

Superconformal and Supersymmetric Constraints to Hadron Masses in Light-Front Holographic QCD

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The Proton Mass: at the Heart of Most Visible Matter

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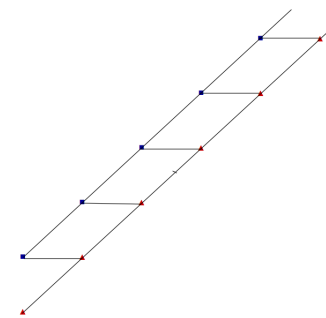
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In collaboration with Stan Brodsky, Alexandre Deur, Hans G. Dosch, Cédric Lorce and Raza Sabbir Sufian

- We use a superconformal algebraic structure to construct relativistic light-front (LF) semiclassical bound-state equations which can be embedded in a higher-dimensional classical gravitational theory
- This approach to hadron physics incorporates basic nonperturbative properties which are not apparent from the chiral QCD Lagrangian:
 - I. Emergence of a mass scale and confinement out of a classically scale-invariant theory
 - II. Occurrence of a zero-mass bound state in the limit of zero quark masses
 - III. Universal Regge trajectories for mesons and baryons
 - IV. Breaking of chirality in the hadron excitation spectrum
 - V. Precise connections between the light meson and nucleon spectra
- Effective theory can be extended to heavy-light hadrons where heavy quark masses break the conformal invariance but the underlying dynamical supersymmetry still holds
- Procedure based on work by de Alfaro, Fubini and Furlan, and Fubini and Rabinovici: a generalized Hamiltonian is constructed as a superposition of superconformal generators which carry different dimensions
- The Hamiltonian remains within the superconformal algebraic structure and leads to unique form of the confinement potential

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1 Superconformal quantum mechanics

[S. Fubini and E. Rabinovici, NPB **245**, 17 (1984)]

- SUSY QM [E. Witten (1981)] contains two fermionic generators Q and Q^\dagger with anticommutation relations

$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$

and the Hamiltonian $H = \frac{1}{2}\{Q, Q^\dagger\}$ which commutes with the fermionic generators $[Q, H] = [Q^\dagger, H] = 0$ closing the graded-Lie $sl(1/1)$ algebra

- Since $[Q^\dagger, H] = 0$ the states $|E\rangle$ and $Q^\dagger|E\rangle$ have identical eigenvalues E , but for a zero eigenvalue we can have the trivial solution $|E = 0\rangle = 0$

- In matrix notation

$$Q = \begin{pmatrix} 0 & q \\ 0 & 0 \end{pmatrix}, \quad Q^\dagger = \begin{pmatrix} 0 & 0 \\ q^\dagger & 0 \end{pmatrix}, \quad H = \frac{1}{2} \begin{pmatrix} q q^\dagger & 0 \\ 0 & q^\dagger q \end{pmatrix}$$

with

$$q = -\frac{d}{dx} + \frac{f}{x}, \quad q^\dagger = \frac{d}{dx} + \frac{f}{x}$$

for a conformal theory and f is dimensionless

- Conformal graded-Lie algebra has in addition to the Hamiltonian H and supercharges Q and Q^\dagger , a new operator S related to the generator of conformal transformations K

$$S = \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix}, \quad S^\dagger = \begin{pmatrix} 0 & 0 \\ x & 0 \end{pmatrix}$$

- Find enlarged algebra (Superconformal algebra of Haag, Lopuszanski and Sohnius (1974))

$$\begin{aligned} \frac{1}{2}\{Q, Q^\dagger\} &= H, & \frac{1}{2}\{S, S^\dagger\} &= K, \\ \{Q, S^\dagger\} &= f - B + 2iD, & \{Q^\dagger, S\} &= f - B - 2iD \end{aligned}$$

where $B = \frac{1}{2}\sigma_3$, and the generators of translation, dilatation and the special conformal transformation H , D and K

$$\begin{aligned} H &= \frac{1}{2} \left(-\frac{d^2}{dx^2} + \frac{f^2 + 2Bf}{x^2} \right) \\ D &= \frac{i}{4} \left(\frac{d}{dx}x + x\frac{d}{dx} \right) \\ K &= \frac{1}{2}x^2 \end{aligned}$$

satisfy the conformal algebra

$$[H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK$$

- Following F&R define a fermionic generator R , a linear combination of the generators Q and S

$$R_\lambda = Q + \lambda S$$

which generates a new Hamiltonian

$$G_\lambda = \{R_\lambda, R_\lambda^\dagger\}$$

where by construction

$$\{R_\lambda, R_\lambda\} = \{R_\lambda^\dagger, R_\lambda^\dagger\} = 0, \quad [R_\lambda, G_\lambda] = [R_\lambda^\dagger, G_\lambda] = 0$$

which also closes under the graded algebra $sl(1/1)$

- The Hamiltonian G_λ is given by

$$G_\lambda = 2H + 2\lambda^2 K + 2\lambda (f - \sigma_3)$$

and leads to the eigenvalue equations

$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f - \lambda + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2} \right) \phi_1 = E \phi_1$$

$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f + \lambda + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2} \right) \phi_2 = E \phi_2$$

2 Superconformal meson-baryon symmetry

[GdT, H.G. Dosch and S. J. Brodsky, PRD **91**, 045040 (2015)]

[H.G. Dosch, GdT, and S. J. Brodsky, PRD **91**, 085016 (2015)]

$$|\phi\rangle = \begin{pmatrix} \phi_{\text{Meson}} \\ \phi_{\text{Baryon}} \end{pmatrix}$$

- Upon the substitutions (slide 6)

$$x \mapsto \zeta$$

$$E \mapsto M^2$$

$$\lambda \mapsto \lambda_B = \lambda_M$$

$$f \mapsto L_M - \frac{1}{2} = L_B + \frac{1}{2}$$

we find the LF bound-state equations

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + \lambda_M^2 \zeta^2 + 2\lambda_M(L_M - 1) \right) \phi_{\text{Meson}} = M^2 \phi_{\text{Meson}}$$

