

# Selected topics in Drell-Yan theory: from twist 3 SSA to exclusive limit

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based on

**PLB690, 519 (2010); EPJC75, 184 (2015);  
PLB751 495 (2015); PoS QNP2012,055(2012);  
AIP Conf.Proc. 1654 (2015) 060007**



# Outline

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- SSA in QCD
- Twist-3 Single Spin asymmetries and DY
- Contour gauge and factor 2
- Gluonic poles and physical fields
- Burkardt sum rules
- Elastic limit: Sivers function and formfactors
- Conclusions



# Single Spin Asymmetries

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- $S(p \times q)$ : “Naïve” T-odd
- Real T-reflection – interchange of initial and final particles
- May lead to the sign change due to phase of  $\langle f | S | i \rangle$ , compensating “naïve” T-oddness
- Phases – mimic T-violation
- Sign of  $i\epsilon$  -measures time direction
- Manifestation of real T(CP) violations in BAU – due to cuts of GUT diagrams!



# “Effective” T-reflection in QCD

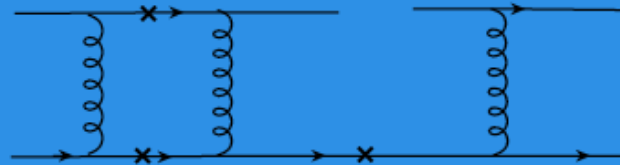
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- Phases = cuts in Soft or Hard variables due to (generalized) optical theorem
- Various manifestations in hadronic physics:
  - No cuts  $\leftrightarrow$  S/H cuts:  $SL(T)FF \leftrightarrow TL(T)FF$
  - Scuts  $\leftrightarrow$  Hcuts:  $SLTFF \leftrightarrow TL(T)FF$  to pairs e.g.  $(\gamma^* \gamma \rightarrow MM) \leftrightarrow (\gamma^* \rightarrow \gamma MM)$
  - Hcuts in scaling region: cuts cancellation (see my talk on Dterm) – sign changes:  $SIDIS \leftrightarrow DY$  (Sivers fctn sign) ;  $DVCS \leftrightarrow TCS$ ;  $DVMP \leftrightarrow$  exclusive  $DY$ ;  
**DDVCS(2 cuts in the same process)**

# Perturbative PHASES IN QCD

QCD factorization: where to borrow imaginary parts?

Simplest way: from short distances - loops in partonic subprocess. Quarks elastic scattering (like  $q - e$  scattering in DIS):

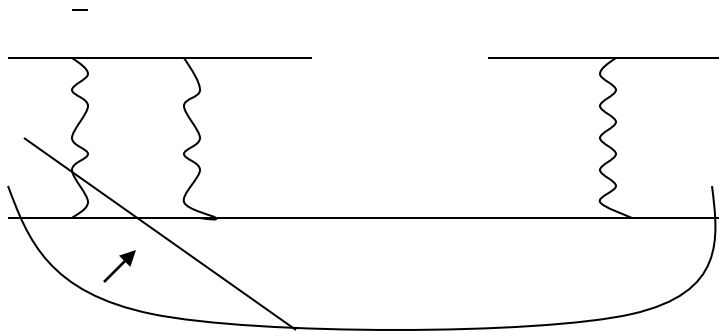


$$A \sim \frac{\alpha_S^{m_{PT}}}{p_T^2 + m^2}$$

Large SSA "...contradict QCD or its applicability"

# Short+ large overlap– twist 3

- Quarks – only from hadrons
- Various options for factorization – shift of SH separation



- New option for SSA: Instead of 1-loop twist 2  
– Born twist 3: Efremov, OT (85, Fermionic poles); Qiu, Sterman (91, GLUONIC poles)

# Gluonic poles and Sivers function



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- Both require Initial or Final State Interaction
- Twist-3: suppression like  $1/Q$
- “Compensated” by gluonic pole ( $\sim$ gauge link) – unsuppressed by  $1/Q$  FSI (Brodsky, Hwang, Schmidt)
- Moment of Sivers function – twist 3 gluonic pole strength (Boer, Mulders, Pijlman)
- TMD – infinite tower of twists projected by the relevant moments (Ratcliffe, OT)



