Microscopic mass models for astrophysics

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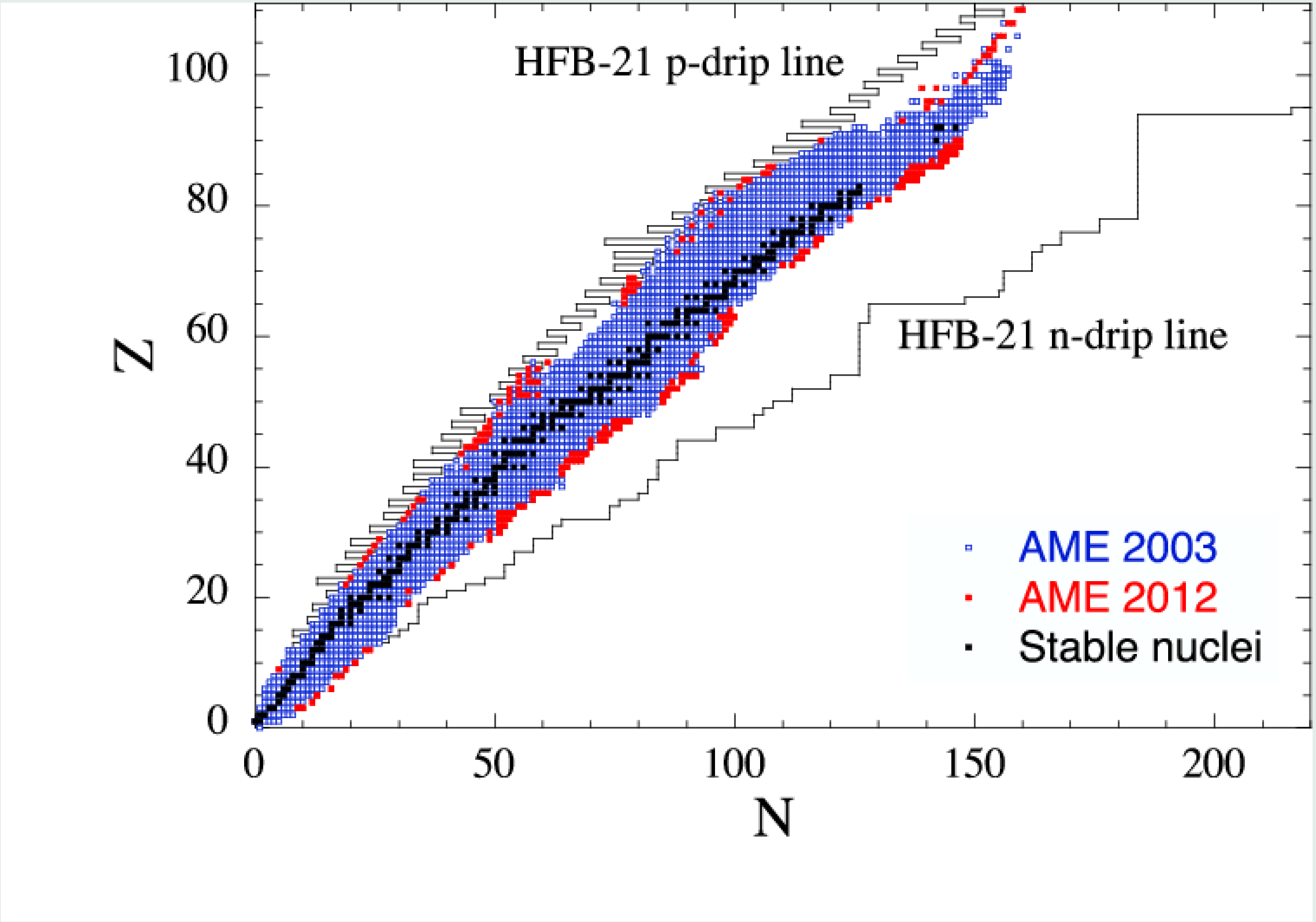

Fig. 1. Nuclear chart

Thousands of nuclei on neutron-rich side remain to be measured

Serious problem for astrophysics:

need masses of nuclei right out to the n-drip line for

i) r-process of nucleosynthesis

ii) EoS of neutron-star outer crust

How to extrapolate from data?

THEORY

Liquid-drop(let) mass models

- all derived from Weizsäcker model of 1935
- parameters fitted to masses: "semi-empirical"
- extensively refined since 1935
- latest form is FRDM - finite-range droplet model

macroscopic-microscopic approach -
shell (including deformation) and pairing
corrections grafted on to liquid-drop picture.

Liquid-drop(let) mass models OK in principle for the bound, finite nuclei of the r-process and outer crust of neutron stars.

However, might question reliability for extrapolation to much larger values of neutron number N.

A further problem arises when we consider neutron stars in more detail.

Outer crust. $\simeq$ 300 meters thick. $0 < n < 1.6 \times 10^{-3}\, n_0$
– n-rich nuclei (+ electrons); within neutron drip line.

