Lattice constraints on the thermal photon rate[1]

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Motivation

(i) Method testing: is it possible to do without MEM in asymptotically free theories?

• "**fancy approach**": Cuniberti method,[2] supplemented by UV from pQCD for removing short-distance singularities: theoretically solid but in practice somewhat fragile, because truncation introduces a parameter.[3]

• "**down-to-earth approach**": Fit a general function in the IR to UV from pQCD: more robust but only a small number of basis functions can be used, so a "good" basis is needed.

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[2] G. Cuniberti, E. De Micheli and G.A. Viano, *Reconstructing the thermal Green functions at real times from those at imaginary times*, cond-mat/0109175.

[3] Perhaps more robust after the reformulation in F. Ferrari, *The Analytic Renormalization Group*, 1602.07355?

Experience so far (continuum data with ~ 1% errors)

	fancy	down-to-earth
mock data[4]	(+)	
light quark diffusion[5]	(+)	
heavy quark diffusion[6]	$-$	+
thermal photon rate[7]	?	+ ← this talk

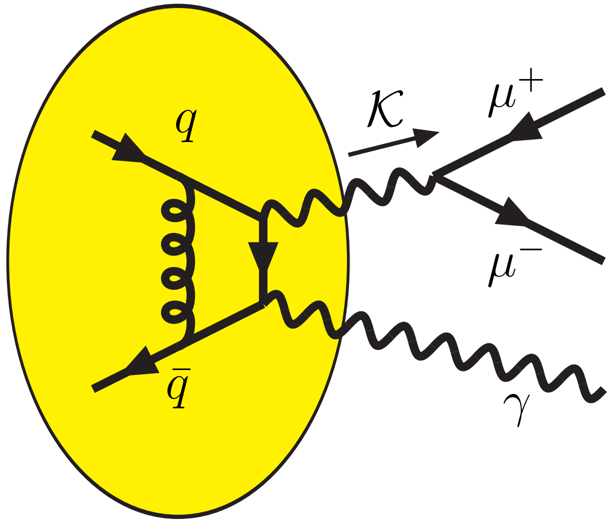
4 Y. Burnier, ML, L. Mether, *A Test on analytic continuation of thermal imaginary-time data*, 1101.5534.

5 Y. Burnier, ML, *Towards flavour diffusion coefficient and electrical conductivity without ultraviolet contamination*, 1201.1994.