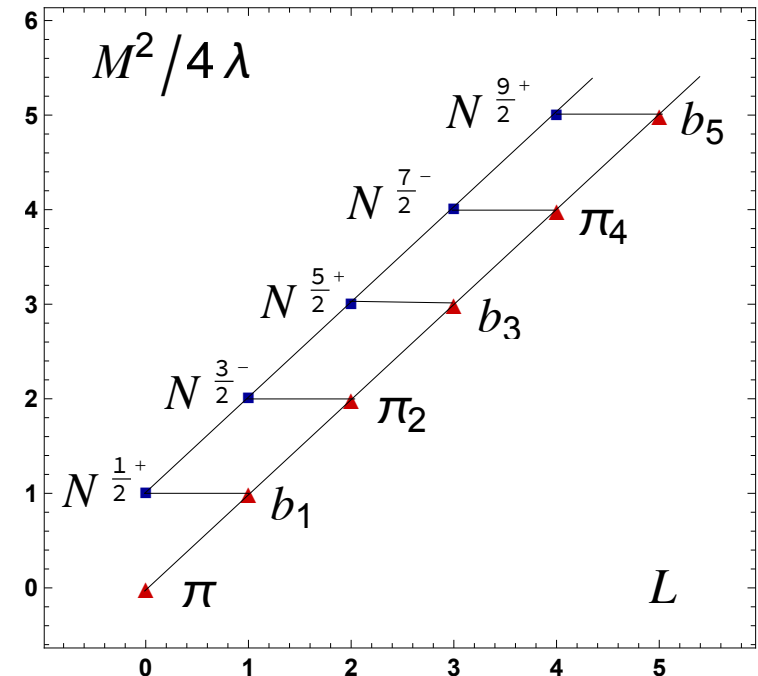
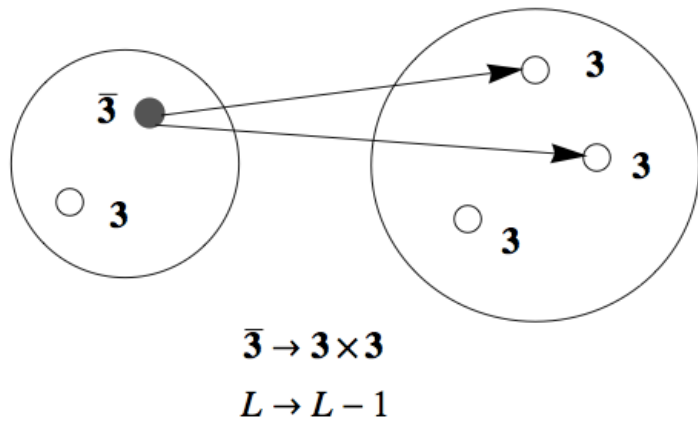
$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_N^2 - 1}{4\zeta^2} + \lambda_B^2 \zeta^2 + 2\lambda_B(L_N + 1) \right) \phi_{\text{Baryon}} = M^2 \phi_{\text{Baryon}}$$

obtained previously from LF holographic QCD (LFHQCD)

S. J. Brodsky, GdT, H. G. Dosch and J. Erlich, *Phys. Rep.* **584**, 1 (2015)

- ζ is the invariant transverse separation between constituents in LF quantization which is identified with the holographic variable z in AdS classical gravity: $\zeta = z$, $\zeta^2 = x(1-x)b_\perp^2$

- Superconformal QM imposes the condition $\lambda = \lambda_M = \lambda_B$ (equality of Regge slopes) and the remarkable relation $\Rightarrow L_M = L_B + 1$
- L_M is the LF angular momentum between the quark and antiquark in the meson and L_B is the relative angular momentum between the active quark and spectator cluster in the baryon
- Special role of the pion as a unique state of zero energy
 $R^\dagger |M, L\rangle = |B, L - 1\rangle, \quad R^\dagger |M, L = 0\rangle = 0$
- Emerging dynamical SUSY from SU(3) color
 (Hadronic SUSY introduced by H. Miyazawa (1966))



- Spin-dependent Hamiltonian to describe mesons and baryons with internal spin (chiral limit)

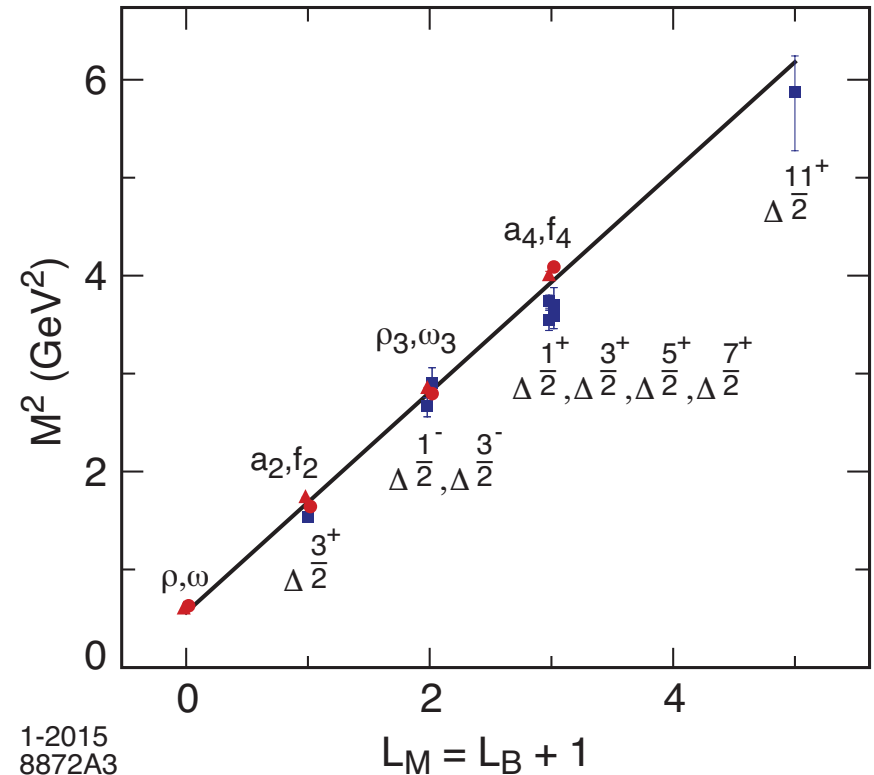
[S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé, PLB **759**, 171 (2016)]

