# Extraction of generalized parton distributions from DVMP

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**Outline:**

- **Introduction: handbag approach, GPDs, subprocess amplitudes**
- **Extraction of GPDs from hard exclusive meson leptoproduction**
- **Applications of the GPDs**
- **DVCS and parton angular momentum**
- **Pion leptoproduction: transversity GPDs and pion pole**
- $\omega$ **production**
- **The exclusive pion-induced Drell-Yan process**
- **Summary**

# Can we apply the asymp. fact. formula ?

rigorous proofs of collinear factorization in generalized Bjorken regime:

for $\gamma_L^* \to V_L(P)$ and $\gamma_T^* \to \gamma_T$ amplitudes $\big(Q^2, W \to \infty,\ x_{\rm B}$ fixed$\big)$

$\mathcal{K} = \int dx K(x,\xi,t)\mathcal{H}(x,\xi,Q^2)$ Radyushkin, Collins et al, Ji et al

possible power corrections not under control $\Longrightarrow$

unknown at which $Q^2$ asymptotic result can be applied

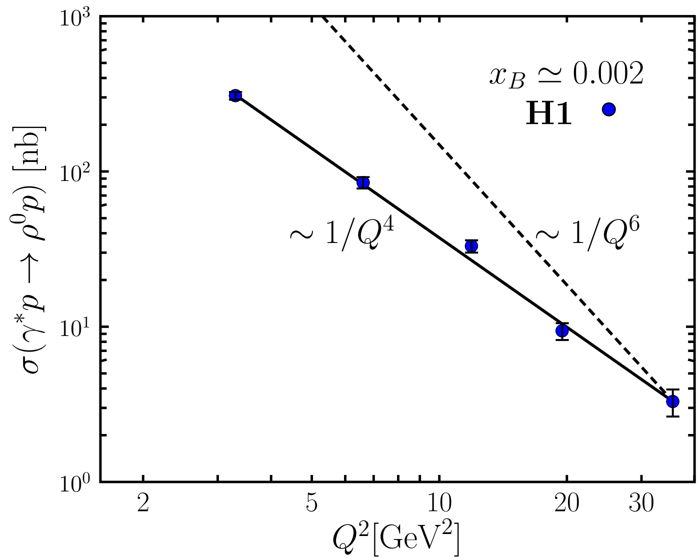
e.g. $\rho^0$ production: $\sigma_L/\sigma_T \propto Q^2$

experiment: $\simeq 2$ for $Q^2 \leq 10\,\mathrm{GeV}^2$

$\gamma_T^* \to V_T$ transitions substantial

$\sigma_L \propto 1/Q^6$ at fixed $x_{\rm B}$

modified by $ln^n(Q^2)$ experiment:




Goloskokov-K (06): take into account transverse size of meson,

i.e. power corrections $1/Q^n$ to subprocess $\gamma_L^* q(g) \to V_L\, q(g)$

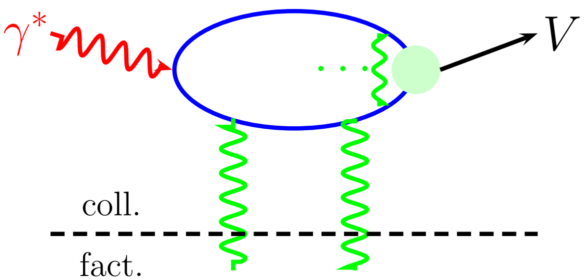
and generalize to other $\gamma^* \to V(P)$ transitions

# The subprocess amplitude for DVMP

mod. pert. approach - quark trans. momenta in subprocess

(emission and absorption of partons from proton collinear to proton momenta)

transverse separation of color sources $\Longrightarrow$ gluon radiation




LO pQCD

$+$ quark trans. mom.

$+$ Sudakov supp.

$\Rightarrow$ asymp. fact. formula

(lead. twist) for $Q^2 \to \infty$

Sudakov factor Sterman et al(93)

$$S(\tau, \mathbf{b}, Q^2) \propto \ln \frac{\ln(\tau Q/\sqrt{2}\Lambda_{\rm QCD})}{-\ln(b\Lambda_{\rm QCD})} + \text{NLL}$$

resummed gluon radiation to NLL $\Rightarrow \exp[-S]$

provides sharp cut-off at $b = 1/\Lambda_{\rm QCD}$

$$\mathcal{H}^M_{0\lambda,0\lambda} = \int d\tau d^2b\, \hat{\Psi}_M(\tau, -\mathbf{b})\, e^{-S} \hat{\mathcal{F}}_{0\lambda,0\lambda}(\bar{x}, \xi, \tau, Q^2, \mathbf{b})$$

$\hat{\Psi}_M \sim \exp[\tau\bar{\tau}b^2/4a_M^2]$ LC wave fct of meson

$\hat{\mathcal{F}}$ FT of hard scattering kernel

e.g. $\propto 1/[k_\perp^2 + \tau(\bar{x}+\xi)Q^2/(2\xi)] \Rightarrow$ Bessel fct

similar (but only for HERA kinematics): leading-log appr., color dipole model

Frankfurt et al (96), Nemcik et al (97),...

unintegrated gluon GPD Martin et al (99)

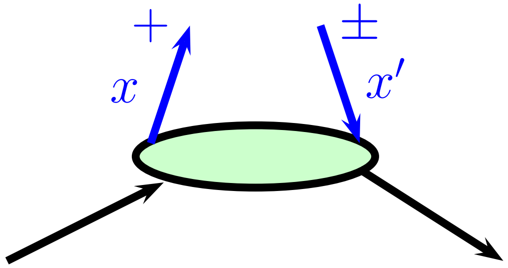
# GPDs

Müller et al (94), Ji(97), Radyushkin (97)

GPDs: $K = K(\bar{x}, \xi, t)$

$K = H, E, \widetilde{H}, \widetilde{E}, H_T, E_T, \widetilde{H}_T, \widetilde{E}_T$

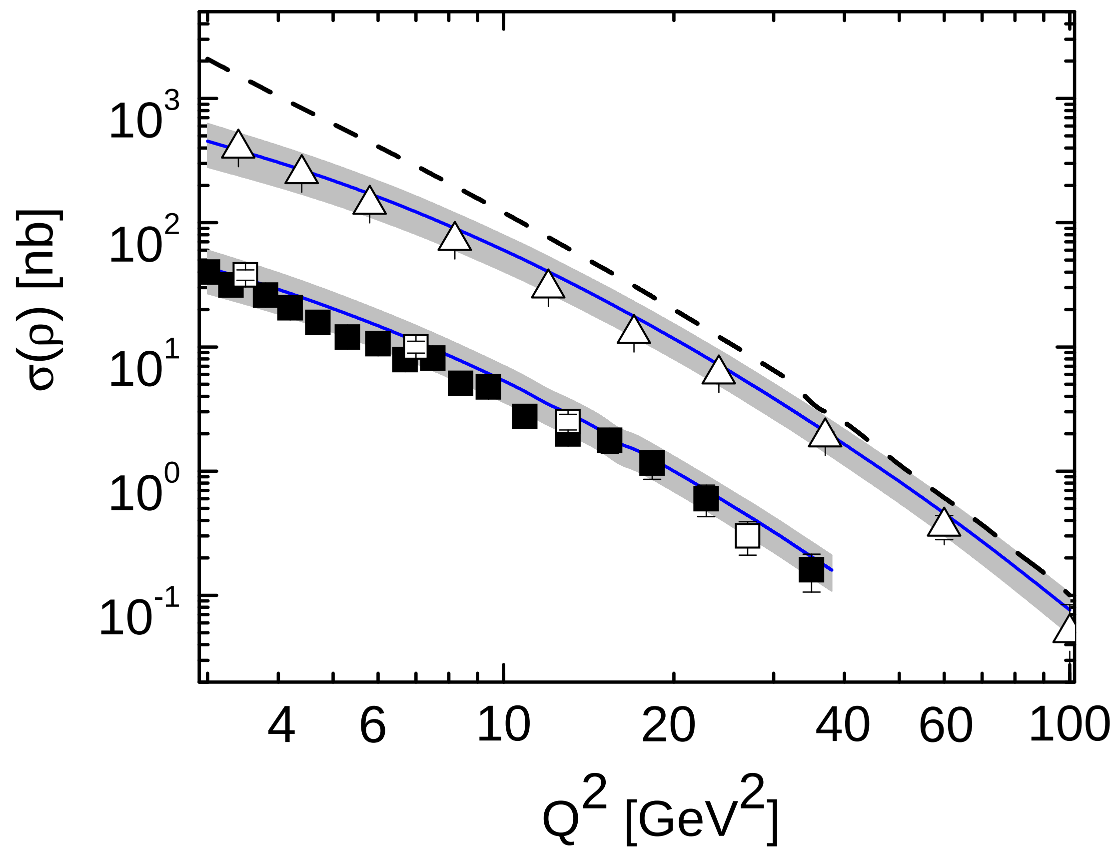
$x = \frac{\bar{x}+\xi}{1+\xi} \quad x' = \frac{\bar{x}-\xi}{1-\xi}$



