Model independent approaches of time-like electromagnetic transitions

B. Sarantsev

HISKP Uni-Bonn (Germany)

PNPI (Russia)

October 2016

General structure of the single–meson electro-production amplitude in c.m.s. of the reaction is given by

$$J_\mu = i\mathcal{F}_1\tilde{\sigma}_\mu + \mathcal{F}_2(\vec{\sigma}\vec{q})\frac{\varepsilon_{\mu ij}\sigma_i k_j}{|\vec{k}||\vec{q}|} + i\mathcal{F}_3\frac{(\vec{\sigma}\vec{k})}{|\vec{k}||\vec{q}|}\tilde{q}_\mu + i\mathcal{F}_4\frac{(\vec{\sigma}\vec{q})}{\vec{q}^2}\tilde{q}_\mu$$
$$+ i\mathcal{F}_5\frac{(\vec{\sigma}\vec{k})}{|\vec{k}|^2}k_\mu + i\mathcal{F}_6\frac{(\vec{\sigma}\vec{q})}{|\vec{q}||\vec{k}|}k_\mu \qquad \mu = 1, 2, 3,$$

where $\vec{q}$ is the momentum of the nucleon in the πN channel and $\vec{k}$ the momentum of the nucleon in the γN channel calculated in the c.m.s. of the reaction. The σ_i are Pauli matrices.

$$\tilde{\sigma}_\mu = \sigma_\mu - \frac{\vec{\sigma}\vec{k}}{|\vec{k}|^2}k_\mu \qquad \mu = 1, 2, 3$$

$$\tilde{q}_\mu = q_\mu - \frac{\vec{q}\vec{k}}{|\vec{k}||\vec{q}|}k_\mu = q_\mu - z\,k_\mu$$

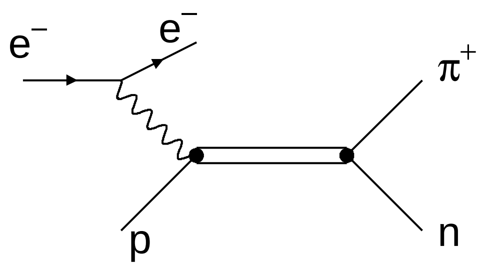
$$J_0 k_0^\gamma = J_\mu k_\mu^\gamma$$

The functions $\mathcal{F}_i$ have the following angular dependence:

$$\begin{aligned}
\mathcal{F}_1(z) &= \sum_{L=0}^{\infty} [LM_L^+ + E_L^+]P'_{L+1}(z) + [(L+1)M_L^- + E_L^-]P'_{L-1}(z), \\
\mathcal{F}_2(z) &= \sum_{L=1}^{\infty} [(L+1)M_L^+ + LM_L^-]P'_L(z) \, , \\
\mathcal{F}_3(z) &= \sum_{L=1}^{\infty} [E_L^+ - M_L^+]P''_{L+1}(z) + [E_L^- + M_L^-]P''_{L-1}(z) \, , \\
\mathcal{F}_4(z) &= \sum_{L=2}^{\infty} [M_L^+ - E_L^+ - M_L^- - E_L^-]P''_L(z) \, , \\
\mathcal{F}_5(z) &= \sum_{L=0}^{\infty} [(L+1)S_L^+ P'_{L+1}(z) - LS_L^- P'_{L-1}(z)] \, , \\
\mathcal{F}_6(z) &= \sum_{L=1}^{\infty} [LS_L^- - (L+1)S_L^+]P'_L(z)
\end{aligned}$$

Here L corresponds to the orbital angular momentum in the πN system, $P'_L(z)$, $P''_L(z)$ are derivatives of Legendre polynomials $z = (\vec{k}\vec{q})/(|\vec{k}||\vec{q}|)$.

Electro-production of pseudoscalar mesons



$$A = \omega^* J_\mu \omega' \bar{u}(k_f)\gamma_\mu u(k_i)\frac{e}{q^2}$$

$$|A|^2 = J_\mu J_\nu^* \frac{e^2}{2Q^4}\left(2K_\mu K_\nu + \frac{q^2}{2}g_{\mu\nu} - \frac{1}{2}q_\mu q_\nu + ih\varepsilon_{\mu\nu\alpha\beta}q_\alpha K_\beta\right)$$

$$\frac{d\sigma}{d\Omega_f d\varepsilon_f d\Omega_\pi} = \Gamma\frac{d\sigma_v}{d\Omega_\pi} \qquad \Gamma = \frac{\alpha}{2\pi^2}\frac{\varepsilon_f}{\varepsilon_i}\frac{|k_\gamma|}{Q^2}\frac{1}{1-\varepsilon}$$

$$\frac{d\sigma_v}{d\Omega_\pi} = \frac{d\sigma_T}{d\Omega_\pi} + \varepsilon_L\frac{d\sigma_L}{d\Omega_\pi} + \left[2\varepsilon_L(1+\varepsilon)\right]^{\frac{1}{2}}\frac{d\sigma_{TL}}{d\Omega_\pi}\cos\Phi_\pi + \varepsilon\frac{d\sigma_{TT}}{d\Omega_\pi}\cos 2\Phi_\pi$$
$$+ \quad h\left[2\varepsilon_L(1-\varepsilon)\right]^{\frac{1}{2}}\frac{d\sigma_{TL'}}{d\Omega_\pi} + h(1-\varepsilon^2)^{\frac{1}{2}}\frac{d\sigma_{TT'}}{d\Omega_\pi}$$

ε_i, k_i,ε_f, k_f- momenta of the initial and final electrons ($K = \frac{1}{2}(k_i + k_f)$). $\vec{q}$ and Θ_e are evaluated in the lab. frame. h is the helicity of the incoming electron. Amaldi et a1 1979, Donnachie and Shaw 1978

Electro-production of pseudoscalar mesons

let us introduce:

$$H_{\mu\nu} = \frac{4\pi}{W^2} \sum_{ij} J_\mu^{i*} J_\nu^j$$

then at $\Phi_\pi = 0$**:**

$$\frac{d\sigma_T}{d\Omega_\pi} = \frac{|\vec{k}_{\pi N}|}{|\vec{k}_{\gamma^* N}|} \frac{H_{xx} + H_{yy}}{2} \qquad \frac{d\sigma_L}{d\Omega_\pi} = \frac{|\vec{k}_{\pi N}|}{|\vec{k}_{\gamma^* N}|} H_{zz}$$