$$G = \{R_\lambda^\dagger, R_\lambda\} + 2\lambda S \quad S = 0, 1$$

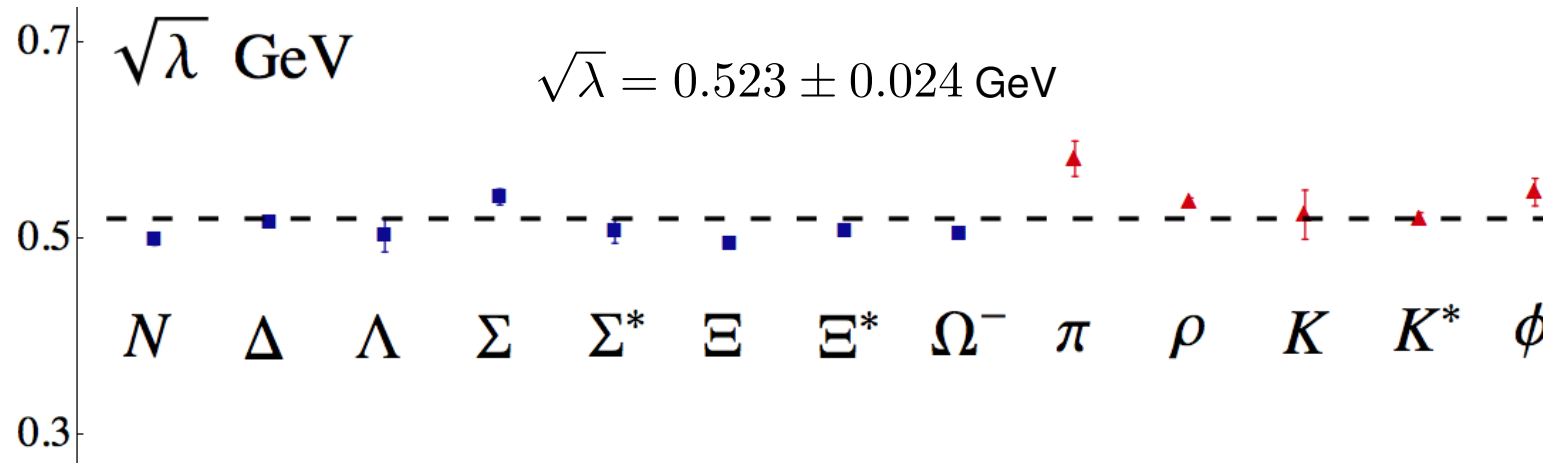
- Mesons : $M^2 = 4\lambda (n + L_M) + 2\lambda S$
 Baryons : $M^2 = 4\lambda (n + L_B + 1) + 2\lambda S$
- Superconformal meson-baryon partners
 ($\sqrt{\lambda} = 0.53 \text{ GeV}$)
- Quadratic mass correction for light quark masses

$$\Delta M^2[m_1, \dots, m_n] = \frac{\lambda^2}{F} \frac{dF}{d\lambda}$$

with $F[\lambda] = \int_0^1 \dots \int dx_1 \dots dx_n e^{-\frac{1}{\lambda} \left(\sum_{i=1}^n \frac{m_i^2}{x_i} \right)} \delta\left(\sum_{i=1}^n x_i - 1\right)$



- How universal is the semiclassical approximation based on superconformal QM and its LF holographic embedding? [S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé, PLB 759, 171 (2016)]



Best fit for hadronic scale $\sqrt{\lambda}$ from the different light hadronic sectors including radial and orbital excitations

- Frame-independent decomposition of the quadratic masses for light hadrons in the chiral limit:

$$M_H^2/\lambda = \underbrace{(2n + L_H + 1)}_{\text{kinetic}} + \underbrace{(2n + L_H + 1)}_{\text{potential}} + \underbrace{2(L_H + s) + 2\chi}_{\text{contribution from AdS and superconformal algebra}}$$

contribution from 2-dim light-front harmonic oscillator

Here n is the radial excitation number, L_H the LF angular momentum, s is the total spin of the meson or the cluster in the baryon, $\chi = -1$ for mesons and $\chi = +1$ for baryons

3 Supersymmetry across the heavy-light hadronic spectrum

[H.G. Dosch, GdT, and S. J. Brodsky, Phys. Rev. D **92**, 074010 (2015), Phys. Rev. D **95**, 034016 (2017)]

- For light quark masses we apply superconformal dynamics and treat quark masses as perturbations: confinement scale remains universal
- For light quark masses decoupling of transverse degrees of freedom (LF variable ζ) from longitudinal ones (LF longitudinal momentum fraction x) [GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]
- Heavy quark mass breaks conformal symmetry but needs not break supersymmetry since it can stem from the dynamics of color confinement: confinement scale depend on the mass of the heavy quark
- Light quarks present in heavy-light hadrons: system still ultrarelativistic and described by LF relativistic bound-state equations
- If separation of kinematic and dynamic variables also persist for heavy-light hadrons, then the holographic embedding constrains specific form of the confinement potential
- Heavy quark effective theory (HQET) determines the dependence of the confinement scale on the heavy quark mass

- SUSY LF bound-state equations for relativistic heavy-light bound states

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + U_M(\zeta) \right) \phi_{Meson} = M^2 \phi_{Meson}$$

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_N^2 - 1}{4\zeta^2} + U_M(\zeta) \right) \phi_{Baryon} = M^2 \phi_{Baryon}$$

where

$$U_{M,B}(\zeta) = V^2(\zeta) \mp V'(\zeta) + \frac{2L_{M,B} \mp 1}{\zeta} V(\zeta)$$

and the superpotential V is a priori unknown

- Embedding in AdS₅ [Phys. Rev. D **95**, 034016 (2017)]