# What has been done?

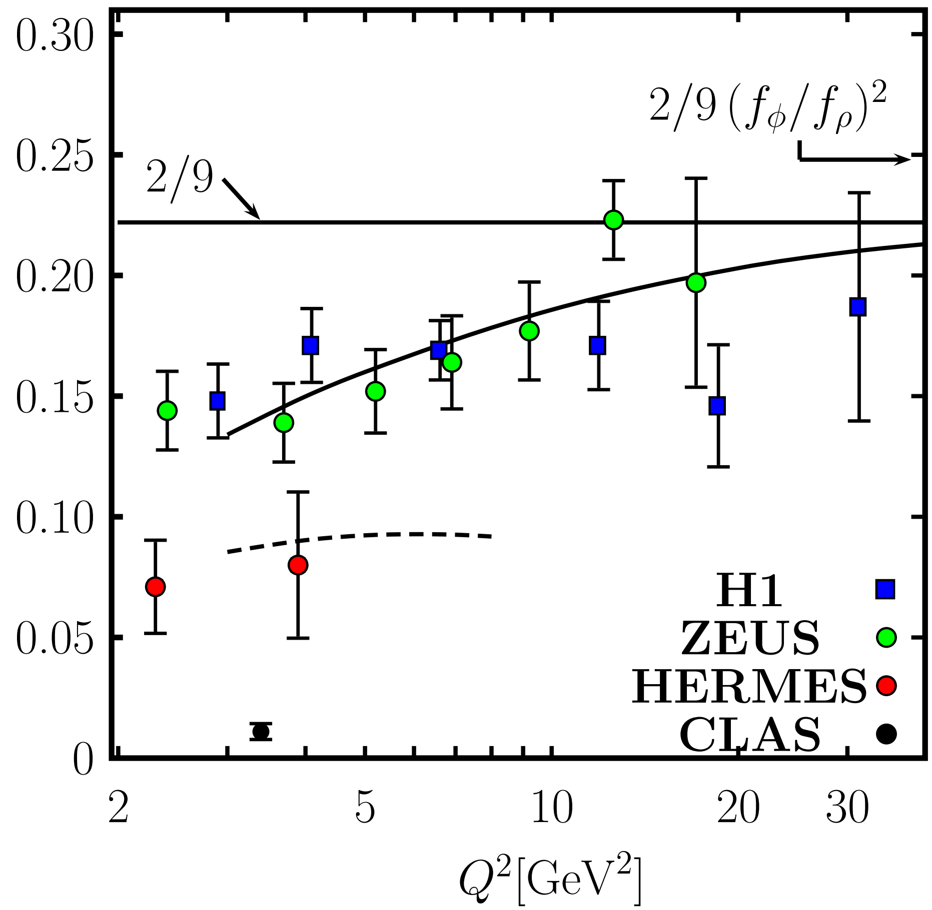
- analysis of FF with help of sum rules (DFJK(04), update: Diehl-K 1302.4604) using CTEQ6 (ABM11, DSSV11) PDFs, fixes $H, E, \widetilde{H}$ for valence quarks
- analysis of $d\sigma_L/dt$ for $\rho^0$ and $\phi$ production Goloskokov-K, hep-ph/0611290
  data from H1, ZEUS, E665, HERMES
  for $Q^2 \gtrsim 3\,\mathrm{GeV}^2$ and $W \gtrsim 4\,\mathrm{GeV}$ ($\xi \lesssim 0.1$, $-t \lesssim 0.5\,\mathrm{GeV}^2$)
  fixes $H$ for sea quarks and gluons for given $H^{\mathrm{val}}$
  ($E$ negligible, others don't contr.) (only free parameters $a_V$)
- analysis of $\pi^+$ production, Goloskokov-K, 0906.0460
  $d\sigma/dt$ and $A_{UT}$ data from HERMES ($W \simeq 4\,\mathrm{GeV}$, $Q^2 \simeq 2-5\,\mathrm{GeV}^2$)
  evidence for strong contr. from $\gamma_T^*$ ($H_T$)
  fixes pion pole and $H_T^{(3)}$ (no clear signal for $\widetilde{E}$)
- $\pi^0$ cross section and $\eta/\pi^0$ cross section ratio from CLAS (large skewness!), SDME and $A_{UT}$ for $\rho^0$ prod. HERMES, Goloskokov-K, 1106.4897, 1310.1472
  fixes $H_T$ and $\bar{E}_T = 2\widetilde{H}_T + E_T$ for valence quarks
- $\widetilde{H}, \widetilde{E}, H_T, \bar{E}_T$ for gluons and sea quarks unknown as yet, $E$ see below
  $E_T, \widetilde{E}_T$ unknown

# Result for $\rho^0$ production




$W = 90$ and 75 GeV (divided by 10) solid (open) symbols: H1 (ZEUS)

leading-twist dominance for $Q^2 \gtrsim 60\,\mathrm{GeV}^2$




$$\sigma_L(\phi)/\sigma_L(\rho^0)$$

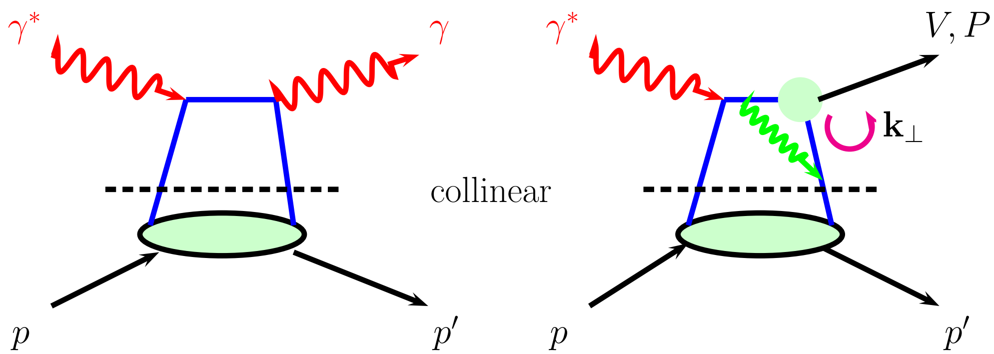
suppression due to
- transverse size parameter
- SU(3) breaking in sea
- and valence quarks for HERMES and CLAS

# Applications

exploiting universality: our set of GPDs allows for parameter free calculations of other hard exclusive reactions (except of possible wave fct effects)

- $\nu_l p \to l P p$ Kopeliovich et al, 1401.1547, 1401.6547
  Schmidt-Siddikov, 1501.04306
  V-A structure leads to different combinations of GPDs no data
- timelike DVCS Pire et al, 1407.0713 no data
- $\pi^- p \to l^+ l^- n$ Goloskokov-K, 1506.04619 no data
- $\gamma^* p \to \omega p$ Goloskokov-K, 1407.1141
  compared with SDMEs from HERMES(14) and $A_{UT}$ from HERMES(15)
  prominent role of pion pole, $\pi \to \omega$ trans. form factor
- DVCS K-Moutarde-Sabatie, 1210.6975
  Müller et al in 1108.1713
  compared to data from Jlab, HERMES, H1, ZEUS
  good agreement with small skewness data, less good with Jlab data