6 A. Francis et al, *Non-perturbative estimate of the heavy quark momentum diffusion coefficient*, 1508.04543.

7 J. Ghiglieri, O. Kaczmarek, ML, F. Meyer, *Lattice constraints on the thermal photon rate*, 1604.07544.

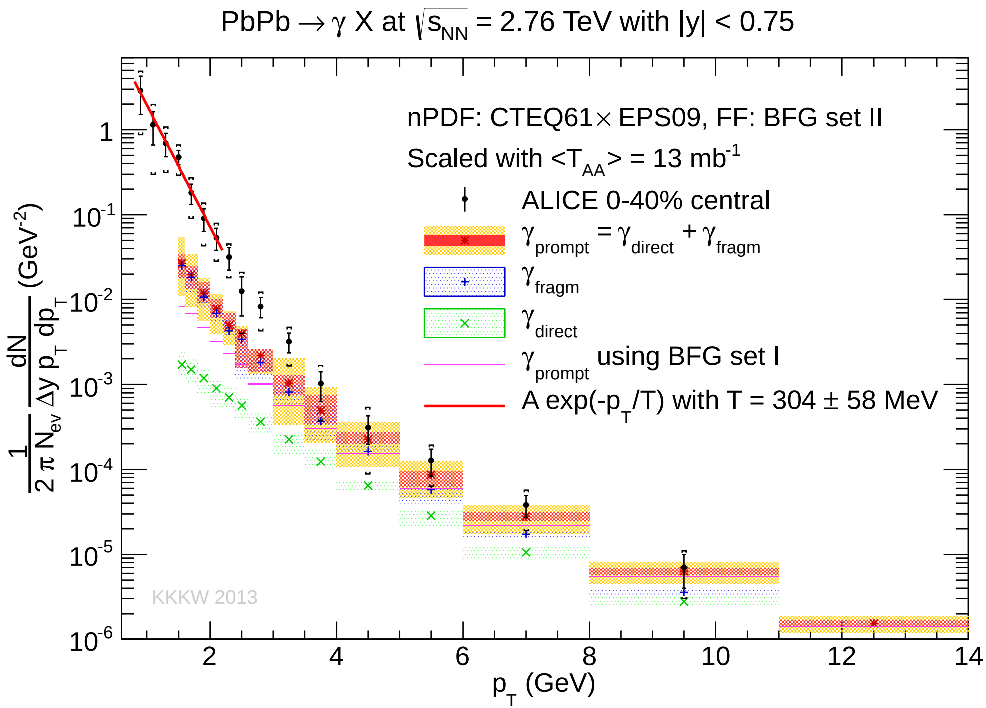
(ii) Physics



Consider thermally produced photons at momenta $k \equiv |\mathbf{k}| \sim$ 1 GeV, as well as $\mu^+\mu^-$ or e^+e^- pairs with an invariant mass

$$M^2 \equiv \mathcal{K}^2 \equiv \omega^2 - k^2 \; .$$

ALICE/LHC photon data for photons:[8]



8 M. Klasen, C. Klein-Bösing, F. König and J.P. Wessels, *How robust is a thermal photon interpretation of the ALICE low-p_T data?*, 1307.7034.

Formalism

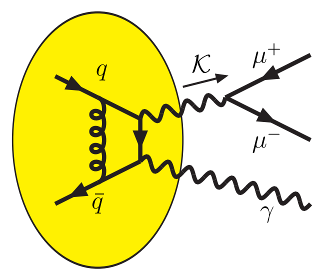
Photon production rate

Through a linear response analysis in α_{em}, a general expression can be derived for the photon production rate (per volume in the thermodynamic limit):

$$\frac{\mathrm{d}\Gamma_\gamma(\mathbf{k})}{\mathrm{d}^3\mathbf{k}} = \frac{2\alpha_{\mathrm{em}}\chi_{\mathrm{q}}}{3\pi^2} n_{\mathrm{B}}(k) D_{\mathrm{eff}}(k) + \mathcal{O}(\alpha_{\mathrm{em}}^2) \, .$$

Here $n_{\mathrm{B}}(k) \equiv 1/(e^{\beta k} - 1)$ is the Bose distribution, and $\chi_{\mathrm{q}} \sim T^2$ is a quark-number susceptibility which is easy to measure with lattice QCD / compute with pQCD.

The relation applies to all orders in α_{s}.



The strong interactions are hidden in $D_{\text{eff}}(k)$

The "effective diffusion coefficient" is defined as

$$D_{\text{eff}}(k) \equiv \begin{cases} \dfrac{\rho_{\rm V}(k, \mathbf{k})}{2\chi_{\rm q} k} & , \quad k > 0 \\ \lim\limits_{\omega \to 0^+} \dfrac{\rho_{\rm V}(\omega, \mathbf{0})}{3\chi_{\rm q}\omega} & , \quad k = 0 \end{cases} .$$

Hydrodynamics shows that $\lim_{k\to 0} D_{\text{eff}}(k) = D$ (cf. below).

Vector spectral function:

$$\rho_{\rm V}(\omega, \mathbf{k}) \equiv \int_{\mathcal{X}} e^{i(\omega t - \mathbf{k}\cdot\mathbf{x})} \langle \frac{1}{2} [V^\mu(t, \mathbf{x}) \, , \, V_\mu(0)] \rangle_{\rm c} \, ,$$

$$V^\mu \equiv \bar\psi \gamma^\mu \psi \, .$$

General structure of ρ_{V}

(i) Perturbation theory ($\equiv$ pQCD)

