New Skyrme energy density functional for a better description of charge-exchange resonances

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Landau Fermi liquid theory in nuclear and many body systems.
ECT* Trento, May 22nd-26th 2017

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Spin and Isospin excitations in Nuclei

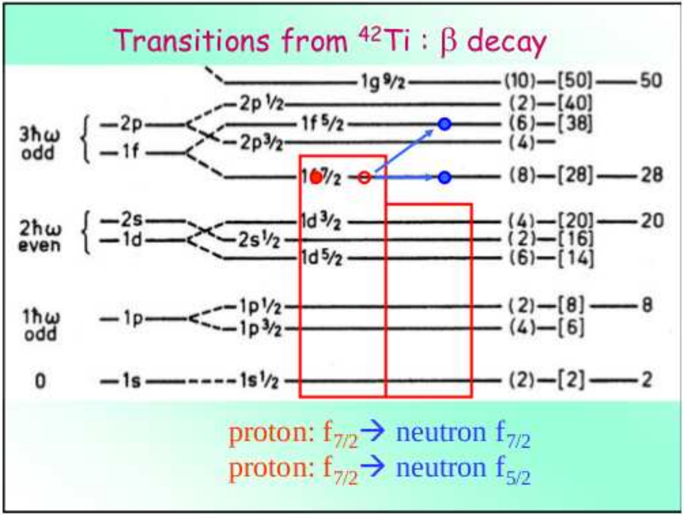
- **Nucleons** are fermions charac. by their spin and isospin
- **Nucleons** with spin (isospin) may **change their state** in **phase**: spin-scalar S=0 modes (isospin-scalar T=0 modes); or **out of phase**: spin-vector S=1 modes (isospin-vector T=1 modes)
- They can be **excited by strong probes** (charge-exhange reactions) and they can **decay via the weak interaction** (axial-vector current couples to the spin and induces $\beta-$decay processes)

One of the most important nuclear excitation modes is the

- **Gamow Teller Resonance** which is a pure **spin-isospin mode** (i.e., from a theoretical picture, it is excited by an operator $\hat{O} \sim \sigma\tau$)

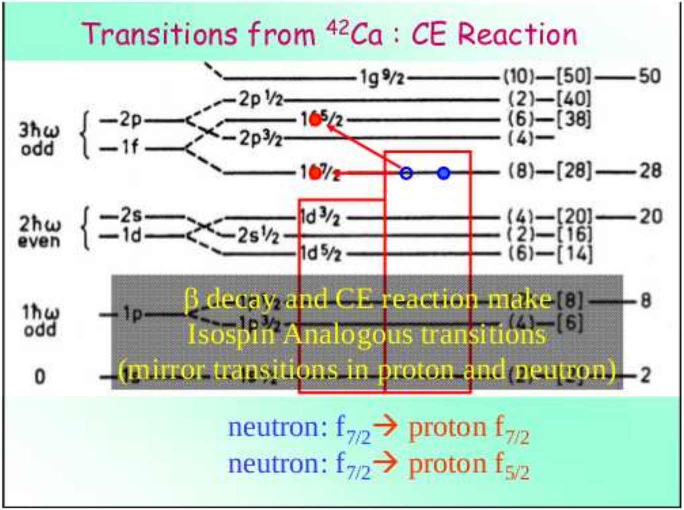
Spin-isospin modes of excitation (such as the **GTR**) give **direct information** on the spin-isospin channel of the **effective interaction**

Example: β−decay transition



Courtesy of Y. Fujita; taken from his lectures http://www.mi.infn.it/~colo/lectures/lectures.html

Example: Gamow Teller transition



Courtesy of Y. Fujita; taken from his lectures http://www.mi.infn.it/~colo/lectures/lectures.html

Therefore,

- allowed **GT** transitions mainly detemine **β-decay half-lives**
- **GT** transitions determine **weak interaction rates** essential role in the **core-collapse dynamics** of massive stars leading to supernova explosion
- In neutron-rich environment, **neutrino-induced nucleosynthesis** may take place via **GT** processes
- **GT** matrix elements are necessary for the study of **double-β-decay**
- may be useful in the **calibration of detectors** used to measure neutrinos that reach the Earth

Some comments on the nuclear many-body problem:

- **Many-body** calculations based on **NN scattering data** in the vacuum are **not conclusive** yet:
  - **different predictions** are found **depending** on the **approach**
  - **EoS** and (only very recently) **few groups** in the world are able to perform extensive calculations for **light and medium mass nuclei**
- Based on effective interactions, **Nuclear Energy Density Functionals** are **successful** **(but still not perfect)** in the description of **masses, nuclear sizes, deformations, Giant Resonances,...**

Nuclear Energy Density Functionals:

Main types of successful EDFs derived from the mean-field approximation

- **Relativistic H o HF models,** based on Lagrangians where effective (heavy) mesons carry the interaction.

$$
\begin{aligned}
\mathcal{L}_{int} \quad &= \bar{\Psi}\Gamma_\sigma(\bar{\Psi},\Psi)\Psi\Phi_\sigma \quad + \bar{\Psi}\Gamma_\delta(\bar{\Psi},\Psi)\boldsymbol{\tau}\Psi\Phi_\delta \\
&-\bar{\Psi}\Gamma_\omega(\bar{\Psi},\Psi)\gamma_\mu\Psi A^{(\omega)\mu} \quad -\bar{\Psi}\Gamma_\rho(\bar{\Psi},\Psi)\gamma_\mu\tau\Psi A^{(\rho)\mu} \\
&-e\bar{\Psi}\hat{Q}\gamma_\mu\Psi A^{(\gamma)\mu}
\end{aligned}
$$

- **Non-relativistic HF models,** based on Hamiltonians where effective interactions are proposed and tested:

$$V^{\rm eff}_{\rm Nucl} = V^{\rm long-range}_{\rm attractive} + V^{\rm short-range}_{\rm repulsive} + V_{\rm SO}$$

- Fitted **parameters contain** (important) **correlations beyond the mean-field**
- Nuclear energy functionals are **phenomenological** $\rightarrow$ **not directly connected to any NN** (or NNN) **interaction**

Drawbacks on current EDFs ???

On the one side,

- **we expect** that the **H(F)+RPA** method based on nuclear effective interactions of the **Skyrme, Gogny or Relativistic** (can be understood as an **approximate realization of an EDF**) $\Rightarrow$ **reasonable description of g.s. energy and density of the system** (<one-body operators>)

On the other side,

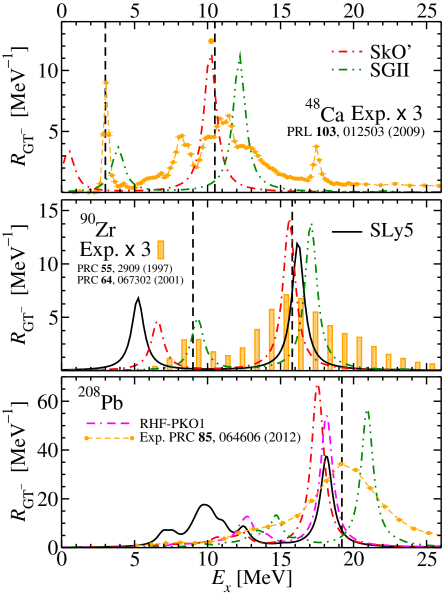
- there are still some **open problems** ... but we will concentrate here on how to

  improve the spin-isospin properties of our EDF

Motivation: Gamow Teller Resonance

The E_x is not properly described in H(F)+RPA

- **SGII**[a]: earliest attempt to give a quantitative description of the GTR
- **SkO′**[b]: accurate in ground state finite nuclear properties and improves the GTR
- **PKO1**[c]: relativistic HF, reasonable GTR still not perfect
- Relativistic H[d]: residual interaction modified *ad-hoc*



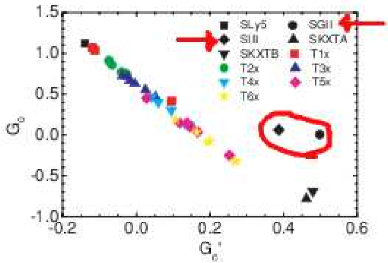
[a] PLB **106**, 379 (1981), [b] PRC **60**, 014316 (1999), [c] PRL **101**, 122502 (2008), [d] PRC 69, 054303

Motivation: which gs properties are important for describing the E_x^{GTR}?

