The double-distribution parametrization of GPDs

P. Kroll

Universitaet Wuppertal

workshop on 'Nucleon and nuclear structure through di-lepton production'

Trento, October 2016

Outline:

- **Definition of double-distribution parametrization**
- **Ansaetze for the zero-skewness GPDs**
- **Properties of valence quark GPDs**
- **The gluon GPD and vector-meson production**
- **Summary**

Definition of the DD parametrization

double distribution representation Mueller *et al* (94), Radyushkin (99)

$$K^i(x,\xi,t) = \int_{-1}^{1} d\rho \int_{-1+|\rho|}^{1-|\rho|} d\eta\, \delta(\rho+\xi\eta-x) \left[K^i(\rho,\xi=0,t) w_i(\rho,\eta)\right] + D_i\, \Theta(\xi^2-\bar{x}^2)$$

weight fct $w_i(\rho,\eta) \sim [(1-|\rho|)^2 - \eta^2]^{n_i}$ $(n_g = n_{\rm sea} = 2, n_{\rm val} = 1$, generates ξ dep.)
zero-skewness GPD $K^i(\rho,\xi=0,t)$

D-term: only for H and E $(D_H = -D_E)$ for gluons and quark singlet
no D-term for valence quarks in GK neglected

advantage: polynomiality and reduction formulas automatically satisfied
positivity bounds have to be checked numerically

Mueller et al: partial-wave expansions of GPDs (in conformal p.w. and in t-channel SO(3) p.w.)

Ansätze for zero-skewness GPDs

simplest ansatz: VGG(98), Freund et al (01)

e.g. $H^q(x, \xi = 0, t) = c_q q(x) F_1^q(t)$ PDF times form factor

respects reduction formula and sum rules $\int dx H^q(x, \xi = 0, t) = F_1^q(t)$

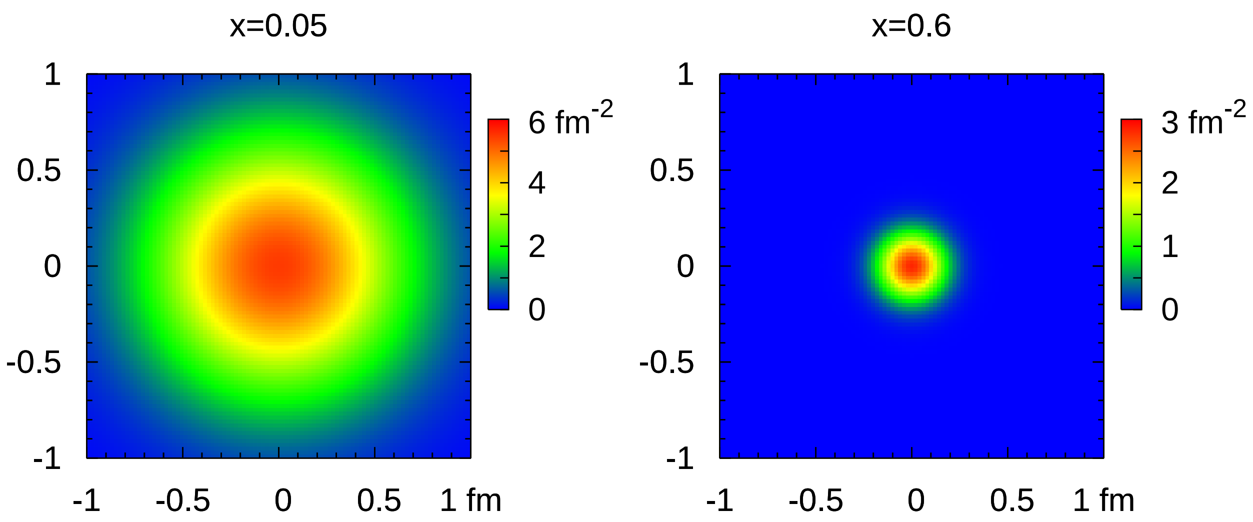
Fourier transformation $\Delta \to b_\perp$

Burkhardt(00,03): $q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{b}\mathbf{\Delta}_\perp} H^q(x, \xi = 0, t = -\Delta_\perp^2)$

density interpretation: $q(x, \mathbf{b}_\perp)$ is the probability to find a quark q with long. momentum fraction x at transverse distance $\mathbf{b}_\perp$

$\mathbf{b}_\perp$ transverse distance between struck parton and hadron's center of momentum $\sum x_i \mathbf{b}_{\perp i} = 0$; $(\sum x_i = 1)$

partons with large (small) x_i must (can) have small (large) $\mathbf{b}_{\perp i}$