# Twist 3 and gauge invariance

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- Required to provide GI set of diagrams, maintain EMGI for  $g_2$  (80's: Efremov, OT; Bukhvostov, Kuraev, Lipatov; Balitsky, Braun)
- $S_T \rightarrow \Delta_T$  : EMGI for DVCS – twist 3 GPDs (talk of S. Liuti) first appeared (Anikin, Pire, OT'98; Polyakov, Kivel et al...); twist 4 - talk of V. Braun

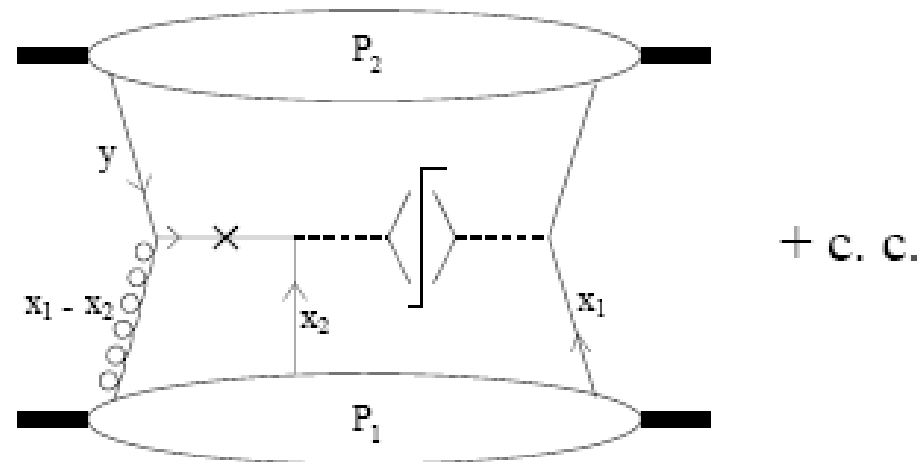


# SSA due to gluonic pole in DY

- TM integrated DY with one transverse polarized beam – unique SSA – gluonic pole (Hammon, Schaefer, OT)

$$A = g \frac{\sin 2\theta \cos \phi \left[ T(x, x) - x \frac{dT(x, x)}{dx} \right]}{M [1 + \cos^2 \theta] q(x)}$$

- Denominator to be modified due to density matrix positivity



# Contour gauge in DY:

(Anikin, OT, Phys.Lett. B690 (2010) 519-525)

- Motivation of contour gauge –  $[-\infty^-, 0^-] = 1$   
elimination of link  $[-\infty^-, 0^-] = Pexp\left\{-ig \int_{-\infty}^0 dz^- A^+(0, z^-, \vec{0}_T)\right\}$

- Field  $A^\mu(z) = \int_{-\infty}^{\infty} d\omega^- \theta(z^- - \omega^-) G^{+\mu}(\omega^-) + A^\mu(-\infty)$

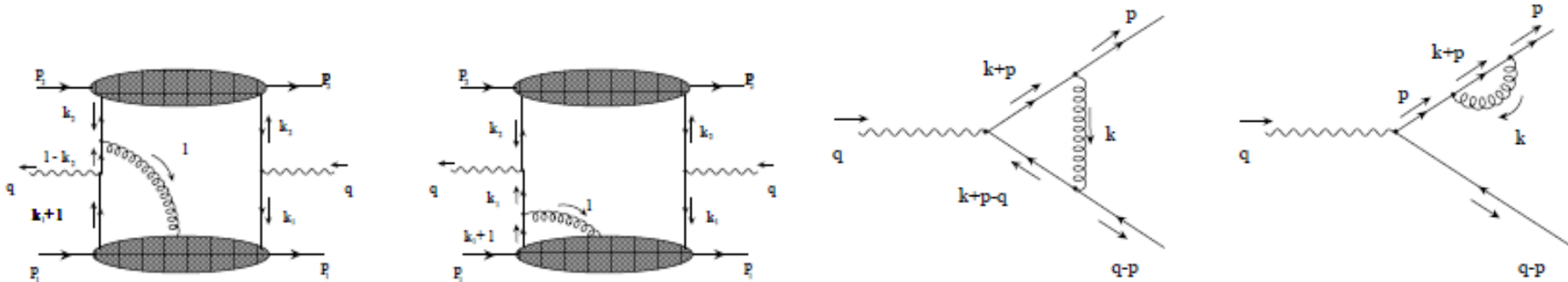
- EOM: Gluonic pole appearance  $B^V(x_1, x_2) = \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon}$

- cf naïve expectation  $B^V(x_1, x_2) = \frac{\mathcal{P}}{x_1 - x_2} T(x_1, x_2)$

- How to check that?