The study[a] of the GTR and the spin-isospin Landau-Migdal parameter G_0' using several Skyrme sets,

- concluded that G_0' **is not the only important quantity** in determining the **excitation energy of the GTR**
- **spin-orbit splittings** also influences the GTR

- Empirical indications[b] suggest that $G_0' > G_0 > 0$
- **Not a very common feature** within available Skyrme forces[c]



[a] M. Bender, J. Dobaczewski, J. Engel, and W. Nazarewicz, Phys. Rev. C **65**, 054322 (2002); [b] T. Wakasa, M. Ichimura, and H. Sakai, Phys. Rev. C **72**, 067303 (2005); T. Suzuki and H. Sakai, Phys. Lett. B **455**, 25 (1999),[c] Li-Gang Cao, G. Colo, and H. Sagawa, Phys. Rev. C **81**, 044302 (2010)

Landau-Migdal vs Skyrme parameters

- The p-h interaction is derived as the second functional derivative of the total energy with respect to density at the Fermi surface. Matrix elements of the form:

$$\langle \vec{k}_1 \vec{k}_2 | V | \vec{k}_1 \vec{k}_2 \rangle = \sum_{\alpha=1,\tau,\sigma,\tau\cdot\sigma} \frac{\delta \mathcal{H}}{\delta \rho_\alpha \delta \rho_\alpha} =$$
$$N_0^{-1} (F + F' \boldsymbol{\tau_1} \boldsymbol{\tau_2} + G \boldsymbol{\sigma_1} \boldsymbol{\sigma_2} + G' \boldsymbol{\tau_1} \boldsymbol{\tau_2} \boldsymbol{\sigma_1} \boldsymbol{\sigma_2})$$

* $N_0 = 2k_F m^* / \hbar^2 \pi^2$ is the density of states per energy at the Fermi surface
* Parameters are functions of $\vec{k}_1$ and $\vec{k}_2$

* $\vec{k}_1$ and $\vec{k}_2$ are taken at the Fermi surface (Landau parameters are only functions of the angle between them and the Fermi momentum $\Rightarrow F = \sum_l F_l P_l(\cos\theta)$).

- One can do the same functional derivative with Skyrme and compare both expressions to find the relation between the Landau-Migdal and the Skyrme parameters (here just two of them):

$$\mathbf{G_0} N_0 = -\frac{1}{4} t_0 + \frac{1}{2} t_0 x_0 - \frac{1}{8} t_1 k_F^2 + \frac{1}{4} t_1 x_1 k_F^2 + \frac{1}{8} t_2 k_F^2 + \frac{1}{4} t_2 x_2 k_F^2 - \frac{1}{24} t_3 \rho^\alpha + \frac{1}{12} t_3 x_3 \rho^\alpha$$
$$\mathbf{G_0'} N_0 = -\frac{1}{4} t_0 - \frac{1}{8} t_1 k_F^2 + \frac{1}{8} t_2 k_F^2 - \frac{1}{24} t_3 \rho^\alpha$$

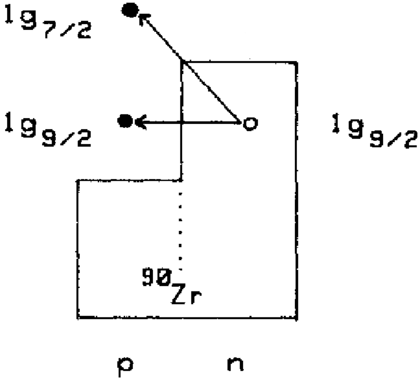
Empirical constraints on G_0 and G_0'

- **Gamow-Teller Resonance** using RPA based on the Woods-Saxon potential have been studied and the **Landau-Migdal parameters estimated by comparing experiment** with theoretical calculations in Refs. [T. Wakasa, M. Ichimura, and H. Sakai, Phys. Rev. C **72**, 067303 (2005) and T. Suzuki and H. Sakai, Phys. Lett. B **455**, 25 (1999)].
  - Landau-Migdal parameter G_0' **dominates the excitation energy in the GTR**
- In our fit, **we do not use the obtained values as pseudodata** because **our theoretical framework is different and the results are associated to different** m^* (our sp energies are based on HF calculations instead of a Wood-Saxon potential).
- **We use** the empirical result in which a **hierarchy** between spin and spin-isospin parameters is suggested:

$$G_0' > G_0 > 0$$

Why spin-orbit splittings are important?