Inner crust. $\simeq$ 500 meters thick. $1.6 \times 10^{-3}\, n_0 < n < 0.4\, n_0$
– nuclear clusters floating in neutron vapour (+ electrons); beyond neutron drip line.

Core. 10 km radius (roughly). n up to about $6\, n_0$
– homogeneous gas of n and p (+ electrons). About 97 % n at n around n_0; other particles towards centre.

Electrical neutrality everywhere assured by electron gas:beta-equilibrated with nucleons.

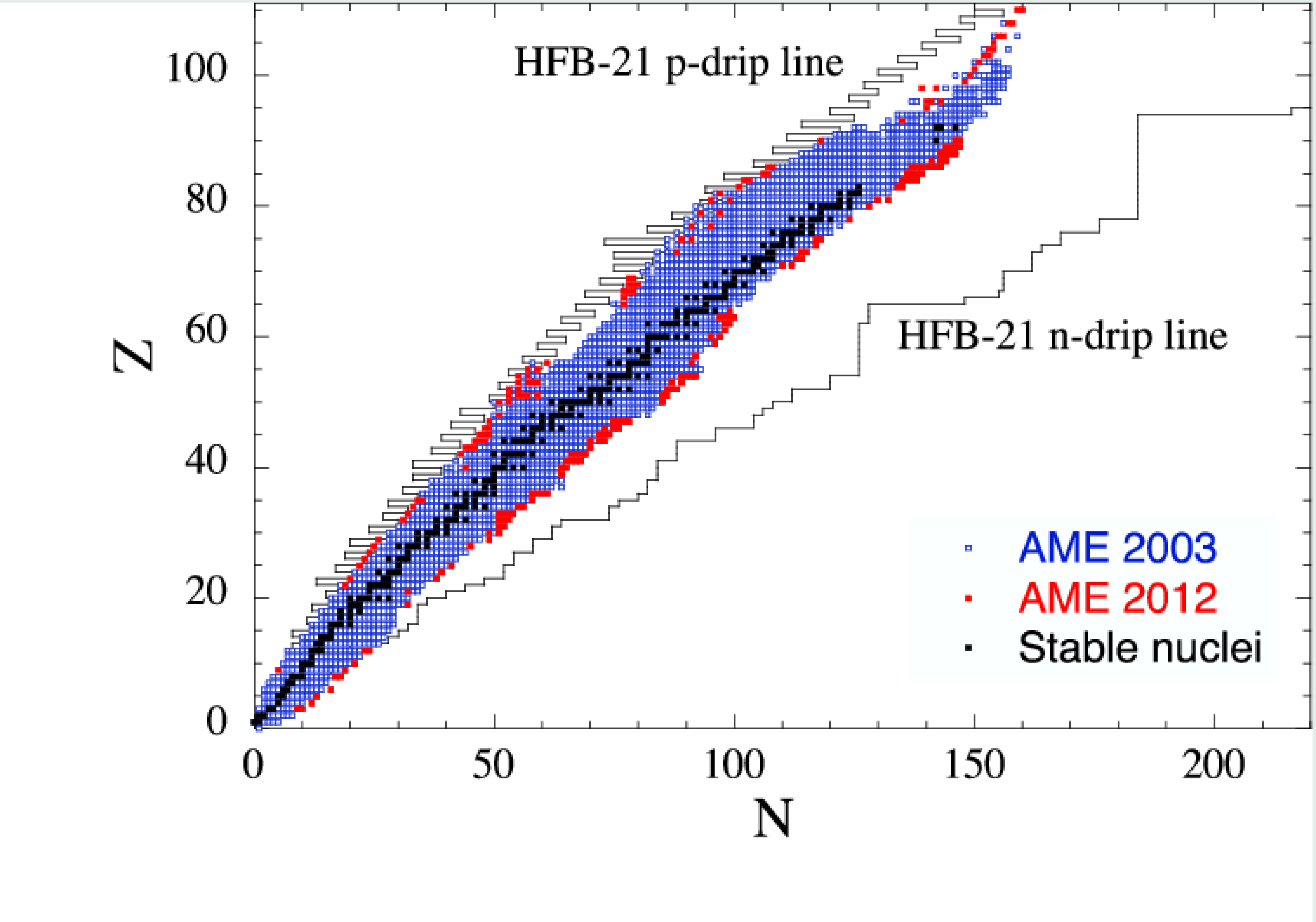

Fig. 1. Nuclear chart

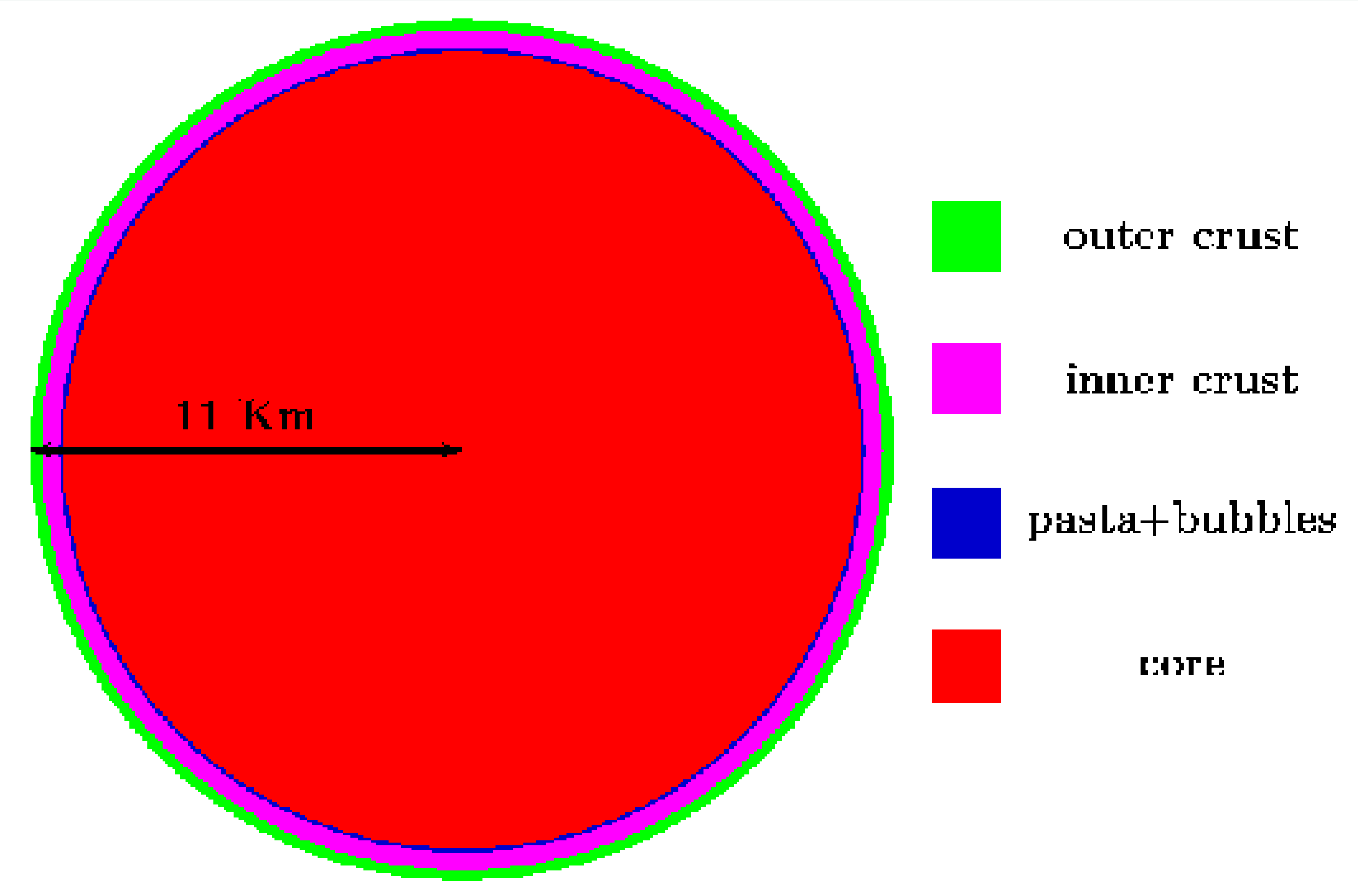

Fig. 2. Neutron star.

Seek unified treatment of all parts of neutron star (as well as r-process nuclei).

Have to extrapolate beyond neutron drip line.

But liquid-drop models cannot do this very well.

Need a microscopic approach.

Hartree-Fock-Bogoliubov mass models

- Microscopic effects (shell-model and pairing) incorporated on same footing as macroscopic effects completely self-consistently.

- Based on *effective* forces: free parameters that are fitted not to two- and three-nucleon data but to mass data themselves.

– semi-empirical tradition of Weizsäcker

We require our mass models to give best possible fit to data, and to extrapolate masses as reliably as possible beyond the known region out to neutron drip line.

Effective force underlying mass model will also be used for calculation of inner crust and core.

