Diagonal and transitional matrix elements from low-energy Coulomb excitation and the link to deformation parameters

Magda Zielińska
IRFU/SPhN, CEA Saclay, France

- What do we measure?
- How we extract matrix elements from the data?
- How they can be related to deformation parameters?

Why do we like Coulomb excitation?

- studies no longer limited to stable or long-lived nuclei
- beam energies at exotic beam facilities perfect for Coulomb excitation (2-5 MeV/A)
- high cross sections (excitation of 2_1^+: barns)
- practical at the neutron-rich side

- direct measurement of quadrupole moment including sign – ideal tool to study shape coexistence
- B(E2) as a measure of collectivity - studies around magic numbers
- easy way to access non-yrast states and study their properties

What we can get from a Coulex experiment?

- observation of new excited levels, selective population of collective states
  - first excited state in ^{80}Zn (J. Van de Walle et al, PRL 99 (2007) 142501)
  - rotational band in ^{97}Rb (C. Sotty, G. Georgiev, to be published)
- B(E2) and B(M1) values between low-lying states, as well as B(E1)'s, B(E3)'s; in rare cases B(E4)
- relative signs of matrix elements
- for complex level schemes up to 50 ME's!
- signs and magnitudes of static E2 moments of excited states

Basic facts about Coulex

- Due to the purely electromagnetic interaction the nucleus undergoes a transition from state $|i\rangle$ to $|f\rangle$.

- Then it decays to the lower state, emitting a γ-ray (or a conversion electron).

- The matrix elements $\langle f||M(E2)||i\rangle$ describe the excitation and decay pattern $\rightarrow$ they are directly connected with γ-ray intensities observed in the experiment.

- In the intrinsic frame of the nucleus they are related to the deformation parameters.

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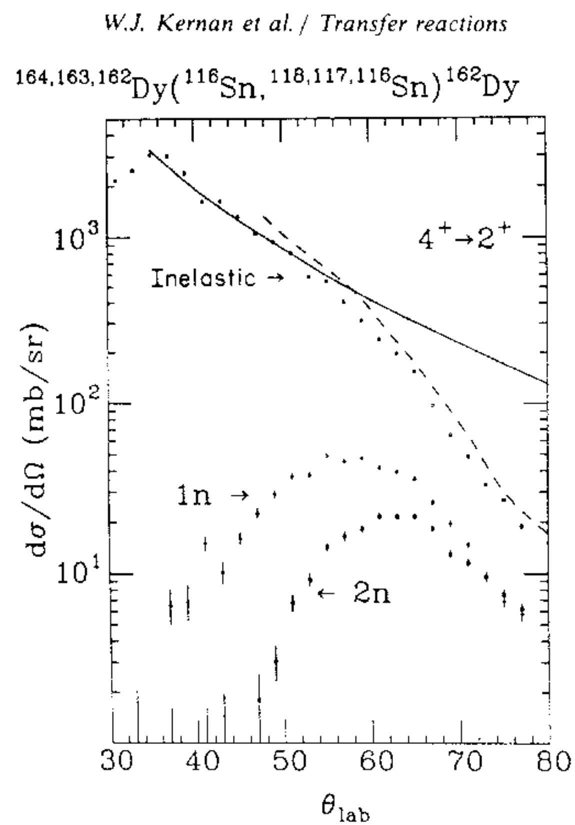
Safe energy

- Cline's "safe energy" criterion: purely electromagnetic interaction if the distance between nuclear surfaces is greater than 5 fm

$$d_{min} = 1.25 \cdot (A_p^{1/3} + A_t^{1/3}) + 5.0 \quad [\mathrm{fm}]$$

- empirical criterion based on systematic studies of inelastic and transfer cross-sections at beam energies of few MeV/A



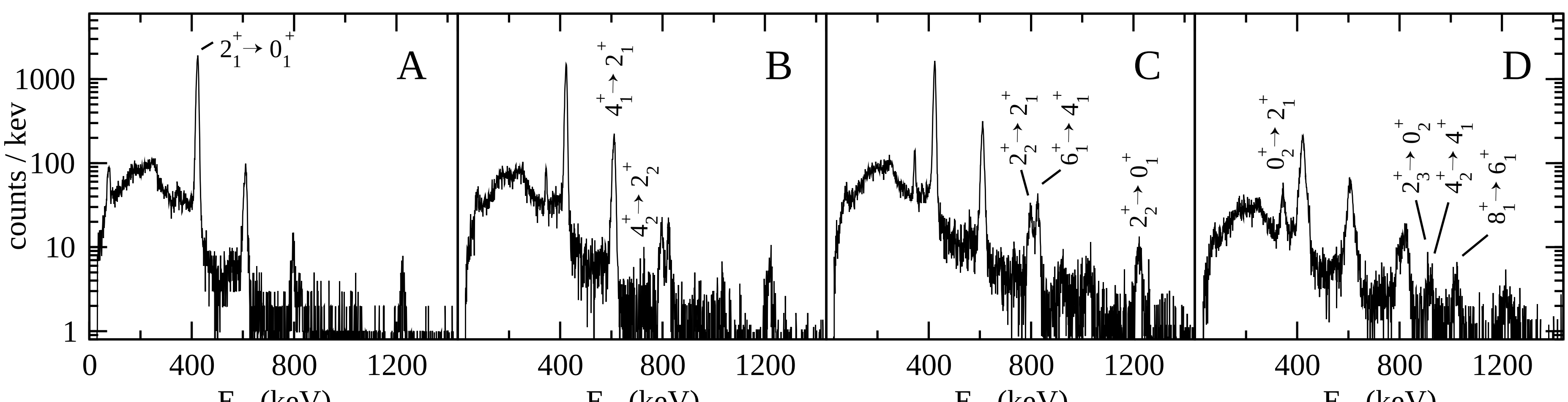
- other criteria established for high-energy Coulex

- one-neutron sub-barrier transfer recently observed in Coulex of ^{42}Ca on ^{208}Pb

Experiment step by step

- velocity vectors of reaction partners (from scattering angle and energy or TOF measured by particle detectors)
  - selection of Coulomb excitation events (high beam energy, exotic beam experiments, experiments with oxide targets...)
  - identification target-projectile
  - description of the excitation process (dependence on θ)
  - Doppler correction of gamma rays
  - possibility to study particle-gamma correlations

- γ-ray intensities following Coulex as a function of CM scattering angle



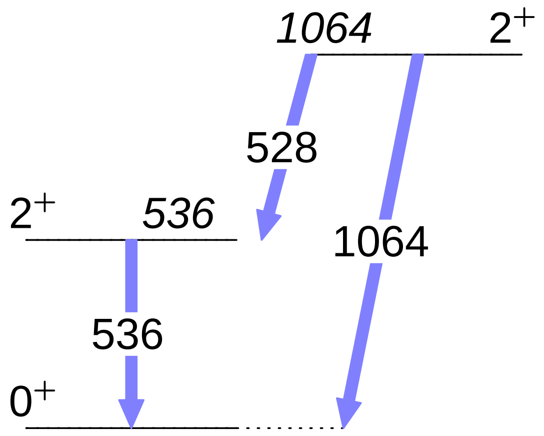
Once we have gamma-ray intensities...

...to convert them to cross section normalisation is needed

- known B(E2) in the studied nucleus
- known B(E2) in the reaction partner
- Rutherford cross section (technically diffucult so less accurate)

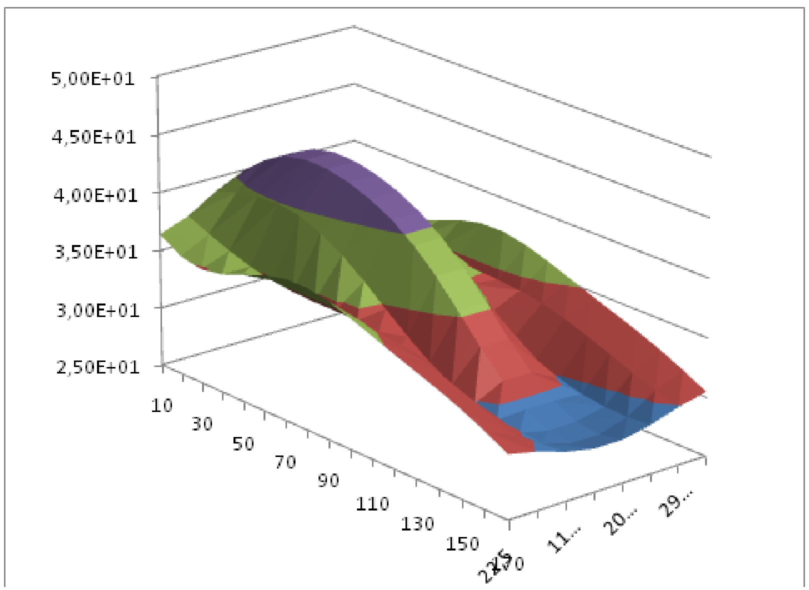
Final step: extraction of individual electromagnetic matrix elements from measured gamma-ray intensities

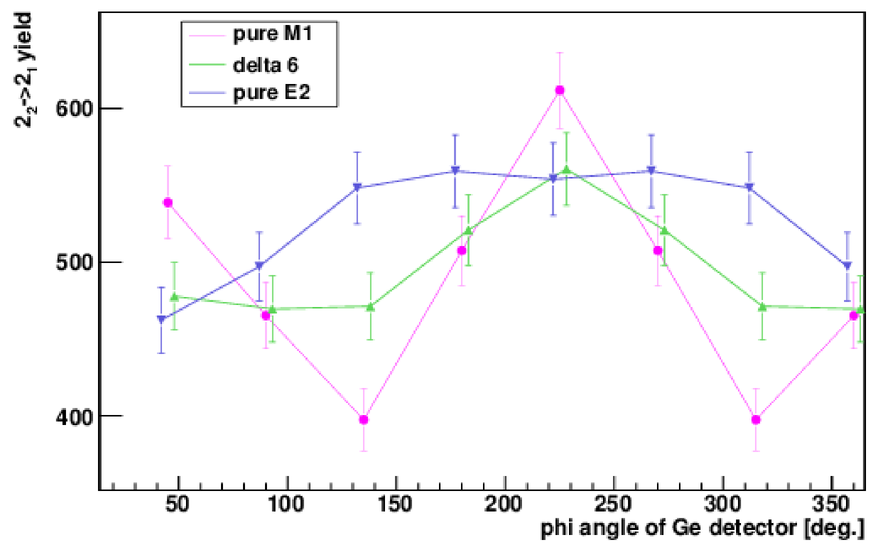
- simple cases (rare) : first/second order perturbation theory
- most cases too complicated: multiple Coulomb excitation
- excited states populated indirectly via intermediate states
- excitation probability of a given state may depend on many matrix elements
- set of coupled equations for excitation amplitudes – solved numerically: dedicated analysis codes