# DVCS



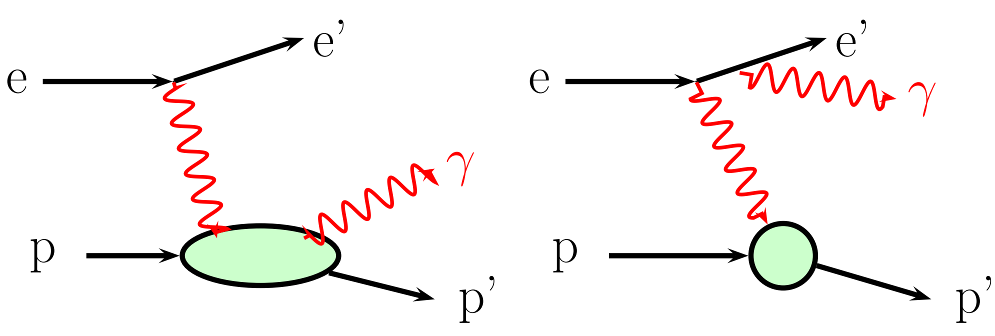

leading-twist, LO accuracy

collinear for consistency

NLO: gluon GPDs contribute as well

with $H$ most of the DVCS observables can be computed

parameter-free computation



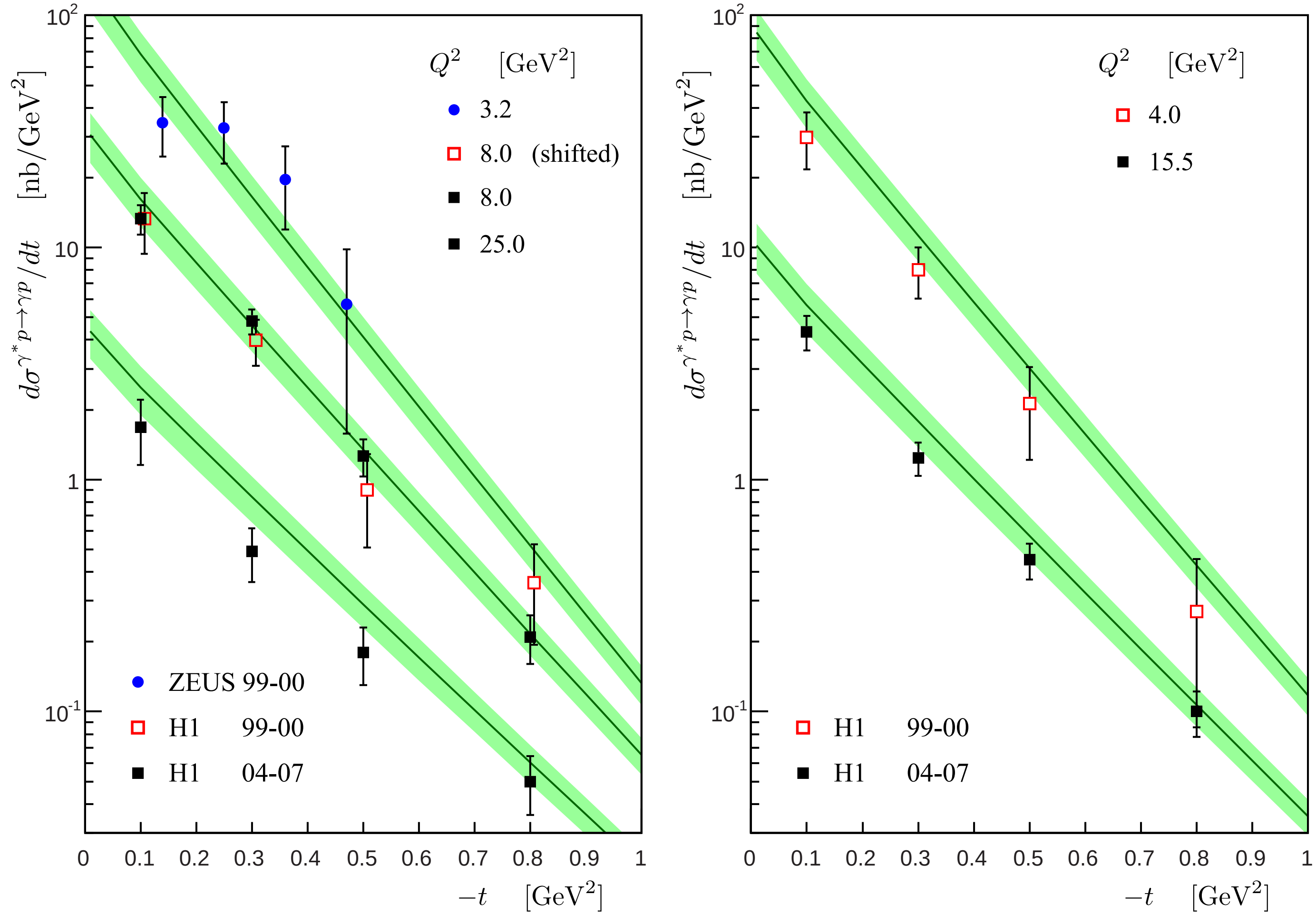

$$d\sigma(lp \to lp\gamma) = d\sigma_{BH} + d\sigma_I + d\sigma_{DVCS}$$

$$d\sigma_i \propto \sum_{n=0}^{3} \left[c_n^i \cos(n\phi) + s_n^i \sin(n\phi)\right]$$

DVCS convolutions

$$\mathcal{K} = e_u^2 \mathcal{K}^u + e_d^2 \mathcal{K}^d + e_s^2 \mathcal{K}^s$$

# DVCS at HERA




$W \simeq 90\,\mathrm{GeV}$ data from ZEUS, H1 K-Moutarde-Sabatie(13)

leading-twist accuracy

# $E$ for gluons and sea quarks

$E$ for valence quarks from FF analysis Diehl-K(13)

Teryaev(99): sum rule (Ji's s.r. and momentum s.r. of DIS) at $t = \xi = 0$

$$\int_0^1 dx x e_g(x) = e_{20}^g = -\sum e_{20}^{a_v} - 2\sum e_{20}^{\bar{a}}$$

valence term very small

$\Rightarrow$ 2nd moments of gluon and sea quarks cancel each other almost completely

(holds approximately for other moments too provided GPDs don't have nodes)

positivity bound for FTs forbids large sea $\Longrightarrow$ gluon small too (Diehl-Kugler(07))

$\frac{b^2}{m^2}\left(\frac{\partial e_s(x,b)}{\partial b^2}\right)^2 \leq s^2(x,b) - \Delta s^2(x,b)$

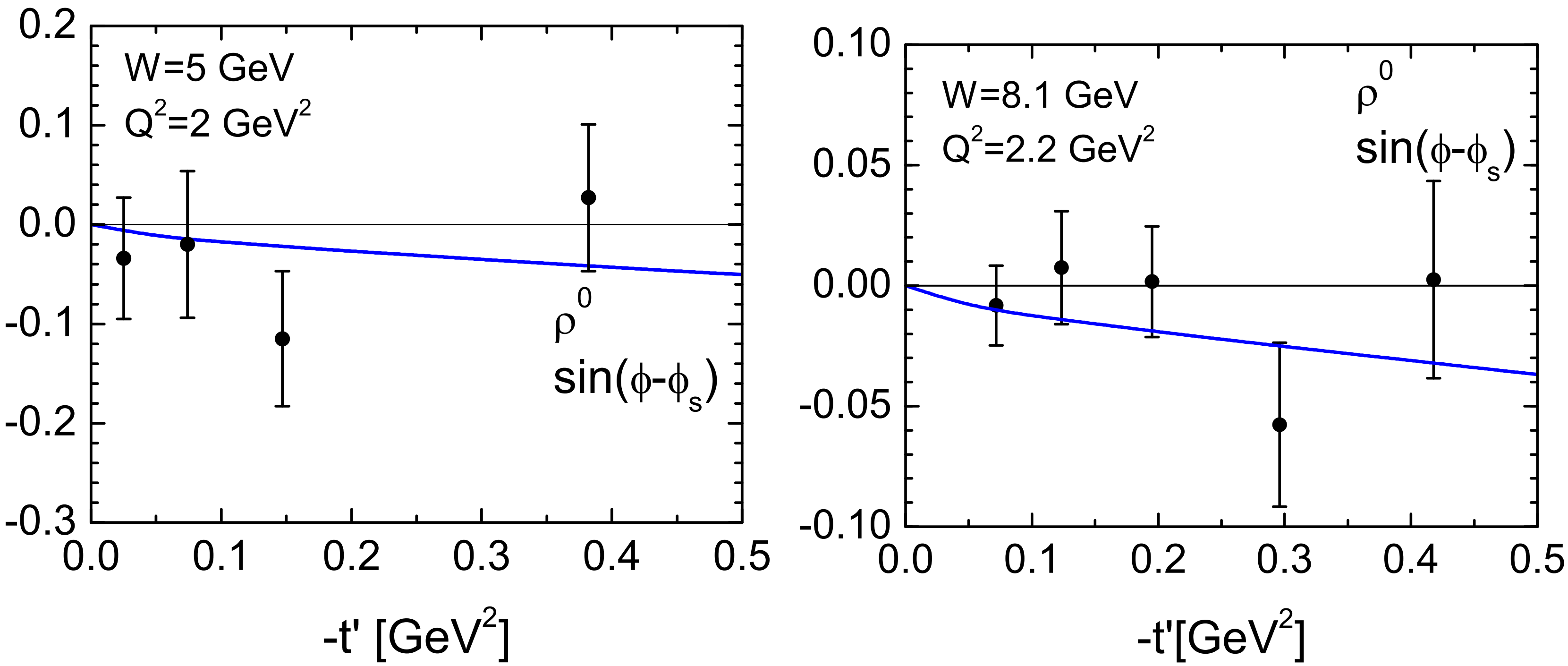
parameterization as described: $\beta_e^s = 7$, $\beta_e^g = 6$ Regge-like parameters as for $H$