# Gauge invariance

- EM GI (experience from  $g_2, DVCS$ ) – 2 contributions



- SSA in on shell PT – only one diagram for GI
- NP tw3 analog - GI only if GP absent
- GI** with GP – “phase without cut” ( in subprocess)



# Analogs/implications

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- Analogous pole – in gluonic GPDs
- Braun, Ivanov, Schafer, Szymanowski'04 – universal GI result for on-shell quark target for different prescriptions in different ways
- Analogous diagram for GI – Boer, Qiu(04)
- Besides consistency proof – factor **2** for asymmetry (missed before)

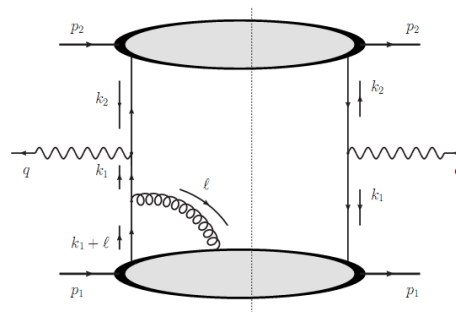
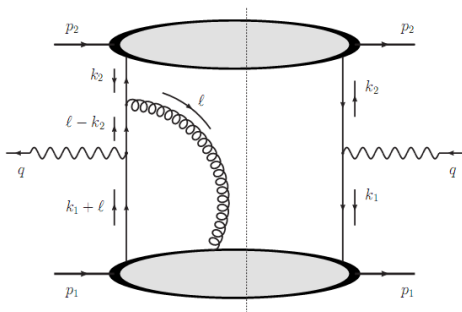
$$\bar{W}_{\mu\nu}^{\text{GI}} = \bar{W}_{\mu\nu}^{(1)} + \bar{W}_{\mu\nu}^{(2)} = -\frac{2}{q^2} \varepsilon_{\nu S T p_1 p_2} Z_\mu \bar{q}(y_B) T(x_B, x_B) \quad \hat{p}_{1\mu} \implies \hat{p}_{1\mu} - q_\mu \frac{\hat{p} \cdot q}{Q^2} = \frac{p_{1\mu} - p_{2\mu}}{2} .$$

$$Z_\mu = \hat{p}_{1\mu} - \hat{p}_{2\mu}$$

# Feynman gauge (Anikin, OT'15)

- Standard

$$\overline{W}_{\mu\nu}^{(\text{Stand.})} + \overline{W}_{\mu\nu}^{(\text{Stand.}, \partial_{\perp})} = \bar{q}(y) \left\{ \begin{aligned} & -\frac{p_{1\mu}}{y} \varepsilon_{\nu ST-p_2} \int dx_2 \frac{x_1 - x_2}{x_1 - x_2 + i\epsilon} B^{(1)}(x_1, x_2) \\ & - \left[ \frac{p_{2\nu}}{x_1} \varepsilon_{\mu ST-p_2} + \frac{p_{2\mu}}{x_1} \varepsilon_{\nu ST-p_2} \right] x_1 \int dx_2 \frac{B^{(2)}(x_1, x_2)}{x_1 - x_2 + i\epsilon} \\ & + \frac{p_{1\mu}}{u} \varepsilon_{\nu ST-p_2} \int dx_2 \frac{B^{(\perp)}(x_1, x_2)}{x_1 - x_2 + i\epsilon} \end{aligned} \right\}, \quad (14)$$