Leading order (LO) at $M \neq 0$:[9]

$$\rho_{\rm V}(\omega,\mathbf{k}) \;=\; \frac{N_{\rm c} T M^2}{2\pi k}\Big\{\ln\Big[\frac{\cosh(\frac{\omega+k}{4T})}{\cosh(\frac{\omega-k}{4T})}\Big] - \frac{\omega\,\theta(k-\omega)}{2T}\Big\}\;.$$

Leading-log order (LL) at $M = 0$:[10]

$$\rho_{\rm V}(k,\mathbf{k}) \;=\; \frac{\alpha_{\rm s} N_{\rm c} C_{\rm F} T^2}{4}\ln\Big(\frac{1}{\alpha_{\rm s}}\Big)\big[1-2n_{\rm F}(k)\big] + \mathcal{O}(\alpha_{\rm s}T^2)\;.$$

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[9] e.g. G. Aarts and J.M. Martínez Resco, *Continuum and lattice meson spectral functions at nonzero momentum and high temperature*, hep-lat/0507004.

[10] J.I. Kapusta, P. Lichard and D. Seibert, *High-energy photons from quark-gluon plasma versus hot hadronic gas*, PRD 44 (1991) 2774; R. Baier, H. Nakkagawa, A. Niégawa and K. Redlich, *Production rate of hard thermal photons and screening of quark mass singularity*, ZPC 53 (1992) 433.

Current status

LO at $M = 0.$[11] (only numerical result)

NLO at $M = 0.$[12] (only numerical result)

NLO at $M \sim gT.$[13] (only numerical result)

NLO at $M \sim \pi T.$[14] (only numerical result)

N^4LO at $M \gg \pi T.$[15] (analytic result)

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[11] P.B. Arnold, G.D. Moore and L.G. Yaffe, *Photon emission from ultrarelativistic plasmas*, hep-ph/0109064; *Photon emission from quark gluon plasma: Complete leading order results*, hep-ph/0111107.

[12] J. Ghiglieri *et al*, *Next-to-leading order thermal photon production in a weakly coupled quark-gluon plasma*, 1302.5970.

[13] J. Ghiglieri and G.D. Moore, *Low Mass Thermal Dilepton Production at NLO in a Weakly Coupled Quark-Gluon Plasma*, 1410.4203.

[14] ML, *NLO thermal dilepton rate at non-zero momentum*, 1310.0164.

[15] S. Caron-Huot, *Asymptotics of thermal spectral functions*, 0903.3958; P.A. Baikov, K.G. Chetyrkin and J.H. Kühn, *Order* α_s^4 *QCD Corrections to* Z *and* τ *Decays*, 0801.1821.

(ii) Hydrodynamic regime

For $k \lesssim \alpha_s^2 T$ the general theory of statistical fluctuations applies,[16] and permits for a "hydrodynamic" prediction:[17]

$$\frac{\rho_V(\omega,\mathbf{k})}{\omega} = \Big(\frac{\omega^2 - k^2}{\omega^2 + D^2 k^4} + 2\Big)\chi_{\mathsf{q}} D \; .$$

Here $D \equiv \lim_{k\to 0} D_{\text{eff}}(k)$ is the diffusion coefficient, and χ_{q} is the quark number susceptibility, parametrizing the constant correlator $\langle V^0(\tau,\mathbf{0}) V^0(0,\mathbf{0})\rangle = \chi_{\mathsf{q}} T$.

Note: ρ_V can be negative for $\omega < k$.

16 Cf. e.g. E.M. Lifshitz and L.P. Pitaevskii, *Statistical Physics, Part 2*, §88-89.

17 Cf. e.g. J. Hong and D. Teaney, *Spectral densities for hot QCD plasmas in a leading log approximation*, 1003.0699.

(iii) AdS/CFT[18]

Nice for qualitative insight.

In the IR this reproduces the hydrodynamic prediction, with the specific values $D = 1/(2\pi T)$ and $\chi_{\rm q} = N_{\rm c}^2 T^2/8$.

One can also try to judge when hydrodynamics applies: the spectral function is close to hydrodynamics for $k \lesssim 0.5/D$, and becomes negative at the smallest ω for $k \lesssim 1.07/D$.

AdS/CFT makes a prediction beyond the hydrodynamic regime as well, however this needs not have any relation to QCD.