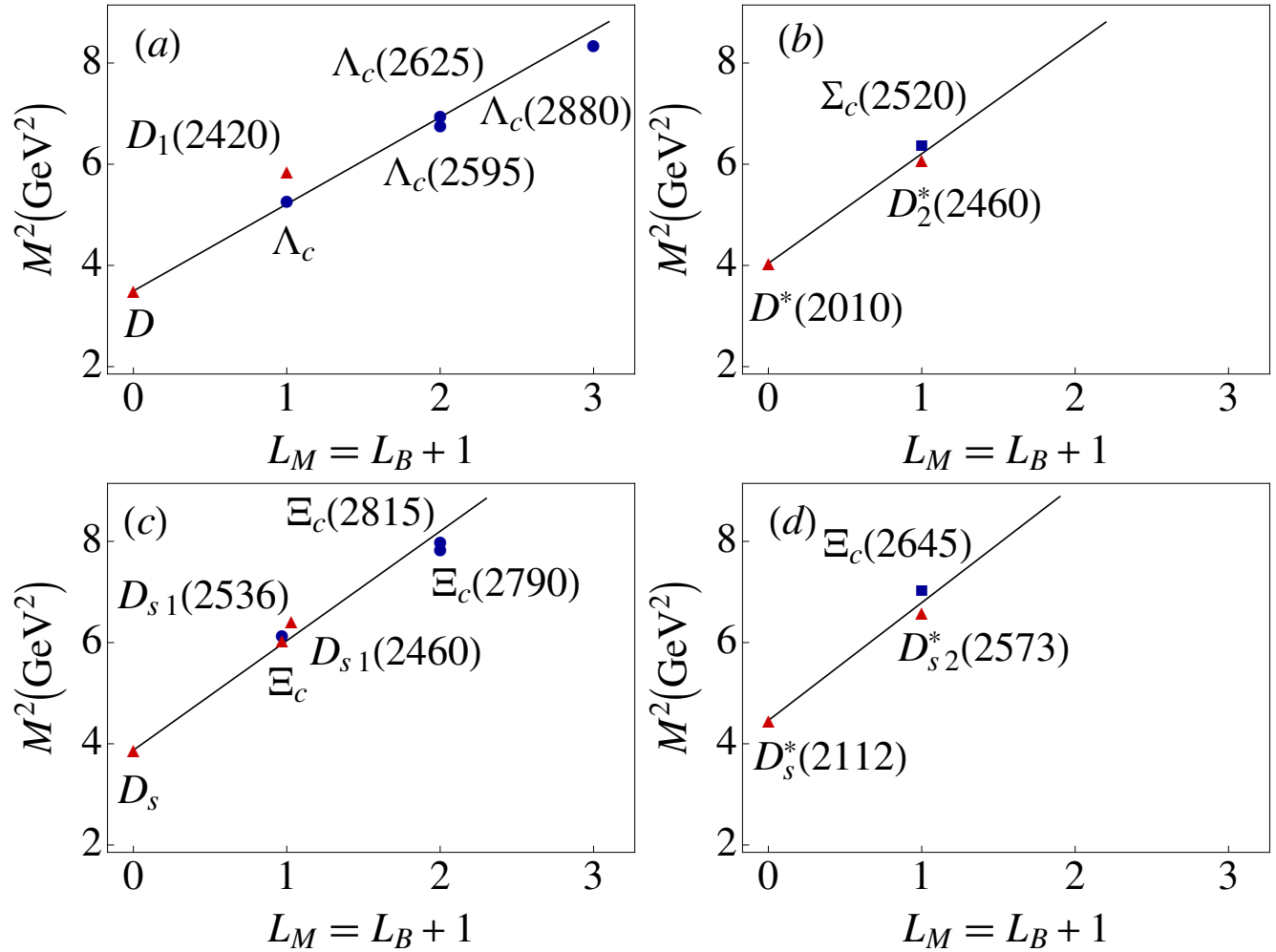
$$V(\zeta) = \frac{1}{2} \left(\lambda\zeta\sigma(\zeta) + \frac{\lambda^2\zeta^2\sigma'(\zeta)}{\lambda^2\zeta^2\sigma(\zeta) + 2(L_M - 1)\lambda} \right)$$

where $\sigma(\zeta)$ is an arbitrary function

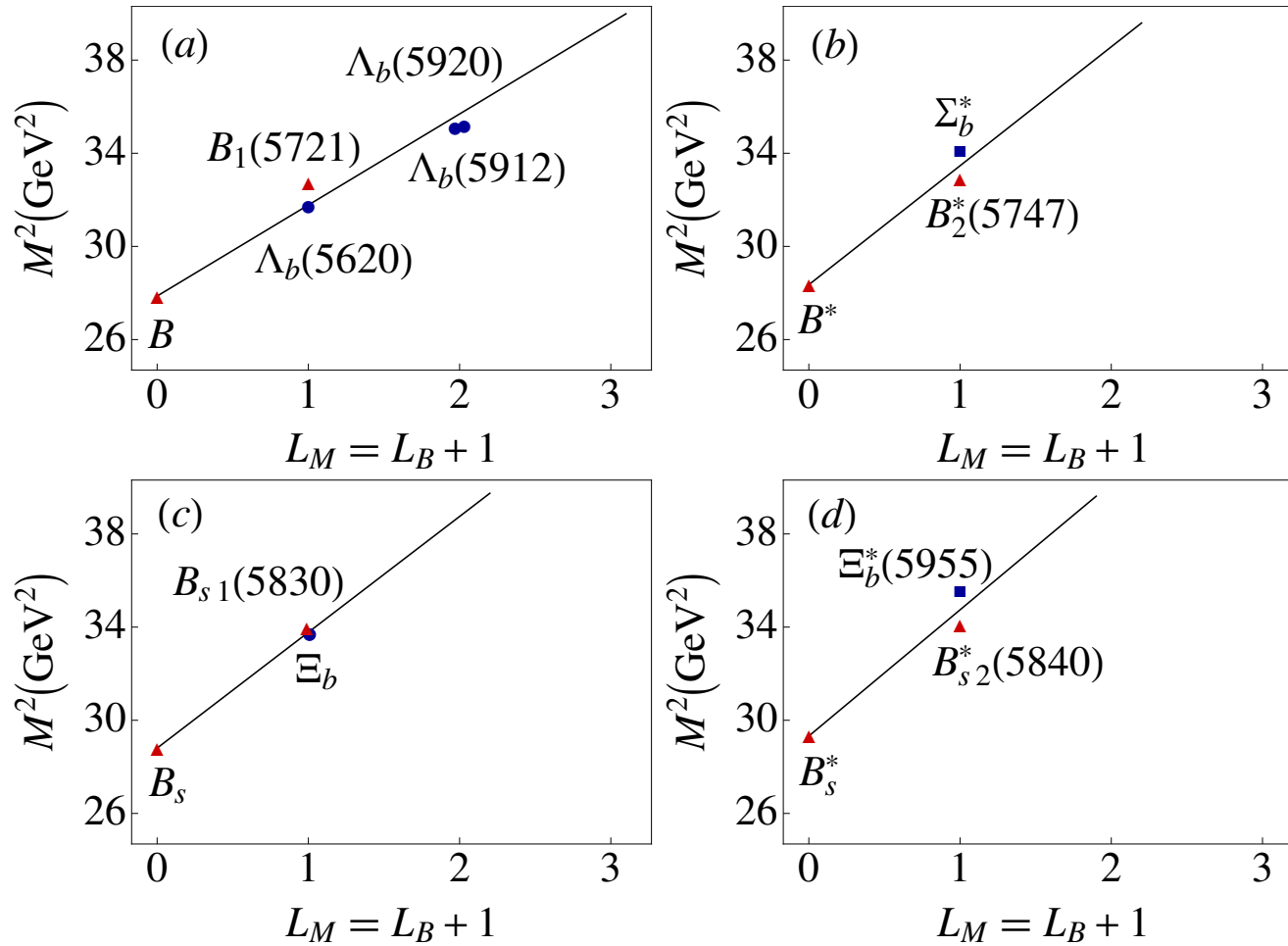
- If embedding is free of kinematical quantities $\sigma'(\zeta) = 0$ and $\sigma = \text{const} \equiv 2A$ with