$e_i = N_i x^{-\alpha_g(0)}(1-x)^{\beta_i}$ flavor symm. sea for $E$ assumed

$N_s$ fixed by saturating bound ($N_s = \pm 0.155$), $N_g$ from sum rules

for $\xi \neq 0$ input to double distribution ansatz

# $A_{UT}^{\sin(\phi-\phi_s)}$ for $\rho^0$ production




data: HERMES(08) COMPASS(12)

theor. result: Goloskokov-K(09)

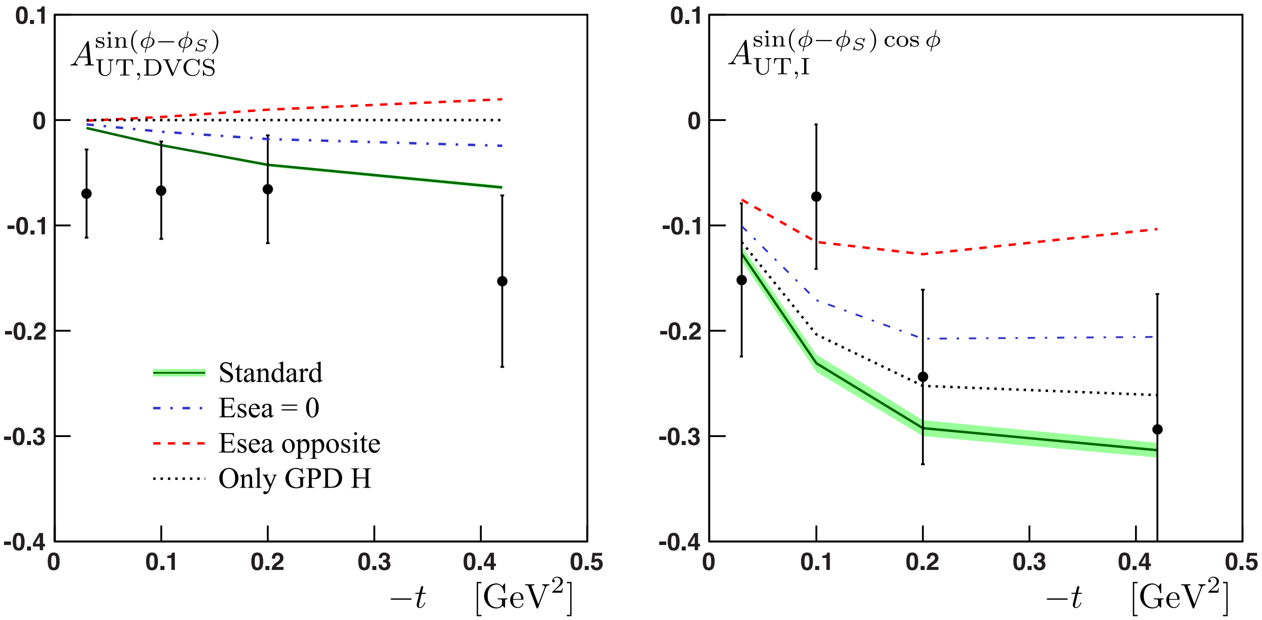
$$A_{UT}^{\sin(\phi-\phi_s)} \sim \mathrm{Im}\big[\mathcal{E}^*\mathcal{H}\big]$$

gluon and sea contr. from $E$ cancel to a large extent

dominated by valence quark contr. from $E$

($\phi_s$ orientation of target spin vector; $\phi$ azimuthal angle between lepton and hadron plane)

# Target asymmetry in DVCS




data: HERMES(08)
$\langle Q^2 \rangle \simeq 2.5\,\mathrm{GeV}^2$
$\langle x_{\rm B} \rangle \simeq 0.09$

theory: KMS(12)

$A_{UT,DVCS}^{\sin(\phi-\phi_s)} \sim \mathrm{Im}\big[\mathcal{E}^*\mathcal{H}\big]$
no cancellation between
sea and gluon
$\Rightarrow \mathcal{E}^{\rm sea}$ seen

from BH-DVCS interference
separate contr. from
$\mathrm{Im}\,\mathcal{H}$ and $\mathrm{Im}\,\mathcal{E}$

negative $\mathcal{E}^{\rm sea}$ favored in both cases

$\mathcal{E}^g \geq 0$ Koempel et al(11) transverse target polarisation in $J/\Psi$ photo- and electroproduction, dominated by gluonic GPDs

# Application: Angular momenta of partons

Ji's sum rule for 2nd moments of $H$ and $E$

$$J^a = \frac{1}{2}\left[q_{20}^a + e_{20}^a\right] \qquad J^g = \frac{1}{2}\left[g_{20} + e_{20}^g\right] \qquad (\xi = t = 0)$$

$q_{20}^a, g_{20}$ from ABM11 (NLO) PDFs $\qquad (a = u, d, s, \bar{u}, \bar{d}, \bar{s})$

$e_{20}^{a_v}$ from form factor analysis Diehl-K. (13):

$e_{20}^s \simeq 0 \ldots - 0.026$ from analysis of $A_{UT}$ in DVCS and pos. bound

$e_{20}^g$ from sum rule for $e_{20}$ ($e_{20}^g \simeq -6e_{20}^s$)

(Goloskokov-K (09), K. 1410.4450)

$$\begin{aligned} J^{u+\bar{u}} &= 0.261 \ldots 0.235\,; \\ J^{d+\bar{d}} &= 0.035 \ldots 0.009\,; \\ J^{s+\bar{s}} &= 0.017 \ldots - 0.009\,; \\ J^g &= 0.187 \ldots 0.265\,; \end{aligned}$$