$x - b_\perp$ $(x - t)$
correlation required

fact. ansatz
obsolete

Regge-like ansatz

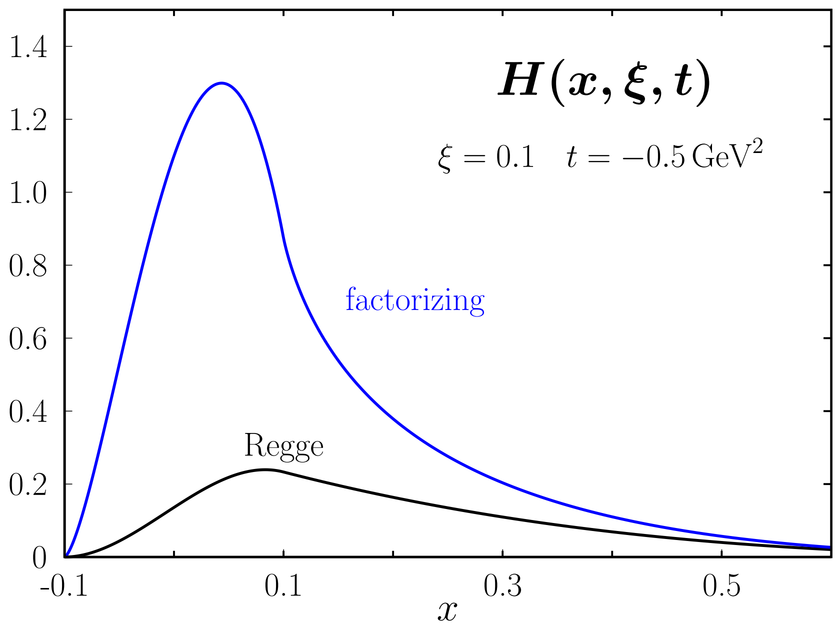
frequently used now

$$H^q(x,\xi=0,t) = q(x)e^{tf_q(x)} \qquad f_q(x) = B_q + \alpha'_q \ln(x)$$

at small x: $q \sim x^{-\alpha(0)} \Longrightarrow H^q \sim x^{-\alpha(t)}$ $\qquad (\alpha(t) = \alpha(0) + t\alpha')$

standard Regge trajectory - $H^q \sim 1/\sqrt{x}$ at $t \simeq 0$ $\qquad$ (fact. ansatz - for all t)

$\sim \sqrt{x}$ at $t \simeq -1\,\mathrm{GeV}^2$ $\qquad (\alpha = 0.5 + t\,\mathrm{GeV}^{-2})$

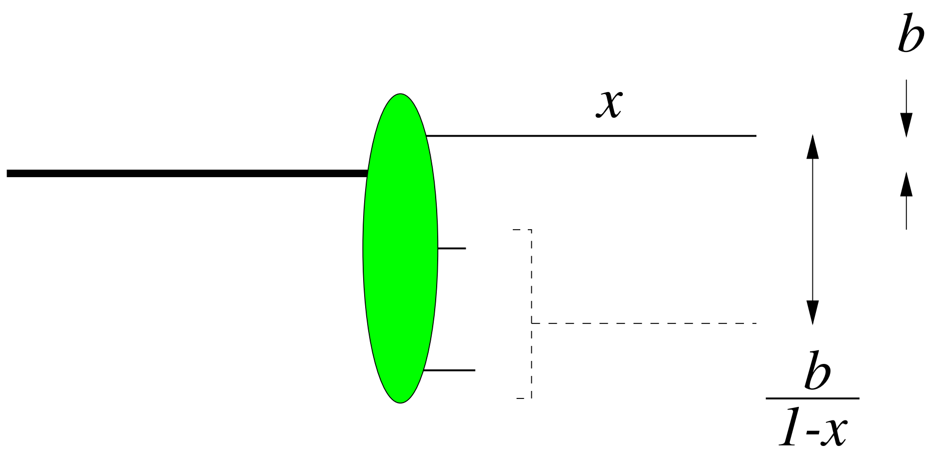


for $t = 0$ same GPDs

Fourier transform of H

$q(x, \mathbf{b}_\perp) = \frac{1}{4\pi} \frac{q(x)}{f_q(x)} \exp\left[-b^2/4f_q(x)\right]$ and $< b_\perp^2 >_x^q = 4f_q(x)$

distance between active parton and cluster of spectators
(rough estimate of proton radius)
$d_q(x) = \frac{\sqrt{<b_\perp^2>_x^q}}{1-x} = \frac{2\sqrt{f_q}}{1-x} \rightarrow 1/(1-x)$ for $x \rightarrow 1$!



Regge-like ansatz only suitable at small x, i.e. at small $-t$

unphysical at large x

Further improvement

used in analysis of nucleon form factors at $\xi = 0$ DFJK04, Diehl-K(13)

$$F_i^{p(n)} = e_{u(d)} F_i^u + e_{d(u)} F_i^d\,, \qquad F_i^a = \int_0^1 dx K_v^a(x, \xi = 0, t)$$