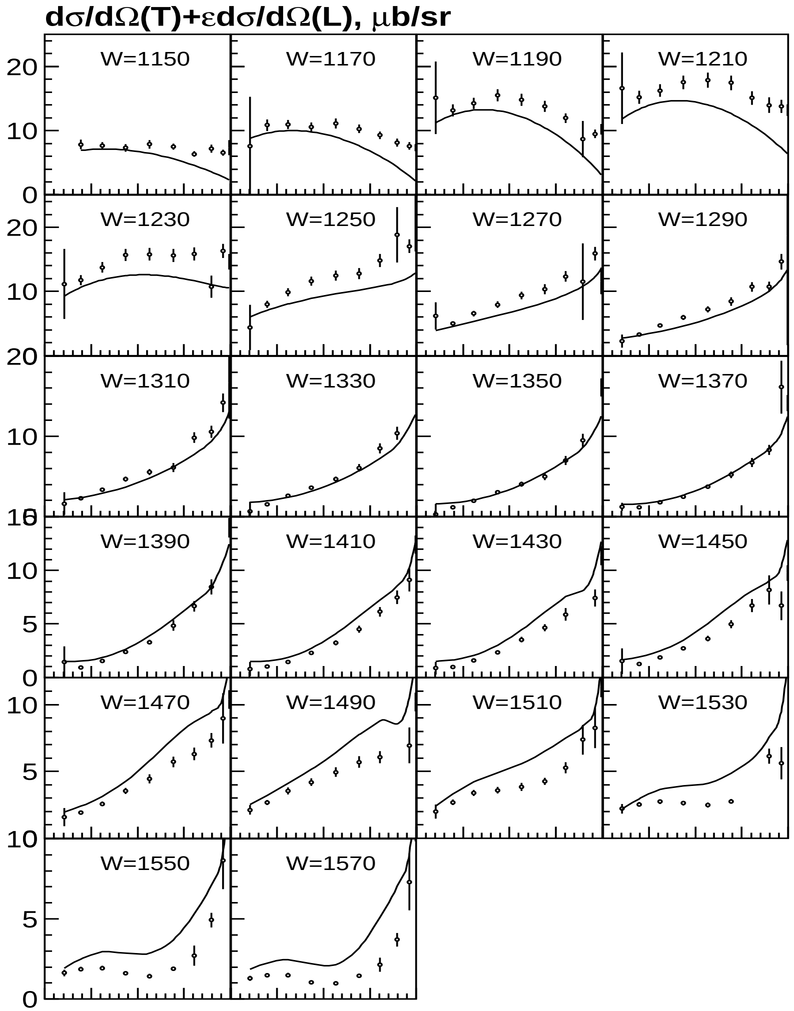
$$\frac{d\sigma_{TT}}{d\Omega_\pi} = \frac{|\vec{k}_{\pi N}|}{|\vec{k}_{\gamma^* N}|} \frac{H_{xx} - H_{yy}}{2} \qquad \frac{d\sigma_{TL}}{d\Omega_\pi} = \frac{|\vec{k}_{\pi N}|}{|\vec{k}_{\gamma^* N}|} \left(- \mathbf{Re} H_{xz} \right)$$

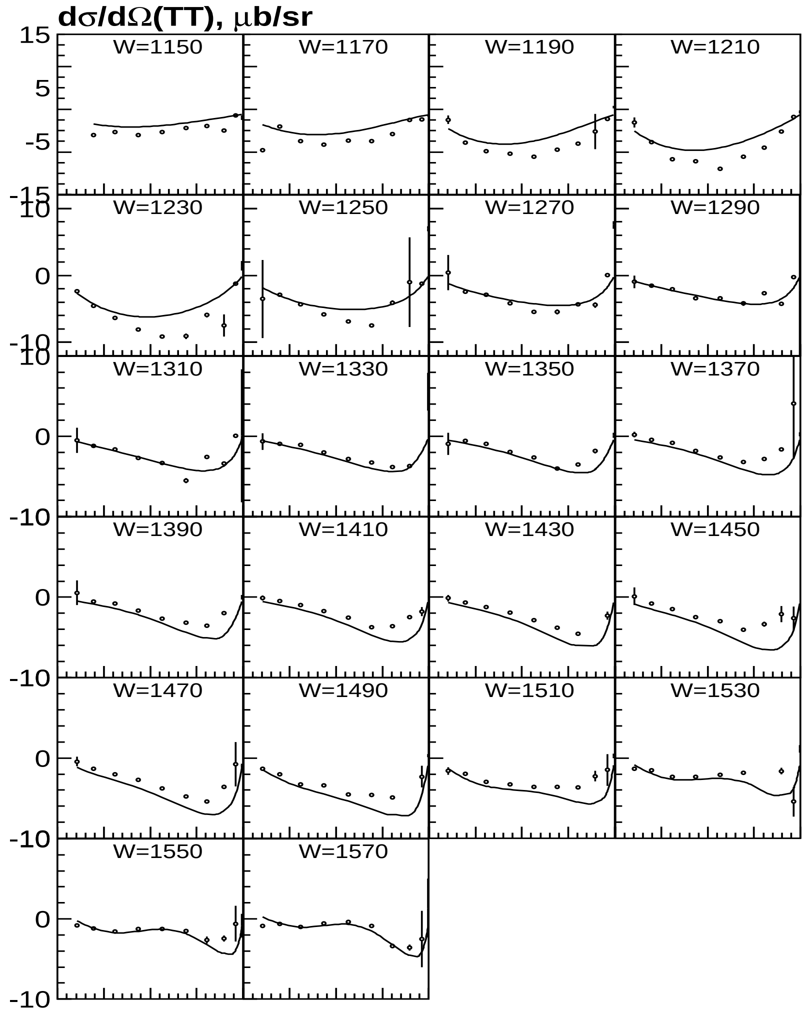
and at $\Phi_\pi = 90^o$**:**

$$\frac{d\sigma_{TL'}}{d\Omega_\pi} = \frac{|\vec{k}_{\pi N}|}{|\vec{k}_{\gamma^* N}|} \mathbf{Im} H_{yz}$$

Dieter Drechsel and Lothar Tiator, J. Phys. G: Nucl. Part. Phys. 18 (1992) 449-497.