$E_x^1 \approx$
$\epsilon_{\pi 1g_{7/2}} - \epsilon_{\nu 1g_{9/2}} + \epsilon_{ph}^1$
$E_x^2 \approx$
$\epsilon_{\pi 1g_{9/2}} - \epsilon_{\nu 1g_{9/2}} + \epsilon_{ph}^2$
$\Delta E_x \approx \Delta\epsilon_{\pi 1g} + \Delta\epsilon_{ph}$



Schematic picture of single-particle transitions involved in the Gamow Teller Resonance of ^{90}Zr. Transitions excited by $\sigma\tau_-$ operator.

F. Osterfeld, Rev. Mod. Phys. **64**, 491 (1992)

Skyrme Model

Hamiltonian[a]

Includes **central tensor terms (J^2 terms)** due to the coupling of tensor and spin and gradients terms and **two spin-orbit parameters** (same as SkO and some SkI forces)

$$\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\text{eff}} + \mathcal{H}_{\text{fin}} + \mathcal{H}_{\text{SO}} + \mathcal{H}_{\text{sg}} + \mathcal{H}_{\text{Coul}}$$

$$\begin{aligned}
\mathcal{K} &= \hbar^2\tau/2m \\
\mathcal{H}_0 &= (1/4)t_0[(2+x_0)\rho^2 - (2x_0+1)(\rho_n^2+\rho_p^2)] \\
\mathcal{H}_3 &= (1/24)t_3\rho^\alpha[(2+x_3)\rho^2 - (2x_3+1)(\rho_n^2+\rho_p^2)] \\
\mathcal{H}_{\text{eff}} &= (1/8)[t_1(2+x_1)+t_2(2+x_2)]\tau\rho \\
&\quad + (1/8)[t_2(2x_2+1)-t_1(2x_1+1)](\tau_n\rho_n+\tau_p\rho_p) \\
\mathcal{H}_{\text{fin}} &= (1/32)[3t_1(2+x_1)-t_2(2+x_2)](\nabla\rho)^2 \\
&\quad - (1/32)[3t_1(2x_1+1)+t_2(2x_2+1)][(\nabla\rho_n)^2+(\nabla\rho_p)^2] \\
\mathcal{H}_{\text{SO}} &= (1/2)W_0\mathbf{J}\cdot\nabla\rho + (1/2)W_0'(\mathbf{J}\cdot_{\mathbf{n}}\nabla\rho_n + \mathbf{J}_{\mathbf{p}}\cdot\nabla\rho_p) \\
\mathcal{H}_{\text{sg}} &= -(1/16)(t_1x_1+t_2x_2)\mathbf{J}^2 + (1/16)(t_1-t_2)({\mathbf{J}_{\mathbf{n}}}^2+{\mathbf{J}_{\mathbf{p}}}^2)
\end{aligned}$$

[a] E. Chabanat et al., Nucl. Phys. A **635**, 231 (1998); E. Chabanat *et al.*, ibid. **643**, 441 (1998)

Fitting Protocol

χ^2 definition: $$\chi^2 = \frac{1}{N_{data}} \sum_i^{N_{data}} \frac{(\mathcal{O}_i^{theo.} - \mathcal{O}_i^{data})^2}{(\Delta\mathcal{O}_i^{data})^2}$$

Landau-Migdal parameters in infinite nuclear matter G_0 and G_0' fixed to **0.15** and **0.35**, respectively, at ρ_0.

Table: Data and *pseudo*-data $\mathcal{O}_i$, adopted errors for the fit $\Delta\mathcal{O}_i$ and selected finite nuclei and EoS.