Dirac (Pauli) ff: $K = H(E)$ (normalization from $\kappa_q = \int_0^1 dx E_v^q(x, \xi = t = 0)$)

axial form factor: $\widetilde{H}$

profile fct: $f_q = (B_q + \alpha'_q \ln 1/x)(1-x)^3 + A_q x(1-x)^2$

valence quark GPDs H, E at $\xi = 0$ parameters fixed from fits to form factors

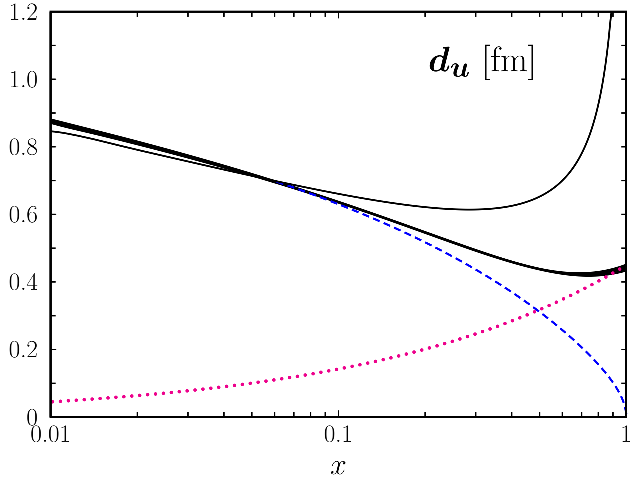
(for E in addition: $e_i = N_i x^{\alpha_i}(1-x)^{\beta_i}$)

$$d_q(x) = \frac{2\sqrt{f_q(x)}}{1-x} \to 2\sqrt{A_q}$$

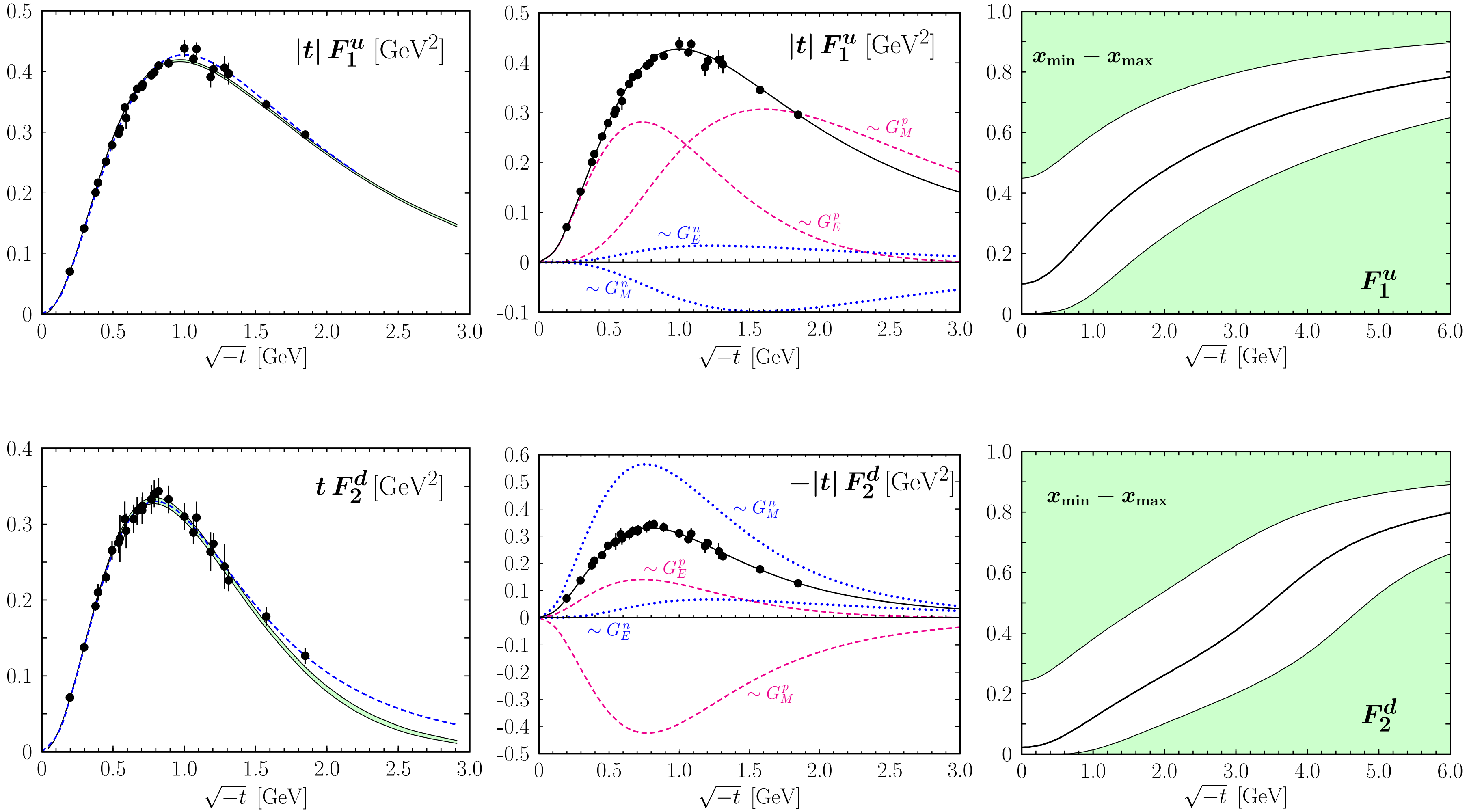
for $x \to 1$

Regge-type term, A term, full profile fct

Regge-like profile fct can (only) be used at small x (small $-t$)



The flavor form factors



fit to the flavor FFs
(interpolated FF data)

