A Poincaré Covariant Nucleon Spectral Function for A=3 nuclei within the Light-front Hamiltonian Dynamics

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- *Neutron transverse-momentum distributions and polarized* $^3\mathrm{He}$ *within light-front Hamiltonian dynamics.* Few-body systems, **54**, 1079 (2013). E. Pace, S. Scopetta, A. Del Dotto, M. Rinaldi, & G.S.
- *A Light-Front Approach to the* $^3\mathrm{He}$ *Spectral Function.* Few-Body Systems, (2014). S. Scopetta, A. Del Dotto, L. Kaptari, E. Pace, M. Rinaldi, & G.S.

Outline

1. Still struggling with the A=3 Spectral Function
2. Poincaré covariance and Few-Nucleon systems
3. Bakamjian-Thomas construction and the Light-Front Relativistic Hamiltonian
4. The Light-Front Nucleon Spectral Function
5. EMC effect in ^{3}He
6. Conclusions & Perspectives

Why are we still struggling with the A=3 Spectral Function?

Forthcoming 12 GeV Experiment at TJLAB

- DIS regime, e.g.

  Hall A, *http : //hallaweb.jlab.org/12GeV/*

  MARATHON Coll. E12-10-103 (Rating A): *MeAsurement of the* F_{2n}/F_{2p}, *d/u RAtios and A=3 EMC Effect in Deep Inelastic Electron Scattering Off the Tritium and Helium MirrOr Nuclei*

  Hall C, *https : //www.jlab.org/Hall − C/*

  J. Arrington, et al PR12-10-008 (Rating A^-): *Detailed studies of the nuclear dependence of* F_2 *in light nuclei*

- SIDIS regime, e.g.

  Hall A, *http : //hallaweb.jlab.org/12GeV/*

  H. Gao et al, PR12-09-014 (Rating A): *Target Single Spin Asymmetry in Semi-Inclusive Deep-Inelastic* $(e, e'\pi^{\pm})$ *Reaction on a Transversely Polarized* $^3\mathrm{He}$ *Target*

  J.P. Chen et al, PR12-11-007 (Rating A): *Asymmetries in Semi-Inclusive Deep-Inelastic* $(e, e'\pi^{\pm})$ *Reactions on a Longitudinally Polarized* $^3\mathrm{He}$ *Target*

- The Standard Model of Few-Nucleon Systems, where nucleon and pion degrees of freedom are taken into account, has achieved a very high degrees of sophistication. Nonetheless, in order to extend the realm of applications, it is necessary to retain all the general principles compatible with a theory where a fixed number of constituents is acting.

- For instance, if we are interested in processes involving nucleons with high 3-momentum and we need high precision for determining relevant physical quantities, like i) the nucleon structure functions (unpolarized and polarized), inside the nucleon, ii) the nucleon TMD, or even iii) signatures of short-range correlations, then one should try to fulfill, as much as possible, relativistic constraints, such as the ones dictated by the covariance with respect the Poincaré Group, $\mathcal{G}_P$.

- At least, one should carefully deal with the boosts of the nuclear states, $|\Psi_{init}\rangle$ and $|\Psi_{fin}\rangle$!

Poincaré covariance and Few-Nucleon systems

Aim and Tool: To describe a Few-Nucleon system through a Poincaré covariant formalism, in order to obtain an approach where both wave functions and operators transform accordingly to the extended Poincaré group (4D translations + Lorentz group + parity and time reversal)

General principles to be implemented

★ Extended Poincaré covariance

$$[P^\mu, P^\nu] = 0, \quad [M^{\mu\nu}, P^\rho] = -\imath(g^{\mu\rho} P^\nu - g^{\nu\rho} P^\mu),$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = -\imath(g^{\mu\rho} M^{\nu\sigma} + g^{\nu\sigma} M^{\mu\rho} - g^{\mu\sigma} M^{\nu\rho} - g^{\nu\rho} M^{\mu\sigma})$$

$\mathcal{P}$ and $\mathcal{T}$ have to be taken into account !