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[18] G. Policastro, D.T. Son and A.O. Starinets, *From AdS / CFT correspondence to hydrodynamics*, hep-th/0205052; S. Caron-Huot *et al*, *Photon and dilepton production in supersymmetric Yang-Mills plasma*, hep-th/0607237.

(iv) Hadronic models, χPT, ...

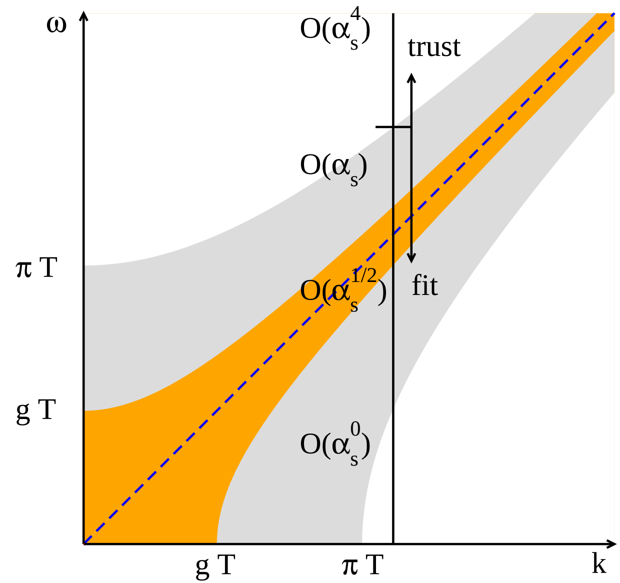
The hadronic side of low temperatures is also of substantial interest, because of the so-called photon v_2 puzzle.

Non-perturbative approach

What can we do with lattice?

$$G_{\rm V}(\tau,\mathbf{k}) = \int_0^\infty \frac{{\rm d}\omega}{\pi}\,\rho_{\rm V}(\omega,\mathbf{k})\,\frac{\cosh[\omega(\frac{\beta}{2}-\tau)]}{\sinh[\frac{\omega\beta}{2}]}\;, \quad \beta \equiv \frac{1}{T}\;.$$

Down-to-earth idea ($g \equiv \sqrt{4\pi\alpha_{\rm s}}$):



Polynomial interpolation (assuming analyticity, $V \to \infty$)

We pick a point above which pQCD should apply, for instance

$$\omega_0 \simeq \sqrt{k^2 + (\pi T)^2}\ ,$$

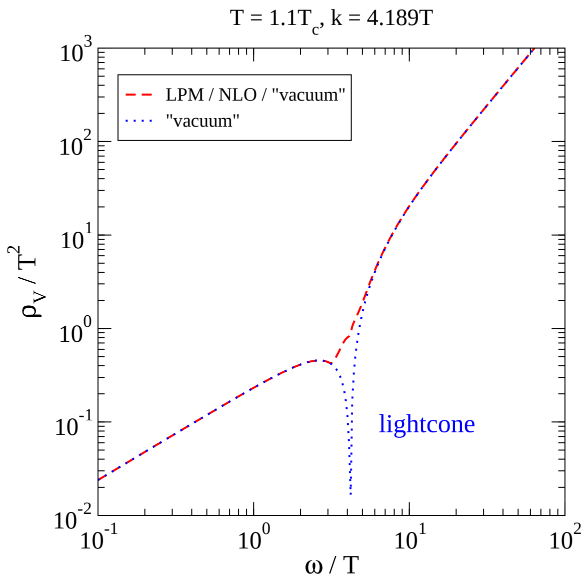
and use that to fix two coefficients:

$$\rho_{\rm V}(\omega_0, \mathbf{k}) \ \equiv \ \beta\ , \quad \partial_\omega \rho_{\rm V}(\omega_0, \mathbf{k}) \ \equiv \ \gamma\ .$$