$\mathcal{O}_i$	$\Delta\mathcal{O}_i$	
B	1.00 MeV	^{40,48}Ca, ^{90}Zr, ^{132}Sn and ^{208}Pb
r_c	0.01 fm	^{40,48}Ca, ^{90}Zr and ^{208}Pb
ΔE_{SO}	$0.04\times\mathcal{O}_i$	$\pi 1g$ in ^{90}Zr and $\pi 2f$ in ^{208}Pb
$e_n(\rho)$	$0.20\times\mathcal{O}_i$	R. B. Wiringa *et al.*, PRC **38**, 1010 (1988)

Skyrme Aizu Milano interaction: SAMi

Parameter set and nuclear matter properties:

Table: SAMi parameter set and saturation properties with the estimated standard deviations inside parenthesis

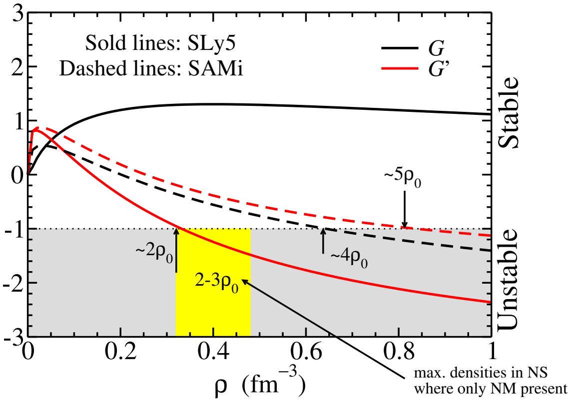
	value(σ)			value(σ)	
t_0	−1877.75(75)	MeV fm^3	ρ_∞	0.159(1)	fm^{-3}
t_1	475.6(1.4)	MeV fm^5	e_∞	−15.93(9)	MeV
t_2	−85.2(1.0)	MeV fm^5	m^*_{IS}	0.6752(3)	
t_3	10219.6(7.6)	MeV fm$^{3+3\alpha}$	m^*_{IV}	0.664(13)	
x_0	0.320(16)		J	28(1)	MeV
x_1	−0.532(70)		L	44(7)	MeV
x_2	−0.014(15)		K_∞	245(1)	MeV
x_3	0.688(30)		G_0	0.15	(fixed)
W_0	137(11)		G'_0	0.35	(fixed)
W'_0	42(22)				
α	0.25614(37)				

SAMi: spin and spin-isospin instabilities in NM

Qualitative test

Imposing that spin and isospin dof at the Fermi surface are stable under generalized deformations [S.-O. Bäckman *et al.*, Nucl. Phys. A **321**, 10 (1979)]

$$1 + G_0 > 0 \qquad 1 + G_0' > 0$$



Equation of State: SAMi vs *ab*–initio calculations

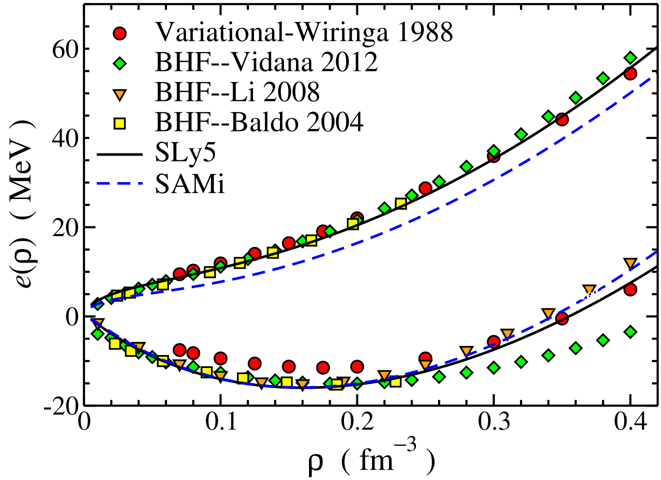


Figure: Neutron and symmetric matter EoS as predicted by the HF SAMi (dashed line) and SLy5 (solid line) interactions and by the benchmark microscopic calculations of R. B. Wiringa *et al.*, PRC **38**, 1010 (1988) (circles). State-of-the-art BHF calculations are shown by diamonds I. Vidaña, private communication, triangles Z. H. Li *et al.*, Phys. Rev. C **77**, 034316 (2008) and squares M. Baldo *et al.*, Nucl. Phys. A **736**, 241 (2004).