★ ★ Macroscopic locality (≡ cluster separability): i.e. *observables associated with different space-time regions commute in the limit of large spacelike separation, rather than for arbitrary (μ-locality) spacelike separations* (Keister-Polyzou, Adv. Nucl. Phys. **21**, 225 (1991))

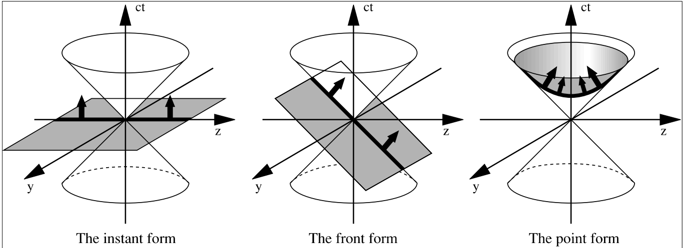
Relevant for taking advantage of the study of two-nucleon systems, when $A \geq 3$ systems are investigated (modulo the presence of many-body interactions).

Adopted Tool: The Dirac Relativistic Hamiltonian Dynamics in Light-front form

Light-front Relativistic Hamiltonian Dynamics

A reasonable compromise:

i) fulfilling Poincaré covariance in a non perturbative way and with the most symmetric initial surface; ii) embedding the whole successful non relativistic phenomenology (trivial separation of global motion); iii) affordable numerical calculations; iv) fixed number of constituents; v) large class of allowed interactions; vi) DIS and SIDIS are sitting on the Light-cone.



$$\tilde{x}^0 = ct,\quad \tilde{x}^1 = x,\quad \tilde{x}^2 = y,\quad \tilde{x}^3 = z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\tilde{x}^0 = ct+z,\quad \tilde{x}^1 = x,\quad \tilde{x}^2 = y,\quad \tilde{x}^3 = ct-z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

$$\tilde{x}^0 = \tau \;,\; ct = \tau\cosh\omega \qquad \tilde{x}^1 = \omega \;,\; x = \tau\sinh\omega\sin\theta\cos\phi$$
$$\tilde{x}^2 = \theta \;,\; y = \tau\sinh\omega\sin\theta\sin\phi \qquad \tilde{x}^3 = \phi \;,\; x = \tau\sinh\omega\cos\theta$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\tau^2 & 0 & 0 \\ 0 & 0 & -\tau^2\sinh^2\omega & 0 \\ 0 & 0 & 0 & -\tau^2\sinh^2\omega\sin^2\theta \end{pmatrix}$$

After S.J. Brodsky, H.C. Pauli and S.S Pinsky, Phys. Rep. **301**, 299 (1998).

Bakamjian-Thomas construction and LFHD

For finite degrees of freedom, an explicit construction of the 10 Poincaré generators, in presence of interactions, was given by Bakamjian and Thomas (PR 92 (1953) 1300).

The essential features of the BT construction are: i) the dynamical generators of $\mathcal{G}_P$, i.e. the ones governing the evolution in a suitable variable called time, are expressed in terms of M, the mass operator of the interacting system, and ii) only M contains the interaction.

For the LFHD, BT construction is achieved via four *simple* steps

- First : construct the 10 generators, $\{P_0^-, P^+, \vec{P}_\perp, J_3, \vec{F}_{0\perp}, K_3, \vec{E}_\perp\}$ for the **non interacting system, i.e. each generator results from the trivial sum on the generators of the constituents**. Recall that $P^\pm = P_0 \pm P_z$
- Second: after defining an intrinsic frame, choose 10 auxiliary operators, $\{M_0, P^+, \vec{P}_\perp, K_3, \vec{E}_\perp, \vec{j}_{0LF}\}$. The non interacting mass, M_0, and the angular momentum, $\vec{j}_{0LF}$ in the intrinsic frame, are given by

$$M_0^2 = P_0^- P^+ - |\vec{P}_\perp|^2 \qquad (0, \vec{j}_{0LF}) = \left[B_{LF}^{-1}\left(\frac{P_0}{M_0}\right)\right]^\mu_\nu \left(\frac{P_0}{M_0}\right) \frac{W_0^\nu}{M_0}$$

$[B_{LF}^{-1}]^\mu_\nu$ is a LF boost, and W_0^ν is the Pauli-Lubanski 4-vector ($W_0^2 = M_0^2\ |\vec{j}_{0LF}|^2$)

NB the commutation rules of the Poincaré generators imply the ones of the auxiliary operators (and viceversa)

- Third : add to M_0 an interaction V that commutes with $\{P^+, \vec{P}_\perp, K_3, \vec{E}_\perp, \vec{j}_{0LF}\}$. Then, the set $\{M, P^+, \vec{P}_\perp, K_3, \vec{E}_\perp, \vec{j}_{0LF}\}$ have the same commutation rules of the non interacting set (i.e. the set with M_0).