Skyrme force 16 parameters

$$
\begin{aligned}
v_{ij} = & \; t_0(1+x_0P_\sigma)\delta(\boldsymbol{r}_{ij}) \\
& + \frac{1}{2}t_1(1+x_1P_\sigma)\frac{1}{\hbar^2}\left[p_{ij}^2\,\delta(\boldsymbol{r}_{ij}) + \delta(\boldsymbol{r}_{ij})\,p_{ij}^2\right] \\
& + t_2(1+x_2P_\sigma)\frac{1}{\hbar^2}\boldsymbol{p}_{ij}.\delta(\boldsymbol{r}_{ij})\,\boldsymbol{p}_{ij} + \frac{1}{6}t_3(1+x_3P_\sigma)\,n(\boldsymbol{r})^\alpha\,\delta(\boldsymbol{r}_{ij}) \\
& + \frac{1}{2}t_4(1+x_4P_\sigma)\frac{1}{\hbar^2}\left[p_{ij}^2\,n(\boldsymbol{r})^\beta\,\delta(\boldsymbol{r}_{ij}) + \delta(\boldsymbol{r}_{ij})\,n(\boldsymbol{r})^\beta\,p_{ij}^2\right] \\
& + t_5(1+x_5P_\sigma)\frac{1}{\hbar^2}\boldsymbol{p}_{ij}.n(\boldsymbol{r})^\gamma\,\delta(\boldsymbol{r}_{ij})\,\boldsymbol{p}_{ij} \\
& + \frac{\mathrm{i}}{\hbar^2}W_0(\boldsymbol{\sigma_i}+\boldsymbol{\sigma_j})\cdot\boldsymbol{p}_{ij}\times\delta(\boldsymbol{r}_{ij})\,\boldsymbol{p}_{ij}
\end{aligned}
$$

Pairing force

n-n and p-p only $\quad (T = |T_z| = 1)$

$$v_q^{\text{pair}}(\boldsymbol{r_i}, \boldsymbol{r_j}) = v^{\pi\, q}[\rho_q(\boldsymbol{r})]\ \delta(\boldsymbol{r}_{ij})$$

$v[\rho_n, \rho_p]$ is a functional of both neutron and proton densities, calculated analytically at each point in the nucleus to reproduce the ${}^1S_0$ pairing gaps of INM of the appropriate density and charge asymmetry, as determined by many-body calculations with realistic two- and three-nucleon forces.

Fine-tuning of the strengths in the form of the four global renormalization parameters $f^{\pi\, q}$, which allow the overall strength to be slightly different for neutrons than for protons, and which also permit each of these strengths to depend on whether there is an even or odd number of nucleons of given charge type.

+ cutoff parameter $\quad \Rightarrow \quad$ 5 pairing parameters

Wigner terms

If Skyrme and pairing forces are only ingredients then serious underbinding (≈ 2 MeV) for $|N - Z| \leq 2$.

Two possible sources:

i) n-p $T = 0$ pairing. Attractive, strongly peaked at $N = Z$.

ii) Wigner supermultiplet theory. In real nucleus isospin T conserved (approximately).
Ground state: $T = |T_z| = |N - Z|$.
Wigner (1937): there should be an energy term $T(T+1)$ - positive. In g. s. we have

$$T(T+1) = (N-Z)^2 + |N-Z|$$

We represent the two together by

$$E_W = V_W \exp\left\{-\lambda\left(\frac{N-Z}{A}\right)^2\right\} + V'_W|N-Z|\exp\left\{-\left(\frac{A}{A_0}\right)^2\right\}$$

Isospin is a good quantum number only for light nuclei, so we put in exponential damping w. r. t. A in second term.

Fits always give $A_0 \approx 30$.

First term is more important.

Coulomb exchange dropped.

B. A. Brown, Phys. Rev. C **58**, 220 (1998).

Leads to distinct improvement,

esp. mirror nuclei (~ 0.7 MeV)

Originates in neglected Coulomb correlations, vacuum polarization and charge-symmetry breaking.

Correction for spurious collective motion.

Must subtract spurious collective energy.

$$E_{coll} = E_{rot}^{crank}[b \ \tanh(c|\beta_2|) + d|\beta_2| \ \exp\{-l(|\beta_2| - \beta_2^0)^2\}]$$

E_{rot}^{crank}: cranking-model value of the rotational correction

β_2: quadrupole deformation

First term: rotational correction

Second term: takes account of the *deformation dependence* of the vibrational correction

5 parameters

Odd nuclei

(odd-A and odd-odd)

Equal-filling approximation

(second great time-saver)

Constraints from real forces on effective forces

Infinite, i.e., homogeneous, nuclear matter (INM) only many-body system other than very light nuclei ($A \leq 16$) that can be reliably calculated *ab initio* from realistic N-N and N-N-N forces. Require:

- i) Pairing strength reproduces pairing gap of INM of appropriate density and neutron/proton ratio.

- ii) Skyrme parameters reproduce EoS of pure neutron matter at high densities.

This improves the reliability of the mass predictions for highly neutron-rich nuclei.

Moreover, such forces are well adapted to the calculation of the EoS of the inner crust and core of neutron stars with the HFB method (or approximations thereto).

While the fit to neutron matter is appropriate to the the highly neutron-rich environment, the mass fit takes into account:

i) presence of protons (inner crust and core)

ii) inhomogeneities (inner crust)

Also constrain Skyrme force to give for the isoscalar effective mass at equilibrium density n_0 the realistic value of

$$M_s^* = 0.8M$$

No constraint on isovector effective mass M_v^*, but it always comes out to be slightly smaller than isoscalar effective mass, as required by both experiment and realistic many-body calculations.