Finite Nulcei: spherical double-magic nuclei

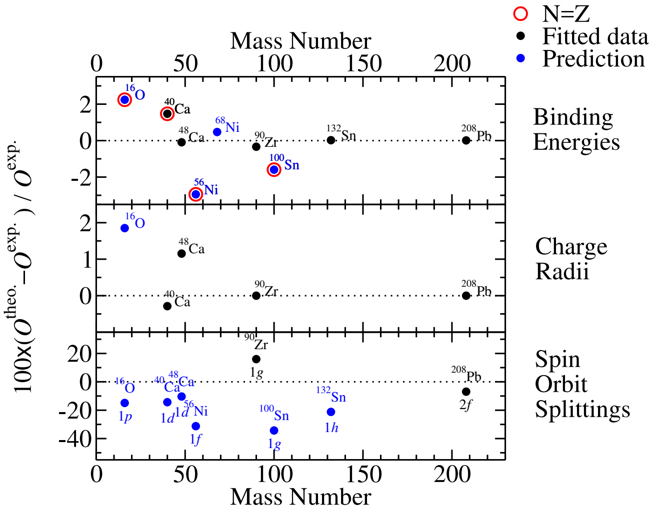


Figure: Finite nuclei properties as predicted by the HF SAMi (black circles) and some predictions (blue circles) for spherical double-magic nuclei. Experimental data taken from Refs. G. Audi *et al.*, NPA **729**, 337 (2003), I. Angeli, ADNDT **87**, 185 (2004), M. Zalewski *et al.*, PRC **77**, 024316 (2008)

Giant Monopole and Dipole Resonances in ^{208}Pb

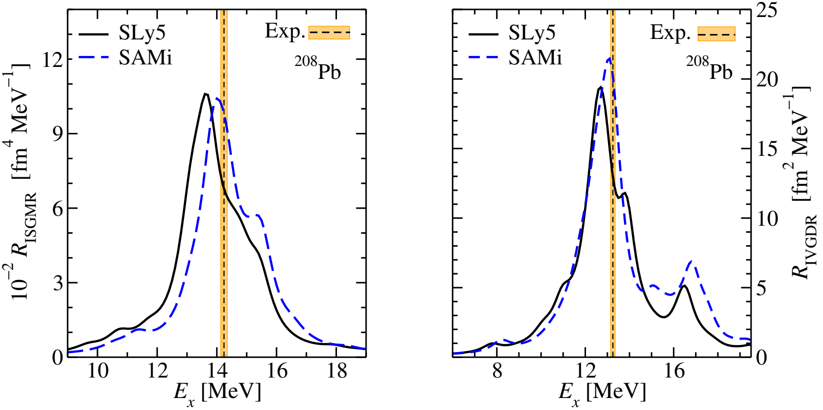
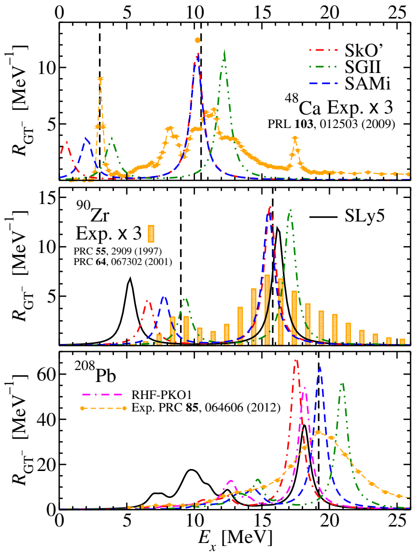


Figure: Strength function at the relevant excitation energies in ^{208}Pb as predicted by SLy5 and the SAMi interaction for GMR and GDR. A Lorentzian smearing parameter equal to 1 MeV is used. Experimental data for the centroid energies are also shown: $E_c(GMR) = 14.24 \pm 0.11$ MeV [D. H. Youngblood, et al., Phys. Rev. Lett. **82**, 691 (1999)] and $E_c(GDR) = 13.25 \pm 0.10$ MeV [N. Ryezayeva et al., Phys. Rev. Lett. **89**, 272502 (2002)].