  NB Two possibilities: $M^2 = M_0^2 + U$ (for the two-body case it allows one to embed the NR phenomenology, easily) or $M = M_0 + V$

- Fourth : invert the second step, starting from $\{M = M_0 + V, P^+, \vec{P}_\perp, K_3, \vec{E}_\perp, \vec{j}_{0LF}\}$ and obtain 10 Poincaré generators, that fulfill the correct commutation rules, but contain the interaction.

NB $|\vec{j}_{0LF}|^2$ and the third component of $\vec{j}_{0LF}$ can be used for labeling the states !!

NB NB BT construction holds for an interacting system with a finite number of dof and it is not unique.

A first lesson

- BT construction provides a viable tool for obtaining the Poincaré generators for an interacting system, with finite degrees of freedom.
- The key ingredient is the mass operator, Casimir of $\mathcal{G}_P$, that contains the interaction, and generates the dependence upon the interaction of the three dynamical generators.in LFHD, namely P^- and the LF transverse rotations $\vec{F}_\perp$.
- The interaction, $U = M_0 V + V M_0 + V^2$ or V, must commutes with all the kinematical generators, and in addition with the non interacting spin. These constraints lead to the independence upon the global motion, as in the non relativistic case and the property that the BT angular momentum is a conserved quantity.
- The full theory must fulfills the macroscopic locality, as well. This property can be implemented by using interaction-dependent, unitary operators: the packing operators (Sokolov, Theor. Mat. Fiz. **36** (1978) 355). Their effects should be small, and therefore they will be neglected in what follows (but in principle to be investigated).

The BT Mass operator for A=3 nuclei - I

$$M_{BT}(123) = M_0(123) + V^{BT}_{12,3} + V^{BT}_{23,1} + V^{BT}_{31,2} + V^{BT}_{123}$$

where

- $M_0(123) = \sqrt{m^2 + k_1^2} + \sqrt{m^2 + k_2^2} + \sqrt{m^2 + k_3^2} = \sqrt{M_0^2(ij) + p_\ell^2} + \sqrt{m^2 + p_\ell^2}$ is the free mass operator, with i) $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$, and ii) $\mathbf{p}_\ell$ are Jacobi momenta wrt the pair (ij), viz

$$\mathbf{p}_1 = \frac{2}{3}\left[\mathbf{k}_1 - \frac{(\mathbf{k}_2 + \mathbf{k}_3)}{2}\right]$$

- $V^{BT}_{ij,\ell} = \sqrt{M_0^2(ij) + v^{BT}_{ij} + p_\ell^2} - \sqrt{M_0^2(ij) + p_\ell^2}$ is the two-body interaction in a A=3 system, and v^{BT}_{ij} the two-body interaction in a A=2 system, fulfilling the proper commutation rules.

  The structure of $V^{BT}_{ij,\ell}$, is suggested by the analysis of a two-body interacting system + a free third particle. One can naturally write

$$M_{1,23} = \sqrt{M_0^2(23) + v^{BT}_{23} + p_1^2} + \sqrt{m^2 + p_1^2} =$$

$$= M_0(123) + \left[\sqrt{M_0^2(23) + v^{BT}_{23} + p_1^2} - \sqrt{M_0^2(23) + p_1^2}\right]$$