Gamma-particle angular correlations

- feasible at several thousands of counts in a given gamma line
- determination of E2/M1 mixing ratios
- determination of spin of a decaying level
- distribution in phi usually more conclusive than in theta

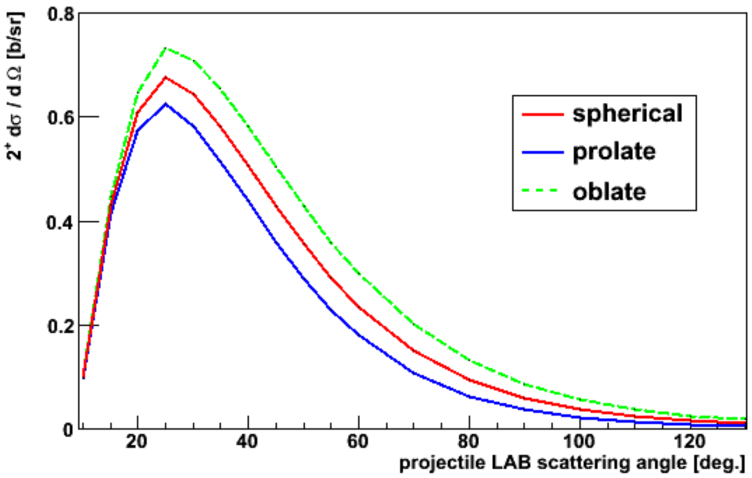


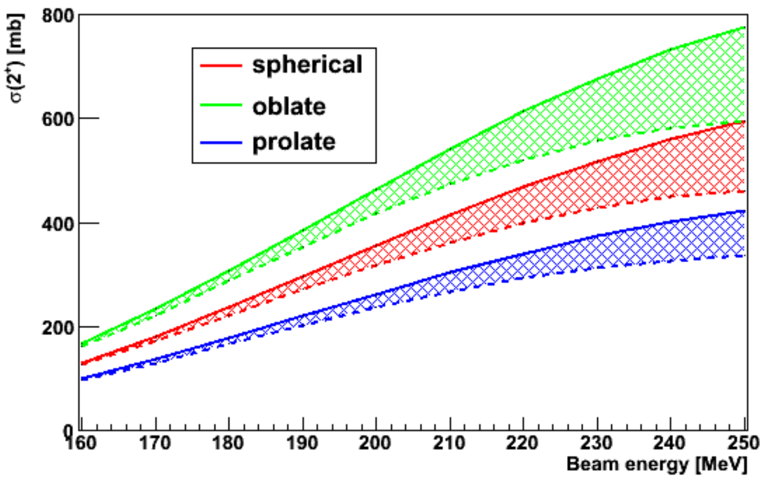


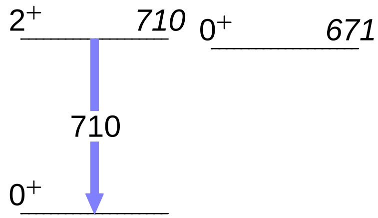
- the distributions are attenuated due to deorienation (recoil in vacuum) – possibility to measure g factors

Reorientation effect

- influence of the quadrupole moment of the excited state on its excitation cross-section: double excitation where "intermediate" states are the m substates of the state of interest
- dependence on scattering angle and beam energy
- however, influence of double-step excitation of other states may have the same effect (depending on $\frac{\Delta E}{E}$)







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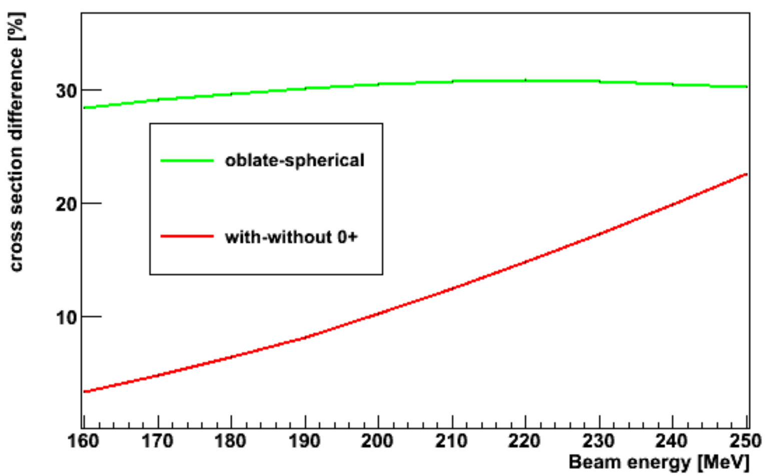
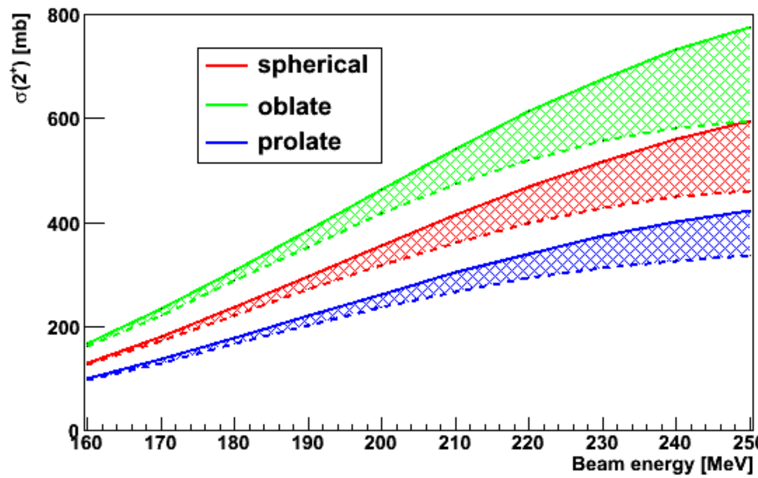
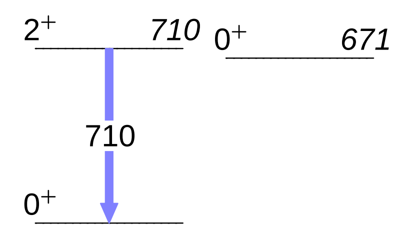
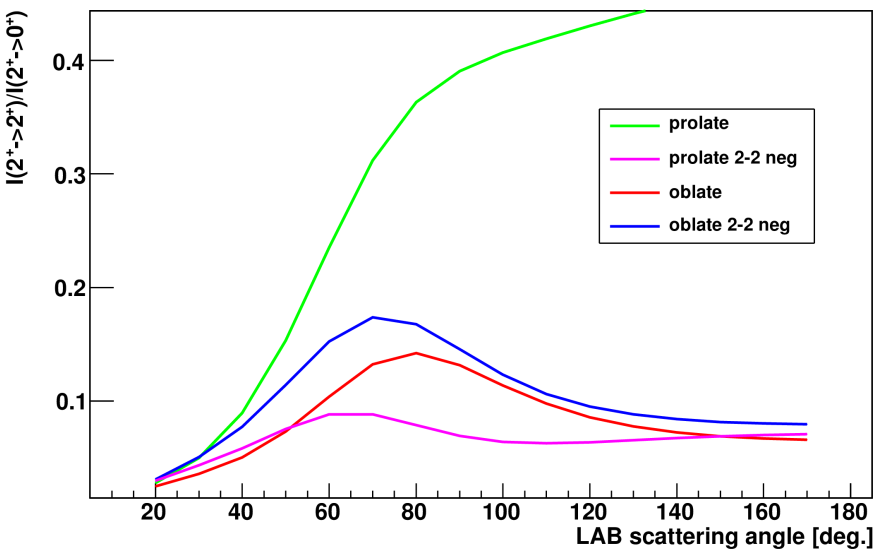


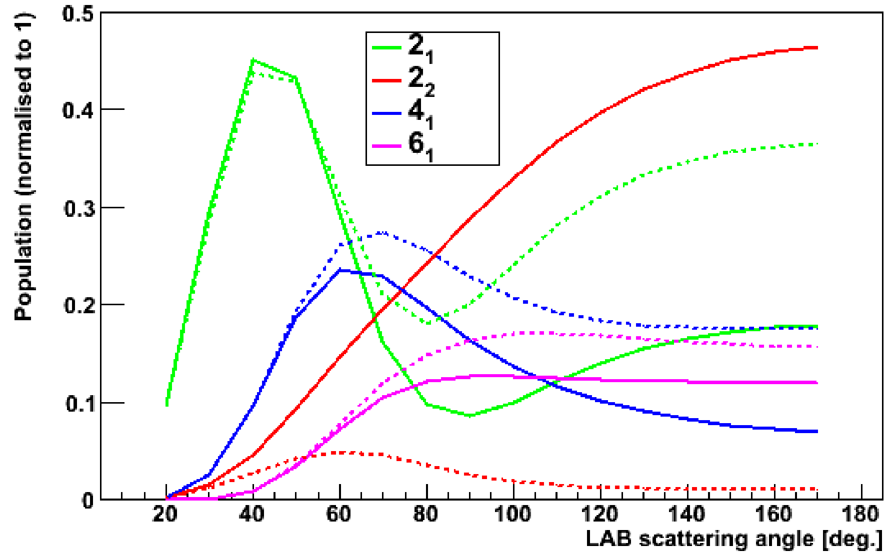




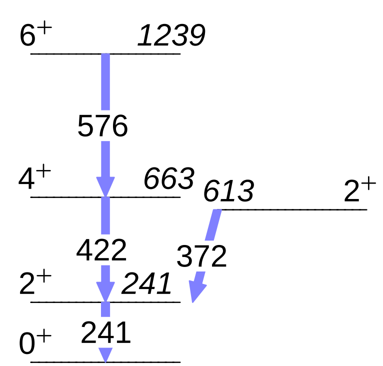
Multi-step excitation and relative signs

