New Perspectives on n-$\bar{n}$ Oscillations

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Based, in part, on...
SG and Xinshuai Yan (UK), Phys. Rev. D93, 096008 (2016) [arXiv:1602.00693];
SG and Ehsan Jafari (UK), Phys. Rev. D91, 096010 (2015) [arXiv:1408.2264v2];
and on work in collaboration with Xinshuai Yan (UK).

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Motivation for Studies of n-$\bar{n}$ Oscillations

The Standard Model (SM) leaves many questions unanswered.
Most notably it cannot explain **the cosmic baryon asymmetry, dark matter, or dark energy.**

$\mathcal{B}$ violation plays a role in at least one of these puzzles.

Although $\mathcal{B}$ violation appears in the SM (sphalerons), we know nothing of its pattern at accessible energies.

Do processes occur with $|\Delta\mathcal{B}| = 1$ or $|\Delta\mathcal{B}| = 2$ or both?
The SM conserves $\mathcal{B} - \mathcal{L}$, but does Nature?

Severe limits on nucleon decay ($|\Delta\mathcal{B}| = 1$) exist, but the origin of $|\Delta\mathcal{B}| = 2$ processes can be completely distinct.

[Marshak and Mohapatra, PRL, 1980; Babu and Mohapatra, PLB, 2001 & 2012; Arnold, Fornal, and Wise, PRD, 2013]

If neutron-antineutron oscillations are observed (a "background free" signal!), then $\mathcal{B} - \mathcal{L}$ is **broken**, and **we have discovered physics beyond the SM.**

Establishing that $\mathcal{B} - \mathcal{L}$ is broken also supports a "see-saw" explanation of the ν mass. [Minkowski, 1977; Gell-Mann, Ramond, & Slansky, 1979; Yanagida, 1980; Mohapatra & Senjanovic, 1980]

Windows on BSM Physics from n-$\bar{n}$ Oscillations

It has long been thought that n-$\bar{n}$ oscillations could shed light on the mechanism of

- Baryogenesis [Kuzmin, 1967]
- Neutrino mass [Mohapatra and Marshak, 1980]

In contrast to proton decay, n-$\bar{n}$ probes new physics at "intermediate" energy scales. The two processes can be generated by **d=6** and **d=9** operators, respectively.

Crudely, $\Lambda_{p\,decay} \geq 10^{15}\,\mathrm{GeV}$ and $\Lambda_{n\bar{n}} \geq 10^{5.5}\,\mathrm{GeV}$.

n-$\bar{n}$ oscillations have been discussed in **many** different contexts.
Some examples... $\Longrightarrow$ **Note talks by Z. Berezhiani, P. S. Bhupal Dev.**

- TeV-Scale Seesaw + Quark-Lepton Unification [Babu, Dev, Mohapatra, 2009; Babu, Dev, Fortes, Mohapatra, 2013]
- SO(10) GUT-Scale Seesaw + TeV sextets [Babu, Mohapatra, 2012]
- TeV-Scale Extra Dimensions [Dvali, Gabadadze; Nussinov, Shrock, 2002; Winslow, Ng, 2010]
- Supersymmetry/Superstring [Mohapatra and Valle, 1986; Goity, Sher, 1994]
- Spontaneous Low Scale $\not{B}$ Violation [Berezhiani, 2015]

On $\mathcal{B} - \mathcal{L}$ Violation

The possibility of $\mathcal{B} - \mathcal{L}$ breaking is more commonly considered in the context of the fundamental nature of the massive neutrino.

A massive neutrino could be a **Majorana** particle with its mass generated by the $d = 5$ operator $(v^2/\Lambda)\nu_L^T C \nu_L$ (N.B. $\mathcal{B} - \mathcal{L}$ is broken!). [Weinberg, 1979]
A massive neutrino could also be a **Dirac** particle, with its mass generated by the Higgs mechanism (N.B. enter the right-handed neutrino!)

A massive neutrino could also get its mass from terms of both types. Even if the Dirac mass were to dominate, the mass eigenstates would be Majorana.

[Gribov and Pontecorvo, 1969; Bilenky and Pontecorvo, 1983]
Although a Majorana mass term breaks $\mathcal{B} - \mathcal{L}$, other sources of $\mathcal{B} - \mathcal{L}$ violation could operate.
Nevertheless, the observation of neutrinoless $\beta\beta$ decay ($|\Delta\mathcal{L}| = 2$) would reveal that the neutrino is Majorana, that the neutrino is its own antiparticle.