- and non-standard  $\overline{W}_{\mu\nu}^{(\text{Non-stand.})} = \bar{q}(y) \frac{p_{2\mu}}{x_1} \varepsilon_{\nu ST-p_2} \times$  pieces

$$\int dx_2 \left\{ B^{(1)}(x_1, x_2) + B^{(2)}(x_1, x_2) \right\}$$

# EM GI – gluon poles not in PARTICULAR structures

## ■ Correlators

$$B^{(1)}(x_1, x_2) = \frac{T(x_1, x_2)}{x_1 - x_2 + i\varepsilon} \Leftarrow \mathcal{F}_2 \left[ \langle p_1, S^T | \bar{\psi}(\eta_1) \gamma^+ A^T(z) \psi(0) | S^T, p_1 \rangle \right],$$

$$B^{(2)}(x_1, x_2) \Leftarrow \mathcal{F}_2 \left[ \langle p_1, S^T | \bar{\psi}(\eta_1) \gamma^\perp A^+(z) \psi(0) | S^T, p_1 \rangle \right],$$

$$B^{(\perp)}(x_1, x_2) \Leftarrow \mathcal{F}_2 \left[ \langle p_1, S^T | \bar{\psi}(\eta_1) \gamma^+ (\partial^\perp A^+(z)) \psi(0) | S^T, p_1 \rangle \right].$$

## ■ Gluon-pole free:

$$(B^{(2)}(x_1, x_2) = -\bar{B}^{(2)}(x_2, x_1)), \text{ obeys } B^{(2)}(x, x) = 0.$$

# Possible interpretation (new)

- Decomposition of Gluon Angular momentum (Y. Hatta)

Complete decomposition

Chen, Lu, Sun, Wang, Goldman (2008)  
Wakamatsu (2010)  
Y.H. (2011)

- Field (axial gauge)= physical

$$M_{\text{quark-spin}}^{\mu\nu\lambda} = -\frac{1}{2} \epsilon^{\mu\nu\lambda\sigma} \bar{\psi} \gamma_5 \gamma_\sigma \psi,$$

$$M_{\text{quark-orbit}}^{\mu\nu\lambda} = \bar{\psi} \gamma^\mu (x^\nu i D_{\text{pure}}^\lambda - x^\lambda i D_{\text{pure}}^\nu) \psi,$$

$$M_{\text{gluon-spin}}^{\mu\nu\lambda} = F_a^{\mu\lambda} A_{\text{phys}}^{\nu a} - F_a^{\mu\nu} A_{\text{phys}}^{\lambda a},$$

$$M_{\text{gluon-orbit}}^{\mu\nu\lambda} = F_a^{\mu\alpha} (x^\nu (D_{\text{pure}}^\lambda A_{\alpha}^{\text{phys}})_a - x^\lambda (D_{\text{pure}}^\nu A_{\alpha}^{\text{phys}})_a)$$

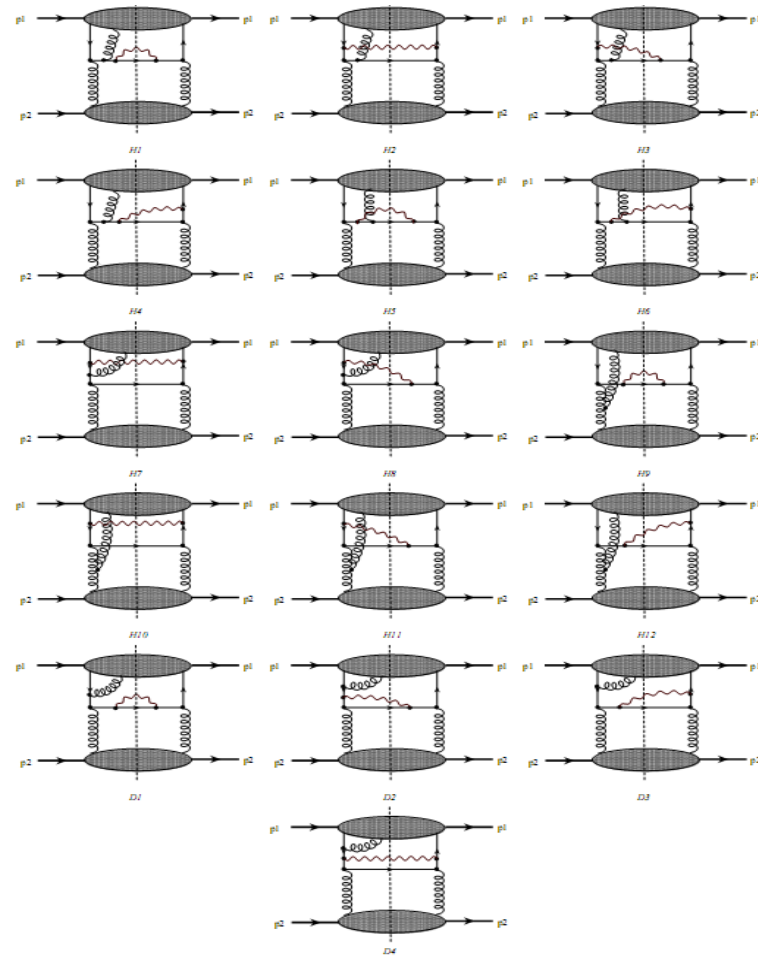
where (my choice)  $A_{\text{phys}}^\mu = \frac{1}{D^+} F^{+\mu}$      $D_{\text{pure}}^\mu = D^\mu - i A_{\text{phys}}^\mu$