• sensitivity of Coulomb excitation data to relative signs of ME's:
result of interference between single-step and multi-step amplitudes
• sign of a product of matrix elements is an observable





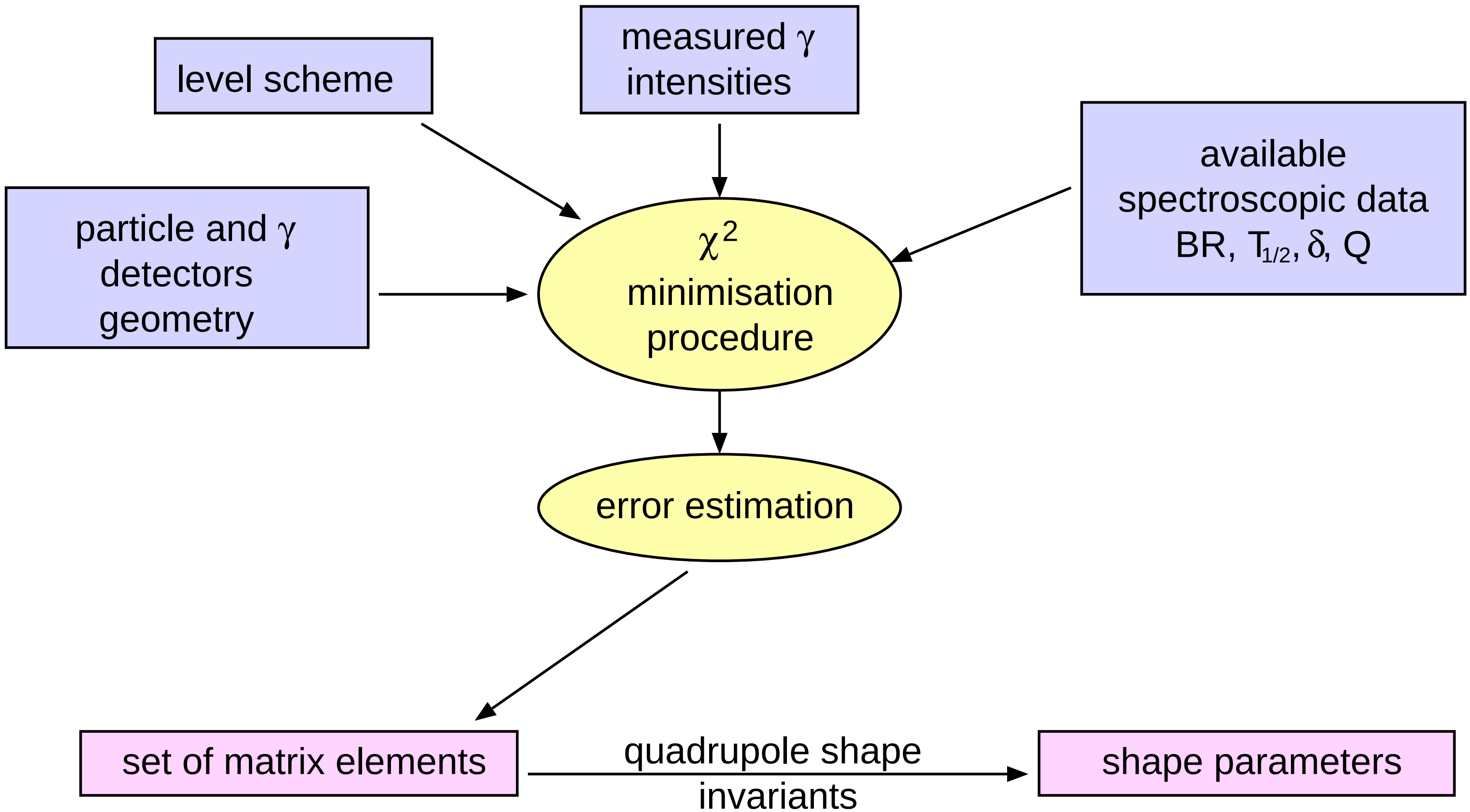
• negative $\langle 2_1^+ \| E2 \| 2_2^+ \rangle$ and $\langle 2_1^+ \| E2 \| 2_1^+ \rangle$: much higher population of 2_2^+ at high CM angles
• calculations for ^{110}Ru on ^{208}Pb



GOSIA code

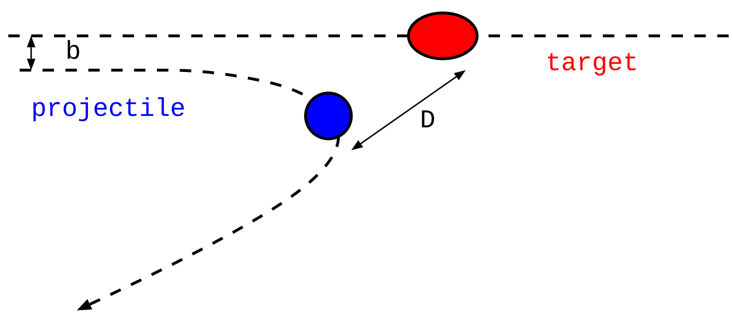
GOSIA: Rochester - Warsaw semiclassical Coulomb excitation least-squares search code

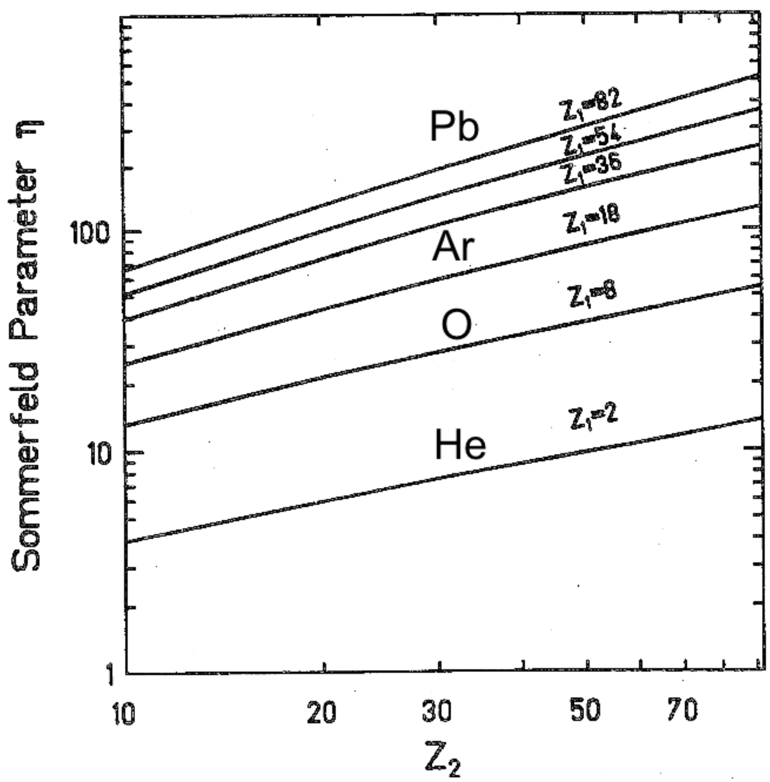
Developed in early eighties by T. Czosnyka, D. Cline, C.Y. Wu (Bull. Am. Phys. Soc. 28 (1983) 745.) and continuously upgraded



Approximations used in GOSIA

1. semi-classical approximation
   - trajectories can be described by the classical equations of motion, excitation process is described using quantum mechanics.





$\lambda_{\text{projectile}} \ll \mathrm{D}$
$\Rightarrow$ Sommerfeld parameter η

$$\eta = \frac{\mathrm{D}}{2\bar{\lambda}} = \frac{\mathrm{Z_p Z_t e^2}}{\hbar \mathrm{v}} \gg 1$$

- condition well fulfilled in heavy-ion induced Coulomb excitation

- semiclassical treatment is expected to deviate from the exact calculation by terms of the order $\sim 1/\eta$

Approximations used in GOSIA

1. semi-classical approximation
   - symmetrisation of the trajectory to take into account the energy transfer
2. limitation to the monopole-multipole term

The excitation process can be described by the time-dependent H:

$$H = H_P + H_T + V(r(t))$$

with $H_{P/T}$ being the free Hamiltonian of the projectile/target nucleus and V(t) being the time-dependent electromagnetic interaction

If the wave function is expressed by eigenfunctions of the free $H_{P/T}$:
$\psi(t) = \sum_n a_n(t)\phi_n$ one gets a set of coupled equations for the time-dependent excitation amplitudes a_n(t)

$$i\hbar\frac{da_n(t)}{dt} = \sum_m \langle\phi_n|V(t)|\phi_m\rangle \exp(i(E_n - E_m)/\hbar)a_m(t)$$

$V(r(t)) = Z_T Z_P e^2/r$ monopole-monopole (Rutherford) term
$+\sum_{\lambda\mu} V_P(E\lambda,\mu) + \sum_{\lambda\mu} V_T(E\lambda,\mu)$ electric multipole-monopole excitation,
$+\sum_{\lambda\mu} V_P(E\lambda,\mu) + \sum_{\lambda\mu} V_T(E\lambda,\mu)$ magnetic excitation (small at low v/c)
+ higher order multipole-multipole terms (neglected – estimated at $\sim$ 0.5 %)

Approximations used in GOSIA

1. semi-classical approximation
   - symmetrisation of the trajectory to take into account the energy transfer
2. limitation to the monopole-multipole term
   other effects taken into account in the description of the excitation process:

- correction for the dipole polarisation effect: quadrupole interaction V(E2) multiplied by a factor

$$1 - d \cdot \frac{E_p A_t}{Z_t^2 (1 + A_p/A_t)} \frac{a}{r}$$

  where d = 0.005 (empirical E1 polarisation strength, from photo-nuclear absorption cross section or GDR energy + dipole sum rule)

  Alder and Winther, Coulomb excitation, appendix J

  important for high-lying levels, high CM angles, heavy beams: ^{104}Ru - 10% change of population of 10^+_γ if effect increased 2 times

- integration over scattering angles covered by particle detectors and incident energy (beam stopping in the target) - changing meshpoints may give an effect of few %, especially for multi-step excitation

Effects taken into account when describing decay

- start from statistical tensors calculated in the excitation stage
  - information on excitation probability and initial sub-state population
- cascade feeding from higher-lying states
- deorientation of the angular distribution (due to recoil in vacuum): Brenn and Spehl two-state model: ^{104}Ru - 2% change of matrix elements if effect increased by 20%
- relativistic transformation of solid angles
- attenuation due to finite size of gamma-ray detectors
- simplified (cylindrical) detector geometry

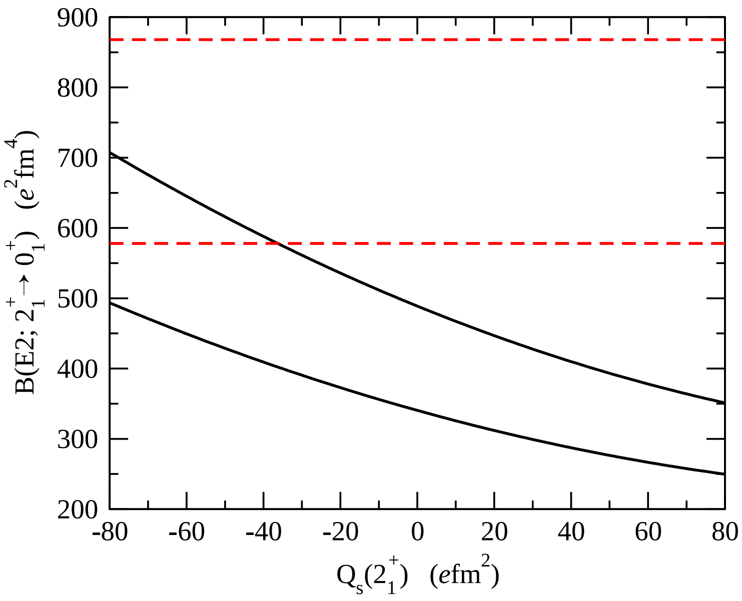
- all approximations have usually an effect $\sim 5\%$ on gamma-ray intensities (often similar to statistical uncertainties, increasing with number of steps needed)
- uncertainties lower than this are rather suspicious (unless they reflect the precision of a lifetime measurement, but the quality of such measurement should also be verified)

Number of parameters versus number of data points

- number of matrix elements coupling low-lying states is higher than number of transitions observed in a Coulex experiment
- some of them have much smaller influence on gamma-ray intensities than the others
- even if dependence of cross-sections on scattering angle can be used, often problem remains underdetermined
  - especially if E1, E3 matrix elements are declared, or for odd nuclei – M1
- additional spectroscopic data needed
  - these data are not used to fix some parameters, but enter the χ^2 function on the equal basis as gamma-ray intensities
- in rare very undetermined cases theoretical relations between the ME's may be used (which couplings are negligible, similar, etc...)

Additional measurements needed for Coulex data analysis...

- lifetime measurements
  - necessary for integral cross-section measurements



A.M. Hurst *et al.*,

Phys. Rev. Lett. 98, 072501 (2007)

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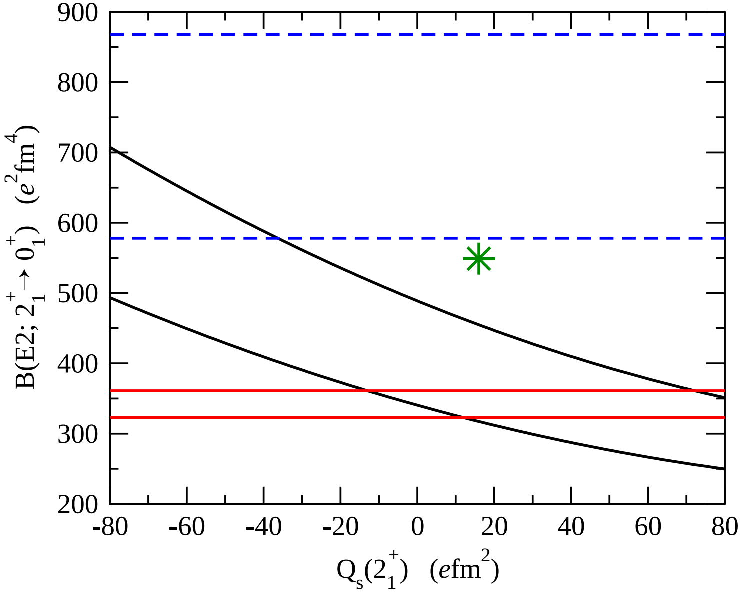

J. Ljungvall *et al.*,
Phys. Rev. Lett. 100, 102502 (2008)

  - increase precision of quadrupole moments/intra-band matrix elements for differential measurements

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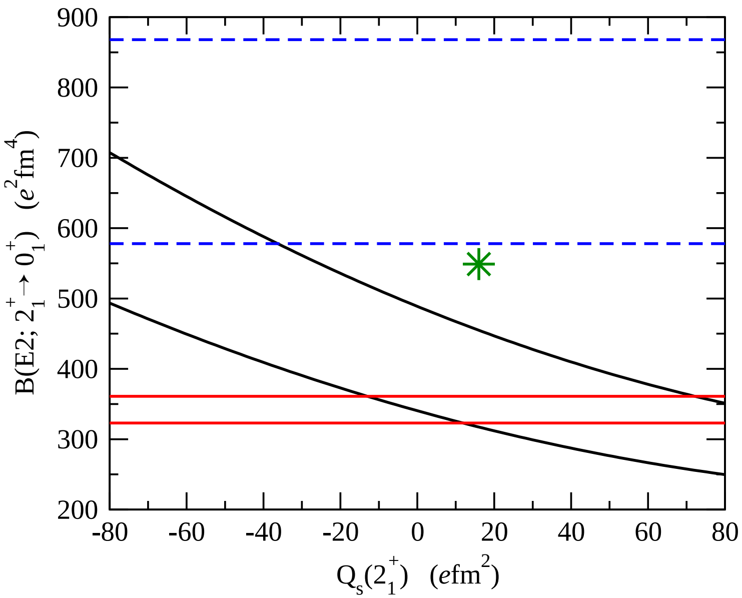
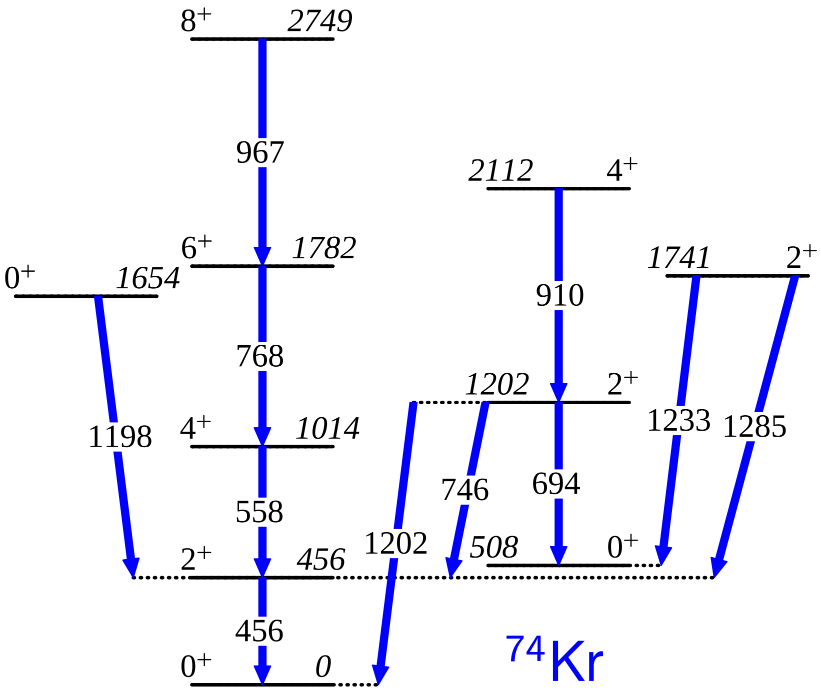


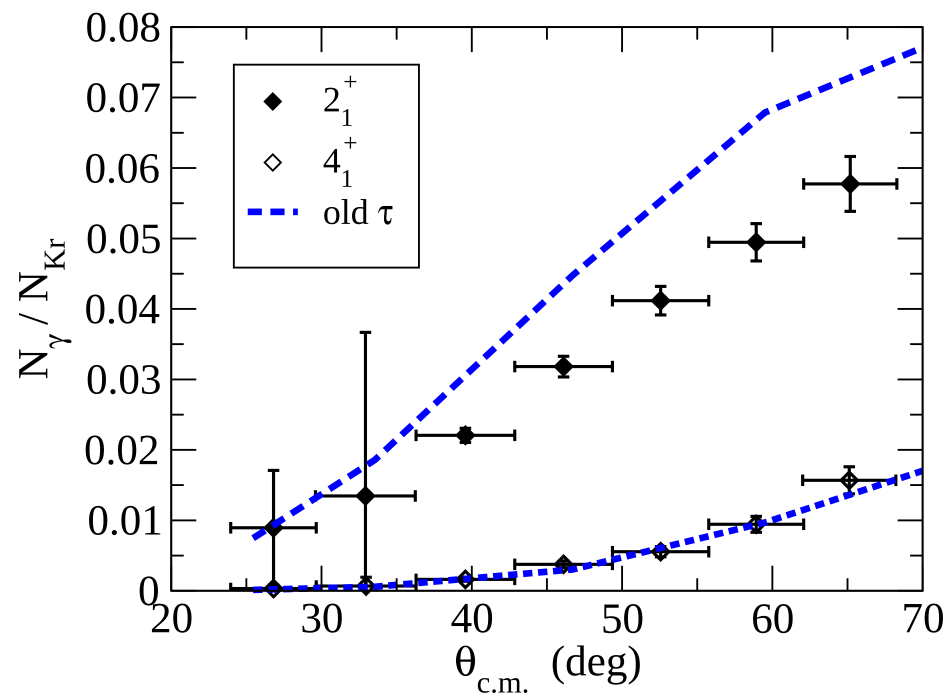
J. Ljungvall *et al.*,
Phys. Rev. Lett. 100, 102502 (2008)