- V^{BT}_{123} is a short-range three-body forces

The BT Mass operator for A=3 nuclei - II

Notice that

$$V^{BT}_{12,3} = \sqrt{M_0^2(12) + v^{BT}_{12} + p_3^2} - \sqrt{M_0^2(12) + p_3^2} =$$

$$= \frac{v^{BT}_{12}}{\sqrt{M_0^2(12) + v^{BT}_{12} + p_3^2} + \sqrt{M_0^2(12) + p_3^2}} \sim$$

$$\frac{4mV^{NR}_{12}}{\sqrt{M_0^2(12) + v^{BT}_{12} + p_3^2} + \sqrt{M_0^2(12) + p_3^2}} \to V^{NR}_{12}$$

For the two-body case, e.g. the deuteron, the Schrödinger Eq. can be rewritten as follows

$$\left[4m^2 + 4k^2 + 4mV^{NR}\right] \; |\psi_D\rangle = \left[4m^2 - 4mB_D\right] |\psi_D\rangle$$

$$\left[M_0^2(12) + 4mV^{NR}\right] \; |\psi_D\rangle = \left[M_D^2 + B_D^2\right] |\psi_D\rangle \sim \; M_D^2 \; |\psi_D\rangle$$

and the identification between v^{BT}_{12} and $4mV^{NR}$ naturally stems out, disregarding correction of the order B_D/M_D

Final remarks: i) the commutation rules impose to V^{BT} analogous properties as the ones of V^{NR}, with respect to the total 4-momentum and total angular momentum; ii) to implement Macro-locality one has to properly choose v_{12}.

The BT Mass operator for A=3 nuclei - III

In the non relativistic framework, it is not taken into account the changes in the two-body interaction when we move from the two-body CM to the three-body CM

The NR mass operator is written as

$$M^{NR} = 3m + \sum_{i=1,3} \frac{k_i^2}{2m} + V_{12}^{NR} + V_{23}^{NR} + V_{31}^{NR} + V_{123}^{NR}$$

NB The operators describing the two- and three-body forces must obey to the commutation rules proper of the Galilean group, leading to the well-known properties like the translational invariance, the independence of total 3-momentum.

Those properties are similar to the ones in BT construction. This allows us to consider the standard non relativistic mass operator as a sensible BT mass operator, and embedding it in a Poincaré covariant approach.

$$M_{BT}(123) = M_0(123) + V_{12,3}^{BT} + V_{23,1}^{BT} + V_{31,2}^{BT} + V_{123}^{BT} \sim M^{NR}$$

From the phenomenological point of view, recall that the 2-body phase-shifts contain the relativistic dynamics, and the Lippmann-Schwinger Eq., like the Schrödinger one, has the suitable structure for the BT construction, and $\Rightarrow$ Poincaré covariance

A second lesson

What has been done till now, within a non relativistic framework, can be re-used in a Poincaré covariant framework, i.e. $V^{BT}_{12,3} \sim v^{NR}_{12}$

But, improvements are possible, e.g. one could directly use

$$V^{BT}_{12,3} = \frac{v^{BT}_{12}}{\sqrt{M_0^2(12) + v^{BT}_{12} + p_3^2} + \sqrt{M_0^2(12) + p_3^2}}$$

with v^{BT}_{12} the two-body interaction that must describe the whole two-nucleon phenomenology (bound + scattering states), in the A=2 CM !

Instant form calculations of the Faddeev equation for both $n - d$ elastic scattering and breakup, with $v^{BT}_{12} \sim$ CD Bonn and TM99 3BF, and the above $V^{BT}_{12,3}$, can be found in Witala et al PRC 83 044001.

Summarizing, we have adopted the standard eigensolutions of M^{NR}, since it is eligible for a Poincaré covariant description of the A=3 nuclei.

The BT Mass operator for A=3 nuclei - IV

To complete the matter: the spin

- within the LFHD, the LF boosts are kinematical ($\equiv$ independent of interaction), and therefore their action is trivial
- Coupling intrinsic spins and orbital angular momenta is easily accomplished within the Instant Form of RHD, since in this form the three generators of the rotations are independent of interaction. This entails that the usual non relativistic machinery (i.e. Clebsch-Gordan coefficients) can be applied
- to embed this machinery in the LFHD one needs unitary operators, the so-called Melosh rotations, R_M, that relate the LF spin wave function and the canonical one. For a (1/2)-particle with LF momentum $\tilde{k} \equiv \{k^+, \vec{k}_\perp\}$

$$|s, \sigma'\rangle_{LF} = \sum_{\sigma} D^{1/2}_{\sigma',\sigma}(R^{\dagger}_M(\tilde{k}))\ |s, \sigma\rangle_{IF}$$

where $D^{1/2}_{\sigma',\sigma}(R^{\dagger}_M(\tilde{k}))$ is the standard Wigner function for the $J = 1/2$ case

