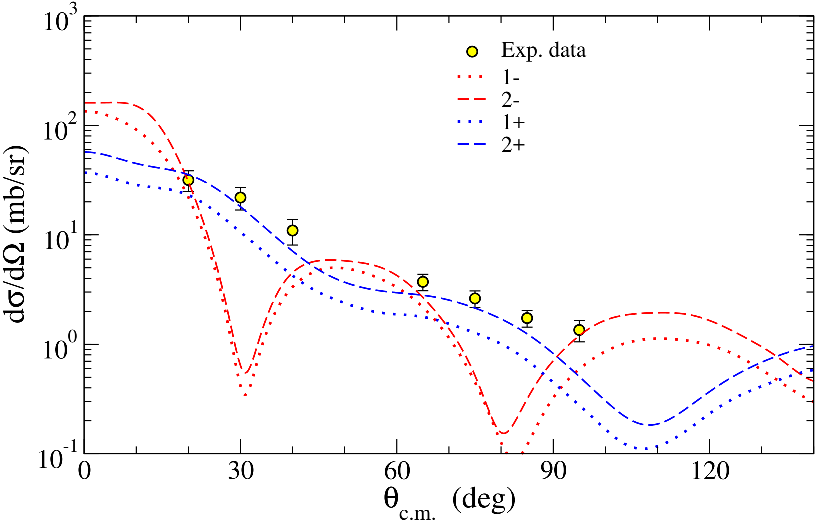
Data from Aksyutina *et al.* [PLB 666 (2008) 430]



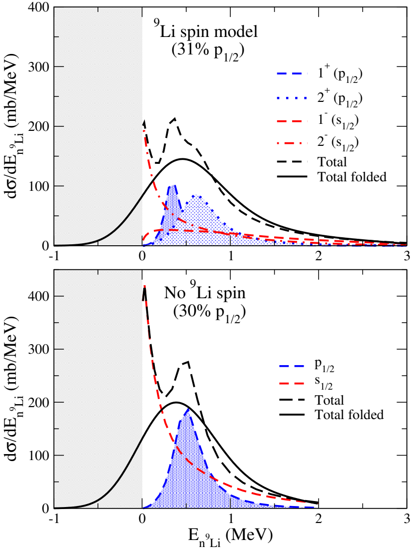
Data from Aksyutina *et al.* [PLB 666 (2008) 430]



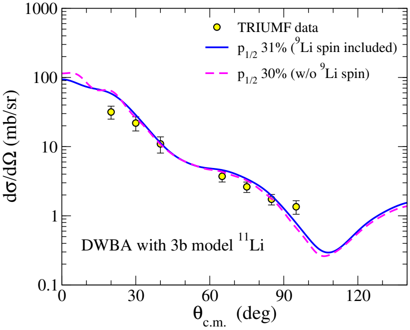
120
100
80
60
40
20
0
-1
0
1
2
3
4
$d\sigma/d\Omega d\varepsilon_{n^9Li}$ (mb/sr*MeV)
ε_{n^9Li} (MeV)
θ=19.4 deg
θ=29.4 deg
θ=74.2 deg



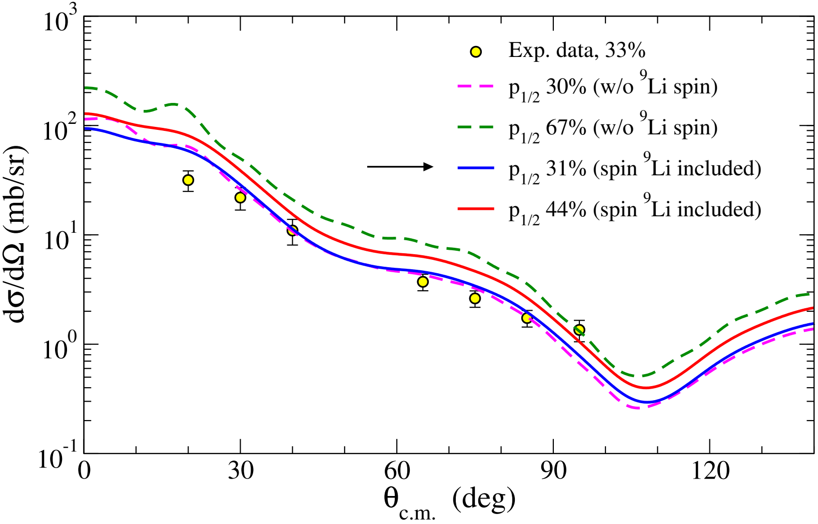
Data from Sanetullaev *et al.* [PLB 755 (2016) 481]



^{9}Li spin model
(31% $p_{1/2}$)
1^+ ($p_{1/2}$)
2^+ ($p_{1/2}$)
1^- ($s_{1/2}$)
2^- ($s_{1/2}$)
Total
Total folded
No ^{9}Li spin
(30% $p_{1/2}$)
$p_{1/2}$
$s_{1/2}$
Total
Total folded
$d\sigma/dE_{n^9Li}$ (mb/MeV)
E_{n^9Li} (MeV)



TRIUMF data
$p_{1/2}$ 31% (^{9}Li spin included)
$p_{1/2}$ 30% (w/o ^{9}Li spin)
DWBA with 3b model ^{11}Li
$d\sigma/d\Omega$ (mb/sr)
$\theta_{c.m.}$ (deg)



Data from Sanetullaev *et al.* [PLB 755 (2016) 481]