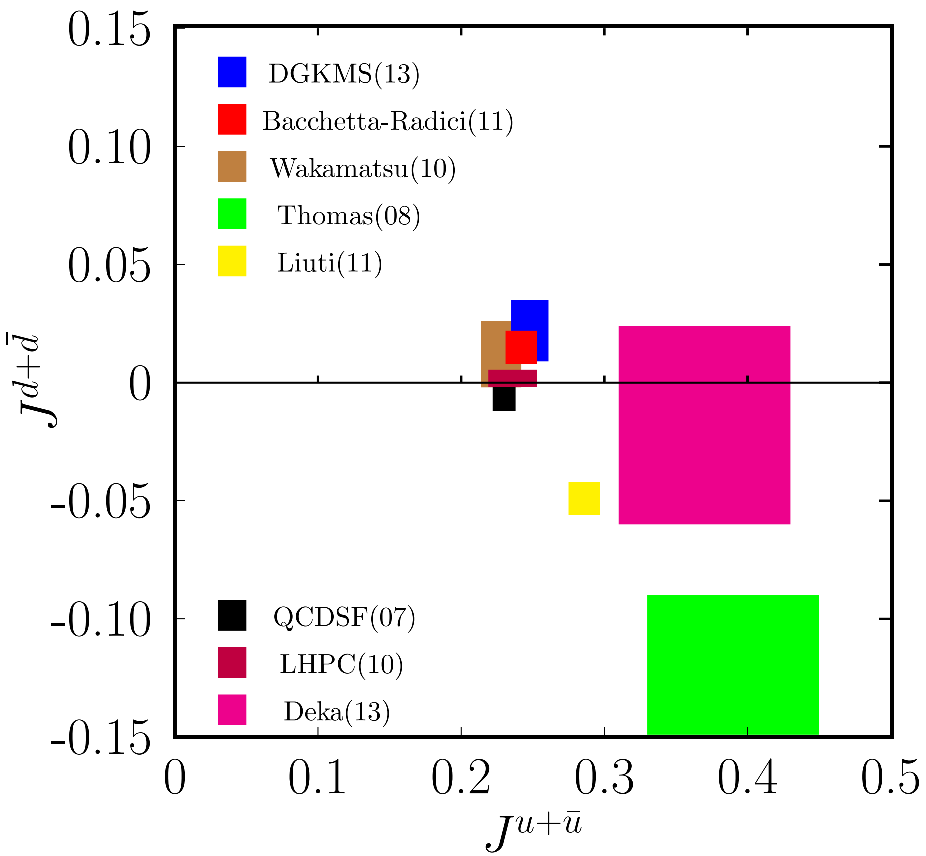
$J^i$ quoted at scale $2\,\mathrm{GeV}$

$\sum J^i = 1/2$

(spin of the proton)

need better determ. of $E^s$ (smaller errors of $A_{UT}$ in DVCS)

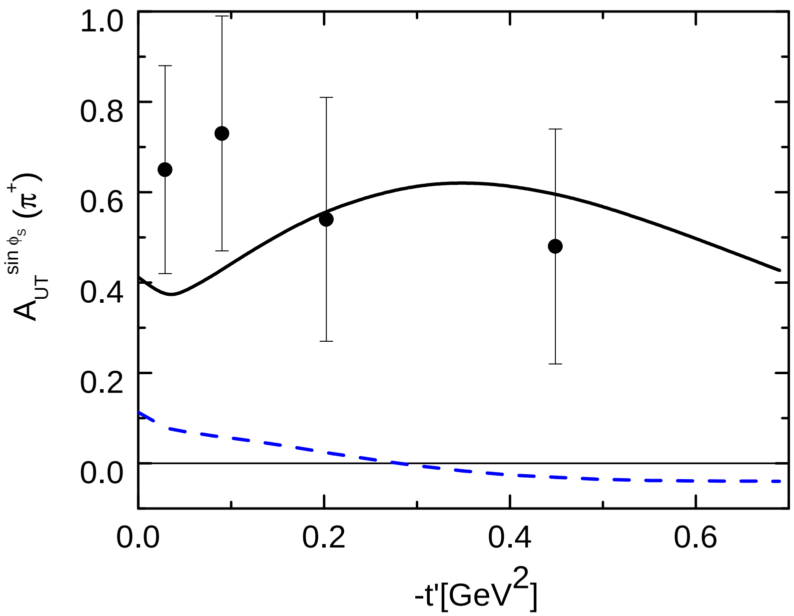
# Comparison with other results

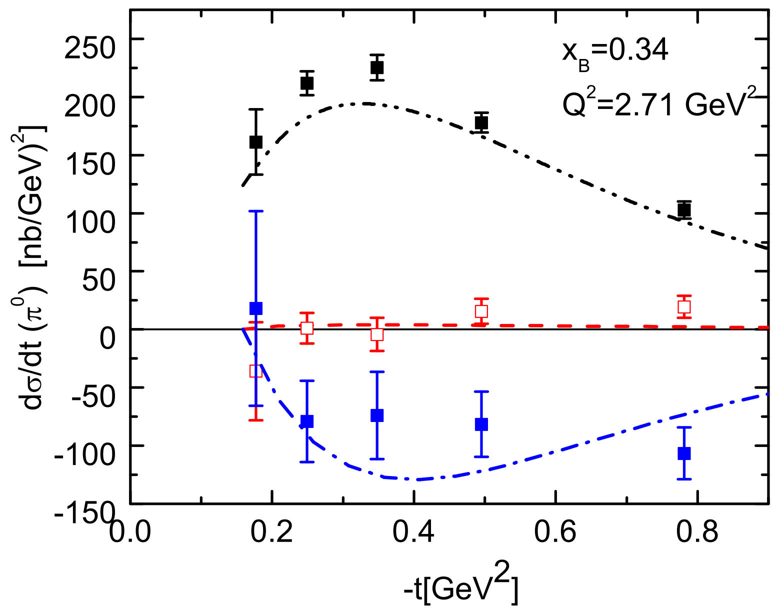



Deka et al (13): $J^g = 0.14 \pm 0.04$

$J^g = 0.187 \ldots 0.265$

# Analysis of pion leptoproduction




HERMES(09)

$Q^2 \simeq 2.5\,\mathrm{GeV}^2$,$W = 3.99\,\mathrm{GeV}$

$\sin\phi_s$ modulation very large

does not vanish for $t' \to 0$

$A_{UT}^{\sin\phi_s} \propto \mathrm{Im}\Big[\mathcal{M}^*_{0-,++}\mathcal{M}_{0+,0+}\Big]$

n-f. ampl. $\mathcal{M}_{0-,++}$ required

CLAS(12)

unsep. cross sec. $d\sigma_T + \epsilon d\sigma_L$

$d\sigma_{LT}$, $d\sigma_{TT}$

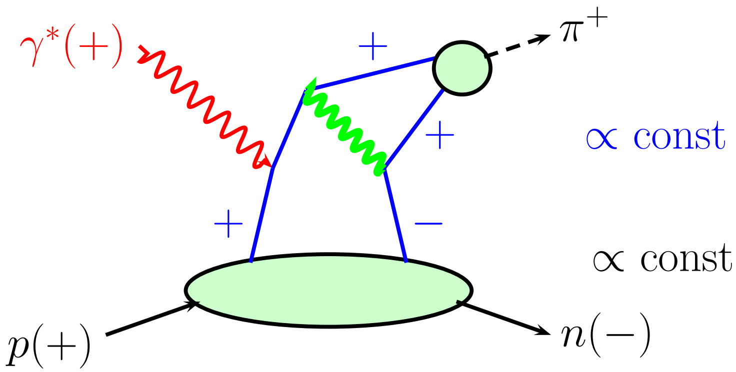
$|d\sigma_{TT}|$ large

flip ampl $\mathcal{M}_{0+,\pm+}$ required

($\propto \sqrt{-t'}$)

**transverse photons are important**

# Transverse photons in the handbag approach




need subprocess amplitude for $\gamma_T^* \to \pi$

non-vanishing for $t \to 0$

there is only one $\mathcal{H}_{0-,++}$

$\Longrightarrow$ requires parton helicity flip

transv. GPDs $\quad H_T, E_T, \widetilde{H}_T, \widetilde{E}_T$

$\mathcal{M}_{0-,++} \propto$ const

goes along with twist-3 w.f.