[Schechter and Valle, PRD, 1982]
The nuclear structure problem in $0\nu\beta\beta$ decay changes with the source of new physics; most effort has been devoted to the study of nuclear matrix elements from Majorana mass terms. [de Gouvêa and Vogel, 2013]

Usual Phenomenology of $n - \bar{n}$ Oscillations

A 2×2 effective Hamiltonian framework for n-$\bar{n}$ mixing

[Marshak and Mohapatra, PLB, 1980; Cowsik and Nussinov, PLB, 1981; Phillips II et al. [NNbar Collaboration], arXiv:1410.1100]

$$\mathcal{H} = \begin{pmatrix} M_n - \mu_n B & \delta \\ \delta & M_n + \mu_n B \end{pmatrix},$$

yields

$$P_{n\to\bar{n}}(t) \simeq \frac{\delta^2}{2(\mu_n B)^2}\left[1 - \cos\left(2\mu_n B t\right)\right]\exp(-\lambda t)$$

so that unless $t \ll 1/(2\mu_n B)$, a nonzero **B** "quenches" n-$\bar{n}$ oscillations.

There have been many studies of n-$\bar{n}$ in "elixir" magnetic fields, all in the 2×2 framework.

[Arndt, Prasad, Riazuddin, PRD 1983; Pusch, Nuov. Cim. 1983; Krsticć, Komarov, Janen, Zenko, PRD 1988; Dubbers, NIM 1989; Kinkel, Z. Phys. C 1992]

Experimentally magnetic fields have been mitigated (to great expense), yielding $P_{n\to\bar{n}}(t) \simeq \delta^2 t^2$ and $\tau_{n\bar{n}} \equiv 1/\delta$ with $\tau_{n\bar{n}} \geq 0.85 \times 10^8\,\mathrm{s}$ at 90% C.L.

[Baldo-Ceolin et al., ZPC, 1994 (ILL)]

However, the n, $\bar{n}$ system in a magnetic field has four (not two!) physical degrees of freedom; we will consider the role of spin explicitly.

n-$\bar{n}$ Oscillations and Nuclear Stability

n-$\bar{n}$ oscillations can be studied in bound or free systems.

New limits on dinucleon decay in nuclei have also recently been established.

[Gustafson et al., Super-K Collaboration, arXiv:1504.0104.]

${}^{16}\mathrm{O}(pp) \to {}^{14}\mathrm{C}\,\pi^+\pi^+$ has $\tau > 7.22 \times 10^{31}$ years at 90% CL.
${}^{16}\mathrm{O}(pn) \to {}^{14}\mathrm{N}\,\pi^+\pi^0$ has $\tau > 1.70 \times 10^{32}$ years at 90% CL.
${}^{16}\mathrm{O}(nn) \to {}^{14}\mathrm{O}\,\pi^0\pi^0$ has $\tau > 4.04 \times 10^{32}$ years at 90% CL.
Note $\tau_{NN} = T_{\mathrm{nuc}}\tau_{n\bar{n}}^2$ with $T_{\mathrm{nuc}} \sim 1.1 \times 10^{25}\mathrm{s}^{-1}$

Large suppression factors appear in all such nuclear studies, making free searches more effective.
In the case of bound n-$\bar{n}$ the suppression is set by

$$\frac{\delta^2}{(V_n - V_{\bar{n}})^2}$$

the difference in nuclear optical potentials. [Dover, Gal, and Richard; Friedman and Gal, 2008]
Now ${}^{16}\mathrm{O}(n-\bar{n})$ has $\tau > 1.9 \times 10^{32}$ years at 90% CL,
yielding $\tau_{n\bar{n}} > 2.7 \times 10^8$ s. [Abe et al., Super-K Collaboration, arXiv:1109.4227.]
Cf. free limit: $\tau_{n\bar{n}} \geq 0.85 \times 10^8\,\mathrm{s}$ at 90% C.L. [Baldo-Ceolin et al., ZPC, 1994 (ILL)]
with future improvements expected.
The nuclear suppression dwarfs that from magnetic fields.

n-$\bar{n}$ Oscillations: the Quark Analog of $0\nu\beta\beta$ Decay?