$$V(\zeta) = \lambda A \zeta$$

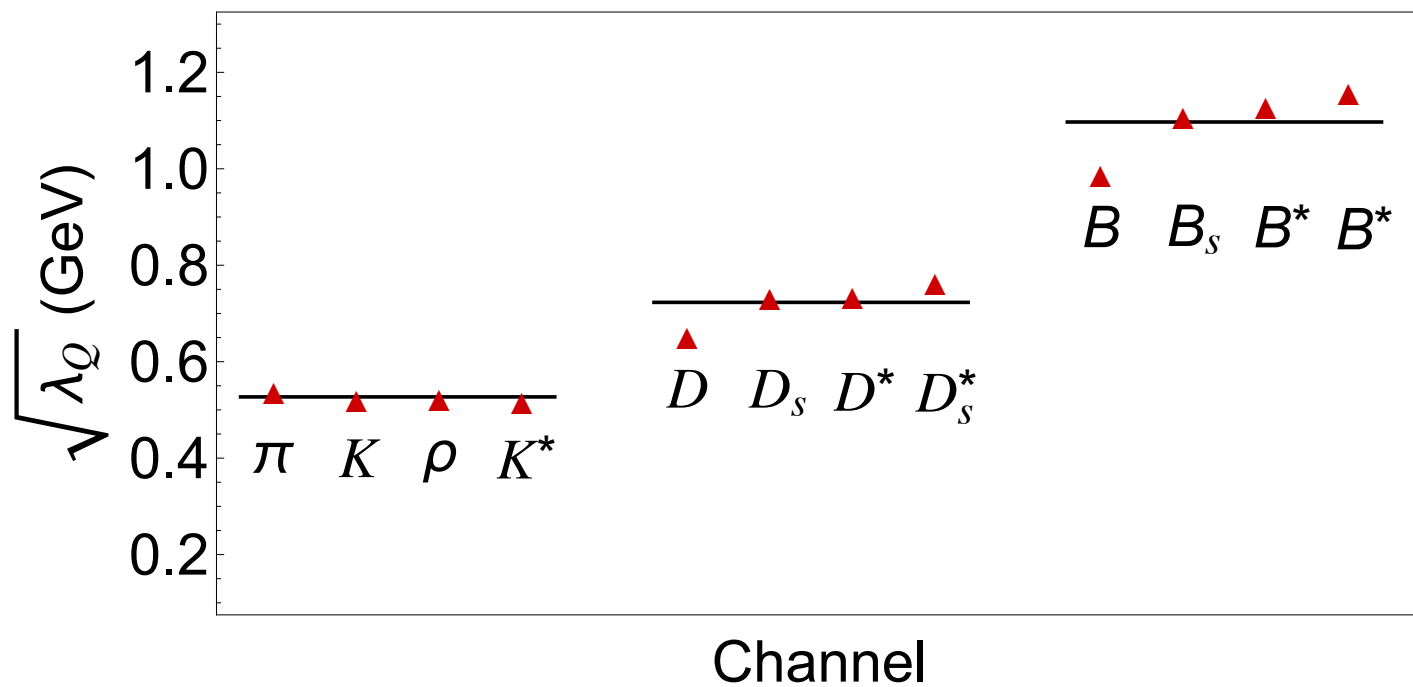
- For strongly broken conformal invariance the potential is still quadratic, $U \sim \zeta^2$, but since A is arbitrary the strength of the potential is not determined



Heavy-light mesons and baryons with one charm quark: $D = q\bar{c}$, $D_s = s\bar{c}$, $\Lambda_c = udc$, $\Sigma_c = qqc$, $\Xi_c = usc$.
 In (a) and (c) $s = 0$ and in (b) and (d) $s = 1$, where s is the total quark spin in the mesons or the spin of the quark cluster in the baryons



Heavy-light mesons and baryons with one bottom quark: $B = q\bar{b}$, $B_s = s\bar{b}$, $\Lambda_c = udb$, $\Sigma_b = qqb$, $\Xi_c = usb$.
 In (a) and (c) $s = 0$ and in (b) and (d) $s = 1$, where s is the total quark spin in the mesons or the spin of the diquark cluster in the baryons



Fitted value of $\sqrt{\lambda_Q}$ for different meson-baryon trajectories indicated by lowest meson state on given trajectory

Scale dependence of λ_Q from heavy quark symmetry

- HQET result for heavy mesons M_M : $\sqrt{M_M} f_M \rightarrow C$
- LFHQCD result for decay constant

$$f_M = \frac{1}{\sqrt{\int_0^1 dx e^{-m_Q^2/\lambda(1-x)}}} \frac{\sqrt{2N_C\lambda}}{\pi} \int_0^1 dx \sqrt{x(1-x)} e^{-m_Q^2/2\lambda(1-x)}$$