PROBLEM:

Many different calculations of neutron matter –

- agree at nuclear and subnuclear densities
- diverge at densities of neutron-star cores

SOLUTION of

Goriely *et al.* Phys. Rev. C **82**, 035804 (2010) –

Fitted three different EoSs of neutron matter, along with data of 2003 Atomic Mass Evaluation of Audi *et al.* (2149 mesured masses with Z and $N \geq 8$)

Set symmetry coeff. $J = 30$ MeV

$$\sigma_{rms} = 0.58 \text{ MeV}$$

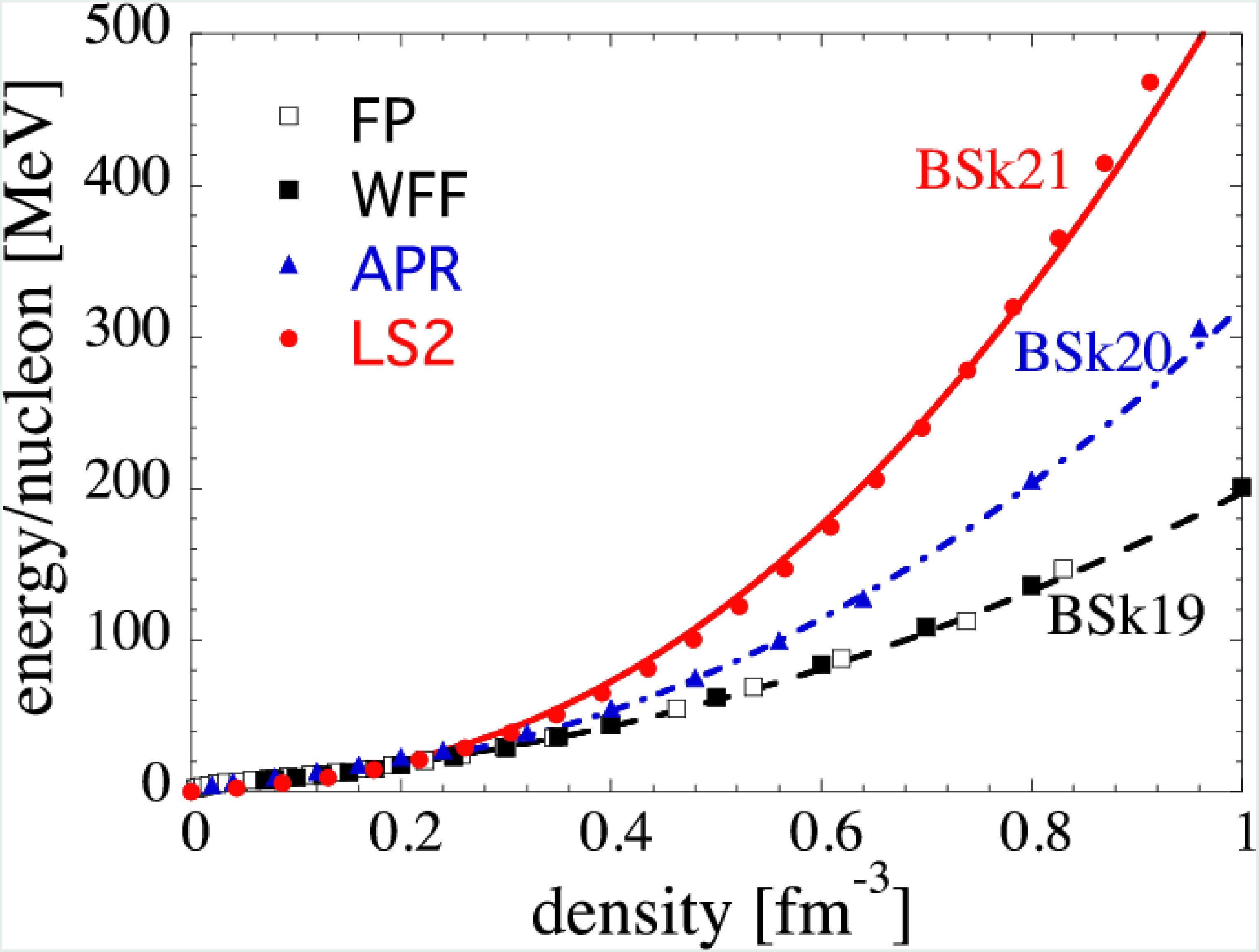

Fig. 3. EoSs of neutron matter.

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Maximum mass of neutron star for a given model –

must solve Tolman-Oppenheimer-Volkoff equations

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{G\rho(r)\mathcal{M}(r)}{r^2}[1+\frac{P(r)}{c^2\rho(r)}][1+\frac{4\pi P(r)r^3}{c^2\mathcal{M}(r)}] \times [1-\frac{2G\mathcal{M}(r)}{c^2 r}]^{-1}$$

$$\mathcal{M}(r) = 4\pi\int_0^r \rho(r')r'^2\mathrm{d}r' \quad .$$

Maximum neutron-star mass for different models.