The SM preserves $\mathcal{B} - \mathcal{L}$, so that the observation of either n-$\bar{n}$ oscillations ($|\Delta\mathcal{B}| = 2$) or of neutrinoless $\beta\beta$ decay ($|\Delta\mathcal{L}| = 2$) would reveal the existence of dynamics beyond the SM.

However, QCD is a gauge theory in SU(3) color $\leftrightarrow$ $\mathbf{3} \neq \mathbf{3}^*$.
Thus n is distinct from $\bar{n}$, and it has a significant magnetic moment.

Certainly the neuton's Dirac mass dominates its measured mass; note $\delta m = (\tau_{n\bar{n}})^{-1} \leq 6 \times 10^{-29}\,\mathrm{MeV}$.

[Baldo-Ceolin et al., ZPC, 1994 (ILL)]

A neutron thus best resembles a pseudo-Dirac neutrino, though its electromagnetic interactions are also well established....

To explore the role of spin in the interplay with SM interactions, we embed the usual phenomenology of n-$\bar{n}$ oscillations in a 4×4 Hamiltonian framework.

[SG and Jafari, 2015]

Effective Hamiltonian Framework

A 4×4 **matrix describes** $\mathcal{H}$ **in this case.**

$\mathcal{H}_{ij}$ with $i, j = 1, \ldots 4$ maps to $n(\mathbf{p}, +)$, $\bar{n}(\mathbf{p}, +)$, $n(\mathbf{p}, -)$, and $\bar{n}(\mathbf{p}, -)$.

Hermiticity and CPT invariance limit its form.

But what is the precise form of the CPT transformation in this case?

Recall from neutrino physics: the discrete symmetry transformations of a theory should not depend on whether it contains Dirac or Majorana fields.

[Kayser and Goldhaber, 1983; Kayser, 1984 — also Carruthers, 1971; Feinberg and Weinberg, 1959]

Consequently the CPT, CP, and C phases of Majorana fields or states are restricted.

[Kayser and Goldhaber, 1983; Kayser, 1984]

Generalizing this to theories of fermions with B-L violation, the phases associated with the discrete symmetry transformations must themselves be restricted.

[SG and Yan, 2016]

Majorana Phase Constraints

For any fermion field

$$\mathbf{C}\psi(x)\mathbf{C}^{-1} = \eta_c C\gamma^0\psi^*(x) \equiv \eta_c i\gamma^2\psi^*(x) \equiv \eta_c\psi^c(x)\,,$$
$$\mathbf{P}\psi(t,\mathbf{x})\mathbf{P}^{-1} = \eta_p\gamma^0\psi(t,-\mathbf{x})\,,$$
$$\mathbf{T}\psi(t,\mathbf{x})\mathbf{T}^{-1} = \eta_t\gamma^1\gamma^3\psi(-t,\mathbf{x})\,,$$

Thus $\mathbf{P}^2\psi(x)\mathbf{P}^{-2} = \eta_p^2\psi(x)$ but $\mathbf{C}^2\psi(x)\mathbf{C}^{-2} = \psi(x)$; $\mathbf{T}^2\psi(x)\mathbf{T}^{-2} = -\psi(x)$

The plane wave expansion of a general Majorana field ψ_m is

$$\psi_m(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}\sqrt{2E}} \sum_s \left\{ f(\mathbf{p},s)u(\mathbf{p},s)e^{-ip\cdot x} + \lambda f^\dagger(\mathbf{p},s)v(\mathbf{p},s)e^{ip\cdot x} \right\}$$

Applying C and noting the Majorana relation,

$$i\gamma^2\psi_m^*(x) = \lambda^*\psi_m(x)$$

yields

$$\mathbf{C}\psi_m(x)\mathbf{C}^{-1} \quad = \quad \eta_c\lambda^*\psi_m(x)$$

$\mathbf{C}f(\mathbf{p},s)\mathbf{C}^{-1} = \eta_c\lambda^* f(\mathbf{p},s)$ and $\mathbf{C}f^\dagger(\mathbf{p},s)\mathbf{C}^{-1} = \eta_c\lambda^* f^\dagger(\mathbf{p},s)$
Since **C** is a unitary operator, taking the adjoint shows $\eta_c^*\lambda$ is real.

Majorana Phase Constraints

Under CP, we find $\eta_p^* \eta_c^* \lambda$ is imaginary, or that η_p^* is imaginary.