- for the nucleon quantities, like the density distribution or the Spectral Function, the Melosh rotations does not produce an extra algebraic burden respect to the Instant form, where one has to carefully deals with the interaction-dependent IF boosts. Schematically

$$O^{LF}_{\sigma''',\sigma} = \sum_{\sigma'',\sigma'} D^{1/2}_{\sigma''',\sigma''}(R^{\dagger}_M)\ O^{IF}_{\sigma'',\sigma'}\ D^{1/2}_{\sigma',\sigma}(R_M)$$

The Light-Front Nucleon Spectral Function

Nucleon Spectral Function: probability distribution to find a nucleon with given 3-momentum, and missing energy inside the nucleus. For a polarized nucleus, one can add the polarization degrees of freedom.

For a polarized nucleus in a NR framework

$$P^N_{\sigma,\sigma',\mathcal{M}_z}(\vec{p},E) = \sum_{f_{(A-1)}} {}_N\langle \vec{p},\sigma;\psi_{f_{(A-1)}}|\psi^A_{J\mathcal{M}_z}\rangle$$

$$\langle \psi^A_{J\mathcal{M}_z}|\psi_{f_{(A-1)}};\vec{p},\sigma'\rangle_N \; \delta(E - E_{f_{(A-1)}} + E_A)$$

- $|\psi^A_{J\mathcal{M}_z}\rangle$: ground state, eigensolution of

$$M^{NR}_A \; |\psi^A_{J\mathcal{M}_z}\rangle = E_A \; |\psi^A_{J\mathcal{M}_z}\rangle$$

- $|\psi_{f_{(A-1)}}\rangle$: a state of the $(A-1)$-nucleon spectator system: fully interacting !

$$M^{NR}_{(A-1)} \; |\psi_{f_{(A-1)}}\rangle = E_{f_{(A-1)}} \; |\psi_{f_{(A-1)}}\rangle$$

- **p** and E are the active nucleon 3-momentum and missing energy, respectively
- NR overlaps ${}_N\langle \vec{p},\sigma;\psi_{f_{(A-1)}}|\psi_{J\mathcal{M}_z}\rangle$ with the same NN interaction in A and $A-1$

LF Nucleon Spectral Function for $^3\mathrm{He}$

$$\mathcal{P}^{\tau}_{\sigma'\sigma}(\xi, \boldsymbol{\kappa}_\perp, \kappa^-, S_{He}) = \left|\frac{\partial \kappa^+}{\partial \xi}\right| \sum\!\!\!\!\!\!\int d\epsilon_S \; \rho(\epsilon_S) \; \delta\left(\kappa^- - M_{He} + \frac{M_S^2 + |\boldsymbol{\kappa}_\perp|^2}{(1-\xi)\; \mathcal{M}_0(1,23)}\right)$$

$$\times \sum_{J_S J_{zS} \alpha} \sum_{T_S \tau_S} {}_{LF}\langle \tau_S, T_S, \alpha, \epsilon_S J_S J_{zS}; \tau\sigma', \tilde{\boldsymbol{\kappa}} | \Psi_0 S_{He}\rangle \; \langle S_{He}, \Psi_0 | \tilde{\boldsymbol{\kappa}}, \sigma\tau; J_S J_{zS} \epsilon_S, \alpha, T_S, \tau_S\rangle_{LF}$$

- $M_S = 2\sqrt{m^2 + m\epsilon_S}$ is the mass of the interacting spectator pair
- $\kappa^+ = \xi \mathcal{M}_0(1,23)$ and

$$\mathcal{M}_0^2(1,23) = \frac{m^2 + |\boldsymbol{\kappa}_\perp|^2}{\xi} + \frac{M_S^2 + |\boldsymbol{\kappa}_\perp|^2}{(1-\xi)}$$

- $\rho(\epsilon_S) \equiv$ density of the two-body states (1 for the bound state, and $m\sqrt{m\epsilon_S}/2$ for the excited ones)
- what about the overlap ${}_{LF}\langle \tau_S, T_S, \alpha, \epsilon_S J_{zS} J_S; \tau\sigma', \tilde{\boldsymbol{\kappa}} | \Psi_0 S_{He}\rangle$?