  - increase precision of quadrupole moments/intra-band matrix elements for differential measurements
- beam composition (isobaric contamination/isomeric ratio)
- beam energy
- conversion coefficients/E0 branchings

Coulomb excitation and lifetime measurements



- subdivision of data in several ranges of scattering angle
- spectroscopic data (lifetimes, branching and mixing ratios)
- least squares fit of ~30 matrix elements (transitional and diagonal)



- results inconsistent with previously published lifetimes
- new RDM lifetime measurement: Köln Plunger & GASP
  ^{40}Ca (^{40}Ca,α2p) ^{74}Kr
  ^{40}Ca (^{40}Ca,4p) ^{76}Kr

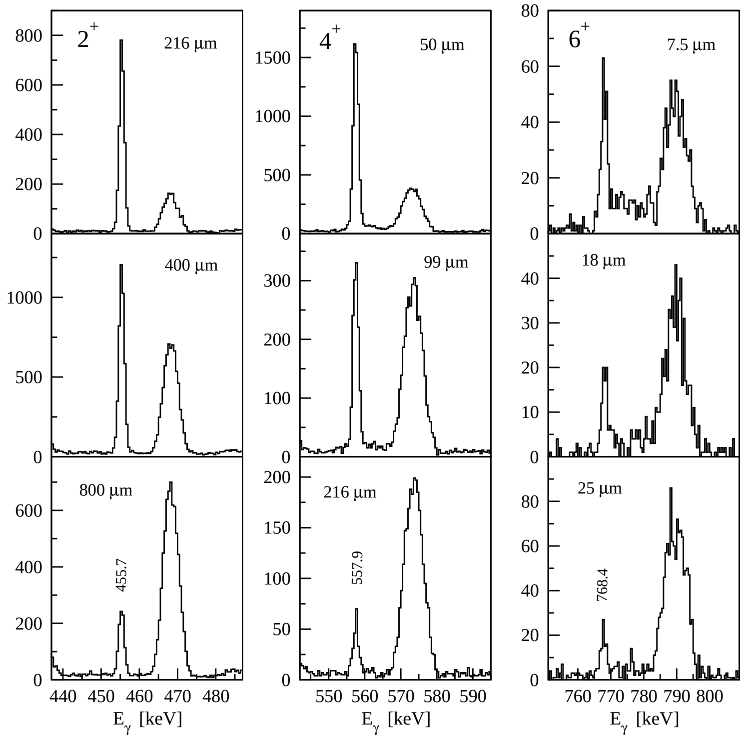
Lifetime measurement

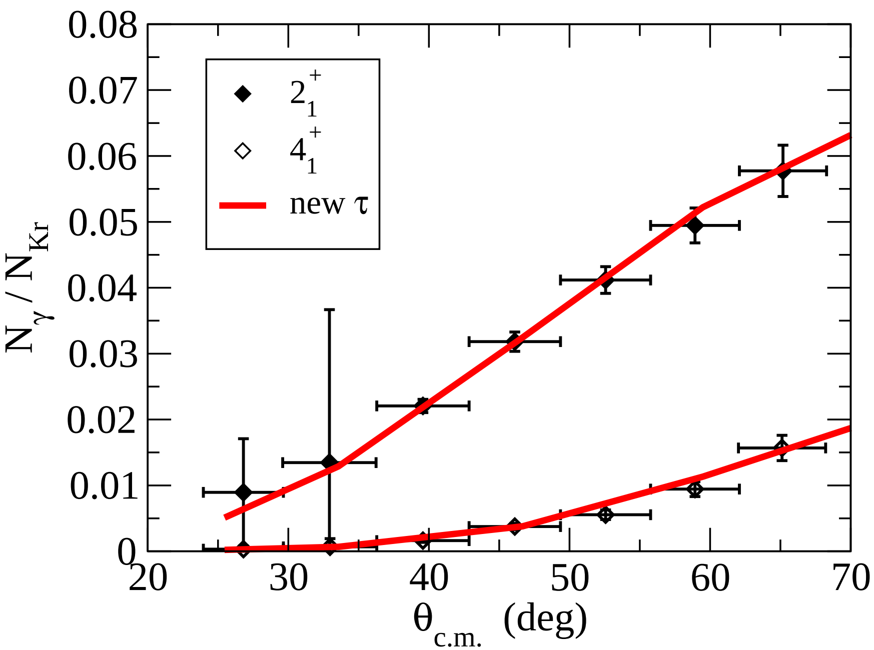
A. Görgen *et al.* EPJ A 26 153 (2005)

		old	new
^{76}Kr	2^+	35.3(10) ps	41.5(8) ps
	4^+	4.8(5) ps	3.87(9) ps

		old	new
^{74}Kr	2^+	28.8(57) ps	33.8(6) ps
	4^+	13.2(7) ps	5.2(2) ps

^{74}Kr, forward detectors (36°)
gated from above





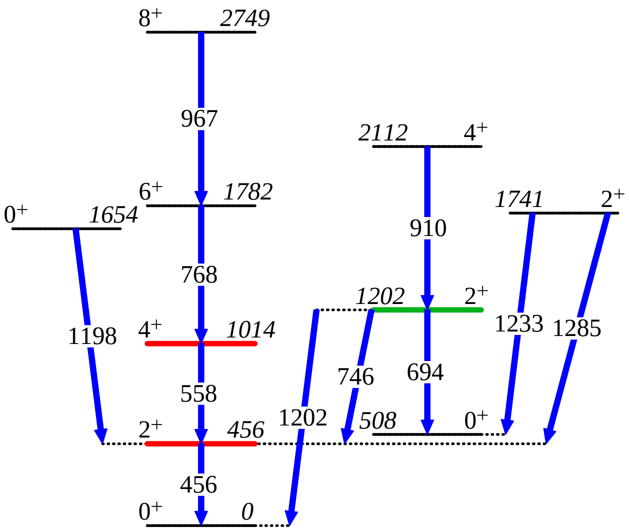
- new lifetimes in agreement with Coulex
- enhanced sensitivity for diagonal and intra-band transitional matrix elements

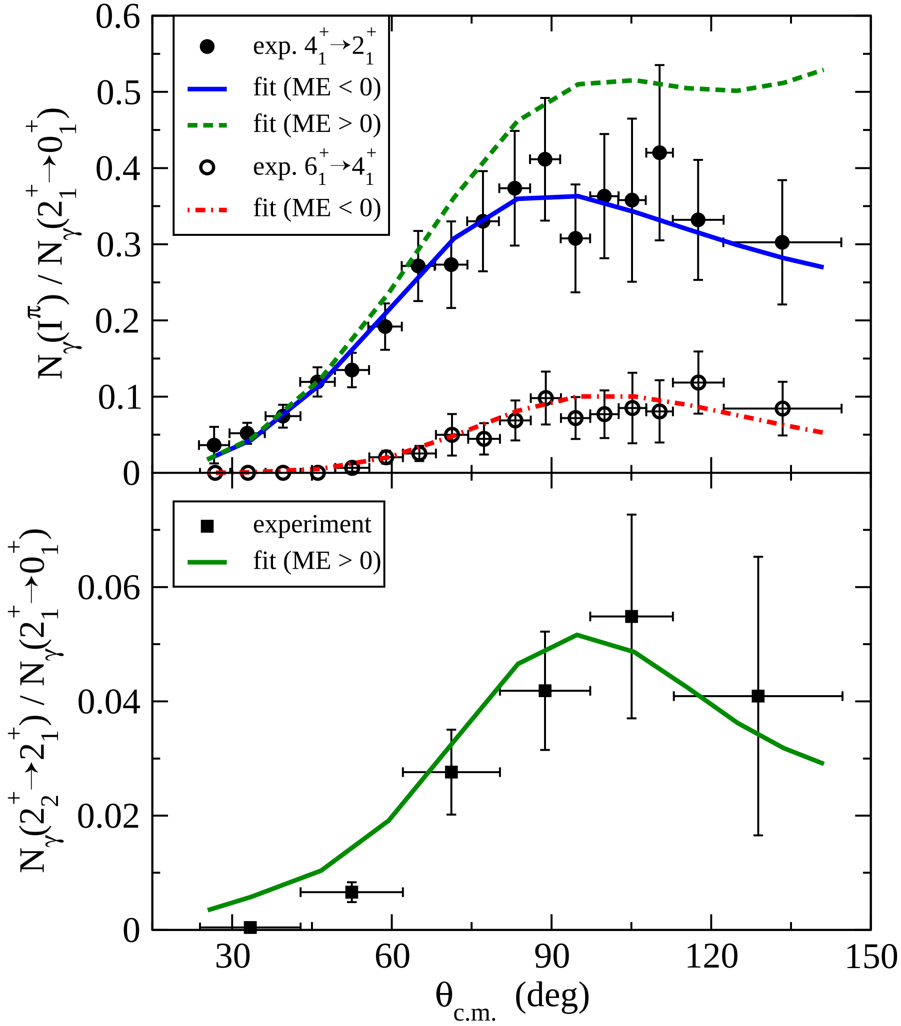
Results: shape coexistence in light Kr isotopes

^{76}Kr: 18 transitional + 5 diagonal ME
^{74}Kr: 14 transitional + 5 diagonal ME

$$\langle 2_1^+ \| E2 \| 2_1^+ \rangle = -\ 0.70^{-0.33}_{-0.30}$$
$$\langle 4_1^+ \| E2 \| 4_1^+ \rangle = -\ 1.02^{+0.59}_{-0.21}$$

$$\langle 2_2^+ \| E2 \| 2_2^+ \rangle = +\ 0.33^{+0.28}_{-0.23}$$





First measurement of diagonal E2 matrix elements using Coulex of radioactive beam

E. Clément *et al.* Phys. Rev. C75, 054313 (2007)

Global vs local minimum

Standard question: is this a unique solution, or maybe a different combination of matrix elements can reproduce the experimental data?