Under T we find that $\eta_t \lambda$ is real, whereas

$$\mathbf{CPT}\psi_m(x)(\mathbf{CPT})^{-1} = -\eta_c \eta_p \eta_t \gamma^5 \psi_m^*(-x)$$

yielding
$\mathbf{CPT} f(\mathbf{p}, s)(\mathbf{CPT})^{-1} = s\lambda^* \eta_c \eta_p \eta_t f(\mathbf{p}, -s)$
$\mathbf{CPT} f^\dagger(\mathbf{p}, s)(\mathbf{CPT})^{-1} = -s\lambda \eta_c \eta_p \eta_t f^\dagger(\mathbf{p}, -s)$

Since **CPT** is antiunitary, $\mathbf{CPT} = KU_{cpt}$, where U_{cpt} denotes a unitarity operator.
We conclude $\eta_c \eta_p \eta_t$ is pure imaginary.
Since η_p is imaginary, $\eta_c \eta_t$ must also be real — but $\eta_c \eta_p$ itself is unconstrained.
Since the phases are unimodular, they impact the discrete symmetry transformation properties of $\mathcal{B}$-$\mathcal{L}$ violating operators only.
Building a Majorana field from Dirac fields yields
$\psi_{m\pm}(x) = \frac{1}{\sqrt{2}}(\psi(x) \pm \mathbf{C}\psi(x)\mathbf{C}^{-1})$ and $\lambda = \pm\eta_c$; all our other conclusions emerge as well.

The prototypical $\mathcal{B} - \mathcal{L}$ violating operator is of form
$\psi^T C\psi + \text{h.c.}$
Since C satisfies $(\sigma^{\mu\nu})^{\mathrm{T}} C = -C\sigma^{\mu\nu}$, this operator is Lorentz invariant. Under **CPT**...

$$\mathcal{O}_1 = \psi^T C\psi + \text{h.c.} \qquad \stackrel{\textbf{CPT}}{\Longrightarrow} -(\eta_c\eta_p\eta_t)^2$$

$$\mathcal{O}_2 = \psi^T C\gamma_5\psi + \text{h.c.} \qquad \stackrel{\textbf{CPT}}{\Longrightarrow} -(\eta_c\eta_p\eta_t)^2$$

$$\mathcal{O}_3 = \psi^T C\gamma^\mu\psi\,\partial^\nu F_{\mu\nu} + \text{h.c.} \qquad \stackrel{\textbf{CPT}}{\Longrightarrow} +(\eta_c\eta_p\eta_t)^2$$

$$\mathcal{O}_4 = \psi^T C\gamma^\mu\gamma_5\psi\,\partial^\nu F_{\mu\nu} + \text{h.c.} \qquad \stackrel{\textbf{CPT}}{\Longrightarrow} -(\eta_c\eta_p\eta_t)^2$$

$$\mathcal{O}_5 = \psi^T C\sigma_{\mu\nu}\psi\, F^{\mu\nu} + \text{h.c.} \qquad \stackrel{\textbf{CPT}}{\Longrightarrow} +(\eta_c\eta_p\eta_t)^2$$

$$\mathcal{O}_6 = \psi^T C\sigma_{\mu\nu}\gamma_5\psi\, F^{\mu\nu} + \text{h.c.} \qquad \stackrel{\textbf{CPT}}{\Longrightarrow} +(\eta_c\eta_p\eta_t)^2$$

The phase constraint $(\eta_c\eta_p\eta_t)^2 = -1$ only flips the sign of the eigenvalue!
The operators do not transform under CPT with definite sign!

Theories of Dirac Fermions with $\mathcal{B}-\mathcal{L}$ Violation

The operators

$$\mathcal{O}_3 = \psi^T C\gamma^\mu\psi\,\partial^\nu F_{\mu\nu} + \text{h.c.} \qquad \overset{\textbf{CPT}}{\Longrightarrow} +(\eta_c\eta_p\eta_t)^2$$

$$\mathcal{O}_5 = \psi^T C\sigma_{\mu\nu}\psi\, F^{\mu\nu} + \text{h.c.} \qquad \overset{\textbf{CPT}}{\Longrightarrow} +(\eta_c\eta_p\eta_t)^2$$

$$\mathcal{O}_6 = \psi^T C\sigma_{\mu\nu}\gamma_5\psi\, F^{\mu\nu} + \text{h.c.} \qquad \overset{\textbf{CPT}}{\Longrightarrow} +(\eta_c\eta_p\eta_t)^2$$

become CPT odd once the phase constraint $(\eta_c\eta_p\eta_t)^2 = -1$ is applied.
They also vanish once the anticommuting nature of the fermion fields is taken into account.
That these operators do not contribute has long been recognized:
The vector, tensor, and axial tensor electromagnetic form factors of Majorana neutrinos vanish.