LF overlaps for ^{3}He from the IF ones

$$
{}_{LF}\langle \tau_S,\, T_S, \alpha, \epsilon_S J_S J_{zS}; \tau\sigma', \tilde{\boldsymbol{\kappa}}|\Psi_0 S_{He}\rangle =
$$

$$
= \sum_{\tau_2,\tau_3} \sum_{\sigma_1''} D^{\frac{1}{2}}[\mathcal{R}_M^\dagger(\tilde{\boldsymbol{\kappa}})]_{\sigma'\sigma_1''} \int d\mathbf{k}_{23} \sum_{\sigma_2,\sigma_3} \sqrt{\frac{\kappa^+ E_{23}}{k^+ E_S}} \sqrt{(2\pi)^3\ k^+ \frac{\partial k_z}{\partial k^+}}\ \times
$$

$$
{}_{IF}\langle \tau_S,\, T_S, \alpha, \epsilon_S J_S J_{zS}|\mathbf{k}_{23}; \sigma_2, \sigma_3; \tau_2, \tau_3\rangle\langle \tau_3, \tau_2, \tau_1; \sigma_3, \sigma_2, \sigma_1''; \mathbf{k}, \mathbf{k}_{23}|\Psi_0 S_{He}\rangle_{IF}
$$

- $\mathbf{k}_\perp = \boldsymbol{\kappa}_\perp$, since the ^{3}He transverse momentum $\mathbf{P}_\perp = 0$ by choice
- given the kinematical nature of the LF boost, one has
  $k^+ = \xi\ M_0(123) = \kappa^+\ M_0(123)/\mathcal{M}_0(1,23)$

  $$\text{with} \qquad M_0^2(123) = \frac{m^2 + |\boldsymbol{\kappa}_\perp|^2}{\xi} + \frac{M_{23}^2 + |\boldsymbol{\kappa}_\perp|^2}{(1-\xi)}$$

  and $M_{23}^2 = 4(m^2 + |\mathbf{k}_{23}|^2)$ is the mass of the spectator pair without interaction !
  Recall that in $\mathcal{M}_0(1,23)$ the spectator pair is interacting, $M_{23} \to M_S$
- $k_z = \frac{1}{2}\ \left[k^+ - \frac{m^2+|\boldsymbol{\kappa}_\perp|^2}{k^+}\right]$, $E_{23} = \sqrt{M_{23}^2 + |\mathbf{k}|^2}$ and $E_S = \sqrt{M_S^2 + |\boldsymbol{\kappa}|^2}$

In the actual calculations, we have approximated the IF overlaps by the NR ones

$$\langle \tau_3, \tau_2, \tau_1; \sigma_3, \sigma_2, \sigma_1''; \mathbf{k}, \mathbf{k}_{23} | \Psi_0 S_{He} \rangle_{IF} \Rightarrow \langle \tau_3, \tau_2, \tau_1; \sigma_3, \sigma_2, \sigma_1''; \mathbf{k}, \mathbf{k}_{23} | \Psi_0 S_{He} \rangle_{NR}$$

$$_{IF}\langle \tau_S, T_S, \alpha, \epsilon_S J_S J_{zS} | \mathbf{k}_{23}; \sigma_2, \sigma_3; \tau_2, \tau_3 \rangle \Rightarrow_{NR} \langle \tau_S, T_S, \alpha, \epsilon_S J_S J_{zS} | \mathbf{k}_{23}; \sigma_2, \sigma_3; \tau_2, \tau_3 \rangle$$