Then the most general polynomial odd in ω takes the form

$$\rho_{\text{fit}} \equiv \frac{\beta\, \omega^3}{2\omega_0^3}\Big(5 - \frac{3\omega^2}{\omega_0^2}\Big) - \frac{\gamma\, \omega^3}{2\omega_0^2}\Big(1 - \frac{\omega^2}{\omega_0^2}\Big) + \sum_{n\geq 0}^{n_{\text{max}}} \frac{\delta_n \omega^{1+2n}}{\omega_0^{1+2n}}\Big(1 - \frac{\omega^2}{\omega_0^2}\Big)^2\ .$$

How does the pQCD result look like?[19,20]



[19] $3T < \omega < 10T$ from J. Ghiglieri and G.D. Moore, *Low Mass Thermal Dilepton Production at NLO in a Weakly Coupled Quark-Gluon Plasma*, 1410.4203 ; $\omega \gtrsim 10T$ from I. Ghisoiu and ML, *Interpolation of hard and soft dilepton rates*, 1407.7955 ; $\omega \gg 10T$ from ML, *NLO thermal dilepton rate at non-zero momentum*, 1310.0164.

[20] J. Ghiglieri and ML, web page *Data for the thermal vector channel spectral function*, `http://www.laine.itp.unibe.ch/dilepton-lattice/`

Lattice details

Imaginary-time observable:

$$G_{\mathrm{V}}(\tau, \mathbf{k}) \equiv \int_{\mathbf{x}} e^{-i\mathbf{k}\cdot\mathbf{x}} \langle V^i(\tau, \mathbf{x})\, V^i(0) - V^0(\tau, \mathbf{x})\, V^0(0) \rangle_{\mathrm{c}} \, .$$

Consider the full G_{V} rather than G^{ii} because this is relevant for dileptons and because much more is known within pQCD.

Momenta are chosen along the lattice axes. With periodic boundary conditions this requires

$$k = 2\pi n T \times \frac{N_\tau}{N_{\mathrm{s}}} \, ,$$

where N_{s} is the number of lattice sites in spatial directions and N_τ that in the time direction. Normally $N_\tau / N_{\mathrm{s}} \lesssim 1/4$ for removing finite-size effects.

Ensemble

β_0	$N_s^3 \times N_\tau$	confs	$T/T_c\|_{t_0}$	k/T
7.192	$96^3 \times 32$	314	1.12	2.094,4.189,6.283
7.544	$144^3 \times 48$	358	1.14	
7.793	$192^3 \times 64$	242	1.15	
7.192	$96^3 \times 28$	232	1.28	1.833,3.665,5.498
7.544	$144^3 \times 42$	417	1.31	
7.793	$192^3 \times 56$	273	1.31	

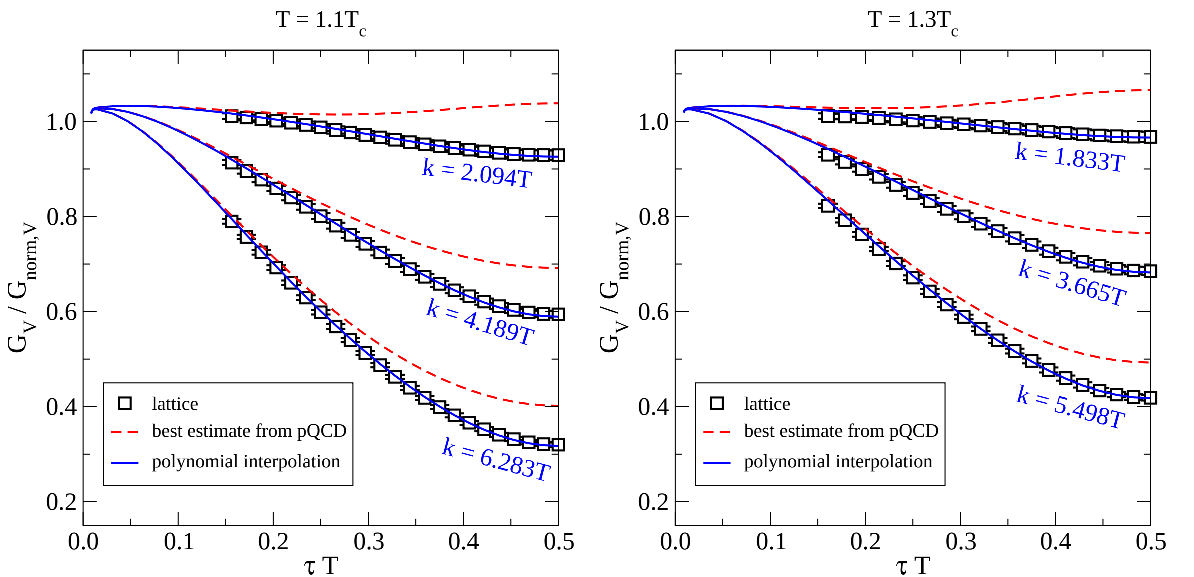
In a separate study finite-V effects have been seen to be small.