Force	$\mathcal{M}_{max}/\mathcal{M}_\odot$
BSk19	1.86
BSk20	2.15
BSk21	2.28

Observation of 2 $\mathcal{M}_\odot$ neutron stars

PSR J1614−2230 $1.97 \pm 0.04\ \mathcal{M}_\odot$ (2010)

PSR J0348+0432 $2.01 \pm 0.04\ \mathcal{M}_\odot$ (2013)

Rules out FP EoS and model BSk19

Nov. 2012: New Atomic Mass Evaluation.

2353 measured masses with Z and $N \geq 8$

so new fits

high densities: constrain generally to LS2 EoS of neutron matter, but also to APR; but NOT to FP.

nuclear densities: see if we can determine symmetry coefficient J from mass fits.

Rms (σ) and mean ($\bar{\epsilon}$) deviations between data and predictions. First pair of lines refers to all the 2353 measured masses M with N and $Z \geq 8$, and second pair to masses M_{nr} of subset of 257 neutron-rich nuclei with $S_n \leq 5.0$ MeV.

	HFB-22	HFB-23	HFB-24	HFB-25	HFB-26
J [MeV]	32	31	30	29	30
Neutron matter	LS2	LS2	LS2	LS2	APR
$\sigma(M)$ [MeV]	0.629	0.569	0.549	0.544	0.564
$\bar{\epsilon}(M)$ [MeV]	-0.043	-0.022	-0.012	0.008	0.006
$\sigma(M_{nr})$ [MeV]	0.817	0.721	0.702	0.791	0.749
$\bar{\epsilon}(M_{nr})$ [MeV]	0.221	0.090	0.011	0.023	0.230