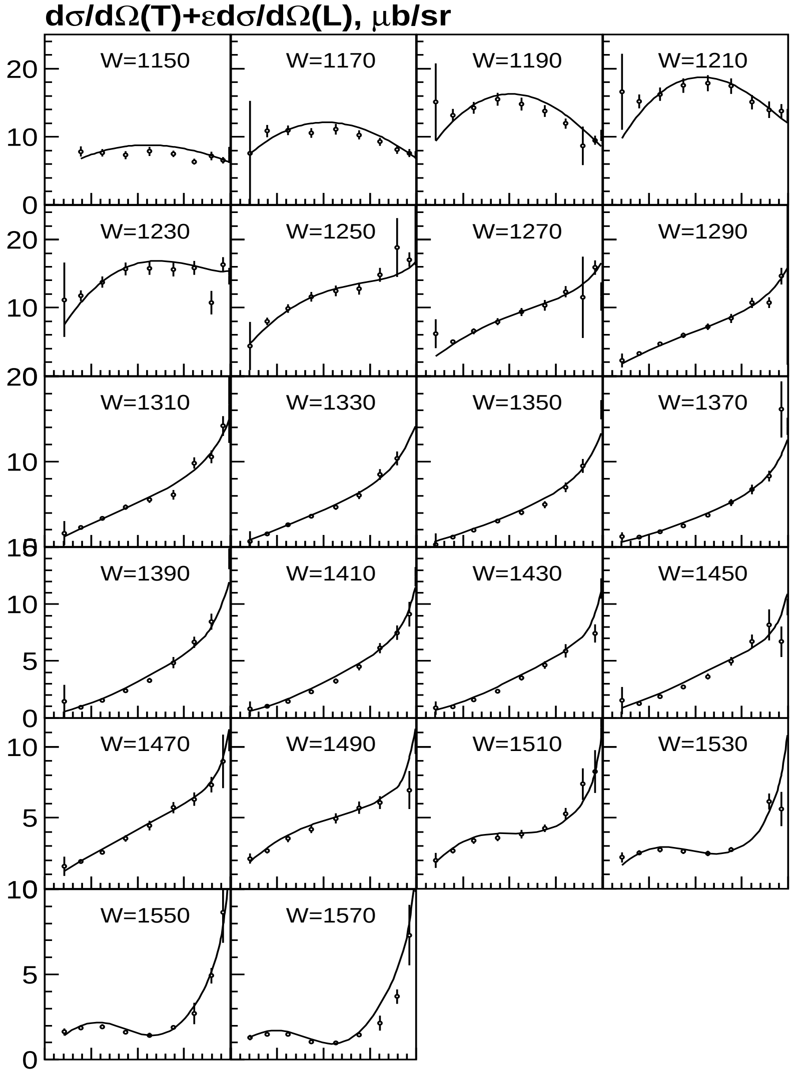
The photoproduction solution and electroproduction data at $Q^2 = -0.3$ GeV^2

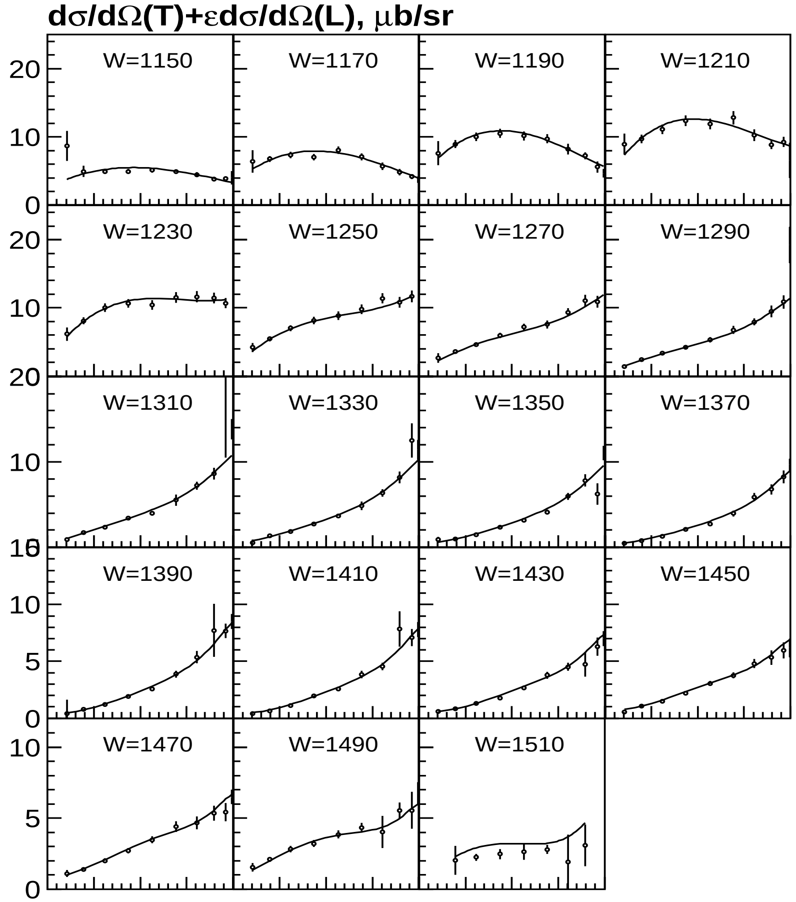




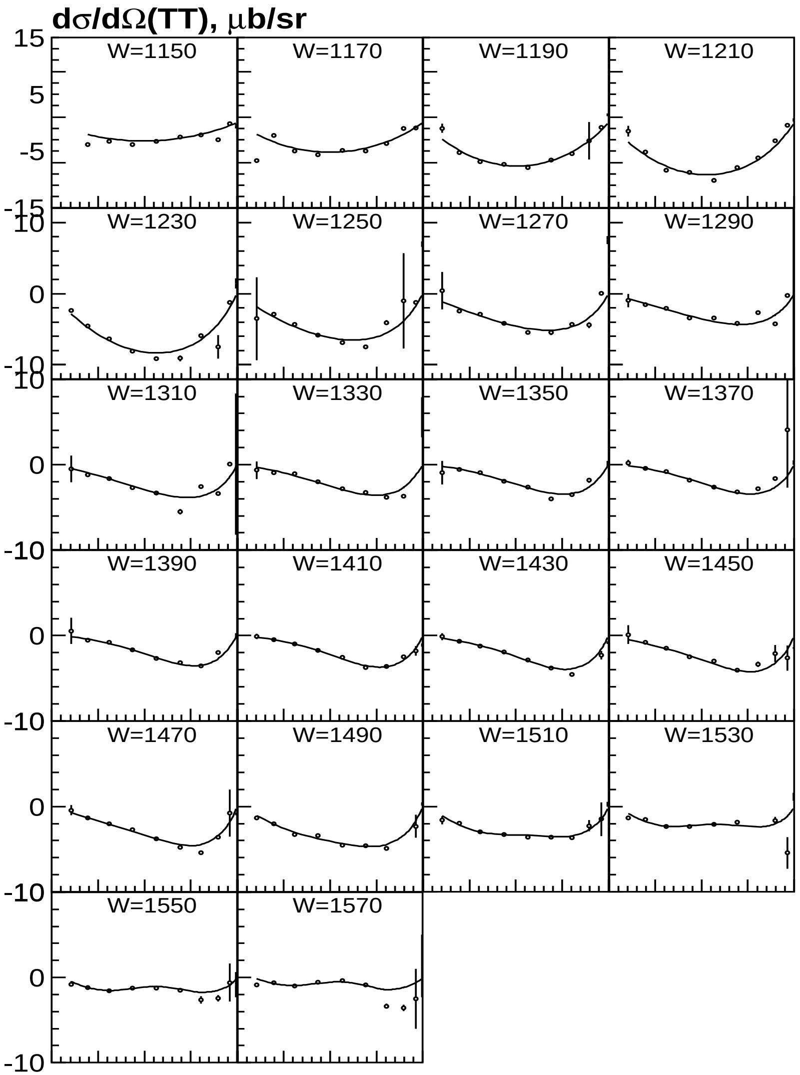
H. Egiyan et al. (CLAS Collaboration) Phys. Rev. C 73, 025204