Genetic Algorithm in GOSIA: JACOB (P. Napiorkowski, HIL Warsaw)

GOSIA:

- often trapped in a local minimum
- various starting points have to be carefully checked (combinations of signs and magnitudes)
- only for very simple cases ”plug and play”

JACOB:

- scan of the χ^2 surface, ”promising” minima localised
- integration procdure repeated for each of them, real solutions identified
- alternative method for error estimation (in development)

Quadrupole sum rules

D. Cline, Ann. Rev. Nucl. Part. Sci. 36 (1986) 683
K. Kumar, PRL 28 (1972) 249

- number of matrix elements obtained from a Coulomb excitation analysis can reach 20-50 (+ for some of them signs are determined)
- quadrupole collectivity produces strong correlations of E2 matrix elements: number of significant collective variables is much lower than the number of matrix elements
- direct comparison of each ME's from experiment and theory is not always conclusive.
- quadrupole invariants provide a syntetic information that can be compared with model predictions.
- electromagnetic multipole operators are spherical tensors $\rightarrow$ products of such operators coupled to angular momentum 0 are rotationally invariant
- in the intrinsic frame of the nucleus, the E2 operator may be expressed by 2 parameters related to charge distribution:

$$\begin{aligned} E(2,0) &= Q\cos\delta \\ E(2,2) = E(2,-2) &= \frac{Q}{\sqrt{2}}\sin\delta \\ E(2,1) = E(2,-1) &= 0 \end{aligned}$$

Quadrupole sum rules

D. Cline, Ann. Rev. Nucl. Part. Sci. 36 (1986) 683
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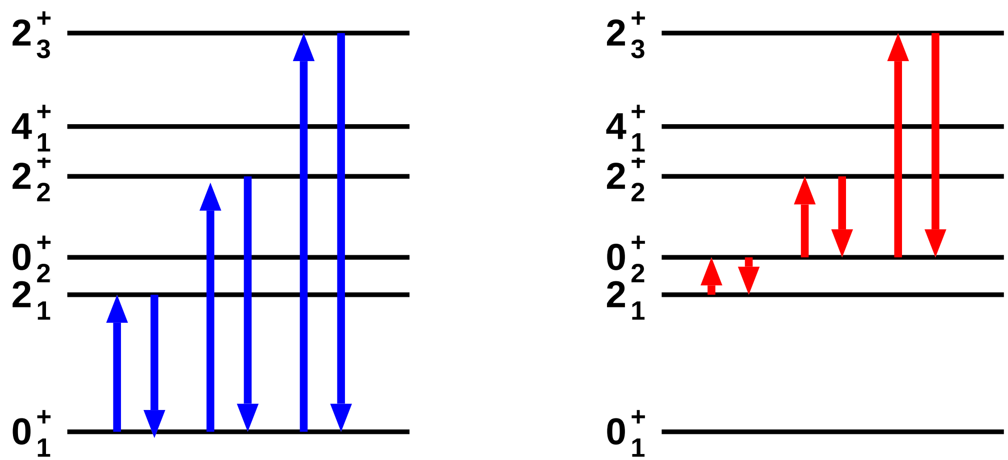
- operator products may be expressed by matrix elements using the intermediate state expansion formula

$$\frac{\langle Q^2\rangle}{\sqrt{5}} = \langle i|\,[\mathrm{E2}\times\mathrm{E2}]^0\,|i\rangle = \frac{1}{\sqrt{(2I_i+1)}}\sum_t \langle i\|\mathrm{E2}\|t\rangle\langle t\|\mathrm{E2}\|i\rangle \left\{ \begin{array}{ccc} 2 & 2 & 0 \\ I_i & I_i & I_t \end{array} \right\}$$

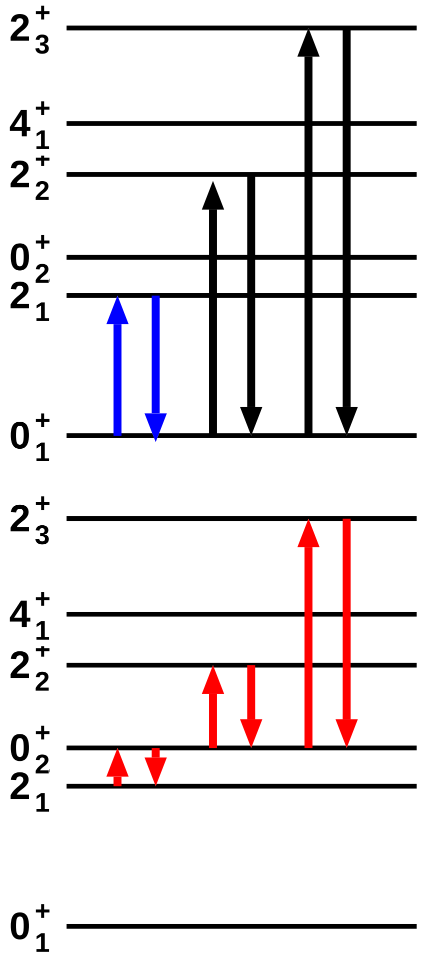


$\langle Q^2\rangle$: overall deformation parameter

Determination to $\langle Q^2 \rangle$: example of ^{100}Mo

K. Wrzosek-Lipska et al, PRC 86 (2012) 064305

state	loop	contribution to $\langle Q^2 \rangle$ [e2b2]
0_1^+	$\langle 0_1^+ \Vert E2 \Vert 2_1^+ \rangle \langle 2_1^+ \Vert E2 \Vert 0_1^+ \rangle$	0.46
	$\langle 0_1^+ \Vert E2 \Vert 2_2^+ \rangle \langle 2_2^+ \Vert E2 \Vert 0_1^+ \rangle$	0.01
	$\langle 0_1^+ \Vert E2 \Vert 2_3^+ \rangle \langle 2_3^+ \Vert E2 \Vert 0_1^+ \rangle$	0.0002
	Total	0.48
0_2^+	$\langle 0_2^+ \Vert E2 \Vert 2_1^+ \rangle \langle 2_1^+ \Vert E2 \Vert 0_2^+ \rangle$	0.26
	$\langle 0_1^+ \Vert E2 \Vert 2_2^+ \rangle \langle 2_2^+ \Vert E2 \Vert 0_2^+ \rangle$	0.10
	$\langle 0_2^+ \Vert E2 \Vert 2_3^+ \rangle \langle 2_3^+ \Vert E2 \Vert 0_2^+ \rangle$	0.25
	Total	0.62



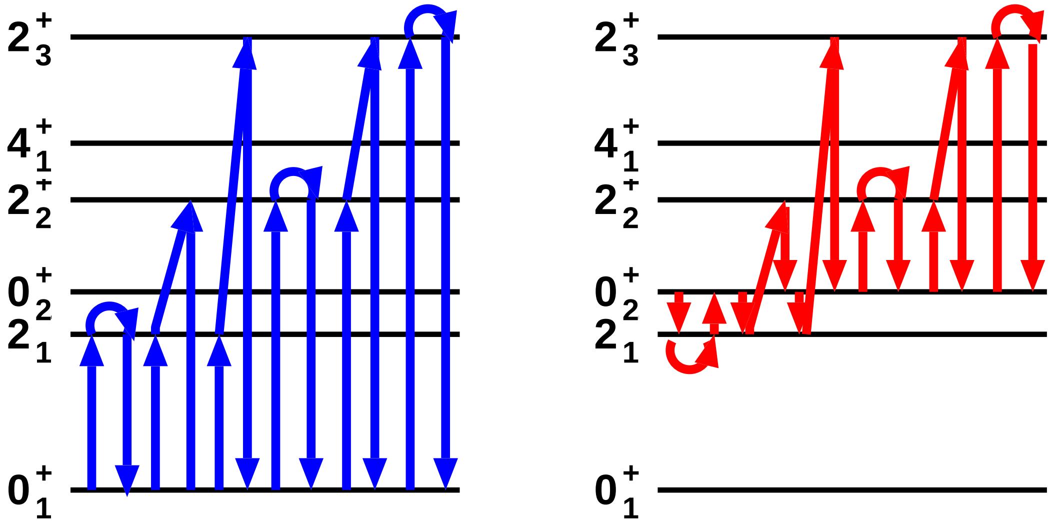
Quadrupole sum rules: triaxiality

D. Cline, Ann. Rev. Nucl. Part. Sci. 36 (1986)

K. Kumar, PRL 28 (1972)

$$\sqrt{\frac{2}{35}}\langle Q^3 \cos 3\delta\rangle = \langle i|\{[E2\times E2]^2 \times E2\}^0|i\rangle$$

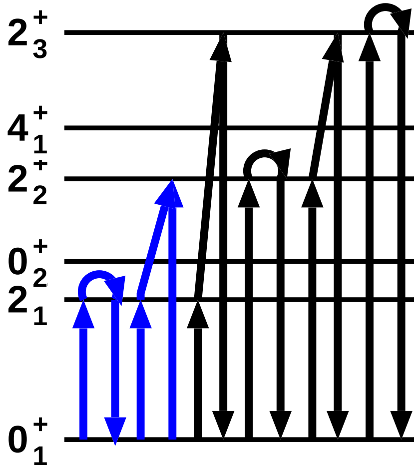
$$= \frac{1}{(2I_i+1)}\sum_{t,u}\langle i\|E2\|u\rangle\langle u\|E2\|t\rangle\langle t\|E2\|i\rangle \begin{Bmatrix} 2 & 2 & 2 \\ I_i & I_t & I_u \end{Bmatrix}$$