Then

$$_{LF}\langle \tau_S, T_S, \alpha, \epsilon_S J_S J_{zS}; \tau\sigma', \tilde{\boldsymbol{\kappa}} | \Psi_0 S_{He} \rangle =$$
$$\sim \sum_{\tau_2,\tau_3} \sum_{\sigma_1''} D^{\frac{1}{2}}[\mathcal{R}_M^\dagger(\tilde{\boldsymbol{\kappa}})]_{\sigma'\sigma_1''} \int d\mathbf{k}_{23} \sum_{\sigma_2,\sigma_3} \sqrt{\frac{\kappa^+ E_{23}}{k^+ E_S}} \times$$
$$_{NR}\langle \tau_S, T_S, \alpha, \epsilon_S J_S J_{zS} | \mathbf{k}_{23}; \sigma_2, \sigma_3; \tau_2, \tau_3 \rangle \langle \tau_3, \tau_2, \tau_1; \sigma_3, \sigma_2, \sigma_1''; \mathbf{k}, \mathbf{k}_{23} | \Psi_0 S_{He} \rangle_{NR}$$

We have also tested the possibility to approximate the dependence upon $|\mathbf{k}_{23}|^2$ that appears in

$$\sqrt{\frac{\kappa^+ E_{23}}{k^+ E_S}} \qquad ,$$

with an average value, obviously different for 2-B and 3-B channels.

Convolution formula for $^3\mathrm{He}$

The nuclear structure function $F_2^A(x_B)$ per nucleon ($x_B = Q^2/2m\nu$) is

$$F_2^A(x_B) = \frac{1}{A}\int_{x_B}^{M_A/m} dz \;\; z \;\left[Z \; F_2^p(\frac{x_B}{z}) \; f_p^A(z) + N \; F_2^n(\frac{x_B}{z}) \; f_n^A(z)\right]$$

with the Light-cone distribution given by

$$f_{p(n)}^A(z) = \int_0^1 d\xi \int d\mathbf{k}_\perp \int d\kappa^- \;\; Tr\left[\mathcal{P}_{p(n)}^{A(LF)}(\xi, \mathbf{k}_\perp, \kappa^-)\right] \; \delta\left(z - \frac{\xi M_A}{m}\right)$$

NB. In the Lab frame, choosing $\mathbf{q}_\perp = 0$

$$q^- \to 2\nu + m x_B$$
$$q^+ \to -m x_B + \frac{(m x_B)^2}{\nu}$$

Normalization and momentum sum rule

From the normalization of the Spectral Function follows

$$\int_0^\infty dz \; f^A_{p(n)}(z) = 1$$

Then one has

$$\mathcal{N}_A = \frac{1}{A}\int_0^\infty dz \; \left[Zf^A_p(z) + (A-Z)f^A_n(z)\right] = 1$$

$$MSR = \frac{1}{A}\int_0^\infty dz \; z \; \left[Zf^A_p(z) + (A-Z)f^A_n(z)\right] = \frac{1}{A}$$

By using the $^3\mathrm{He}$ wave function, corresponding to the NN interaction AV18, that was calculated by Kievsky, Rosati and Viviani (Nucl. Phys. A551, 241 (1993)) we obtained

$$MSR_{calc} = 0.3331$$

Namely, within LFHD normalization and momentum sum rule do not conflict !!

Preliminary Results for $^3\mathrm{He}$ EMC effect

To start, we have calculated the contribution from the 2B channel, i.e. the spectator pair lives in a deuteron state

$$
{}_{LF}\langle \tau_D, T_D, \alpha, \epsilon_D J_D J_{zD}; \tau\sigma', \tilde{\boldsymbol{\kappa}} | \Psi_0 S_{He} \rangle =
$$