(enhanced by chiral condensate

$\mu_\pi = m_\pi^2/(m_u + m_d) \simeq 2\,\text{GeV}$

at scale 2 GeV)

$$\mathcal{M}_{0-++} = e_0\sqrt{1-\xi^2}\int dx \mathcal{H}_{0-++}^{\text{twist}-3} H_T$$

$$\mathcal{M}_{0+\pm+} = -e_0\frac{\sqrt{-t'}}{4m}\int dx \mathcal{H}_{0-++}^{\text{twist}-3} \bar{E}_T$$

(suppr. by $\mu_\pi/Q$ as compared to $L \to L$)

prominent role of transversity GPDs also claimed by Ahmad et al (09)

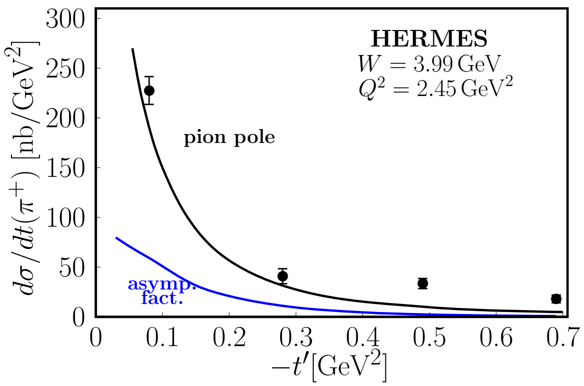
analysis and results different

# The pion pole

leading amplitudes for $Q^2 \to \infty$

$$\mathcal{M}_{0+0+} = \frac{e_0}{2}\sqrt{1-\xi^2}\langle \widetilde{H} - \frac{\xi^2}{1-\xi^2}\widetilde{E}\rangle \qquad \mathcal{M}_{0-0+} = e_0\frac{\sqrt{-t'}}{4m}\xi\langle\widetilde{E}\rangle$$

For $\pi^+$ production - pion pole: (Mankiewicz et al (98), Penttinen et al (99))




$$\widetilde{E}^u_{\rm pole} = -\widetilde{E}^d_{\rm pole} = \Theta(|x|\leq\xi)\frac{mf_\pi g_{\pi NN}}{\sqrt{2}\xi}\frac{F_{\pi NN}(t)}{m_\pi^2-t}\Phi_\pi(\frac{x+\xi}{2\xi})$$

$$\Longrightarrow \frac{d\sigma_L^{\rm pole}}{dt} \sim \frac{-t}{Q^2}\left[\sqrt{2}e_0 g_{\pi NN}\frac{F_{\pi NN}(t)}{m_\pi^2-t}Q^2F_\pi^{\rm pert}(Q^2)\right]^2$$

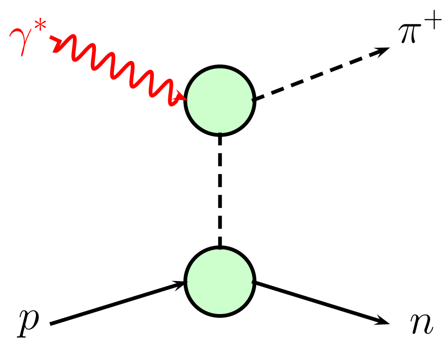
understimates c.s.(blue l.) $\quad F_\pi^{\rm pert.} \simeq 0.3-0.5F_\pi^{\rm exp.}$

($F_\pi$ measured in $\pi^+$ electroproduction at Jlab)

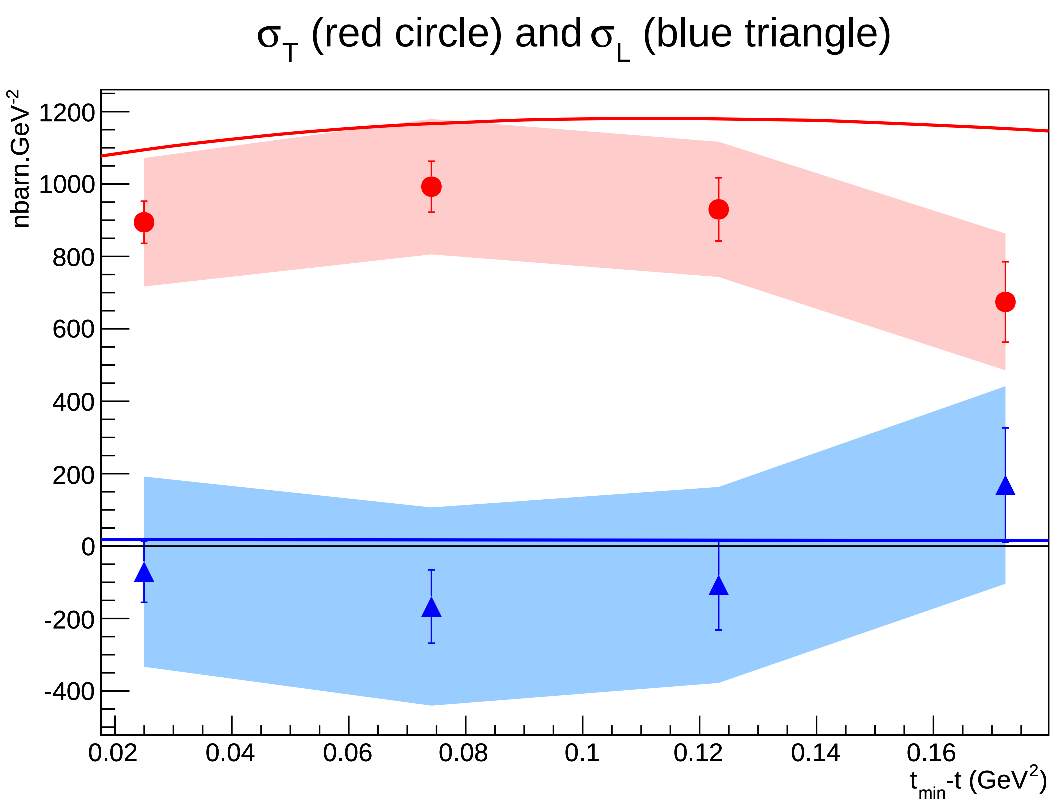
Goloskokov-K(09): $F_\pi^{\rm pert} \to F_\pi^{\rm exp}$

knowledge of the sixties suffices to explain

$\pi^+$ data at small $-t$ and large $Q^2$



# Separated $\pi^0$ cross section




Hall A (15) preliminary

$Q^2 = 1.76\,\mathrm{GeV}^2$

$x_B = 0.36$

predictions from GK(11)

data now out: 1608.01003

$d\sigma_T \gg d\sigma_L$

like expectations for $Q^2 \to 0$

and not for $Q^2 \to \infty$

From pion leptoproduction we learn about $H_T$ and $\bar{E}_T$

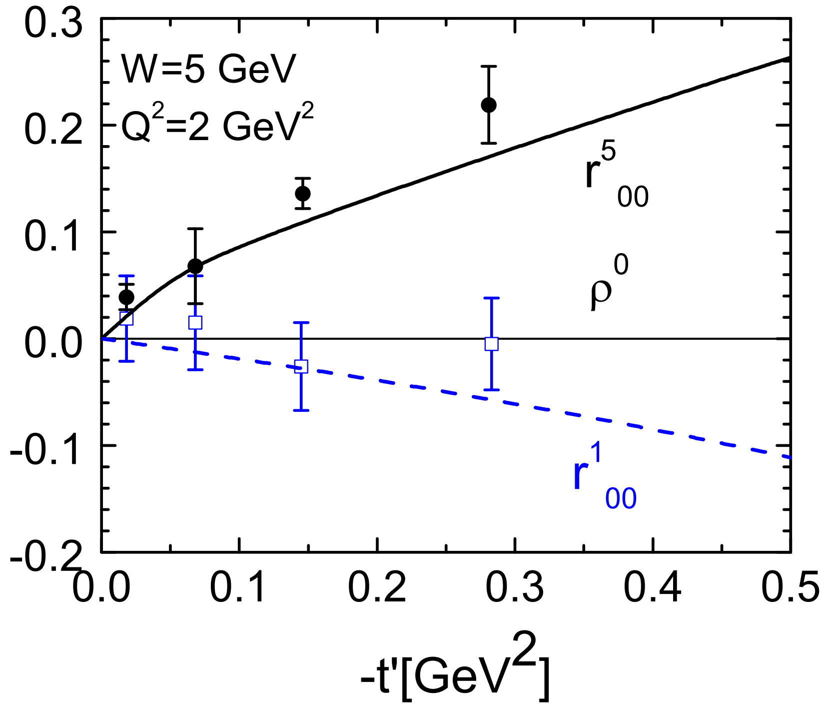
and not about $\widetilde{H}$, $\tilde{E}$

# Transversity in vector meson electroproduction

as for pions: $\gamma_T^* \to V_L$ amplitudes, same subprocess amplitude
except $\Psi_\pi \to \Psi_V$, i.e. $f_\pi \to f_V$, $\mu_\pi/Q \to m_V/Q$