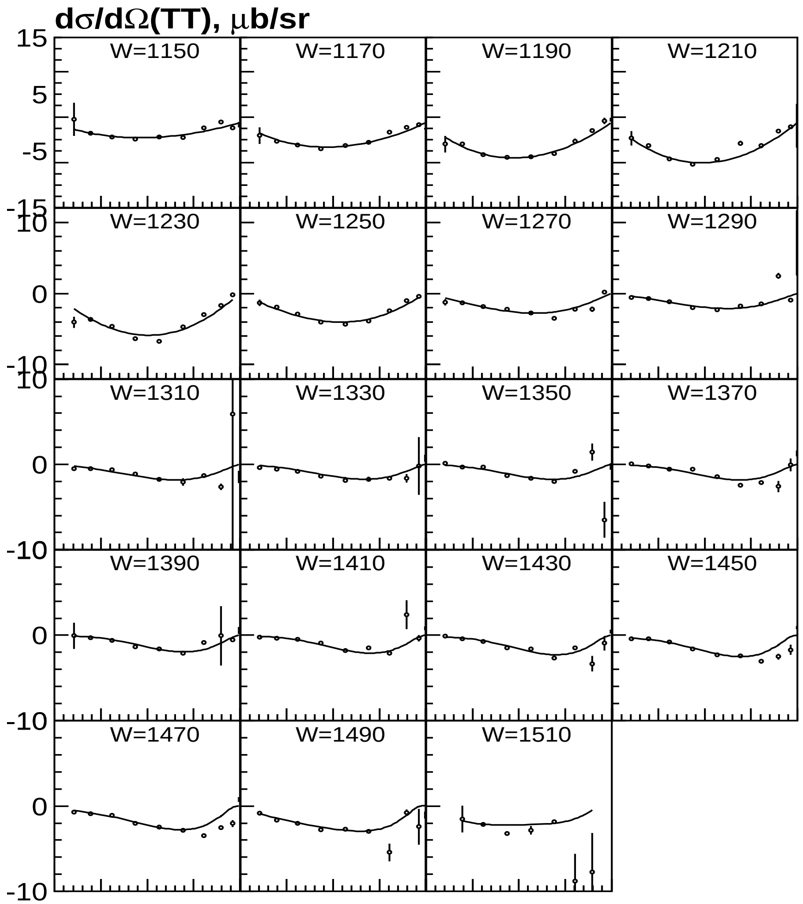
The fit of the $\frac{d\sigma_T}{d\Omega} + \varepsilon \frac{d\sigma_L}{d\Omega}$ electroproduction data at $Q^2 = -0.3$ and $Q^2 = -0.5$ GeV^2