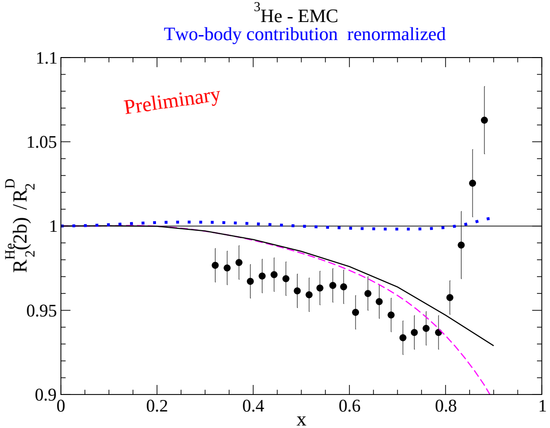
$$
\sim \sum_{\tau_2,\tau_3} \sum_{\sigma_1''} D^{\frac{1}{2}} [\mathcal{R}_M^\dagger(\tilde{\boldsymbol{\kappa}})]_{\sigma'\sigma_1''} \int d\mathbf{k}_{23} \sum_{\sigma_2,\sigma_3} \sqrt{\frac{\kappa^+ E_{23}}{k^+ E_D}} \sqrt{(2\pi)^3 k^+ \frac{\partial k_z}{\partial k^+}} \times
$$

$$
{}_{NR}\langle \tau_D, T_D, \alpha, \epsilon_D J_D J_{zD} | \mathbf{k}_{23}; \sigma_2, \sigma_3; \tau_2, \tau_3 \rangle \langle \tau_3, \tau_2, \tau_1; \sigma_3, \sigma_2, \sigma_1''; \mathbf{k}, \mathbf{k}_{23} | \Psi_0 S_{He} \rangle_{NR}
$$

We used the eigensolutions for deuteron and $^3\mathrm{He}$ obtained from the NN interaction Argonne V18 for evaluating

$$
R_2^A(x) = \frac{A\ F_2^A(x)}{Z\ F_2^p(x) + (A-Z)\ F_2^n(x)}
$$

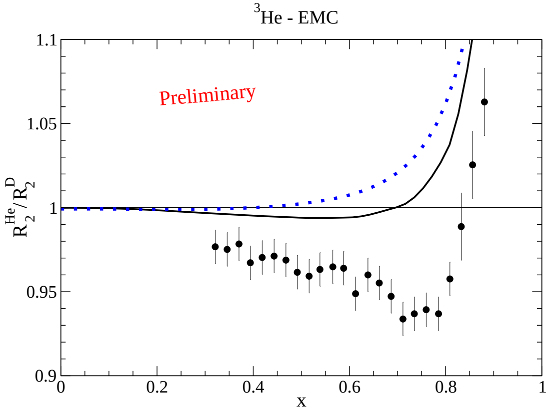
EMC effect in ^{3}He and LFSF



- Solid line: calculation with the LF Spectral Function.
- Dashed line: as the solid line, but with $\sqrt{\bar{k}^2_{23}} = 136.37\ MeV$ for D (AV18).
- Dotted line: LF Momentum Distribution with only two-body contribution

The three curves, have been divided by the probability to find a deuteron in ^{3}He , $\sim 2/3$.

Full calculation of $R_2^{He}(x)/R^D(x)$: 2-body and 3-body



- Solid line: LF Spectral Function, with the exact calculation for the 2-body channel, and an average kinetic energy in the 3-body contribution.
- Dotted line: LF momentum distribution

NB within the LF framework normalization of the Light-cone distribution and momentum sum rule are fulfilled automatically. Big difference from the IF approach !

Conclusions & Perspectives

- A Poincaré covariant description of a A=3 nucleus, based on the Light-front Hamiltonian Dynamics, has been proposed.
- The Bakamjian-Thomas construction of the Poincaré generators has been exploited for including the interaction. This allows one to embed the successful phenomenology for Few-Nucleon systems in a Poincaré covariant framework.
- We have started the evaluation of the Nucleon Spectral function for $^3\mathrm{He}$, by approximating the IF overlaps with their counterpart calculated non relavistically with the AV18 NN interaction
- A first test of our approach is the EMC effect for $^3\mathrm{He}$. At the presnt stage, we have calculated the 2-body contribution to the Nucleon SF with the full expression, while the 3-body one has been evaluated with an average $|\mathbf{k}_{23}|^2$. Encouraging improvements clearly appear after comparing with experimental data. A deceasing of the model dependence in the extraction of F_2^N, from the nuclear medium, is expected.
- Next steps: i) full calculation of the 3-body contribution; evaluation of both will polarized and unpolarized SF.