With such large β_0 we are frozen to the trivial topological sector,[21] but do not expect this to affect the results dramatically.

[21] S. Schaefer *et al.* [ALPHA Collaboration], *Critical slowing down and error analysis in lattice QCD simulations*, 1009.5228.

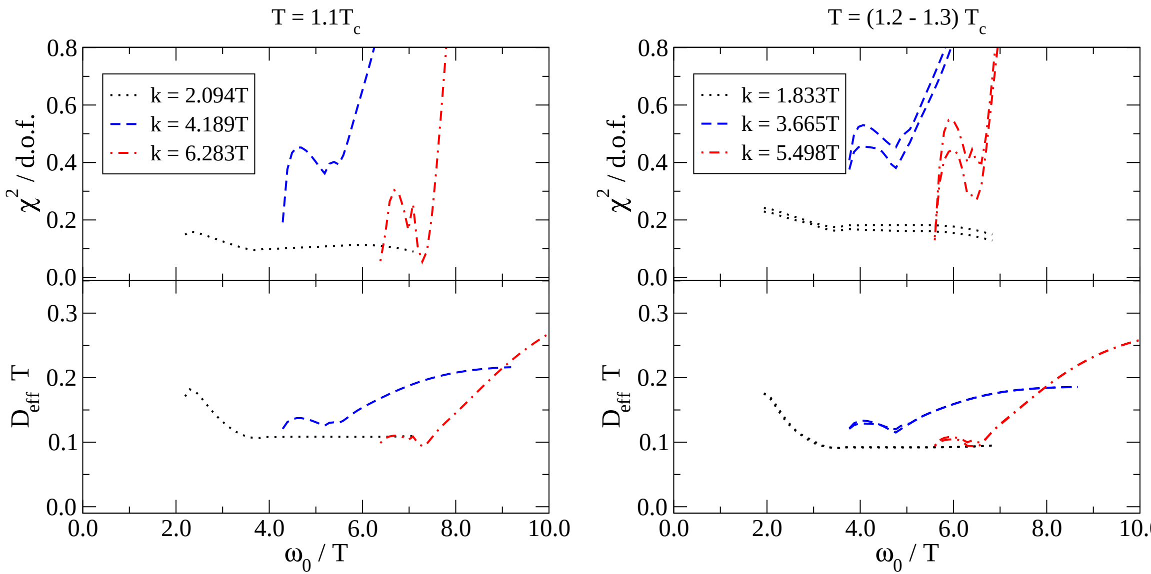
Results

Imaginary-time correlators after continuum extrapolation



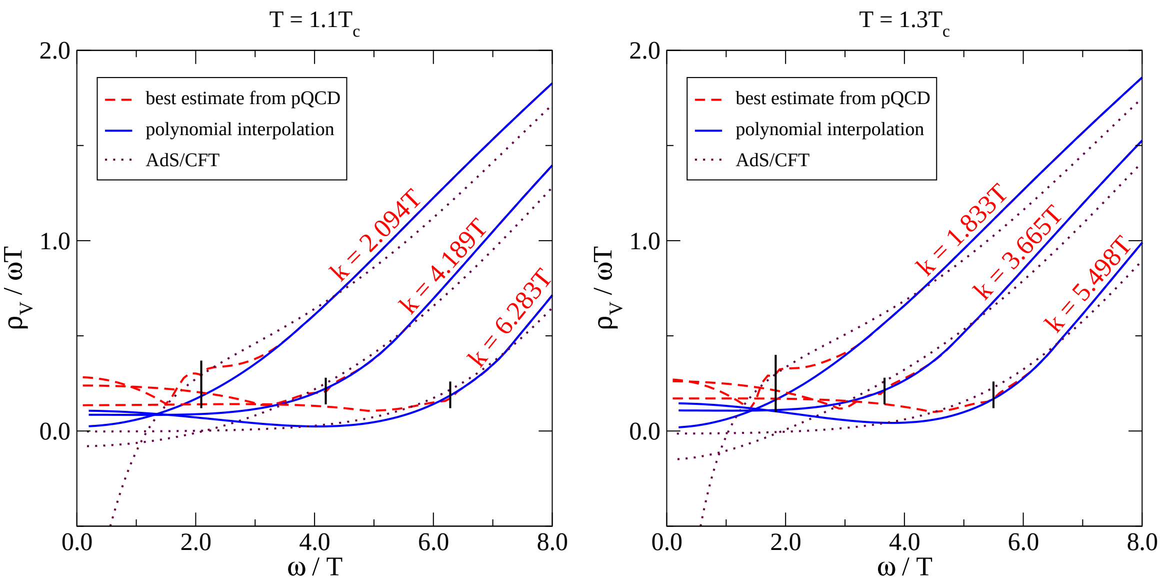