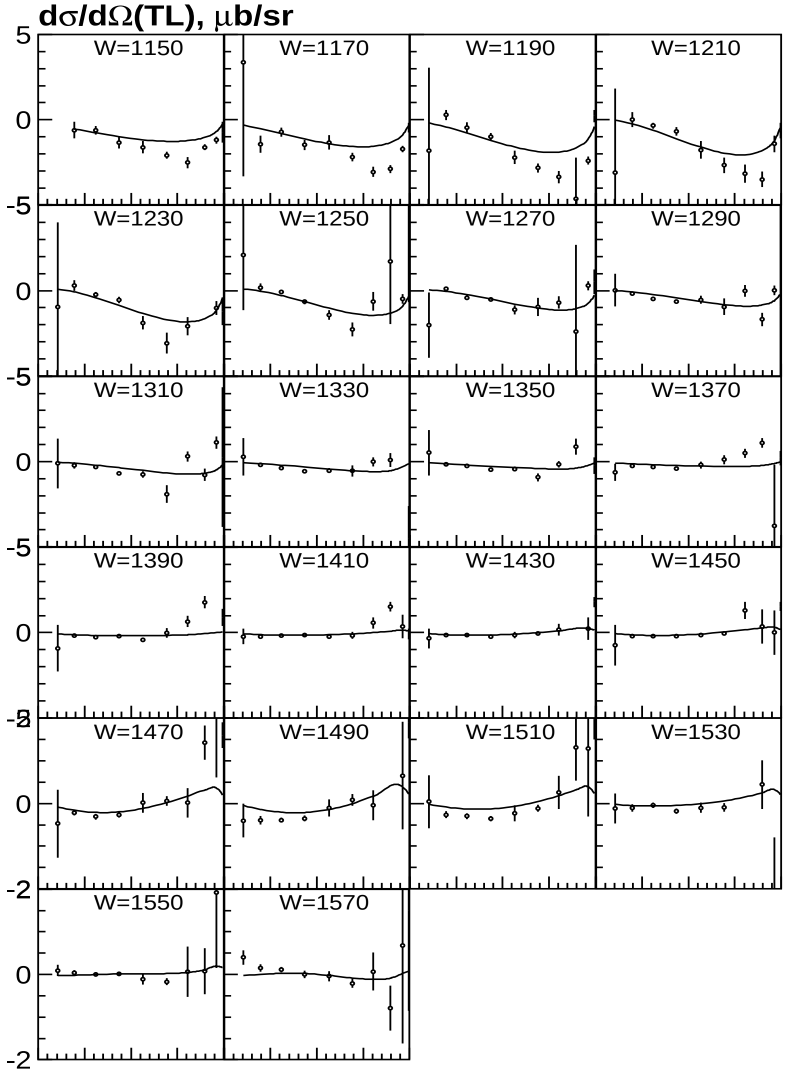


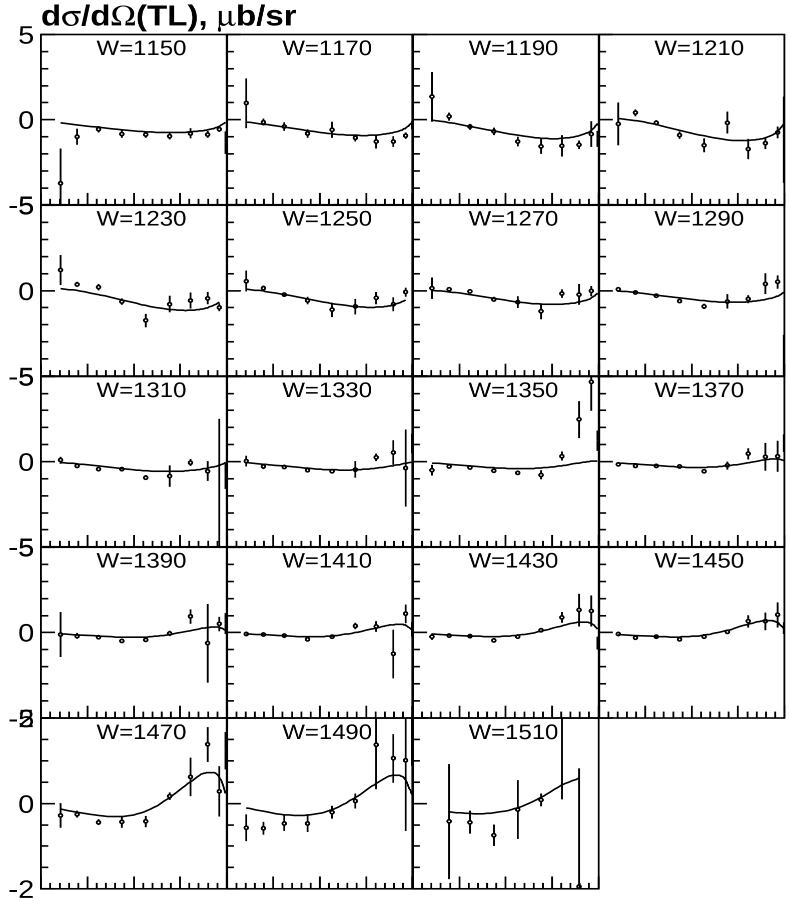
The fit of the $\frac{d\sigma_{TT}}{d\Omega}$ electroproduction data at $Q^2 = -0.3$ and $Q^2 = -0.5$ GeV^2



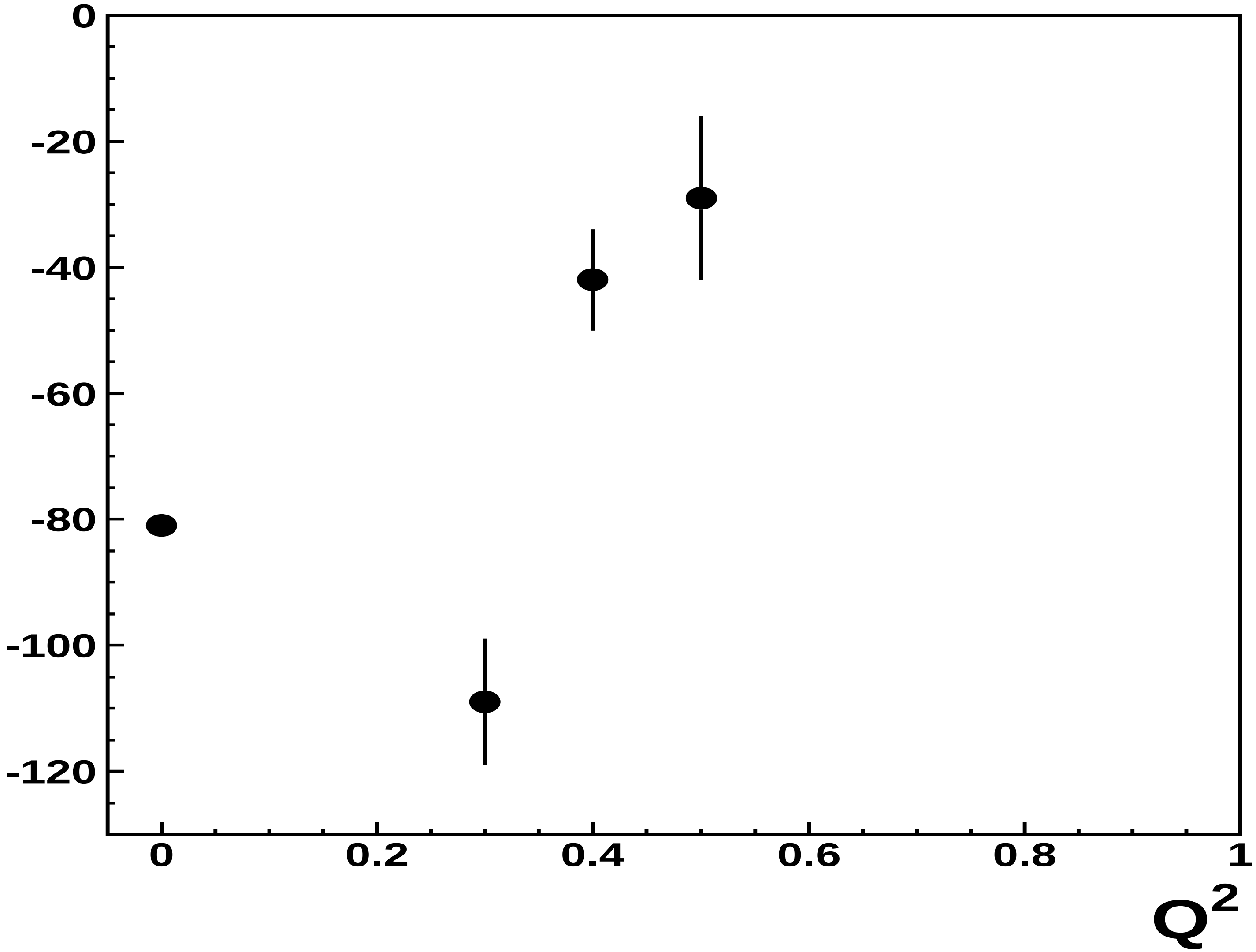


The fit of the $\frac{d\sigma_{TL}}{d\Omega}$ electroproduction data at $Q^2 = -0.3$ and $Q^2 = -0.5$ GeV^2





The $A^{\frac{1}{2}}$ for $N(1440)\frac{1}{2}+$



The $\gamma n \to \pi^- p$ and $\pi^- p \to \gamma n$ amplitudes

The photoproduction amplitude in the c.m.s. of the reaction has a general form:

$$A^i = w^* J^i_\mu w' \varepsilon_\mu \qquad i = 1, 2$$

For a particular partial wave:

$$d\sigma(\gamma n \to \pi^- p) = \frac{(2\pi)^4}{4|k_{\gamma N}|\sqrt{s}} A^2 d\Phi(p_1, \dots p_n) \frac{1}{(2s_1+1)(2s_2+1)}$$

In the case of the two body final state:

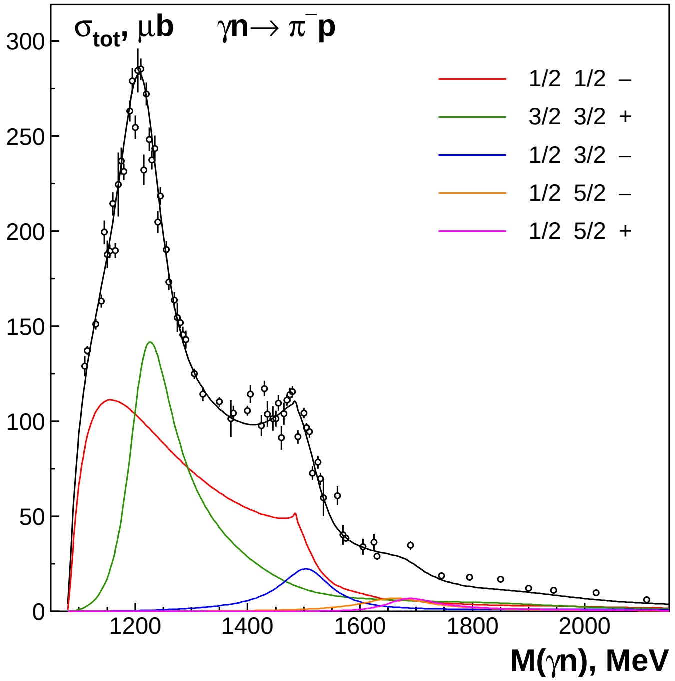
$$\frac{1}{2}(2\pi)^4 d\Phi(p_1, \dots p_n) = \rho_f(s)\frac{dz}{2} \qquad \rho_f(s) = \frac{1}{16\pi}\frac{2|p|}{\sqrt{s}}$$

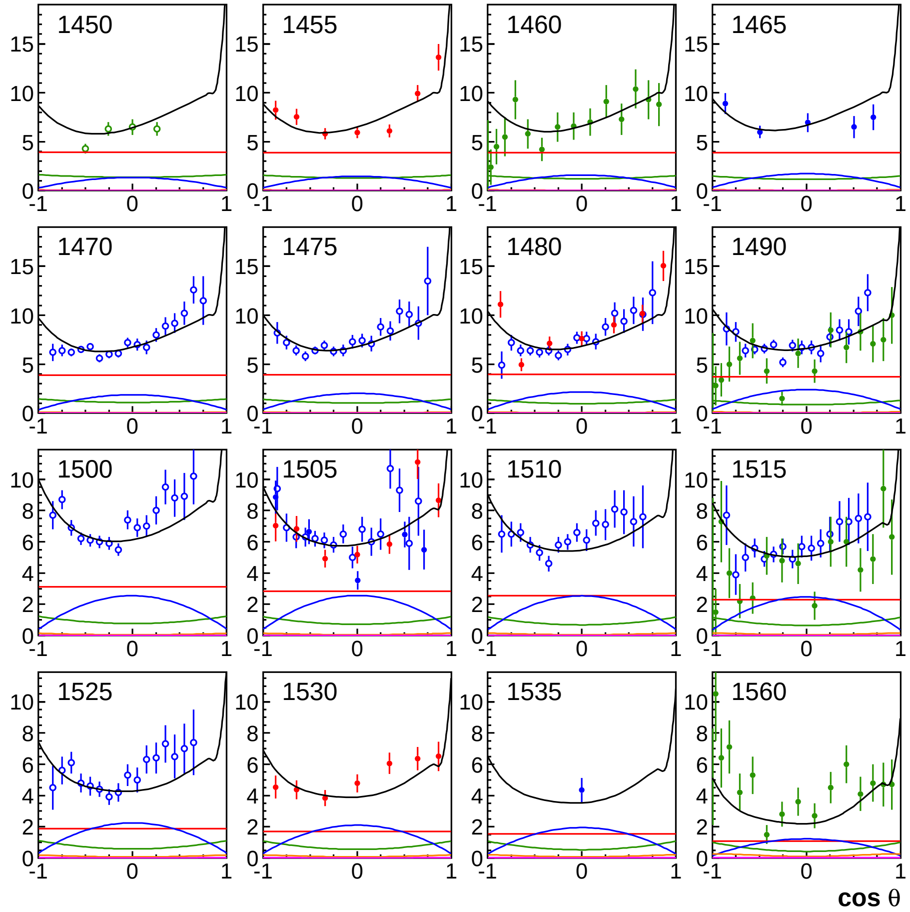
and $|p|$ is the momentum of the final particle in c.m.s. of the reaction.

One can rewrite the cross section in a more symmetrical form using the phase volume of the initial particles $\rho_i(s)$.

$$d\sigma(\gamma n \to \pi^- p) = \frac{1}{4}\frac{4\pi}{|k_{\gamma n}|^2}\rho_i(s)\rho_f(s)\frac{dz}{2} A^2$$

The description of the $\gamma n \to \pi^- p$ reaction (from $\pi^- p \to \gamma n$ reaction)





J.C.Comiso *et al.*, PRD 12, 719 (1975) G.J.Kim *et al.*, PRD 40, 244 (1989)

A.Shafi *et al.*, PRC 70, 035204 (2004) M.T.Tran *et al.*, NPA 324, 301 (1979)

$$\frac{d\sigma}{dz}(\pi^- p \to \gamma n) = 2\frac{k_{\gamma n}^2}{k_{\pi^- p}^2}\frac{d\sigma}{dz}(\gamma n \to \pi^- p)$$

The differential cross section can also written in the form:

$$\frac{d\sigma}{dz}(\pi^- p \to \gamma n) = \frac{|\vec{k}_{\gamma N}|}{|\vec{k}_{\pi^- N}|} H_{\mu\nu}(s,z) \sum_{\Lambda} \varepsilon_{\mu}^{*\Lambda} \varepsilon^{\Lambda}{}_{\nu}$$

where

$$H_{\mu\nu} = \frac{4\pi}{W^2} J_\mu J_\nu^* \qquad \sum_{\Lambda} \varepsilon_{\mu}^{*\Lambda} \varepsilon^{\Lambda}{}_{\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

If z-axis is directed along photon momentum

$$\frac{d\sigma}{dz}(\pi^- p \to \gamma n) = 2\frac{|\vec{k}_{\gamma N}|}{|\vec{k}_{\pi^- N}|} \frac{H_{xx} + H_{yy}}{2}$$