- Poles – only from physical fields

# Direct photon production (Anikin, OT, Eur.Phys.J. C75 (2015) 5, 184)

- Gluonic poles from 4 diagrams
- Standard (H10) also enhanced by factor 2 from nonstandard ones

$$dW(\text{diag.H1}) + dW(\text{diag.H7}) + dW(\text{diag.D4}) = dW(\text{diag.H10}).$$





# Burkardt SR and Energy-Momentum tensor (OT'14)

- Consider twist 3 (= relevant moment of Sivers function being infinite tower of twists) gluonic pole
- EMT forward matrix element: no spin-dependent structure  $p^\rho \epsilon^{\mu\sigma\rho\alpha}$

$$b_V(x_1, x_2) = \frac{i}{M} \int \frac{d\lambda_1 d\lambda_2}{2\pi} e^{i\lambda_1(x_1-x_2)+i\lambda_2 x_2} \epsilon^{\mu\sigma\rho\alpha} \langle p_1, s | \bar{\psi}(0) \hat{n} D_\mu(\lambda_1) \psi(\lambda_2) | p_1, s \rangle \sum \int \int dx_1 dx_2 \frac{T(x_1, x_2)}{x_1 - x_2} = 0$$

- Naively: Valid identically due to symmetry properties implied by T-invariance)
- However: pole should get imaginary part due to EMGI and related contour gauge – analog of force calculation in original Burkardt' derivation

# Pole prescription and Burkardt SR

- Pole prescription (dynamics!) provides (“T-odd”) symmetric part!

$$\sum \int \int dx_1 dx_2 \frac{T(x_1, x_2)}{x_1 - x_2 + i\varepsilon} = 0$$

- SR:

$$\sum \int dx T(x, x) = 0$$

- Gluonic poles for 3 gluon correlators should be related to gluon Sivers function – natural but not yet proved!



# Validity for each flavour

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- Can it be valid separately for each quark flavour: nodes (Boer, Prokudin)?
- Valid if structures forbidden for TOTAL EMT do not appear for each flavour
- Structure contains besides S gauge vector n: GI separation of EMT – forbidden: SR valid separately!