[Schechter and Valle, 1981; Nieves, 1982; Kayser, 1982; Shrock, 1982; Li and Wilczek, 1982; Davidson, Gorbahn, Santamaria, 2006]

Recall flavor-spin neutrino oscillations. The flavor-diagonal ν transition magnetic moment vanishes due to the antisymmetry of fermion exchange.

[Okun, Voloshin, and Vysotsky, 1986 & 1986; Lim and Marciano, 1988]

Theories with $\mathcal{B}-\mathcal{L}$ Violation

If the fermion field is Majorana, we have $i\gamma^2\psi_m^*(x) = \lambda^*\psi_m(x)$ so that $\psi_m^T C = \lambda\bar{\psi}_m$, $C^\dagger\psi_m^* = \lambda^*\gamma^0\psi_m$, $\psi_m^\dagger C^\dagger = -\lambda^*\psi_m^T\gamma^0$, and $C\psi_m = -\lambda\gamma^0\psi_m^*$.

Thus

$$\mathcal{O}_1 = (\lambda+\lambda^*)\bar{\psi}_m\psi_m = -(\lambda+\lambda^*)\psi_m^T\bar{\psi}_m^T$$
$$\mathcal{O}_2 = (\lambda-\lambda^*)\bar{\psi}_m\gamma_5\psi_m = -(\lambda-\lambda^*)\psi_m^T\gamma_5\bar{\psi}_m^T$$
$$\mathcal{O}_3 = (\lambda+\lambda^*)\bar{\psi}_m\gamma^\mu\psi_m j_\mu = (\lambda+\lambda^*)\psi_m^T\gamma^{\mu\,T}\bar{\psi}_m^T j_\mu, \text{ with } j_\mu \equiv \partial^\nu F_{\mu\nu}$$
$$\mathcal{O}_4 = (\lambda-\lambda^*)\bar{\psi}_m\gamma^\mu\gamma_5\psi_m j_\mu = -(\lambda-\lambda^*)\psi_m^T\gamma_5\gamma^{\mu\,T}\bar{\psi}_m^T j_\mu$$
$$\mathcal{O}_5 = (\lambda+\lambda^*)\bar{\psi}_m\sigma^{\mu\nu}\psi_m F_{\mu\nu} = (\lambda+\lambda^*)\psi_m^T(\sigma^{\mu\nu})^T\bar{\psi}_m^T F_{\mu\nu}$$
$$\mathcal{O}_6 = (\lambda-\lambda^*)\bar{\psi}_m\sigma^{\mu\nu}\gamma_5\psi_m F_{\mu\nu} = (\lambda-\lambda^*)\psi_m^T\gamma_5(\sigma^{\mu\nu})^T\bar{\psi}_m^T F_{\mu\nu}$$

Since $\bar{\psi}_m\psi_m = -\psi_m^T\bar{\psi}_m^T$, we see that $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_4$ need not vanish, but that $\mathcal{O}_3$, $\mathcal{O}_5$, $\mathcal{O}_6$ must vanish.

In the case of Dirac fields, this proof does not hold, and we must turn to other methods.

For example, expanding the operators in a free-particle, particle-wave expansion shows that $\mathcal{O}_3$, $\mathcal{O}_5$, $\mathcal{O}_6$ all vanish due to the anticommuting nature of fermion fields.

The analogous quark-level operators also vanish in the MIT bag model.

CP Transformation Properties

The surviving operators transform under CP as

$$\mathcal{O}_1 = \psi^T C \psi + \text{h.c.} \quad \stackrel{\textbf{CP}}{\Longrightarrow} -(\eta_c \eta_p)^2$$

$$\mathcal{O}_2 = \psi^T C \gamma_5 \psi + \text{h.c.} \quad \stackrel{\textbf{CP}}{\Longrightarrow} -(\eta_c \eta_p)^2$$

$$\mathcal{O}_4 = \psi^T C \gamma^\mu \gamma_5 \psi \, \partial^\nu F_{\mu\nu} + \text{h.c.} \quad \stackrel{\textbf{CP}}{\Longrightarrow} -(\eta_c \eta_p)^2$$

where we have left the phase dependence explicit.