$\gamma_T^* \to V_L$ amplitudes of about the same strength as the $\gamma_T^* \to \pi$ ones but
competition with $\mathcal{H}$ (for gluons and quarks) instead with $\widetilde{\mathcal{H}}$ ($|\mathcal{H}| \gg |\widetilde{\mathcal{H}}|$)
$\Longrightarrow$ small transversity effects for vector mesons
to be seen in some of the SDMEs and in spin asymmetries
examples from Goloskokov-K(13,14) no fit

# Spin density matrix elements




SDME from HERMES(09)

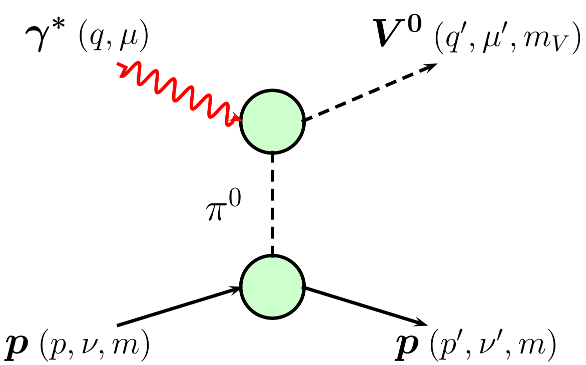
$r^1_{00} \sim -|\bar{\mathcal{E}}_T|^2$ $\qquad$ $r^5_{00} \sim \mathrm{Re}\big[\bar{\mathcal{E}}^*_T \mathcal{H}\big]$

# $\omega$ production

important ingredient: pion pole (as for $\pi^+$ production)

does not contribute to $\gamma_L^* \to V_L$ clearly not leading twist

Goloskokov-K(14)




$$\langle\omega|j_\kappa^{\rm el}(0)|\pi\rangle = e_0 g_{\gamma^*\pi\omega}(Q^2,t)\varepsilon(\kappa,q,\epsilon_\omega,q')$$

large $Q^2$, small $-t$: $g_{\gamma^*\pi\omega}(Q^2,t) \simeq g_{\pi\omega}(Q^2)$

dominant $\gamma_T^* \to V_T$ transitions

(suppressed by $1/Q$ as compared to $\gamma_L^* \to V_L$)

subdominant $\gamma_L^* \to V_T$ (suppressed by $1/Q^2$)

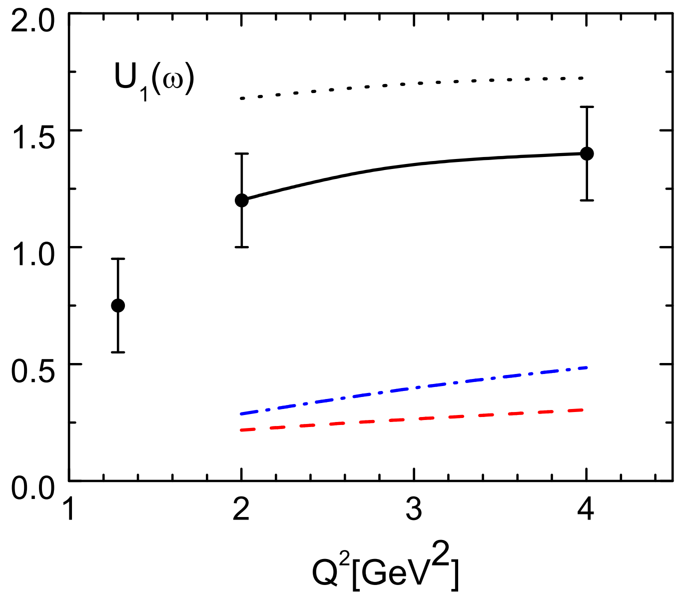
HERMES(14) $\omega$ SDMEs at $W = 4.8\,\mathrm{GeV}$:

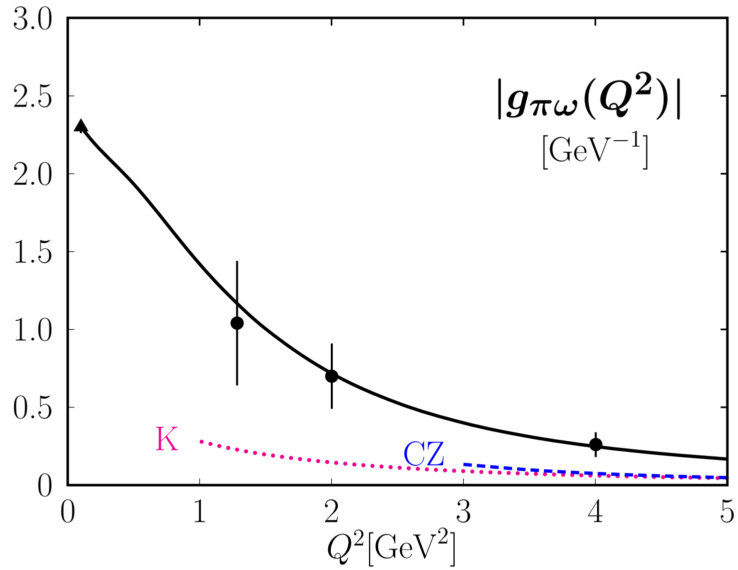
$$U_1 = 1 - r_{00}^{04} + 2r_{1-1}^{04} - 2r_{11}^{1} - 2r_{1-1}^{1} = 2\frac{d\sigma_U}{d\sigma} \qquad d\sigma_U = \frac{U_1}{2-U_1}d\sigma_N$$

$d\sigma_N$: from our GPDs $H$ and $E$ (like $\rho^0$) and small contr. from $H_T$ and $\bar{E}_T$

$d\sigma_U$: pion pole ($\sim g_{\pi\omega}(Q^2)$) and small background ($\widetilde{H}$)

# $\pi - \omega$ form factor




$g_{\pi\omega}$ unknown exp. in space-like region, except at $Q^2 = 0$ from $\omega \to \pi\gamma$ decay

fit $g_{\pi\omega}$ to $U_1$ results consistent with $Q^2 = 0$ value

$$\text{interpolation}: \quad |g_{\pi\omega}| = \frac{2.3\,\text{GeV}^{-1}}{1 + Q^2/a_1^2 + Q^4/a_2^4 = 0} \qquad (a_1 = 2.7\,\text{GeV}\,, a_2 = 1.2\,\text{GeV})$$

sign cannot be fixed from SDME; need spin asymmetries HERMES(15)

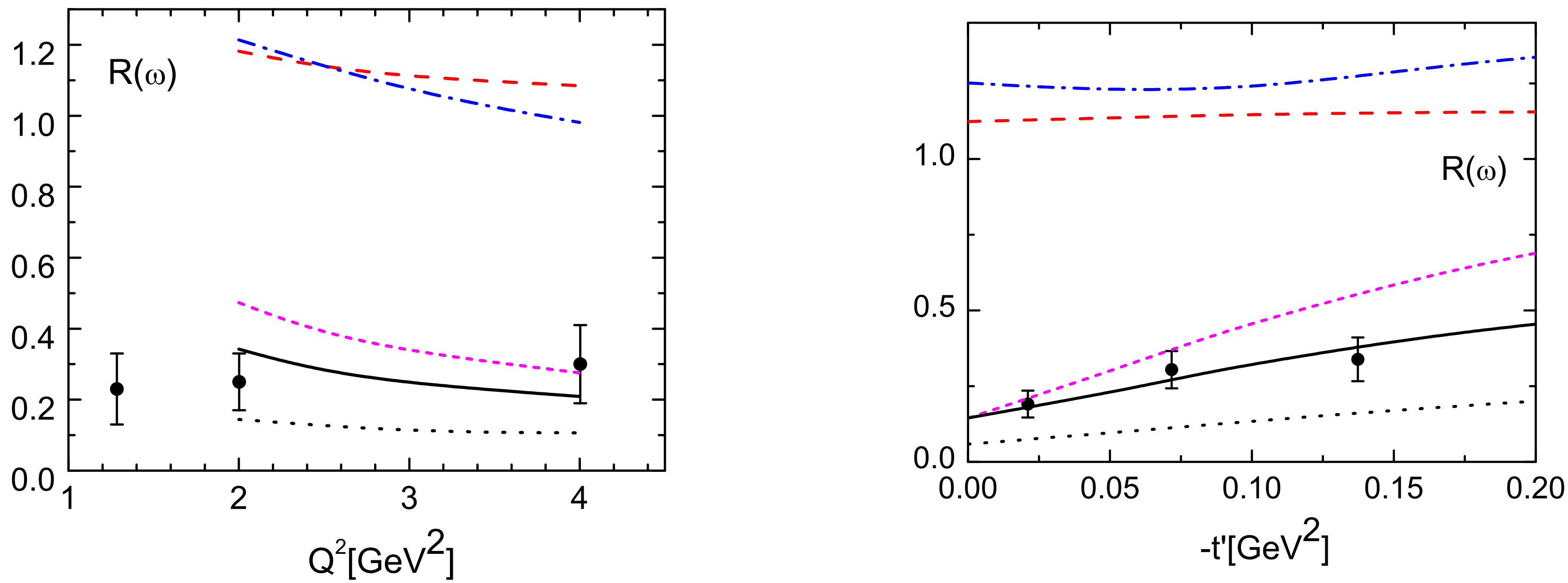
theory: QCD sum rules (only soft) Khodjamirian(99), Braun-Halperin(94)

perturbative QCD to twist-3 accuracy Chernyak-Zhitnitsky(84)