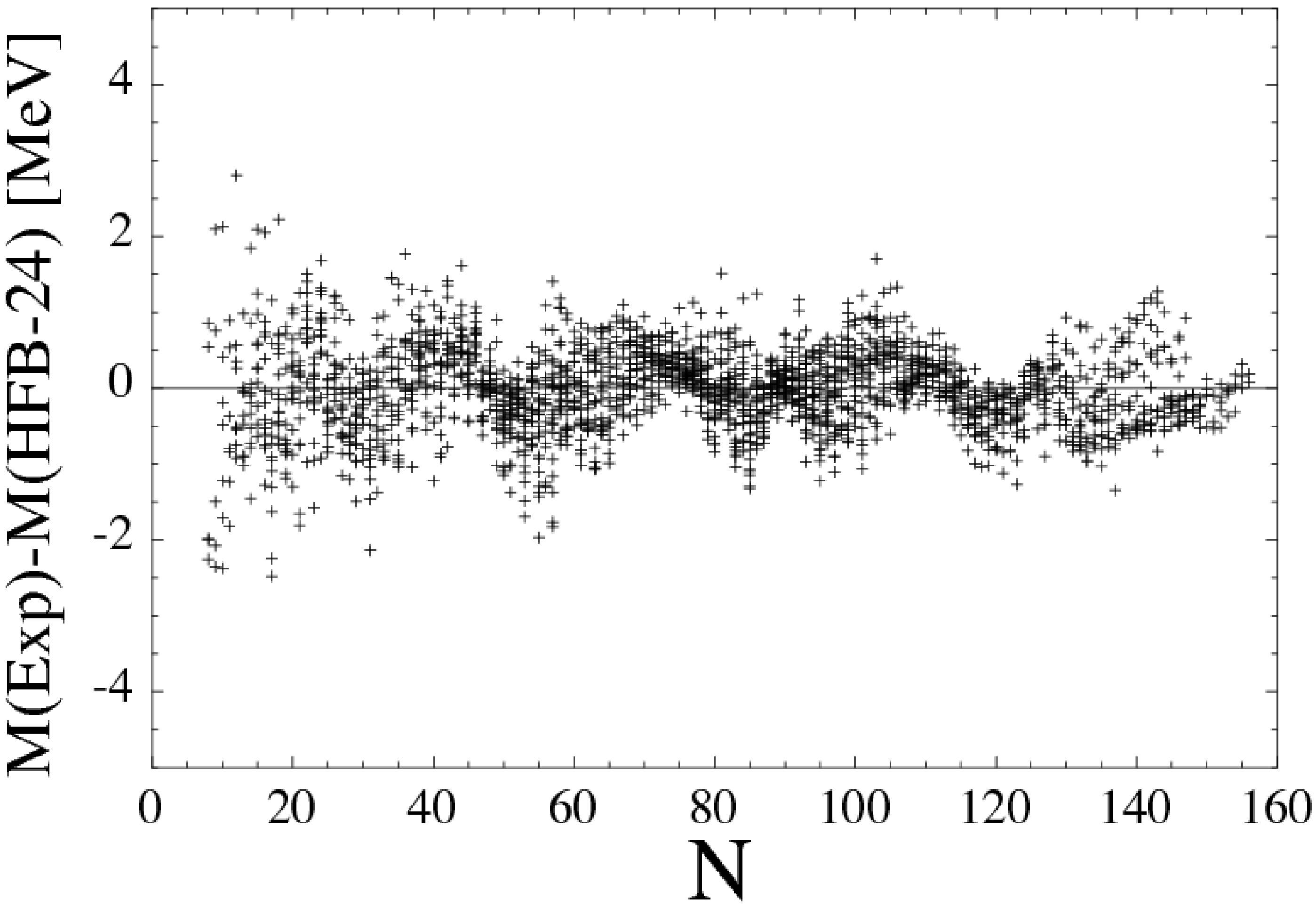

Fig. 4. Deviation of HFB-24 masses from experiment (AME 2012)

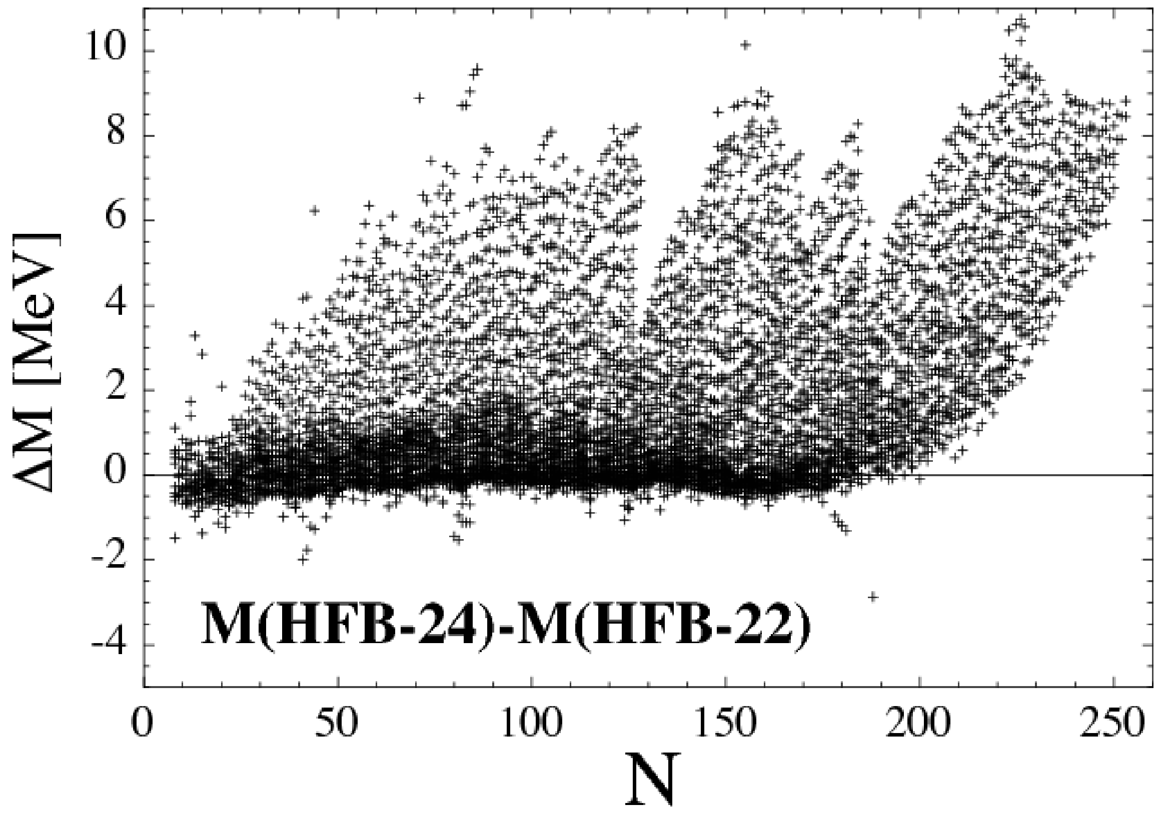

Fig. 5. Differences between HFB-24 and HFB-22 masses.