Employing $\eta_p^2 = -1$, the CP transformation properties remain nevertheless indeterminate — because they are given by η_c^2.

Prompted by the remark that $n^T C n + \text{h.c.}$ breaks CP,

[Berezhiani and Vainshtein, 2015]

explicit examples of the indeterminate CP of $n^T C n + \text{h.c.}$ employing $\psi \to \psi' = e^{i\theta}\psi$, have also been noted.

[Fujikawa and Tureanu, 2015] $\Longrightarrow$ **Note talk by A. Tureanu.**

The noted phase rotation has the effect of changing $\eta_c \to e^{2i\theta}\eta_c$, $\eta_t \to e^{-2i\theta}\eta_t$, with η_p unchanged, in the C, T, and P transformations.

Implications of the CPT Phases

Previously it had been suggested that spin-dependent SM effects involving transverse magnetic fields could help connect n and $\bar{n}$ states of opposite spin and thus evade the need for magnetic field quenching.

[SG and Jafari, 2015]

The success of this suggestion is sensitive to the CPT phase constraint we have discussed.

Fixing the spin quantization axis with $\mathbf{B}_0$ and defining $\omega_0 \equiv -\mu_n B_0$ and $\omega_1 \equiv -\mu_n B_1$, the Hamiltonian matrix in the $|n(+)\rangle$, $|\bar{n}(+)\rangle$, $|\bar{n}(-)\rangle$, $|\bar{n}(-)\rangle$ basis at $t > 0$ is of form

$$\mathcal{H} = \begin{pmatrix} M + \omega_0 & \delta & \omega_1 & 0 \\ \delta & M - \omega_0 & 0 & -\omega_1 \\ \omega_1 & 0 & M - \omega_0 & -\delta\eta_{cpt}^2 \\ 0 & -\omega_1 & -\delta\eta_{cpt}^2 & M + \omega_0 \end{pmatrix},$$

where M is the neutron mass and δ denotes a $n(+) \to \bar{n}(+)$ transition matrix element.

Previously $\eta_{cpt}^2 = 1$ was employed.

[SG and Jafari, 2015]

Example

A static transverse field $\mathbf{B}_1$ is suddenly applied at $t = 0$. [Purcell]

$$\mathcal{H} = \begin{pmatrix} M+\omega_0 & \delta & \omega_1 & 0 \\ \delta & M-\omega_0 & 0 & -\omega_1 \\ \omega_1 & 0 & M-\omega_0 & -\delta \\ 0 & -\omega_1 & -\delta & M+\omega_0 \end{pmatrix}$$

Note $|\delta| \ll |\omega_0|, |\omega_1|$**, but consider** $|\omega_0| \sim |\omega_1|$**.**
Here $\mathcal{P}_{n+\to\bar{n}-}(t) = \mathcal{P}_{n-\to\bar{n}+}(t)$ and $\mathcal{P}_{n+\to\bar{n}+}(t) = \mathcal{P}_{n-\to\bar{n}-}(t)$.

The unpolarized n-$\bar{n}$ transition probability is

$$\begin{aligned} \mathcal{P}_{n\to\bar{n}}(t) &= \delta^2 \left[\frac{\omega_1^2 t^2}{\omega_0^2+\omega_1^2} + \frac{\omega_0^2}{(\omega_0^2+\omega_1^2)^2} \sin^2\left(t\sqrt{\omega_0^2+\omega_1^2}\right) \right. \\ &+ \left. \frac{\omega_0^2\omega_1^2 t}{(\omega_0^2+\omega_1^2)^{5/2}} \left(1 - \sin\left(2t\sqrt{\omega_0^2+\omega_1^2}\right)\right) \right] + \mathcal{O}(\delta^3)\,, \end{aligned}$$

— and the first term is of $\mathcal{O}(1)$ in magnetic fields! There's no quenching!
This arises because $n+ \to \bar{n}-$ and $n- \to \bar{n}+$ can occur.

A Problem — and its Solution.