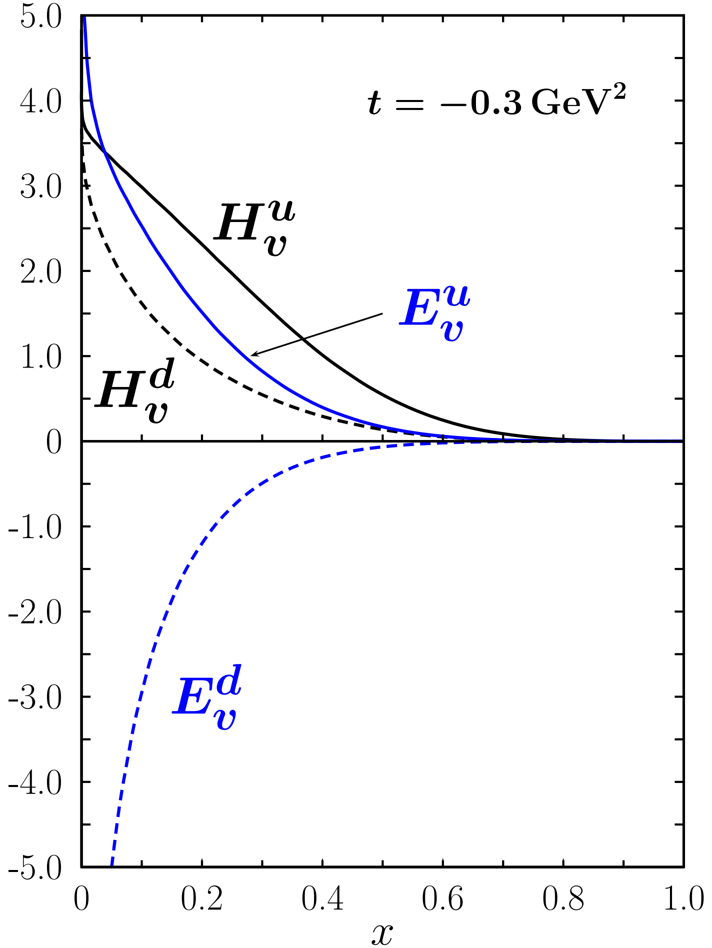
decomposition into
Sachs FFs

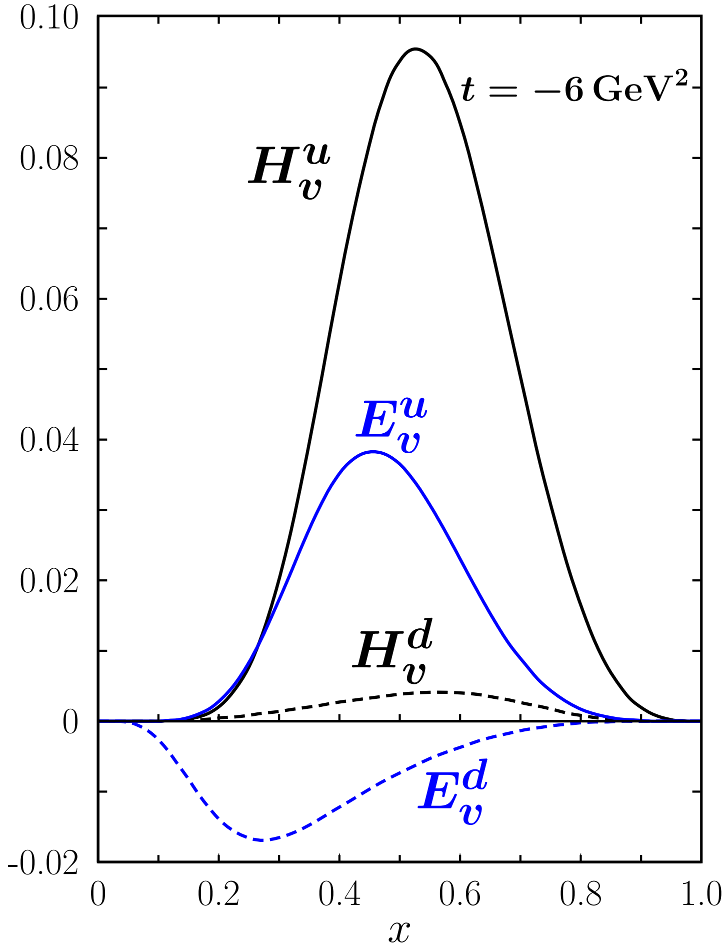
$$\int_{x_{max}(0)}^{1(x_{min})} dx K_v^q(x,t) = 5\% F_i^q(t)$$

$x \leftrightarrow t$ correlation

green bands - GPDs not determined

The GPDs H and E ($\mu = 2\,\mathrm{GeV}$)





$K_v^q \sim x^{-\alpha_q(0)-t\alpha_q'}$ at small x

singular (zero) at small (large) $-t$

makes $x \leftrightarrow t$ correlation obvious

$K_v^q \sim (1-x)^{\beta_q}$ at large x

pronounced peak

position moves to larger x and becomes narrower with increasing $-t$

Large–t behavior of flavor form factors

at large t: dominance of narrow region of large x:

$q_v \sim (1-x)^{\beta_q}$, $f_q \sim A_q(1-x)^2$ (analogously for F_2^q)

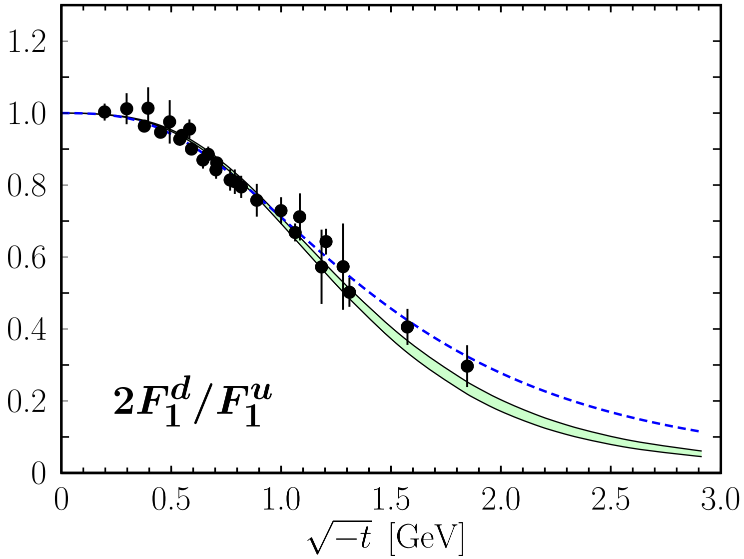
Saddle point method provides $1 - x_s = \left(\frac{2}{\beta_q} A_q |t|\right)^{-1/2}$ $F_1^q \sim |t|^{-(1+\beta_q)/2}$

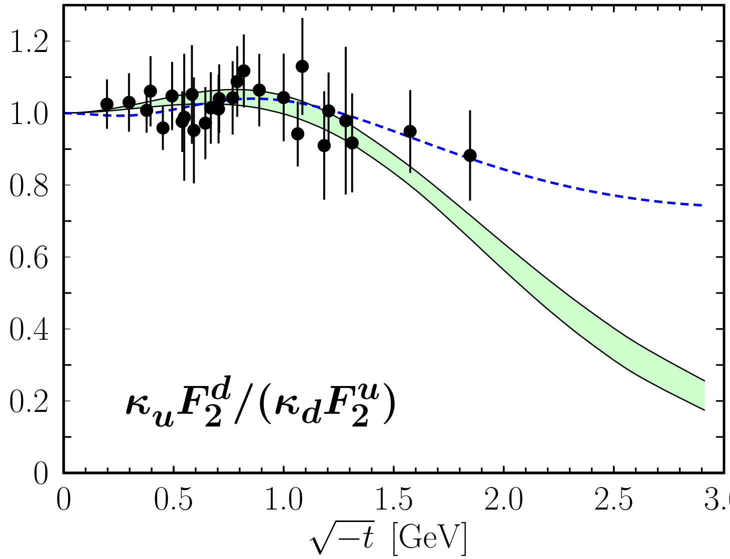
derivation of power law requires that x_s lies in sensitive x-region

active parton carries most of proton momentum while the spectators are soft

region of Feynman mechanism (similar to Drell-Yan)

power laws from wave fct overlaps: Dagaonkar-Jain-Ralston (14)





ABM PDFs: $\beta_u \simeq 3.4$, $\beta_d \simeq 5$, e_v^q: $\beta_u = 4.65$, $\beta_d = 5.25$

The gluon GPD H^g

same type of parametrization as for quarks

consider region of large energies and small x_B: vector-meson production is diffractive, amplitude dominantly imaginary, fed by gluonic (+ sea) GPD H

$$\mathrm{Im}\mathcal{M}^V_{0+0+} \sim H^g(\xi,\xi,t)/\xi$$

provided $xg(x) \sim x^{-\delta_g(Q^2)}$:

$H^g(\xi,\xi,t) = c_g(\delta_g, \alpha'_g, n_g) 2\xi g(2\xi) e^{t(B-\alpha'_g \ln(2\xi))} = c_g H^g(2\xi,0,t)$

$$\Longrightarrow \qquad \mathrm{Im}\mathcal{M}^V_{0+0+} \sim W^{2\alpha_g(t)} \qquad\qquad \alpha_g = 1 + \delta_g + \alpha'_g t$$