Analysis of the virtual photon production

$$A = \sum_{\Lambda} A_\mu^{(H)} \varepsilon_\mu^{(\Lambda)} \varepsilon_\nu^{*(\Lambda)} A_\nu^{(dec)} \frac{e}{Q^2}$$

where $\varepsilon_\mu^{(\Lambda)}$ is polarization vector of vector particle. The amplitude squared can be written as:

$$|A|^2 = \frac{e^2}{Q^4} \sum_{\Lambda\Lambda'} A_\mu^{*(H)} \varepsilon_\mu^{*(\Lambda')} \varepsilon_\nu^{(\Lambda')} A_\nu^{*(dec)} A_\alpha^{(H)} \varepsilon_\alpha^{(\Lambda)} \varepsilon_\beta^{*(\Lambda)} A_\beta^{(dec)} = \frac{e^2}{Q^4} \sum_{\Lambda\Lambda'} \rho_{\Lambda\Lambda'}^{(H)} \rho_{\Lambda\Lambda'}^{(dec)}$$

where

$$\rho_{\Lambda\Lambda'}^{(H)} = A_\mu^{*(H)} \varepsilon_\mu^{*(\Lambda')} A_\alpha^{(H)} \varepsilon_\alpha^{(\Lambda)} \qquad \rho_{\Lambda'\Lambda}^{(dec)} = \varepsilon_\nu^{(\Lambda')} A_\nu^{*(dec)} A_\beta^{(dec)} \varepsilon_\beta^{*(\Lambda)} = \varepsilon_\nu^{(\Lambda')} L_{\nu\beta} \varepsilon_\beta^{*(\Lambda)}$$

Helicity basis (in c.m.s. of the virtual photon)

$$\varepsilon^{(+1)} = \frac{1}{\sqrt{2}}(0, -1, -i, 0) \quad \varepsilon^{(-1)} = \frac{1}{\sqrt{2}}(0, +1, -i, 0) \quad \varepsilon^{(\,0)} = (0, 0, 0, 1)$$

See also article by Enrico Speranza, Miklos Zetenyi and Bengt Friman

The decay of the vector particle into two fermions:

$$A_\mu^{(dec)} = \bar{u}(k_1)\gamma_\mu u(k_2)$$

where $u(k1)$ **and** $u(k2)$ **are bispinors of the final fermions, e.g. electron and positron.**

$$L_{\mu\nu} = -Tr\left[\gamma_\mu(m_e + \hat{k}_1)\gamma_\nu(m_e - \hat{k}_2)\right] = -4\left(g_{\mu\nu}(m_e^2 + k_1 k_2) - k_{1\mu}k_{2\nu} - k_{1\nu}k_{2\mu}\right)$$

In the c.m.s. of the vector particle

$$L_{\mu\nu} = 4\left(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu} - 2g_{\mu\nu}(m^2 + |\vec{k}|^2)\right)$$

$$k_1 = (k_0, |k|\sin\theta\cos\phi, |k|\sin\theta\sin\phi, |k|\cos\theta) \qquad k_2 = (k_0, -k_x, -k_y, -k_z)$$

$$\begin{aligned}
\rho_{00} &= L_{zz} = 8m_e^2 + 4(-2k_z^2 + 2|\vec{k}|^2) = 8m_e^2 + 4|\vec{k}|^2\, 2(1 - cos^2\theta) \\
\rho_{11} &= \frac{1}{2}\left(L_{xx} + L_{yy}\right) = 8m_e^2 + 4(-k_x^2 - k_y^2 + 2|\vec{k}|^2) = 8m_e^2 + 4|\vec{k}|^2\,(1 + cos^2\theta) \\
\rho_{10} &= \frac{1}{\sqrt{2}}\left(-L_{xz} - iL_{yz}\right) = 4\sqrt{2}(k_x k_z + ik_y k_z) = 4|\vec{k}|^2\frac{1}{\sqrt{2}}\sin(2\theta)e^{i\phi}
\end{aligned}$$

$$\rho^{dec} = 4|\vec{k}|^2 \begin{pmatrix} \frac{2m_e^2}{|\vec{k}|^2} + (1 + cos^2\theta) & \sqrt{2}\sin\theta\cos\theta e^{i\phi} & \sin^2\theta e^{2i\phi} \\ \sqrt{2}\sin\theta\cos\theta e^{-i\phi} & \frac{2m_e^2}{|\vec{k}|^2} + 2(1 - cos^2\theta) & -\sqrt{2}\sin\theta\cos\theta e^{i\phi} \\ \sin^2\theta e^{-2i\phi} & -\sqrt{2}\sin\theta\cos\theta e^{-i\phi} & \frac{2m_e^2}{|\vec{k}|^2} + (1 + cos^2\theta) \end{pmatrix}$$

For unpolarized reaction $\rho_{11} = \rho_{-1-1}$, ρ_{1-1} is real and ρ_{10} is imaginary.

$$\begin{aligned} |A|^2 &= 8m_e^2\big(\rho_{00}^{(H)} + 2\rho_{11}^{(H)}\big) + 4|\vec{k}|^2\big[2\rho_{00}^{(H)}(1 - \cos^2\theta) + 2\rho_{11}^{(H)}(1 + \cos^2\theta) \\ &+ 2\sqrt{2}|\vec{k}|^2\sin(2\theta)\cos\phi\,\mathbf{Re}\rho_{10}^{(H)} + 2\sin^2\theta\mathbf{Re}\rho_{1-1}^{(H)}\cos(2\phi)\big] \end{aligned}$$

Taking into account three body final phase volume:

$$\frac{d\sigma_{\pi p \to n e^+ e^-}}{dq^2} = 2\frac{k_{\gamma n}}{k_{\pi^- p}}\frac{\alpha}{4\pi q^4}\sqrt{1 - \frac{4m_e^2}{q^2}}|A|^2\frac{dz}{2}\,\frac{d\cos\theta_e d\Phi_e}{4\pi}$$

If one integrate over electron angle:

$$N = \left(8m_e^2 + \frac{16}{3}|\vec{k}|^2\right)\left[\rho_{00}^{(H)} + 2\rho_{11}^{(H)}\right] = \frac{4}{3}\left(2m_e^2 + q^2\right)\left[\rho_{00}^{(H)} + 2\rho_{11}^{(H)}\right]$$