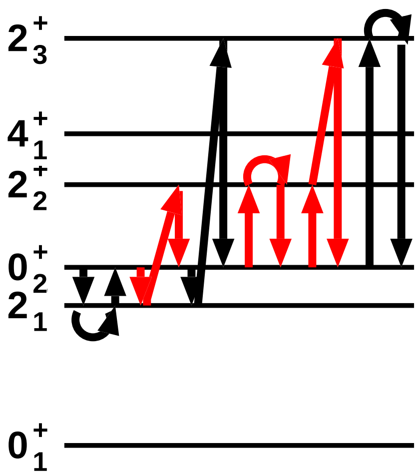


$\langle\cos 3\delta\rangle$: triaxiality parameter

Determination of $\langle \cos 3\delta \rangle$: example of ^{100}Mo

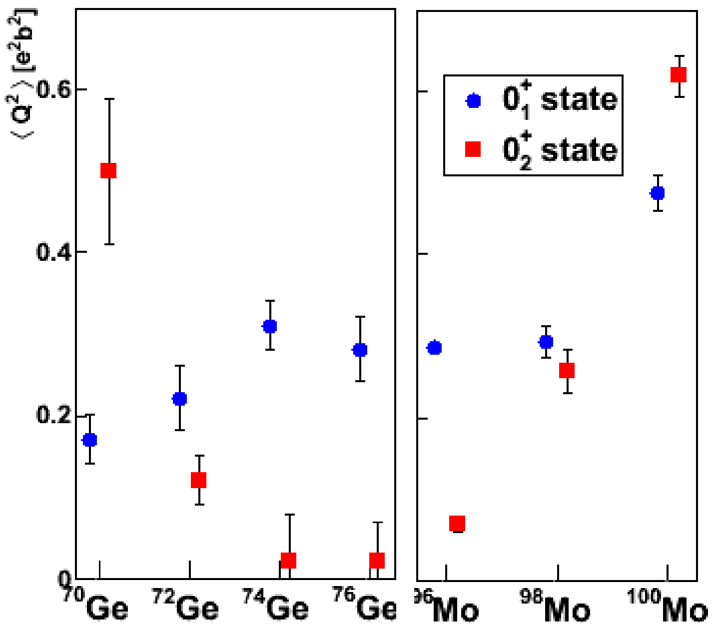
state	loop	contribution to $\langle Q^3 \cos 3\delta \rangle$
0^+_1	$\langle 0^+_1 \Vert E2 \Vert 2^+_1 \rangle \langle 2^+_1 \Vert E2 \Vert 2^+_1 \rangle \langle 2^+_1 \Vert E2 \Vert 0^+_1 \rangle$	-0.154
	$\langle 0^+_1 \Vert E2 \Vert 2^+_1 \rangle \langle 2^+_1 \Vert E2 \Vert 2^+_2 \rangle \langle 2^+_2 \Vert E2 \Vert 0^+_1 \rangle$	0.132
	$\langle 0^+_1 \Vert E2 \Vert 2^+_1 \rangle \langle 2^+_1 \Vert E2 \Vert 2^+_3 \rangle \langle 2^+_3 \Vert E2 \Vert 0^+_1 \rangle$	0.002
	$\langle 0^+_1 \Vert E2 \Vert 2^+_2 \rangle \langle 2^+_2 \Vert E2 \Vert 2^+_2 \rangle \langle 2^+_2 \Vert E2 \Vert 0^+_1 \rangle$	0.013
	$\langle 0^+_1 \Vert E2 \Vert 2^+_2 \rangle \langle 2^+_2 \Vert E2 \Vert 2^+_3 \rangle \langle 2^+_3 \Vert E2 \Vert 0^+_1 \rangle$	-0.001
	$\langle 0^+_1 \Vert E2 \Vert 2^+_3 \rangle \langle 2^+_3 \Vert E2 \Vert 2^+_3 \rangle \langle 2^+_3 \Vert E2 \Vert 0^+_1 \rangle$	-0.0001
	Total	-0.008
0^+_2	$\langle 0^+_2 \Vert E2 \Vert 2^+_1 \rangle \langle 2^+_1 \Vert E2 \Vert 2^+_1 \rangle \langle 2^+_1 \Vert E2 \Vert 0^+_2 \rangle$	-0.09
	$\langle 0^+_2 \Vert E2 \Vert 2^+_1 \rangle \langle 2^+_1 \Vert E2 \Vert 2^+_2 \rangle \langle 2^+_2 \Vert E2 \Vert 0^+_2 \rangle$	-0.31
	$\langle 0^+_2 \Vert E2 \Vert 2^+_1 \rangle \langle 2^+_1 \Vert E2 \Vert 2^+_3 \rangle \langle 2^+_3 \Vert E2 \Vert 0^+_2 \rangle$	-0.04
	$\langle 0^+_2 \Vert E2 \Vert 2^+_2 \rangle \langle 2^+_2 \Vert E2 \Vert 2^+_2 \rangle \langle 2^+_2 \Vert E2 \Vert 0^+_2 \rangle$	0.12
	$\langle 0^+_2 \Vert E2 \Vert 2^+_2 \rangle \langle 2^+_2 \Vert E2 \Vert 2^+_3 \rangle \langle 2^+_3 \Vert E2 \Vert 0^+_2 \rangle$	-0.13
	$\langle 0^+_2 \Vert E2 \Vert 2^+_3 \rangle \langle 2^+_3 \Vert E2 \Vert 2^+_3 \rangle \langle 2^+_3 \Vert E2 \Vert 0^+_2 \rangle$	-0.06
	Total	-0.51

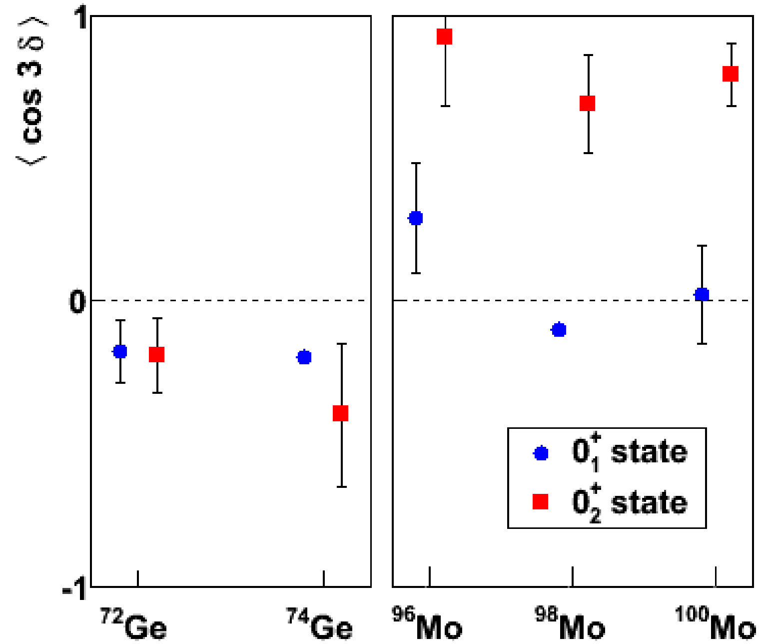




Shape evolution of $^{96-100}$Mo

M. Zielińska et al, Nucl. Phys. A 712 (2002) 3
K. Wrzosek-Lipska et al, PRC 86 (2012) 064305





- Ge isotopes, ^{96}Mo: coexistence of the deformed ground state with a spherical 0_2^+
- ground states of the Mo isotopes triaxial, deformation of 0_2^+ increasing with N
- shape coexistence in ^{98}Mo manifested in a different triaxiality of 0_1^+ and 0_2^+