$$\frac{G_{\text{norm},V}}{6T^3} \equiv \pi(1-2\tau T)\frac{1+\cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + \frac{2\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \,.$$

One-parameter fits (δ_0) as a function of ω_0



Final results are from two-parameter fits (δ_0, δ_1) to a full bootstrap ensemble for the continuum-extrapolated correlator.

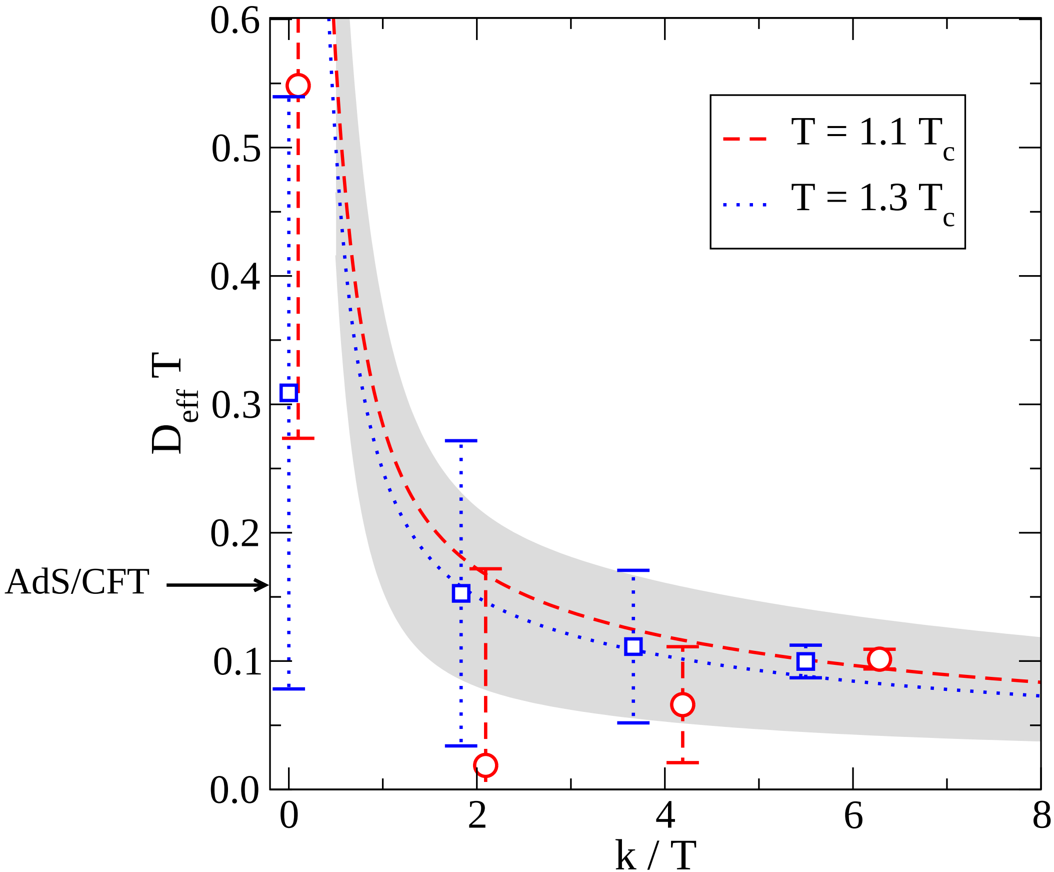
One-parameter best fits for the spectral function