The exact eigenvalues at $t > 0$ are

$$
\begin{aligned}
E_1 &= M - \sqrt{\omega_0^2 + (\delta - \omega_1)^2}\,, \\
E_2 &= M + \sqrt{\omega_0^2 + (\delta - \omega_1)^2}\,, \\
E_3 &= M - \sqrt{\omega_0^2 + (\delta + \omega_1)^2}\,, \\
E_4 &= M + \sqrt{\omega_0^2 + (\delta + \omega_1)^2}
\end{aligned}
$$

The eigenenergies do not depend on just $|\mathbf{B}|$ for $t > 0$.
This is incompatible with rotational invariance.

[Voloshin, priv. comm., 2015; Berezhiani and Vainshtein, 2015]

What's wrong?!

We have noted that consistency in the description of Dirac and Majorana fields requires $\eta_{cpt}^2 = -1$.

Including this sign in our Hamiltonian framework the "Purcell" example changes the outcomes completely.

n-$\bar{n}$ Oscillations and Spin

Upon including $\eta_{cpt}^2 = -1$ the solution of the "Purcell" example yields

- Energy eigenvalues at $t > 0$ that depend on $|\mathbf{B}|$ only
- No $n+ \to \bar{n}-$ or $n- \to \bar{n}+$ transitions
- Quenching of $n\bar{n}$ transitions irrespective of transverse magnetic fields

However, spin-dependent effects may still prove key to realizing n-$\bar{n}$ transitions. Consider

$$\mathcal{O}_4 = \psi^T C\gamma^\mu \gamma_5 \psi\, \partial^\nu F_{\mu\nu} + \mathrm{h.c.}$$

$n(+) \to \bar{n}(-)$ can occur directly because the interaction with the current can flip the spin.

This is concomitant with $n(p_1, s_1) + n(p_2, s_2) \to \gamma^*(k)$, for which only $L = 1$ and $S = 1$ is allowed via angular momentum conservation and Fermi statistics. [Berezhiani and Vainshtein, 2015]

Nuclear stability should also set limits on this source of B-L violation.

[Berezhiani and Vainshtein, 2015]

Here $e + n \to \bar{n} + e$, e.g., so that the experimental concept "$n\bar{n}$ conversion" would be completely different.

Implications of the CP Phases

Physically, too, the appearance of $n - \bar{n}$ oscillations cannot signal CP violation — only one operator mediates an low energy, *en vacuo* $n - \bar{n}$ **transition, and it appears as $|\delta|^2$.**

This is in contradistinction to case of a permanent EDM.

$$\mathcal{H} = -\frac{\mu}{S}\mathbf{S}\cdot\mathbf{B} - \frac{d}{S}\mathbf{S}\cdot\mathbf{E}$$

The appearance of nonzero *d changes* the splitting of spin states in a nonzero magnetic field, so that it is appreciable even if engendered by a single operator.

$\mathcal{B}-\mathcal{L}$ *Violation and Theories of Self-Conjugate Fermions*

We have found that the employing phase constraint $(\eta_c \eta_p \eta_t)^2 = -1$ still yields CPT-odd operators. We emphasize that the operators are Lorentz invariant by construction.

"When you come to the fork in the road, take it." — Yogi Berra

The CPT theorem is not broken, however, because the wrong CPT operators do appear to vanish.

The stature of the proof that they do indeed vanish depends on whether the fermions are Majorana or Dirac. In the latter case, canonical quantization and a Fourier expansion of the fermion field is required, though fermion antisymmetry is still key.

To consider why it should be possible to write down a CPT-odd, Lorentz-invariant operator (even if it does vanish!), we recall theories of self-conjugate particles with half-integer isospin, which are non-local and have anomalous CPT properties. [Carruthers, 1967; Lee, 1967; Fleming and Kazes, 1967; Jin, 1967; Kantor, 1967; Steinmann, 1967; Zumino and Zwanziger, 1967; Carruthers, 1968 & 1968 & 1968 & 1968]

$\mathcal{B}-\mathcal{L}$ Violation and Theories of Self-Conjugate Fermions

In attempting to rationalize the spectral pattern of the low-lying, light hadrons, Carruthers discovered a class of theories for which the CPT theorem does not hold.

[Carruthers, 1967]

The pions form a self-conjugate isospin multiplet (π^+, π^0, π^-), but the kaons form pair-conjugate multiplets (K^+, K^0) and $(\bar{K}^0, K^-)$.