Shape evolution of $^{96-100}$Mo

M. Zielińska et al, Nucl. Phys. A 712 (2002) 3
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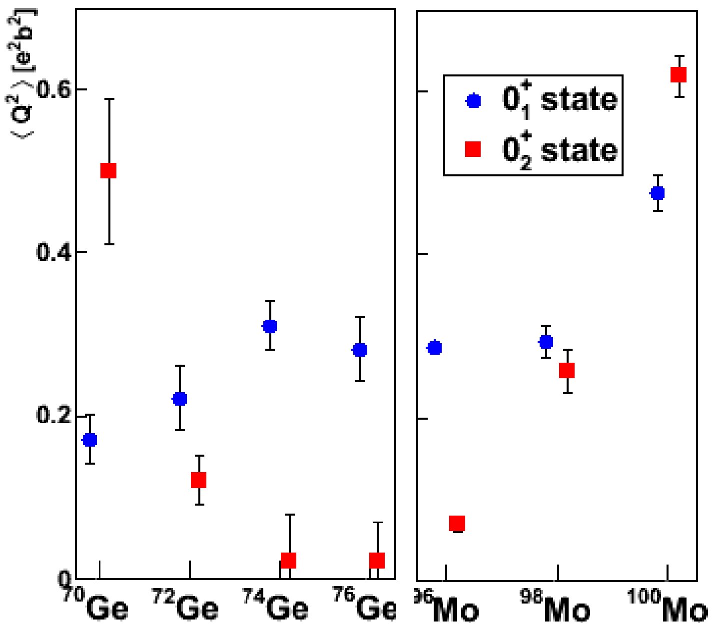
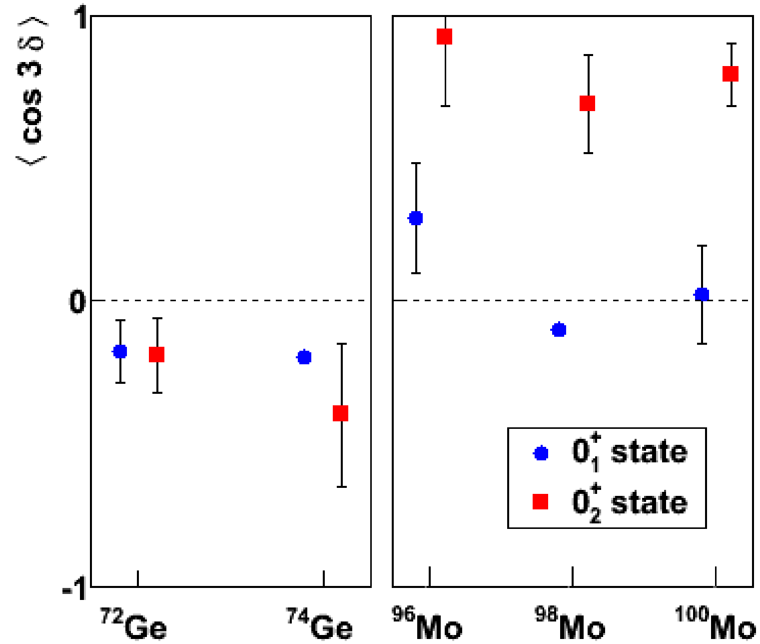
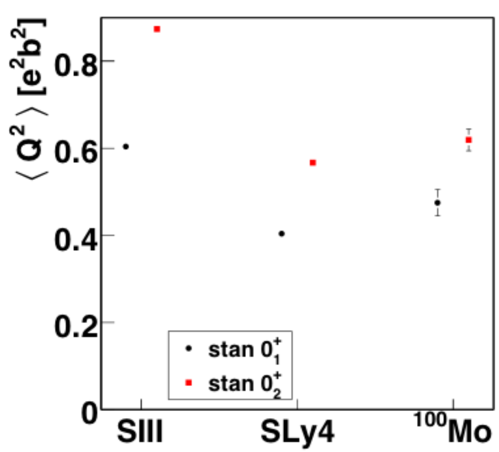
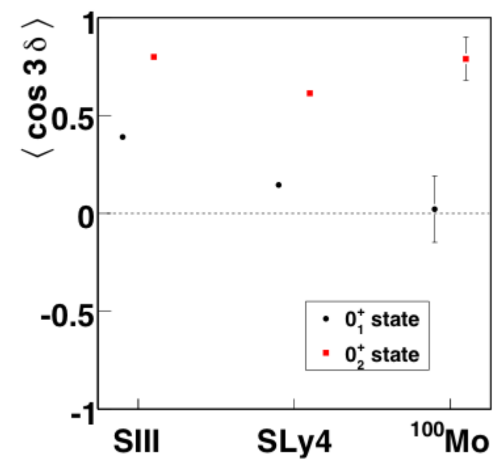




- ^{100}Mo: good agreement with GBH calculations
- ^{96,98}Mo: 0_2^+ band not well described





Calculations by L.Próchniak
(in K. Wrzosek-Lipska et al)

Do we really know all states that should enter the sum?

• especially for the (E2 x E2 x E2), where terms can cancel out – can we say that terms involving higher lying levels (e.g. the 2^+_4 state) do not significantly contribute to the magnitude of the rotational invariant?

• common argument: if such state were coupled to the state in question via a huge E2 matrix element, it would be populated in the experiment

• comparison with GBH calculations for ^{100}Mo: Q^2, $Q^3\cos(3\,\delta)$ calculated directly and from theoretical values of matrix elements, limited to the same 3 intermediate states

$\Rightarrow$ difference below 3% for both 0^+ states

Link to beta and gamma

- relations between the quadrupole invariants and β, γ variables depend on the definition of the collective variables
  - Nilsson model ellipsoidal deformation parameters: formulae given in: J. Srebrny, Nucl. Phys. A 766 (2006) 25

$$Q^2 = (\frac{3}{4\pi}ZR^2)^2(\beta^2+2C\beta^3\cos 3\gamma+C^2\beta^4) \approx q_0^2(\beta^2+2C\beta^3\cos 3\gamma+17C^2\beta^4)+O(\beta^5)$$

$$Q^3\cos 3\delta = (\frac{3}{4\pi}ZR^2)^3(\beta^3\cos 3\gamma + 3C\beta^4 + 3C^2\beta^5\cos 3\gamma + 2C^3\beta^6\cos^2 3\gamma - C^3\beta^6)$$

$$\approx q_0^3(\beta^3\cos 3\gamma + 27C\beta^4 + 3C\beta^4 + 30C^3\beta^6\cos^2 3\gamma + 71C^3\beta^6) + O(\beta^7)$$

where $q_0 = 3/4\pi ZR_0^2$, $C = 1/4\sqrt{5/4\pi}$

Link to beta and gamma

- relations between the quadrupole invariants and β, γ variables depend on the definition of the collective variables
  - Bohr Collective Hamiltonian (K. Wrzosek, PRC 86 (2012) 064305)

$$\langle Q^2 \rangle = q_0^2 \langle \beta^2 \rangle, \langle Q^3 \cos 3\delta \rangle = q_0^3 \langle \beta^3 \cos 3\gamma \rangle$$

(1) values deduced from probability density distributions
(2) values calculated from theoretical ME's

	GBH		
	(1)	(2)	exp
$\overline{\beta}$	0.20	0.20	0.22 $\pm$0.01
$\overline{\gamma}$	27°	27°	29°$\pm$3°
$\overline{\beta}$	0.24	0.24	0.25 $\pm$0.01
$\overline{\gamma}$	18°	17°	10°$\pm$3°

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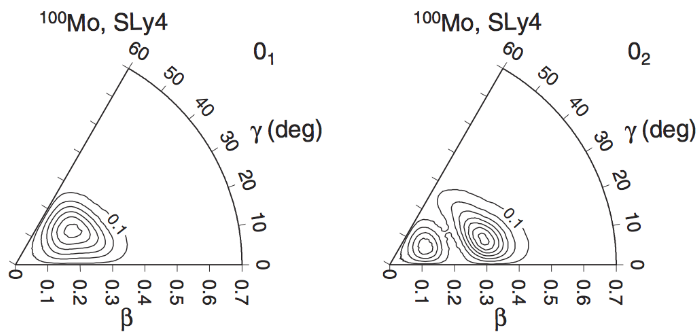
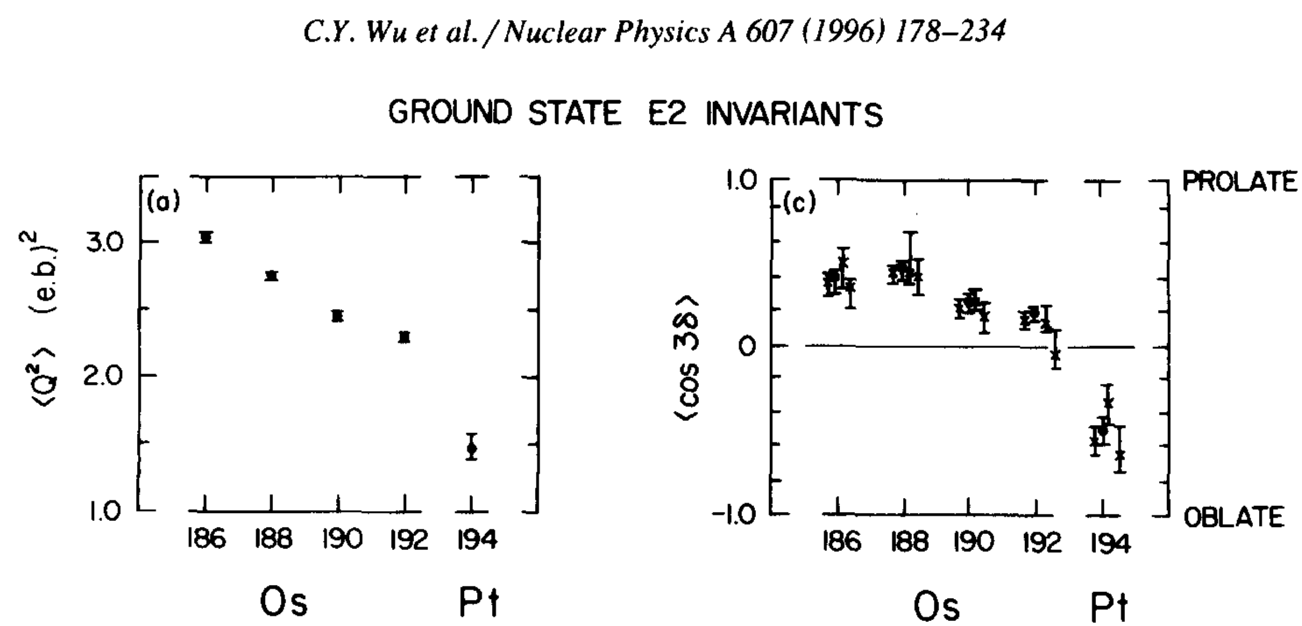


FIG. 15. Probability density [Eq. (26)] for the 0_1^+ and 0_2^+ states for the Skyrme SLy4 interaction. The contour interval is 0.3.

Experimental applications and limitations

- this technique works best for transitional nuclei
- it is safest for 0^+ states (only 2^+ states enter sums)
- for 2^+ – problem how to populate 3^+ states (as shown by the ^{104}Ru case)
- successfully applied to (among others):
  - prolate-oblate shape transition in the chain of $^{186-192}$Os,^{194}Pt,
    C.Y. Wu, Nucl. Phys. A 607 (1996) 178



Experimental applications and limitations

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    C.Y. Wu, Nucl. Phys. A 607 (1996) 178
  - non-axial stiff rotors: ^{168}Er, ^{182,184}W,
    B. Kotliński,Nucl. Phys. A517 (1990) 365, C.Y. Wu, Nucl. Phys. A533 (1991) 359
  - quasi-vibrational ^{104}Ru,
    J. Srebrny, Nucl. Phys. A 766 (2006) 25
  - shape coexistence:
    - $^{70-76}$Ge, M. Sugawara, Eur. Phys. J. A16 (2003) 409 and ref. therein
    - $^{96-100}$Mo, M. Zielińska, Nucl. Phys. A 712 (2002) 3,
      K. Wrzosek, PRC 86 (2012) 064305