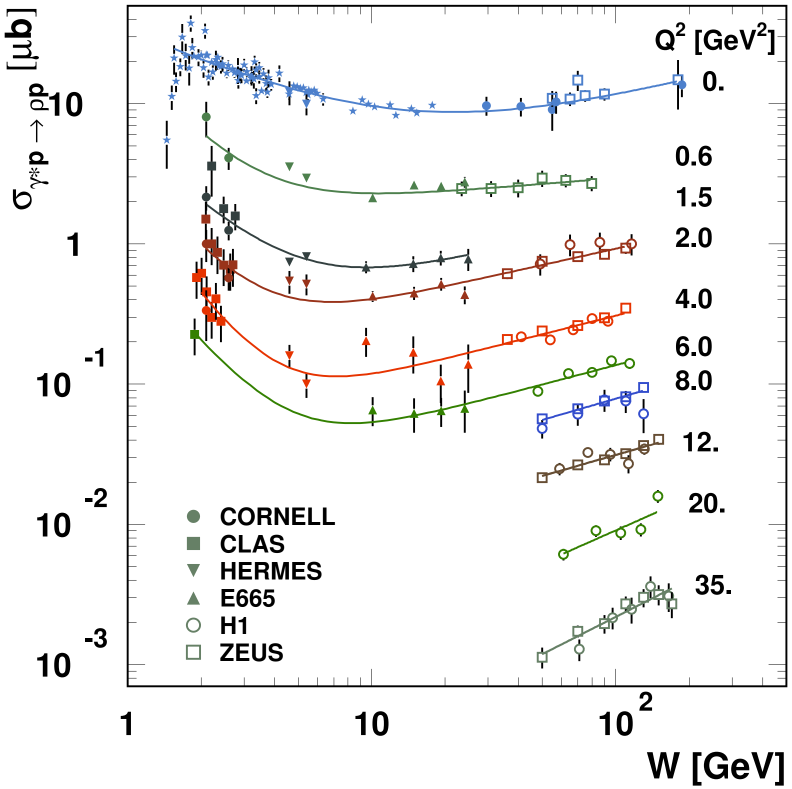
$$\sigma_L = \int_{t_{max}}^{t_{min}} dt \frac{d\sigma_L}{dt} \sim W^{4(\alpha_g(t_{min})-1)} \sim W^{4\delta_g(Q^2)}$$

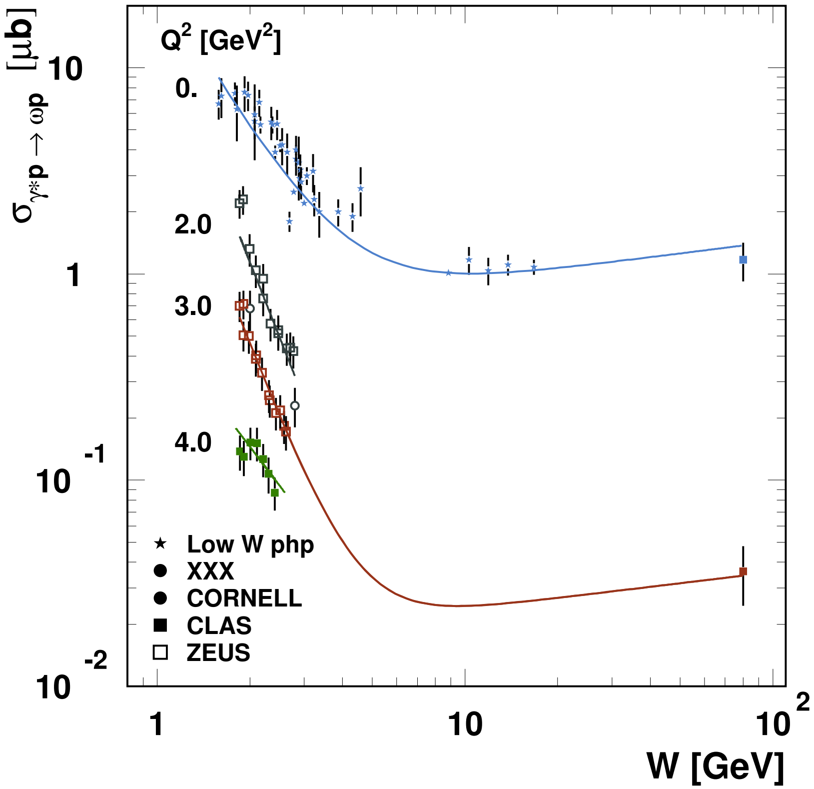
transverse cross section similar $\qquad t_{min} = 4m^2\xi^2/(1-\xi^2) \simeq 0$

close connection to color-dipole model (lead.log.appr) Brodsky et al (94)