# Burkardt SR and Equivalence Principle

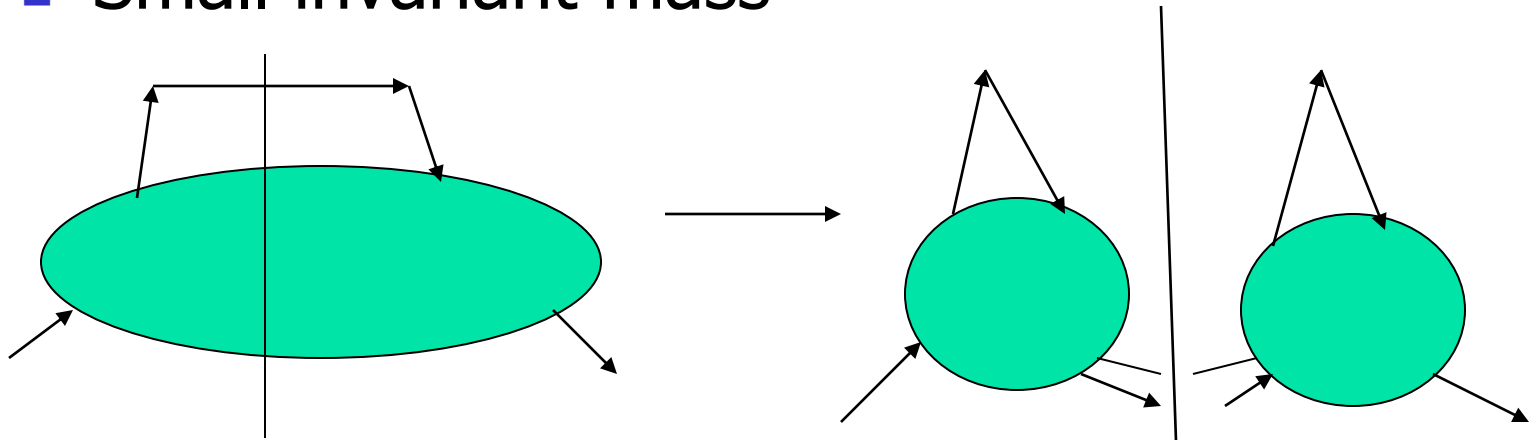


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- Absence of  $s/n$ -dependent part for EMT matrix element
- Coupling to gravity - Equivalence Principle (OT'99)
- Provides another interpretation of Ji's SR and momentum SR (Efremov, OT'82) for tensor spin structure function (OT'99, '08)
- Validity for each flavour – extension of EP; for Ji's SR – only for sum of quarks

# Exclusive limit : DIS and space-like (transitional and elastic) FFs

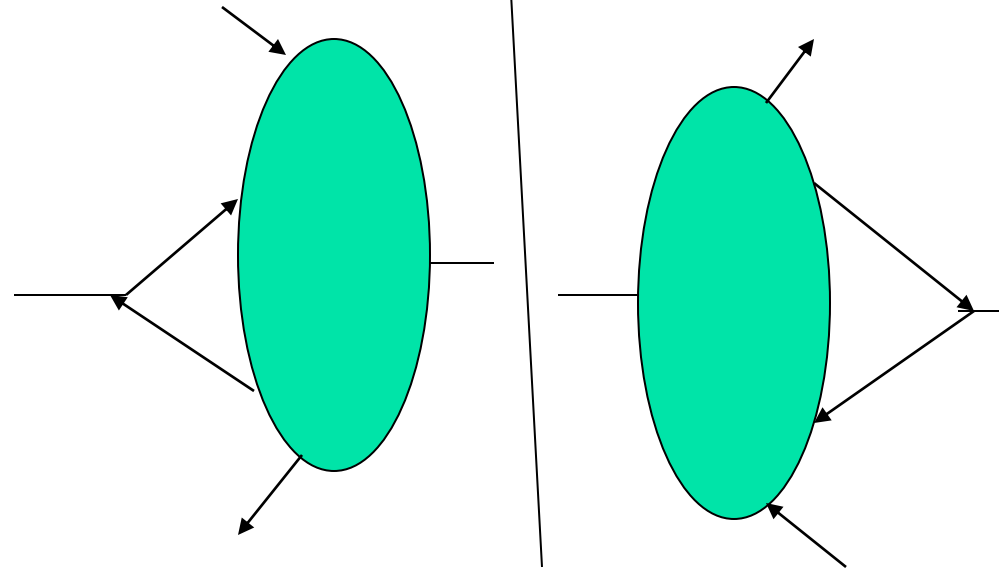
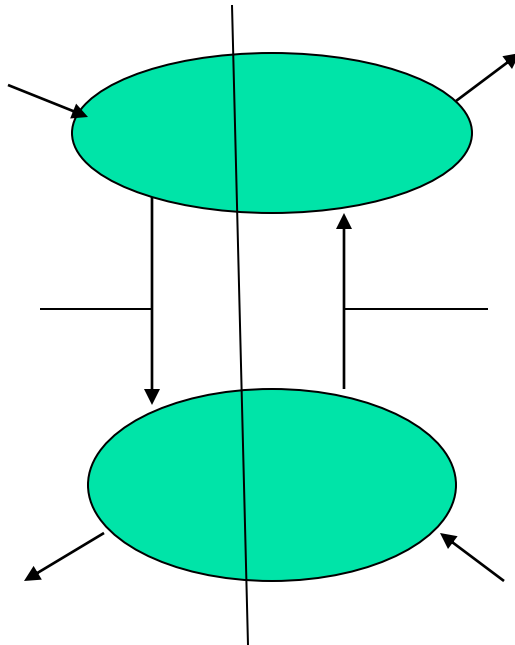
- Small invariant mass