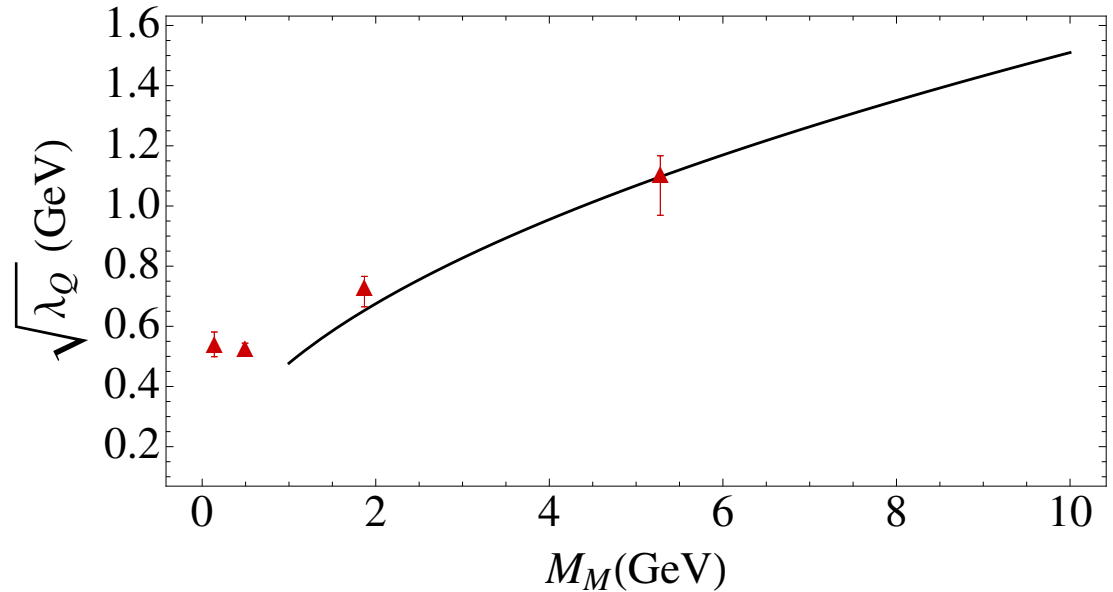
- In the large m_Q limit

$$f_M = \sqrt{\frac{6}{e}} \left(1 + \operatorname{erf}\left(\frac{1}{2}\right)\right) \frac{\lambda^{3/2}}{m_Q^2}$$

- From the HQET relation

$$\lambda_Q = C m_Q, \dim(C) = [\text{Mass}]$$

- Increase of λ_Q with increasing quark mass is dynamically necessary



4 Infrared behavior of the strong coupling in light-front holographic QCD

[S. J. Brodsky, GdT and A. Deur, PRD **81** (2010) 096010]

[A. Deur, S. J. Brodsky and GdT, PLB **750**, 528 (2015); PLB **757**, 275 (2016), arXiv:1608.04933 [hep-ph]]

- Effective coupling $\alpha_{g_1} = g_1^2/4\pi$ defined from an observable: g_1 scheme from Bjorken sum rule

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = 1 - \frac{6}{g_A} \int_0^1 dx g_1^{p-n}(x, Q^2)$$

- Infrared behavior of strong coupling in LFHQCD from Fourier transform of the LF transverse coupling:

$$\alpha_{g_1}^{LFHQCD}(Q^2) = \pi \exp(-Q^2/4\lambda)$$

- Large Q -dependence of α_s is computed from the pQCD β series:

$$Q^2 \frac{d\alpha_s}{dQ^2} = \beta(Q^2) = -(\beta_0 \alpha_s^2 + \beta_1 \alpha_s^3 + \beta_2 \alpha_s^4 + \dots)$$

where coefficients β_i are known up to β_4 in \overline{MS} scheme (five-loops):

- $\alpha_{g_1}^{pQCD}(Q^2)$ expressed as a perturbative expansion in $\alpha_{\overline{MS}}(Q)$:

$$\alpha_{g_1}^{pQCD}(Q^2) = \pi \left[\alpha_{\overline{MS}}/\pi + a_1 (\alpha_{\overline{MS}}/\pi)^2 + a_2 (\alpha_{\overline{MS}}/\pi)^3 + \dots \right]$$

The coefficients a_i are known up to order a_4

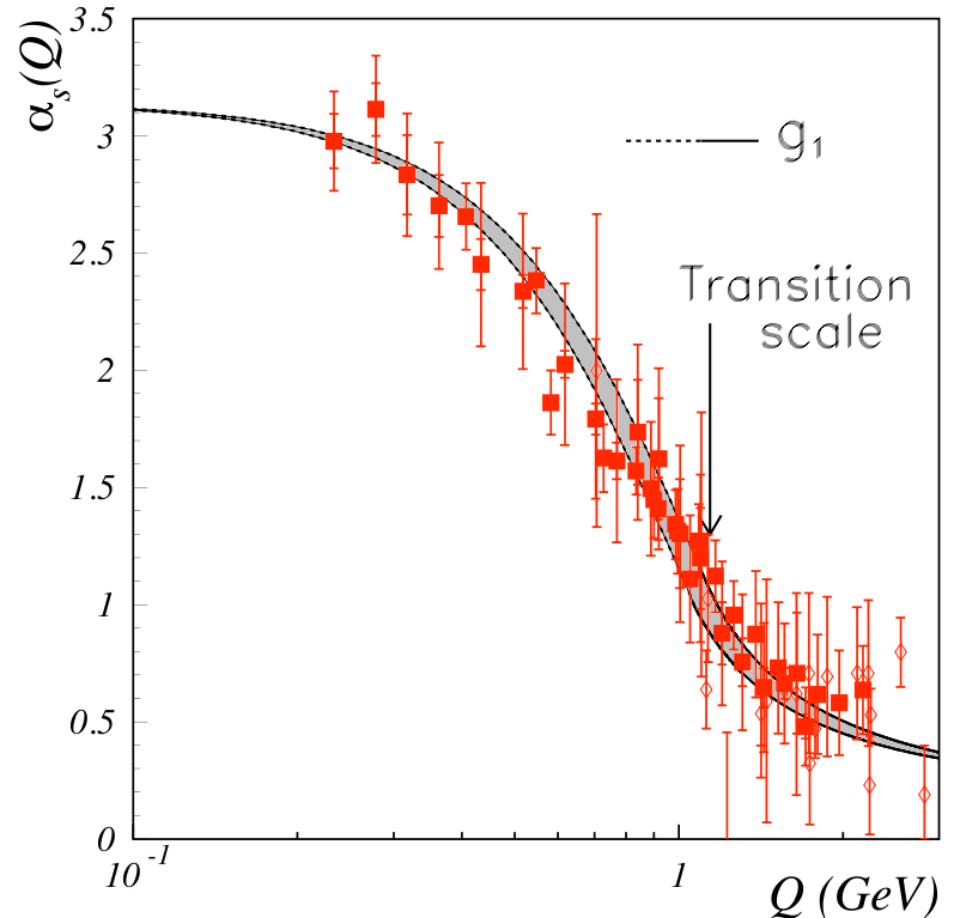
- Λ_{QCD} and transition scale Q_0 from matching perturbative and nonperturbative regimes:
for $\sqrt{\lambda} = 0.523 \pm 0.024$ GeV

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.019 \text{ GeV}$$

(World Average: $\Lambda_{\overline{MS}} = 0.332 \pm 0.019$ GeV)

Transition scale: $Q_0^2 \simeq 1 \text{ GeV}^2$

- Connection between the proton mass $M_p^2 = 4\lambda$ and the fundamental QCD mass scale Λ_{QCD} in any renormalization scheme !