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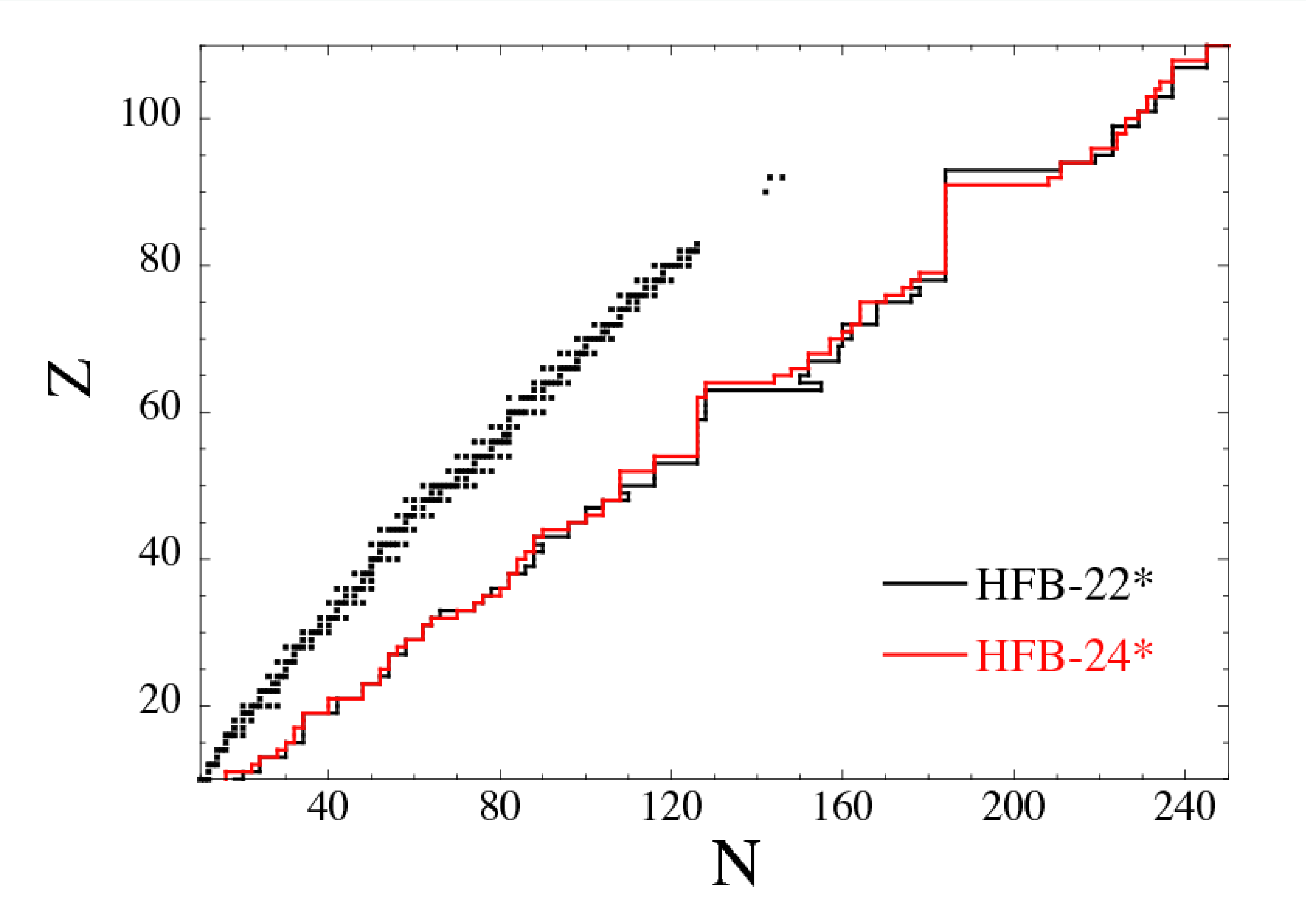

Fig. 6. Neutron drip lines for HFB-22 and HFB-24

Higher-order symmetry parameters.

Energy per nucleon of INM of density n and asymmetry $\eta = (n_n - n_p)/n$

$$e(n, \eta) = e(n, \eta = 0) + e_{sym}(n)\eta^2 + O(\eta^4) \quad ,$$

Expand around equilibrium density n_0 of symmetric INM ($\eta = 0$)

$$e_{sym}(n) = J + \frac{1}{3}L\epsilon + \frac{1}{18}K_{sym}\epsilon^2 + \ldots \quad ,$$

where $\epsilon = (n - n_0)/n_0$.

	BSk22	BSk23	BSk24	BSk25	BSk26
J [MeV]	32.0	31.0	30.0	29.0	30.0
L [MeV]	68.5	57.8	46.4	36.9	37.5
K_{sym} [MeV]	13.0	-11.3	-37.6	-28.5	-135.6

Maximum neutron-star mass for different models.

Force	$\mathcal{M}_{max}/\mathcal{M}_{\odot}$
BSk22	2.26
BSk23	2.27
BSk24	2.28
BSk25	2.22
BSk26	2.15

Maximum mass determined almost completely by EoS of neutron matter at core densities.

J is irrelevant.

Unified treatment of all parts of neutron star

For each of these interactions (BSk22 – 26) and their respective mass tables (HFB-22 – HFB-26) we can now calculate all parts of neutron star.

Also the r-process.

Conclusions

- Developed microscopic mass models that fit 2012 data with precision.
- Best fit for $J = 30$ MeV ($\sigma = 0.549$ MeV).
- These models well adapted to extrapolation to highly neutron-rich environments.
- Can be used for r-process and outer crust of neutron stars.
- Underlying interactions can be used to calculate inner crust and core of neutron stars.
- Generalization to collapsing SN cores ($T > 0$) possible.