difference is skewing effect c_g

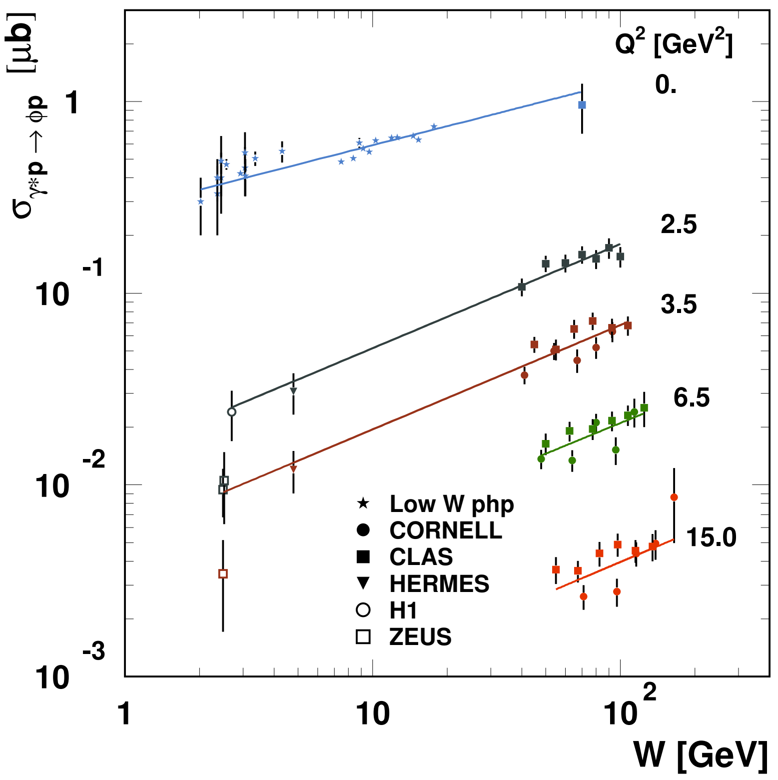
ρ^0 and ω production

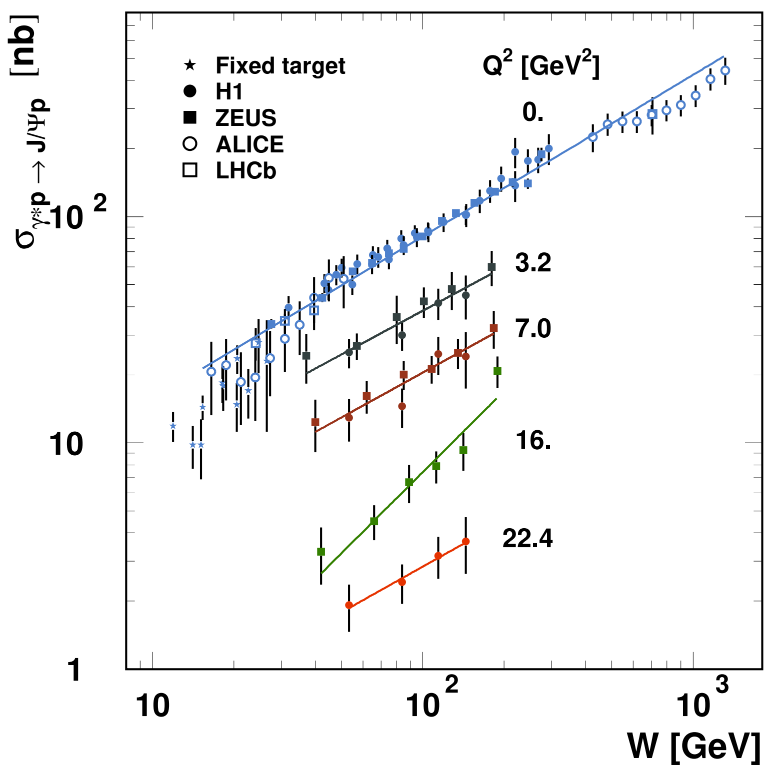




Favart-Guidal-Horn-K., 1511.04535

ϕ and J/Ψ production

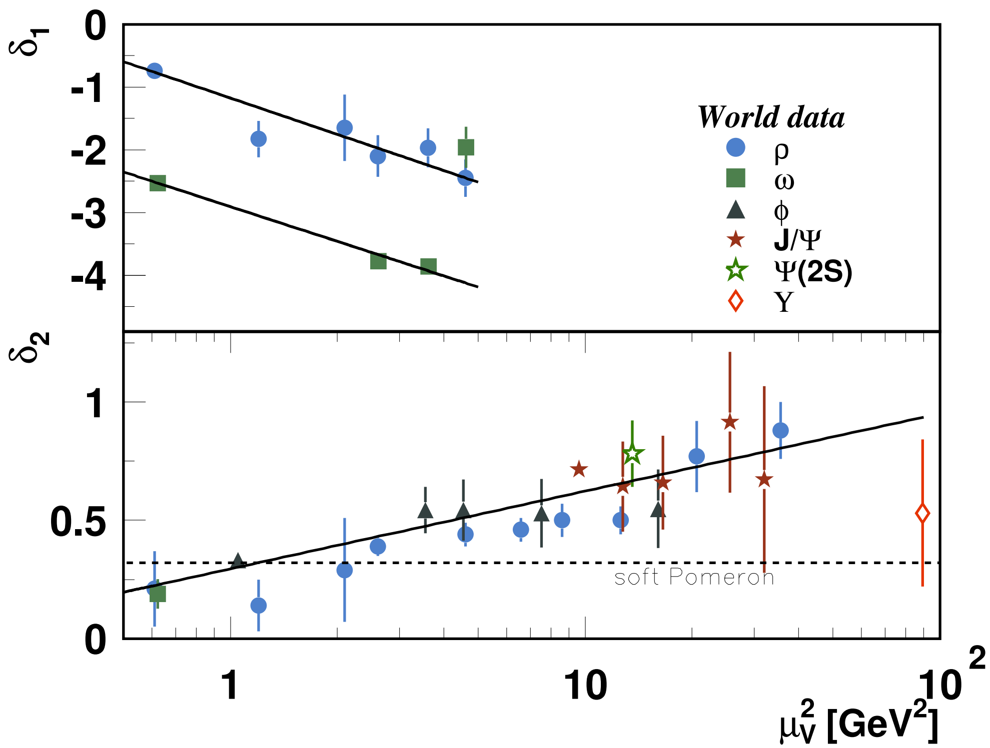




Favart-Guidal-Horn-K., 1511.04535

Fit

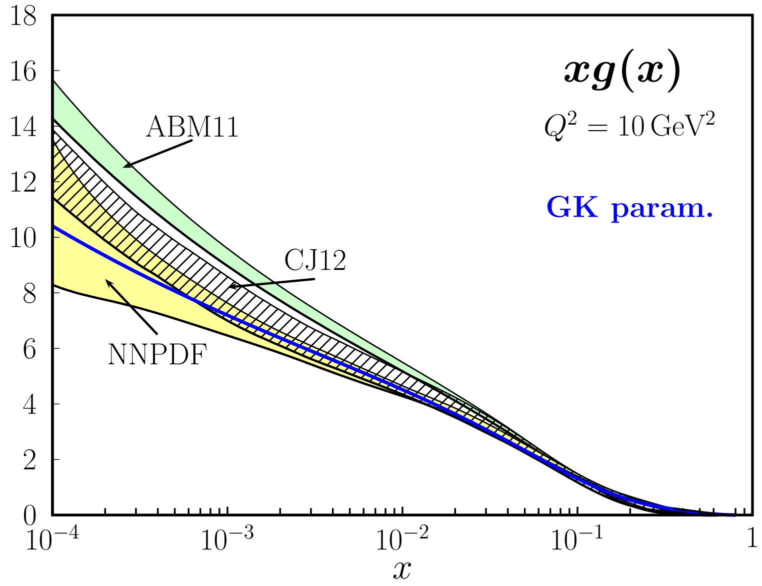
$$\sigma_V(W, \mu_V) = a_1 W^{\delta_1(\mu_V)} + a_2 W^{\delta_2(\mu_V)} \qquad \mu_V^2 = Q^2 + M_V^2$$



$$\delta_2 = 0.31 + 0.13 \ln\left(\mu_V^2/\mathrm{GeV}^2\right) = 4\delta_g$$

signals common underlying dynamics - gluon PDF, GPD

Gluon PDF



$xg(x) = x^{-\delta_g}(1-x)^5 \sum_i c_i x^{i/2}$

GK fitted to CTEQ6

behavior of gluon PDF problematic!

badly determined for $x \lesssim 10^{-3}$

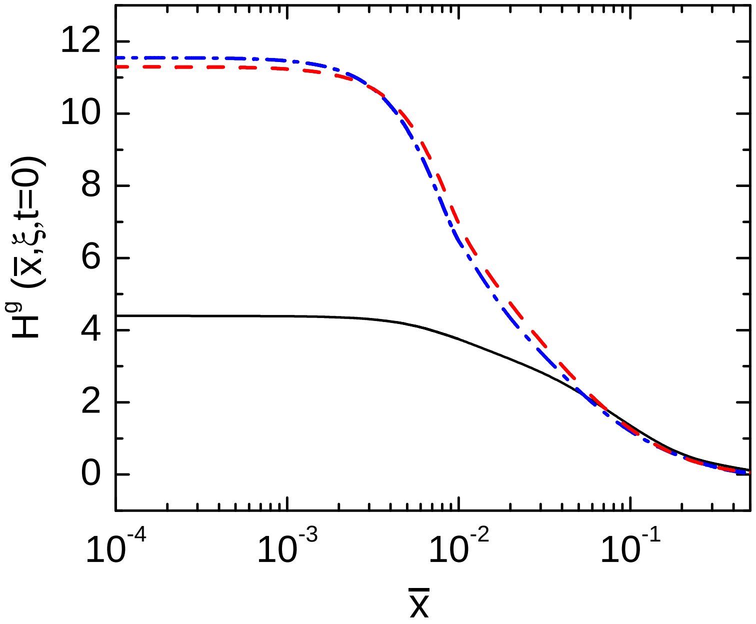
difficult to fit LHC data on

photoproduction of $J/\Psi, \Upsilon$

$x = M^2/W^2 \simeq 10^{-5}$

Jones-Martin-Ryskin-Teubner (15,16): LHC data may be used in PDF analyses

Evolution



$\xi = 0.01$

$Q^2 = 4 \to 40\,\mathrm{GeV}^2$

dashed line: appr. evolution

dash-dotted line: Vinnikov

evolution important only for gluons
(and sea quarks)
other GPDs only used for
$2\,\mathrm{GeV}^2 \lesssim Q^2 \lesssim 5\,\mathrm{GeV}^2$
full evolution (Vinnikov code) appr.
equal to evolution of PDFs
(see also Diehl-Kugler (07))
for sea quarks worse

Summary

- Double-distribution representation of GPDs leads to interesting parametrizations of zero-skewness GPDs. The physical interpretation requires a complicated profile function at large $-t$. Strong $x-t$ correlation
- double distr. ansatz is to be probed against DVMP and DVCS data it turns out that it 'flexible' enough to account for all small ξ data; constraints from positivity, PDFs, form factors used
- future improvements: use of new PDFs, more complicated profile fcts. for all GPDs, D-term, kinematical corrections at low Q^2, low W, large ξ and new data from COMPASS, JLAB12 and EIC?