Gamow Teller Resonance in ^{48}Ca, ^{90}Zr and ^{208}Pb

Operator:
$\sum_{i=1}^{A} \boldsymbol{\sigma}(i)\tau_{\pm}(i)$

Figure: Gamow Teller strength distributions in ^{48}Ca (upper panel), ^{90}Zr (middle panel) and ^{208}Pb (lower panel) as measured in the experiment [T. Wakasa *et al.*, Phys. Rev. C **55**, 2909 (1997), K. Yako *et al.*, Phys. Rev. Lett. **103**, 012503 (2009), A. Krasznaborkay *et al.*, Phys. Rev. C **64**, 067302 (2001), H. Akimune *et al.*, Phys. Rev. C **52**, 604 (1995) and T. Wakasa et al., Phys. Rev. C **85**, 064606 (2012)] and predicted by SLy5, SkO', SGII and SAMi forces.



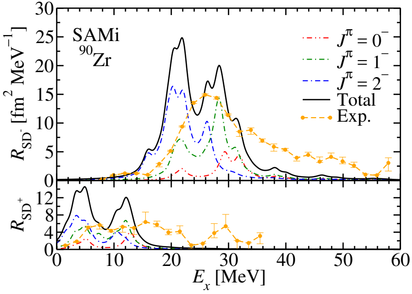
Spin Dipole Resonances in ^{90}Zr and ^{208}Pb

Operator:

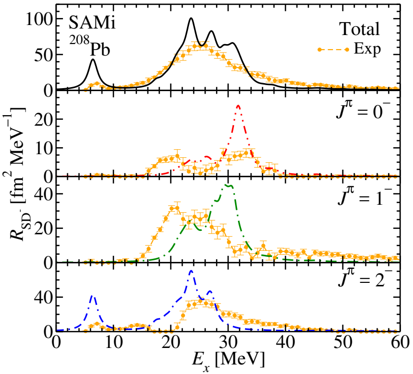
$\sum_{i=1}^{A} \sum_{M} \tau_{\pm}(i) r_i^L \left[Y_L(\hat{r}_i) \otimes \boldsymbol{\sigma}(i) \right]_{JM}$

Sum Rule:

$\int [R_{SD^-}(E) - R_{SD^+}(E)] dE =$
$\frac{9}{4\pi} \left(N \langle r_n^2 \rangle - Z \langle r_p^2 \rangle \right)$



K. Yako *et al.*, Phys. Rev. C **74**, 051303(R) (2006)



T. Wakasa *et al.*, Phys. Rev. C **85**, 064606 (2012)

Conclusions:

- we have **successfully determined a new Skyrme** energy density functional which **accounts** for the most relevant quantities in order to improve the description of **charge-exchange nuclear resonances**:
  - the **hierarchy** and **positive values** of the spin and spin-isospin Landau-Migdal parameters G_0 **and** G_0'
  - the **proton spin-orbit splittings** of different **high angular momenta** single-particle levels
- the **GTR** in ^{48}Ca and the **GTR**, **IAR**, and **SDR** in ^{90}**Zr** and ^{208}**Pb** are predicted with **good accuracy by SAMi**
- **SAMi** does **not deteriorate** the description of other **nuclear observables**
- **applicability in nuclear physics and astrophysics**

Thank you for your attention!

Extra Material

Motivation:

Gamow Teller Resonance II: quenching of the strength

- **Experimentally**, the **GTR** exhausts **60–70%** of the **Ikeda sum rule**: $\int [R_{GT^-}(E) - R_{GT^+}(E)] dE = 3(N - Z)$
- To **explain** the problem, two possibilities that go beyond RPA correlations have been drawn:
  - the effects of the second-order configuration mixing: **2p-2h correlations**
  - within the quark model, a **n(p)** can become a **p(n)** or a $\Delta^+(\Delta^{++})$ under the action of the GT^- operator and since there is **no Pauli blocking for $\Delta-$h excitations** ⇒ it may **contribute to the GTR**.
- The **experimental analysis of ^{90}Zr** ⇒ **quenching** (2/3) has to be **mainly attributed to 2p-2h** coupling and not to $\Delta-$isobar effects much smaller [T. Wakasa *et. al.*, Phys. Rev. C **55**, 2909 (1997)].
- E_x **GTR in nuclei** mainly in the region of several **tens of MeV** and the $\Delta-$h states are hundreds of MeV above the GT ⇒ **hard to excite the Δ** in the nuclear medium.