Carruthers discovered that free theories of self-conjugate bosons with half-integer isospin are nonlocal, that the commutator of two self-conjugate fields with opposite isospin components do not vanish at space-like separations. [Carruthers, 1967]

Moreover, since weak local communitivity fails, CPT symmetry is no longer expected to hold, nor should the CPT theorem of Greenberg apply. [Carruthers, 1968; Streater and Wightman, 2000; Greenberg, 2002]

The neutron and antineutron are members of pair-conjugate $I = 1/2$ multiplets. The quark-level operators that generate $n-\bar{n}$ oscillations would also produce $p-\bar{p}$ oscillations under the isospin transformation $u \leftrightarrow d$, though the latter operators are removed by electric charge conservation....

Although many have studied the impact of external magnetic fields on n-$\bar{n}$ oscillations, our work is the first to incorporate spin in a fundamental way.

We have also analyzed the C, P, and T transformations of fermions with $\mathcal{B} - \mathcal{L}$ violation and have found that the so-called arbitrary phases are not arbitrary. We find $\eta_{cpt}^2 = -1$, as well as $\eta_{ct}^2 = 1$ and $\eta_p^2 = -1$. These phase restrictions are only appreciable in $\mathcal{B} - \mathcal{L}$ violating operators and impact their interplay with SM effects.

A particular $n - \bar{n}$ transition operator coupled to an external electromagnetic current looks promising for practical applications....

"The future ain't what it used to be." — Yogi Berra

Backup Slides

Effective Hamiltonian Framework

In the SM, a **time-dependent** magnetic field **perpendicular** to $\mathbf{B}_0$ can flip the spin. [$\omega_1 \equiv -\mu_n B_1$ and $\mathbf{B}_1 \perp \mathbf{B}_0$]

$$\mathcal{H} = \begin{pmatrix} M+\omega_0 & \delta & \omega_1 & 0 \\ \delta & M-\omega_0 & 0 & -\omega_1 \\ \omega_1 & 0 & M-\omega_0 & -\delta \\ 0 & -\omega_1 & -\delta & M+\omega_0 \end{pmatrix}$$

If $|\omega_1| \sim |\omega_0|$, the "quenching" disappears!

Alternatively, or additionally, BSM operators can act

$$\mathcal{H} = \begin{pmatrix} M+\omega_0 & \delta & 0 & \varepsilon_1 \\ \delta & M-\omega_0 & -\varepsilon_1 & 0 \\ 0 & -\varepsilon_1^* & M-\omega_0 & -\delta \\ \varepsilon_1^* & 0 & -\delta & M+\omega_0 \end{pmatrix}$$

The term ε_1 can have different sources.

The effect of ε_1 is independent of magnetic fields. However, if ε_1 itself is independent of magnetic field, then $\varepsilon_1 \neq 0$ violates angular momentum conservation. [SG and Jafari, arXiv:1408.2264v2]

Such terms can be nonzero if Lorentz symmetry is violated (LV) and can be constrained without magnetic field mitigation. [Babu, Mohapatra, arXiv:1504.01176]

However, if ε_1 depends on external fields, then $\varepsilon_1 \neq 0$ w/o LV.

[SG and Jafari, arXiv:1408.2264v2]

Effective Hamiltonian Framework

At low energies, in the **absence of magnetic fields**, under CPT, Lorentz invariance, and Hermiticity

$$\mathcal{H} = \begin{pmatrix} M & \delta & 0 & 0 \\ \delta^* & M & 0 & 0 \\ 0 & 0 & M & -\delta \\ 0 & 0 & -\delta^* & M \end{pmatrix}$$

N.B. the two sectors decouple

In a **static** magnetic field $\mathbf{B}_0$ [$\omega_0 \equiv -\mu_n B_0$]

$$\mathcal{H} = \begin{pmatrix} M+\omega_0 & \delta & 0 & 0 \\ \delta^* & M-\omega_0 & 0 & 0 \\ 0 & 0 & M-\omega_0 & -\delta \\ 0 & 0 & -\delta^* & M+\omega_0 \end{pmatrix}$$

ω_0 real

Here the magnetic field "quenches" ***n*-$\bar{n}$** **oscillations.**

Idea: Use spin manipulation techniques familiar from the theory of nuclear magnetic resonance to evade magnetic field quenching.