- Nonperturbative β -function from LFHQCD (infrared fixed point $\beta(Q^2 \rightarrow 0) = 0$)

$$\beta(\alpha_s) = Q^2 \frac{d\alpha_s}{dQ^2} = -\pi \frac{Q^2}{4\lambda} e^{-Q^2/4\lambda}$$

- Similar behavior of the IR strong coupling from DSE:

D. Binosi, C. Mezrag, J. Papavassiliou, C. D. Roberts and J. Rodriguez-Quintero, arXiv:1612.04835

5 Constraints from the QCD trace anomaly

(In preparation)

- Why the pion is exactly massless in the chiral limit but the proton massive?
- Not a simple problem in QCD where the pion is a bound state of a quark and anti-quark: It requires an exact cancellation of the kinetic and potential energy
- In LFHQCD masslessness of the pion is guaranteed from the strong constraints imposed by superconformal symmetry
- What is the connection between the universal LFHQCD scale λ and QCD dynamical variables?
- Constraints imposed by the QCD trace anomaly?

$$\Theta_{\mu}^{\mu} = \frac{\beta(\alpha_s)}{2\alpha_s} G_{\rho\sigma}^a G^{a\rho\sigma} + (1 + \gamma_m) m \bar{\psi}\psi$$

with

$$\beta(\alpha_s) = \mu^2 \frac{d\alpha_s}{d\mu^2}, \quad \gamma_m = \mu \frac{d}{d\mu} \log m$$

- The matrix element of $\Theta^{\mu\nu}$ is

$$\langle P | \Theta^{\mu\nu} | P \rangle = 2P^\mu P^\nu$$

- In the chiral limit

$$M^2 = -\frac{1}{4} \frac{\beta(\alpha_s(\mu^2))}{\alpha_s(\mu^2)} \langle P | G^2 | P \rangle_{\mu^2}$$

evaluated at the renormalization scale $\mu^2 = M^2$

- The pion is a special case since we evaluate the β -function at the scale $\mu^2 = 0$ where it vanishes: The quantum contribution from the anomaly is zero and the pion is massless in the chiral limit
- At the proton scale $-P^2 = \mu^2 = M^2$

$$\lambda = \frac{1}{16} \langle P | G^2 | P \rangle_{\mu^2 = M_p^2}$$

with $\alpha_s(\mu^2) = \alpha_s(0) e^{-\mu^2/4\lambda}$

- Fundamental connection between LFHQCD scale λ and the QCD matrix element of G^2 in the proton

6 Nucleon form factors in light-front holographic QCD

[R. S. Sufian, G. F. de Teramond, S. J. Brodsky, A. Deur and H. G. Dosch, PRD **91** (2016) 096010]

- LFHQCD leads to analytic expressions for the hadron FFs which incorporates power-law scaling for a given twist τ from hard scattering and vector dominance at low energy

$$F_\tau(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_{\rho_{n=0}}^2}\right) \left(1 + \frac{Q^2}{M_{\rho_{n=1}}^2}\right) \cdots \left(1 + \frac{Q^2}{M_{\rho_{n=\tau-2}}^2}\right)}$$

expressed as a product of $\tau - 1$ poles along the vector meson Regge radial trajectory

- FF contains a cluster decomposition: hadronic FF factorizes into the $\tau = N - 1$ product of twist-two monopole FFs evaluated at different scales