($W = 4.8\,\text{GeV}$, without pion pole, 8 GeV, dotted 3.5 GeV, $t' = -0.08\,\text{GeV}^2$)

# Longitudinal/transversal separation

$$R = \frac{1}{\epsilon}\frac{r_{00}^{04}}{1-r_{00}^{04}} = \frac{d\sigma(L\to L) + d\sigma(T\to L)/\epsilon}{d\sigma(T\to T) + \epsilon d\sigma(L\to T)}$$



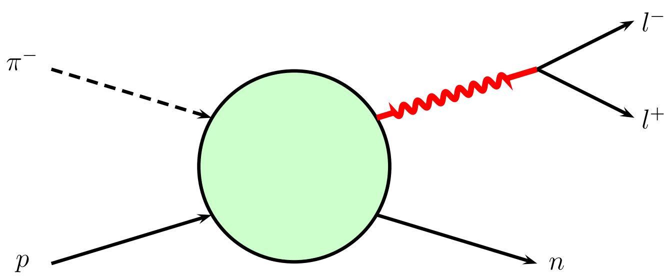

$W = 4.8\,\mathrm{GeV}$, $t' = -0.08\,\mathrm{GeV}^2$ $\qquad$ $Q^2 = 2.42\,\mathrm{GeV}^2$

differs from $d\sigma_L/d\sigma_T$ (dashed line)

by $\gamma_L^* \to V_T$ (pion pole) and $\gamma_T^* \to V_L$ $(H_T, \bar{E}_T)$ transitions

pion-pole contr. in $\rho^0$ production - small but visible

# The exclusive pion-induced Drell-Yan process



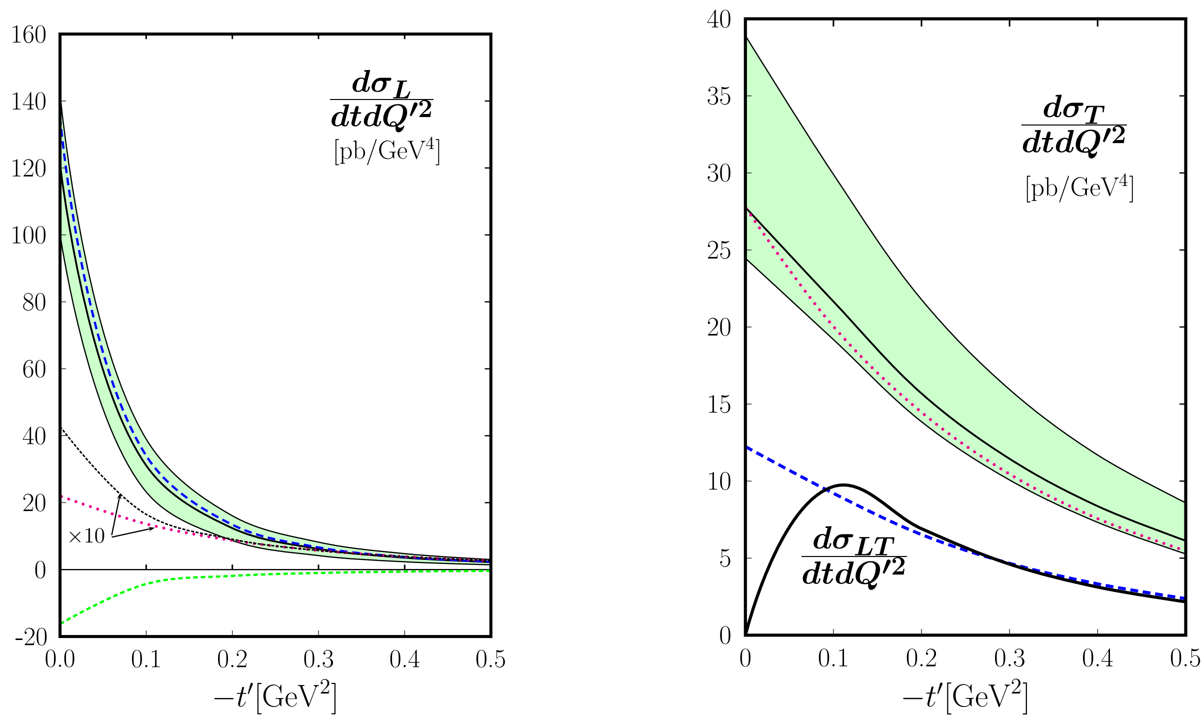

$s - u$ crossed to pion production
same GPDs contribute, $Q^2 \to -Q'^2$

Berger-Diehl-Pire (01): leading-twist, LO analysis of long. cross section (i.e. exploiting asymp. factor. formula) (detailed reanalysis Sawada et al, 1605.00364)

we know that leading-twist analysis of $\pi^+$ production fails with JLAB, HERMES data by order of magnitude

Therefore . . . (Goloskokov-K. 1506.04619)
a reanalyis of the exclusive Drell-Yan process seems appropriate
making use of what we have learned from analysis of pion production

# Estimate of partial cross sections




$Q'^2 = 4\,\mathrm{GeV}^2$ and $s = 20\,\mathrm{GeV}^2$ solid lines with error bands: full result

pion pole, $|\langle \widetilde{H}^{(3)} \rangle|^2$, interference, short dashed: leading-twist contribution

only new element: time-like pion FF:

$Q'^2 > 2\,\mathrm{GeV}^2$: $Q'^2|F_\pi(Q'^2)| = 0.88 \pm 0.04\,\mathrm{GeV}^2$ (CLEO, BaBar, $J/\Psi \to \pi^+\pi^-$)

phase $(\exp[i\delta(Q'^2)])$ from disp. rel. Belicka et al(11) for $Q'^2 < 8.9\,\mathrm{GeV}^2$

$\delta = 1.014\pi + 0.195(Q'^2/\mathrm{GeV}^2 - 2) - 0.029(Q'^2/\mathrm{GeV}^2 - 2)^2$

for $Q'^2 \geq 8.9\,\mathrm{GeV}^2$: $\delta = \pi$, the LO pQCD result

# Summary

- The handbag approach, generalized to transverse photons and with meson size corrections, describes all DVMP data for $Q^2 \gtrsim 2\,\mathrm{GeV}^2$ and $W \gtrsim 4\,\mathrm{GeV}$ for $\rho^0, \omega$ ( $\gtrsim 2\,\mathrm{GeV}$ for $\phi, \pi$)
- From the combined analysis of nucleon form factors, DVMP (and DVCS for $E^{\mathrm{sea}}$) a set of GPDs has been extracted $(H, E, \widetilde{H}, H_T, \bar{E}_T$ for valence quarks, gluon and sea quarks only for $H$ )
- This set of GPDs allows for calculations of other hard exclusive processes (e.g. DVCS, $\omega$ production), to evaluate Ji's sum rule and to study the transverse localization of partons in the proton
- Nothing is perfect - the GPDs need improvements:
  use of new PDFs, more complicated profile fcts. for all GPDs, $D$-term, kinematical corrections at low $Q^2$, low $W$, large $\xi$
  need for new data from COMPASS, JLAB12 and EIC