There is a clear reduction in the spacelike domain.

(Positivity has **not** been imposed.)

Value at the photon point



What did we learn?

(i) Methods

If had:

(a) continuum-extrapolated lattice data

(b) bootstrap ensemble for error estimation

(c) high-order pQCD predictions for UV

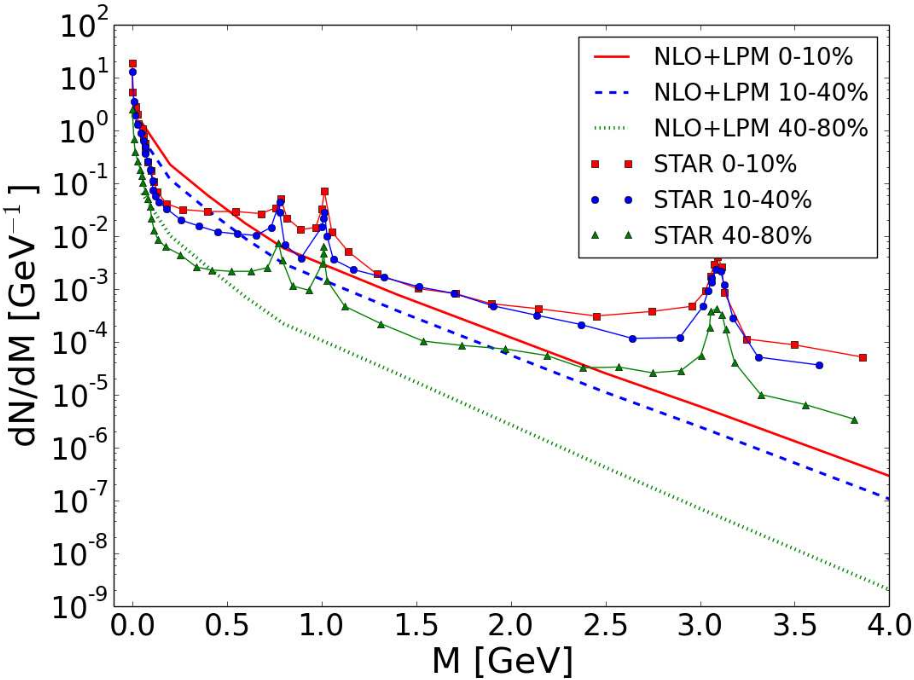
(d) a well-motivated functional basis for IR

(e) a somewhat increased statistical precision

then results might be brought under reasonable control.

But the fancy approach should also be tested!

(ii) Physics: reduction could agree with phenomenology!?[22]



[22] Y. Burnier and C. Gastaldi, *Contribution of next-to-leading order and Landau-Pomeranchuk-Migdal corrections to thermal dilepton emission in heavy-ion collisions*, 1508.06978.

Summary

Large distances $k < 2T$ $\rightarrow$ strong interactions $\rightarrow$ less thermodynamic fluctuations $\rightarrow$ less currents $\rightarrow$ less photons.

The onset of the hydrodynamic regime can be empirically monitored through the k-dependence of $D_{\text{eff}}(k)$.

In principle the results can be implemented in hydrodynamic codes and compared with experimental data for the photon rate.