- May be related to unitarity analyticity and DR (OT'05)
- Relation between  $x \rightarrow 1$  and large  $Q^2$
- pdf  $\sim (FF)^2$
- Elastic(DYW) or Transition(BG)

# Exclusive limit of DY and time-like FFs (OT'14)

- (Proton-antiproton) DY at small  $s - Q^2$  -elastic FF(DYW)



$$(pdf)^2 \sim (\text{Dirac}) (FF)^2$$

- Other beams – baryon number conservation – time-like **transition** FFs (BG)

# Comparing space-like and time-like FFs

- “Duality intervals” - from mass to x-space
- DIS:  $(P+q)^2 = (P_f + \delta P_{DIS})^2 = (M + \mu_{DIS})^2$   $\mu_{DIS} \sim$  pion related scale
- Deviation of  $x_B (\equiv 1 - \delta_{DIS})$  from 1  
$$\delta_{DIS} \sim 2M\mu_{DIS}/Q^2.$$
- DY:  $(P_1 + P_2)^2 = (q + \delta P_{DY})^2$
- Deviation of  $\tau = Q^2/s (\equiv 1 - \delta_{DY})$  from 1  
$$\delta_{DY} \sim 2\mu_{DY}/Q$$



# FFs from duality intervals

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- DIS:  $F_{SL}^2 \sim \int_0^{\delta_{DIS}} d\bar{x} f(\bar{x}) \quad x = 1 - \bar{x}$
- DY:  $F_{TL}^2 \sim \int_0^{\delta_{DY}} d\bar{x}_1 d\bar{x}_2 f(\bar{x}_1) f(\bar{x}_2) \delta(\delta_{DY} - \bar{x}_1 - \bar{x}_2)$
- Proton-antiproton DY –same parton distributions  $f(\bar{x}) = C\bar{x}^a$

$$F_{SL}^2(Q^2) \sim \frac{C}{a+1} \left( \frac{2M\mu_{DIS}}{Q^2} \right)^{a+1} ; F_{TL}^2(Q^2) \sim \frac{C^2}{2(a+1)} \left( \frac{4\mu_{DY}^2}{Q^2} \right)^{a+1}$$

- Pion:  $a=1$  supported





# SL vs TL

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- Same Q-dependence
- Normalization –defined by distribution scale ( $\sim 5$ ) and duality intervals
- Asymptotically coincide – scales close to QCDSR pion duality interval (rather than pion mass) similar (equal?! ) for DIS and DY) !?

# Sivers function and formfactors



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- Relation between Sivers function and AMM known on the level of matrix elements (Brodsky, Schmidt, Burkardt)
- Phase?
- Duality for observables?

# BG/DYW type duality for DY SSA in exclusive limit

- Proton-antiproton DY – valence annihilation – analyticity - cross section is described by Dirac FF squared
- The SSA similar to twist 3 one- due to interference of Dirac and Pauli FF's with a phase shift (Rekalo, Brodsky)
- Exclusive large energy limit;  $x \rightarrow 1$  :  
 $T(x,x)/q(x) \rightarrow \text{Im } F2/F1(Q^2 \sim M^2(1-x))$
- Both directions – estimate of Sivers at large  $x$  and explanation of phases in FF's



# Conclusions

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- SSA in DY : EM GI brings phases without cuts and factor 2 –tests at COMPASS, NICA
- Direct photon production – analogous modification of gluonic poles
- Feynman gauge: Gluonic poles due to physical field only
- Burkardt sum rule with hidden phase: relation to energy momentum-tensor and equivalence principle
- Exclusive limit – transition FFs
- Sivers function  $\sim (\text{Im } F_1 F_2^*)$

# Where to measure?

- J-PARC, COMPASS,..
- NICA@JINR (talk of A.V.Efremov)
- <http://nica.jinr.ru/>

- Recent view