Covariance analysis: χ^2 test

Observables $\mathcal{O}$ are used to calibrate the parameters $\mathbf{p}$ of a given model. The optimum parametrization $\mathbf{p}_0$ is determined by a least-squares fit with the global quality measure,

$$\chi^2(\mathbf{p}) = \sum_{i=1}^{m} \left(\frac{\mathcal{O}_i^{\text{theo.}} - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$

Assuming that the χ^2 is a well behaved (analytical) function in the vicinity of the minimum and that can be approximated by an hyper-parabola,

$$\begin{aligned}
\chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) &\approx \frac{1}{2} \sum_{i,j}^{n} (p_i - p_{0i}) \partial_{p_i} \partial_{p_j} \chi^2 (p_j - p_{0j}) \\
&\equiv \sum_{i,j}^{n} (p_i - p_{0i}) \mathcal{M}_{ij} (p_j - p_{0j})
\end{aligned}$$

where $\mathcal{M}$ is the curvature matrix.

Covariance analysis: χ^2 test

$\mathcal{M}$ provides us access to estimate the errors between predicted observables ($A(\mathbf{p})$),

$$\Delta\mathcal{A} = \sqrt{\sum_{\imath}^{n} \partial_{p_\imath} A \mathcal{E}_{\imath\imath} \partial_{p_\imath} A} \tag{1}$$

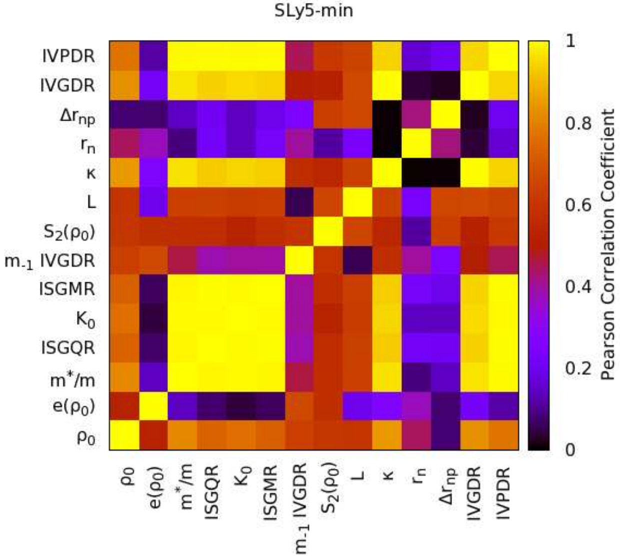
$\mathcal{E} = \mathcal{M}^{-1}$ and the correlations between predicted observables,

$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA} C_{BB}}} \tag{2}$$

where,

$$C_{AB} = \overline{(A(\mathbf{p}) - \overline{A})(B(\mathbf{p}) - \overline{B})} \approx \sum_{\imath\jmath}^{n} \partial_{p_\imath} A \mathcal{E}_{\imath\jmath} \partial_{p_\jmath} B$$

Covariance analysis: SLy5-min as an example



Covariance analysis: SLy5-min as an example

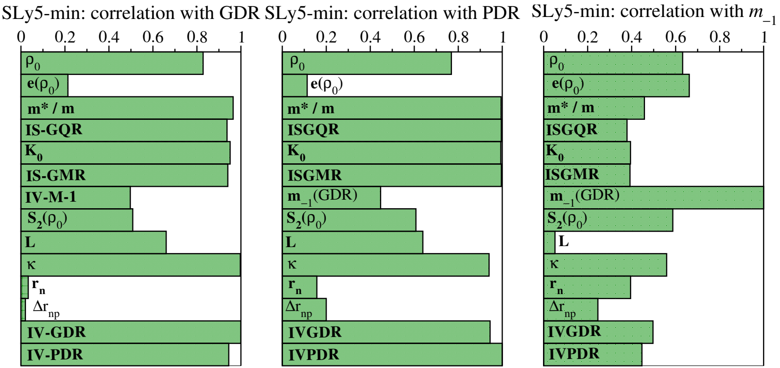


Figure: Pearson product-moment correlation coefficient for the IVGDR (left panel), IVPDR (middle panel) and m_{-1}(IVGDR) (right panel) with all other studied properties as predicted by the covariance analysis of SLy5.