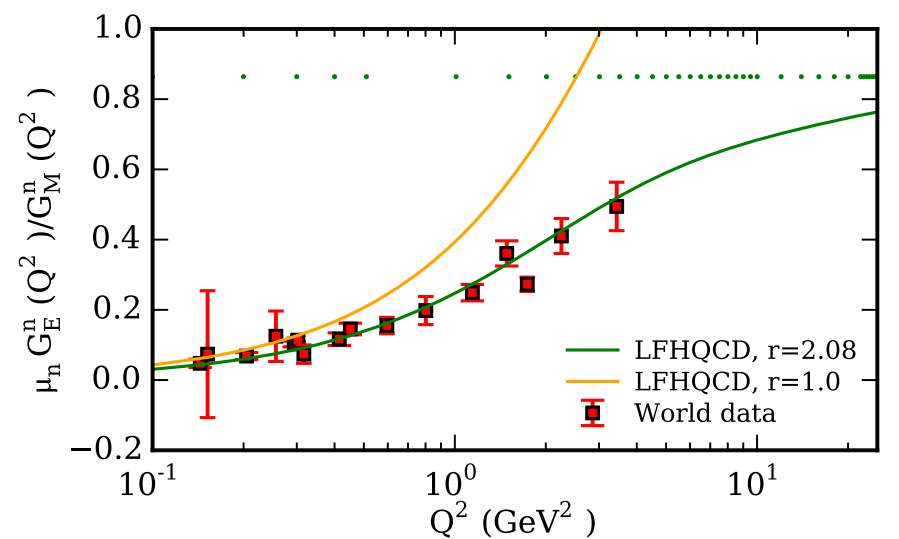
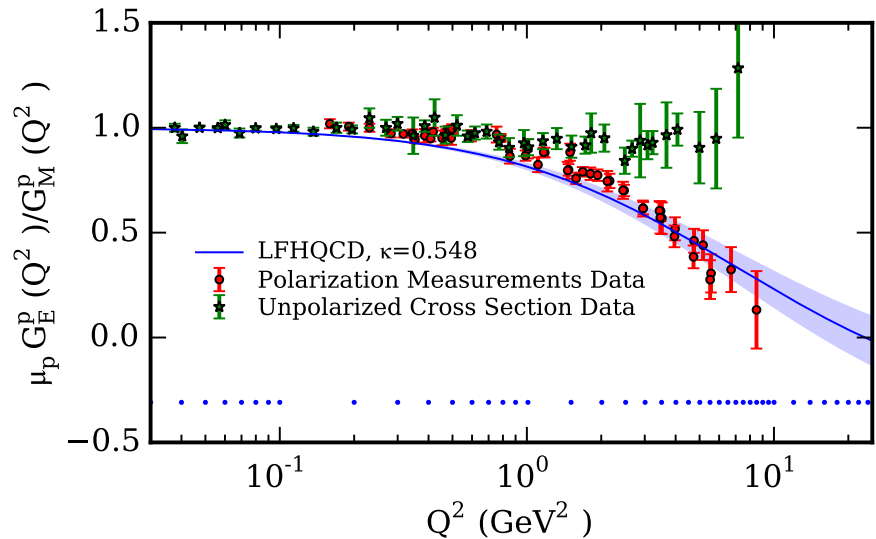
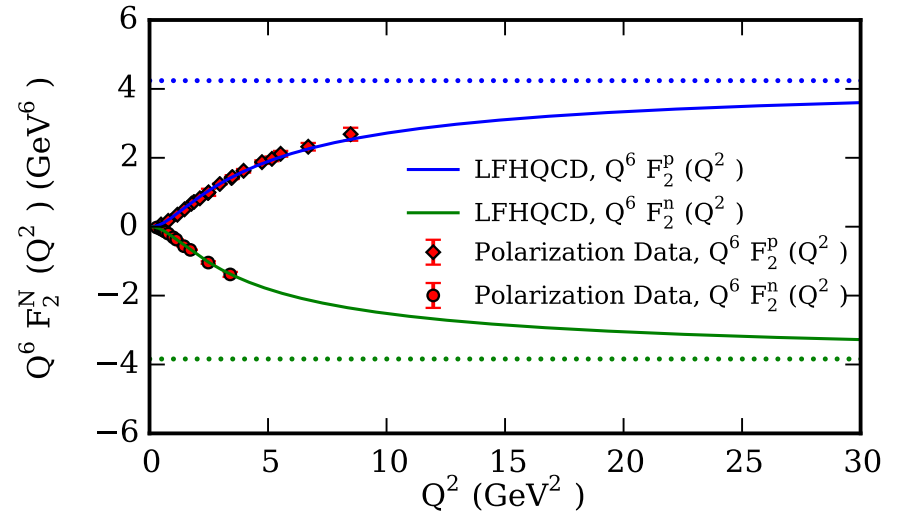
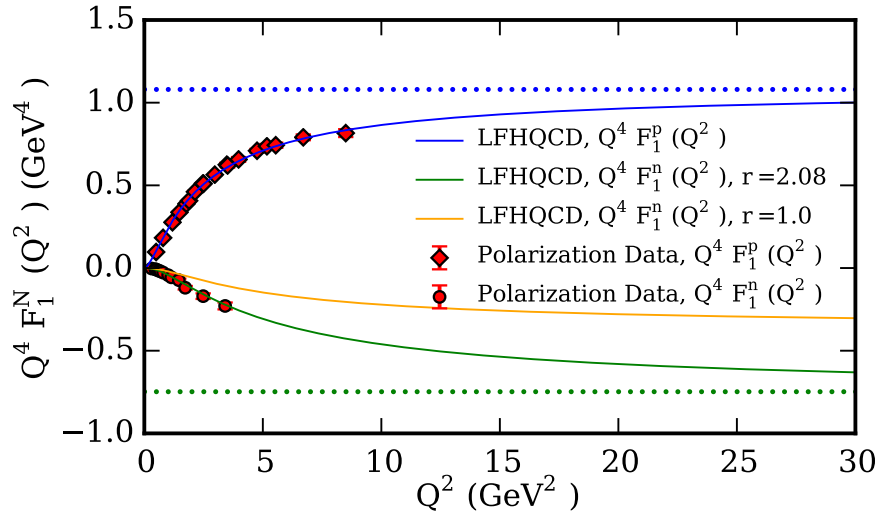
$$F_i(Q^2) = F_{i=2}(Q^2) F_{i=2}\left(\frac{1}{3}Q^2\right) \cdots F_{i=2}\left(\frac{1}{2^{i-1}}Q^2\right)$$

- In the case of a nucleon, for example, the Dirac FF for the twist-3 valence state

$$F_1(Q^2) = F_{i=2}(Q^2) F_{i=2}\left(\frac{1}{3}Q^2\right)$$

is the product of a point-like quark and a diquark-cluster FF consistent with leading-twist scaling,

$$Q^4 F_1(Q^2) \sim \text{const}$$



Comparison of the holographic results with selected world and data and asymptotic predictions

- Free parameters: two parameters for the probabilities of higher Fock states for the Pauli FF and a parameter r for possible SU(6) spin-flavor symmetry breaking effects in the neutron Dirac FF

7 Structure functions in light-front holographic QCD

- Recent progress in computation of nonperturbative structure functions: DAs, GPDs, TMDs ...

T. Gutsche, V. E. Lyubovitskij and I. Schmidt, arXiv:1610.03526

C. Mondal, arXiv:1609.07759

M. C. Traini, arXiv:1608.08410

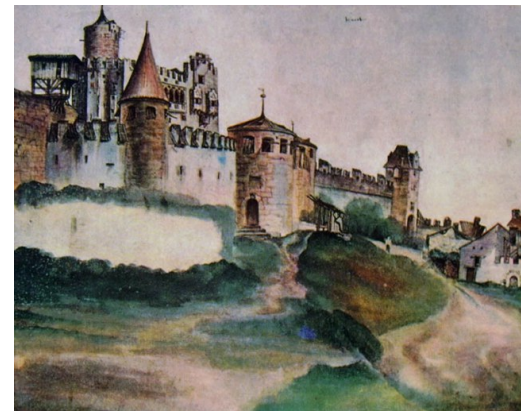
M. Traini, S. Scopetta, M. Rinaldi and V. Vento, arXiv:1609.07242

T. Maji and D. Chakrabarti, arXiv:1702.04557

M. Rinaldi, arXiv:1703.00348

A. Bacchetta, S. Cotogno and B. Pasquini, arXiv:1703.07669

...



Thanks !

Review of LFHQCD, see S. J. Brodsky, GdT, H. G. Dosch and J. Erlich, *Phys. Rept.* **584**, 1 (2015)