For unnormilized density matrix elements:

$$\rho_{11} = \frac{H_{xx} + H_{yy}}{2} \qquad \rho_{00} = H_{zz}$$

$$\rho_{10} = \frac{1}{\sqrt{2}}\big[\mathbf{Re}(-H_{xz} - iH_{yz}\big] = \frac{1}{\sqrt{2}}\big[\mathbf{Re}(-H_{xz}) + \mathbf{Im}H_{yz}\big]$$

$$\rho_{1-1} = \frac{H_{yy} - H_{xx}}{2}$$

For the electroproduction:

$$\frac{d\sigma_T}{d\Omega_\pi} = \frac{|\vec{k}_{\pi N}|}{|\vec{k}_{\gamma^* N}|}\frac{H_{xx} + H_{yy}}{2} \qquad \frac{d\sigma_L}{d\Omega_\pi} = \frac{|\vec{k}_{\pi N}|}{|\vec{k}_{\gamma^* N}|}H_{zz}$$

$$\frac{d\sigma_{TT}}{d\Omega_\pi} = \frac{|\vec{k}_{\pi N}|}{|\vec{k}_{\gamma^* N}|}\frac{H_{xx} - H_{yy}}{2} \qquad \frac{d\sigma_{TL}}{d\Omega_\pi} = \frac{|\vec{k}_{\pi N}|}{|\vec{k}_{\gamma^* N}|}\big(- \mathbf{Re}H_{xz}\big)$$

$$\frac{d\sigma_{TL'}}{d\Omega_\pi} = \frac{|\vec{k}_{\pi N}|}{|\vec{k}_{\gamma^* N}|}\mathbf{Im}H_{yz}$$

It is more standard to fit the differential cross section and normilized matrix elements.

$$\tilde{\rho}_{11} = \frac{1}{2}\frac{H_{xx} + H_{yy}}{H_{xx} + H_{yy} + H_{zz}} \rightarrowtail \frac{1}{2}$$

$$\tilde{\rho}_{10} = \frac{1}{\sqrt{2}}\frac{\left[\mathbf{Re}(-H_{xz}) + \mathbf{Im}H_{yz}\right]}{H_{xx} + H_{yy} + H_{zz}} \rightarrowtail 0$$

$$\tilde{\rho}_{1-1} = \frac{1}{2}\frac{H_{yy} - H_{xx}}{H_{xx} + H_{yy} + H_{zz}} \rightarrowtail \frac{1}{2}\Sigma$$

where Σ is the beam asymmetry.

γN interaction

Photon has quantum numbers $J^{PC}=1^{--}$, proton $1/2^+$. Then in S-wave two states can be formed is $1/2^-$ and $3/2^-$. Then P-wave $1/2^+, 3/2^+$ and $1/2^+, 3/2^+, 5/2^+$.

$$V^{(1+)\mu}_{\alpha_1\ldots\alpha_n} = \gamma^{\perp\perp}_{\mu} i\gamma_5 X^{(n)}_{\alpha_1\ldots\alpha_n}\,, \qquad V^{(1-)\mu}_{\alpha_1\ldots\alpha_n} = \gamma_\xi \gamma^{\perp\perp}_{\mu} X^{(n+1)}_{\xi\alpha_1\ldots\alpha_n}\,,$$
$$V^{(2+)\mu}_{\alpha_1\ldots\alpha_n} = \gamma_\nu i\gamma_5 X^{(n+1)}_{\nu\alpha_1\ldots\alpha_n} g^{\perp\perp}_{\mu\alpha_n}\,, \quad V^{(2-)\mu}_{\alpha_1\ldots\alpha_n} = X^{(n-1)}_{\alpha_2\ldots\alpha_n} g^{\perp\perp}_{\alpha_1\mu}$$
$$V^{(3+)\mu}_{\alpha_1\ldots\alpha_n} = \hat{k} i\gamma_5 X^{(n)}_{\alpha_1\ldots\alpha_n} Z_\mu\,, \qquad V^{(3-)\mu}_{\alpha_1\ldots\alpha_n} = \hat{k}\gamma_\chi X^{(n+1)}_{\chi\alpha_1\ldots\alpha_n} Z_\mu\,, \ .$$

$$Z_\mu = \left((Pk^\gamma)k^\gamma_\mu - (k^\gamma)^2 P_\mu\right)$$

$$X^0 = 1 \qquad X^{(1)}_\mu = k^\perp_\mu = k_\nu g^\perp_{\nu\mu}; \qquad g^\perp_{\nu\mu} = \left(g_{\nu\mu} - \frac{P_\nu P_\nu}{P^2}\right)$$

$$\gamma^{\perp\perp}_\mu = \gamma_\nu g^{\perp\perp}_{\mu\nu} \qquad g^{\perp\perp}_{\nu\mu} = \left(g_{\nu\mu} - \frac{P_\nu P_\nu}{P^2} - \frac{k^\perp_\nu k^\perp_\nu}{k^2_\perp}\right)$$

$$X^{(n)}_{\mu_1\ldots\mu_n} = \frac{2n-1}{n^2}\sum_{i=1}^{n} k^\perp_{\mu_i} X^{(n-1)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_n} - \frac{2k^2_\perp}{n^2}\sum_{\substack{i,j=1\\ i<j}}^{n} g_{\mu_i\mu_j} X^{(n-2)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_{j-1}\mu_{j+1}\ldots\mu_n}$$

For the positive states $J = L + 1/2 \quad (L{=}n)$**:**

$$A^{i+}_\mu \quad = \quad \bar{u}(q_N) X^{(n)}_{\alpha_1\ldots\alpha_n}(q^\perp) F^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n} V^{(i+)\mu}_{\beta_1\ldots\beta_n}(k^\perp) u(k_N)$$

$$\begin{array}{lll}
\mathcal{F}^{1+}_1 = \lambda_n\, P'_{n+1} & \mathcal{F}^{2+}_1 = 0 & \mathcal{F}^{3+}_1 = 0 \\
\mathcal{F}^{1+}_2 = \lambda_n\, P'_n & \mathcal{F}^{2+}_2 = -\frac{\lambda_n}{n}\, P'_n & \mathcal{F}^{3+}_2 = 0 \\
\mathcal{F}^{1+}_3 = 0 & \mathcal{F}^{2+}_3 = \frac{\lambda_n}{n}\, P''_{n+1} & \mathcal{F}^{3+}_3 = 0 \\
\mathcal{F}^{1+}_4 = 0 & \mathcal{F}^{2+}_4 = \frac{\lambda_n}{n}\, P''_n & \mathcal{F}^{3+}_4 = 0 \\
\mathcal{F}^{1+}_5 = 0 & \mathcal{F}^{2+}_5 = 0 & \mathcal{F}^{3+}_5 = +\xi_n\, P'_{n+1} \\
\mathcal{F}^{1+}_6 = 0 & \mathcal{F}^{2+}_6 = 0 & \mathcal{F}^{3+}_6 = -\xi_n\, P'_n
\end{array}$$

where

$$\lambda_n = \frac{\alpha_n}{2n+1}(|\vec{k}||\vec{q}|)^n \chi_i \chi_f \qquad \chi_{i,f} = \sqrt{m_{i,f} + k_{0i,f}}$$

$$\xi_n = k^2_\perp (Pk^\gamma) \frac{\alpha_n}{2n+1}(|\vec{k}||\vec{q}|)^n \chi_i \chi_f$$

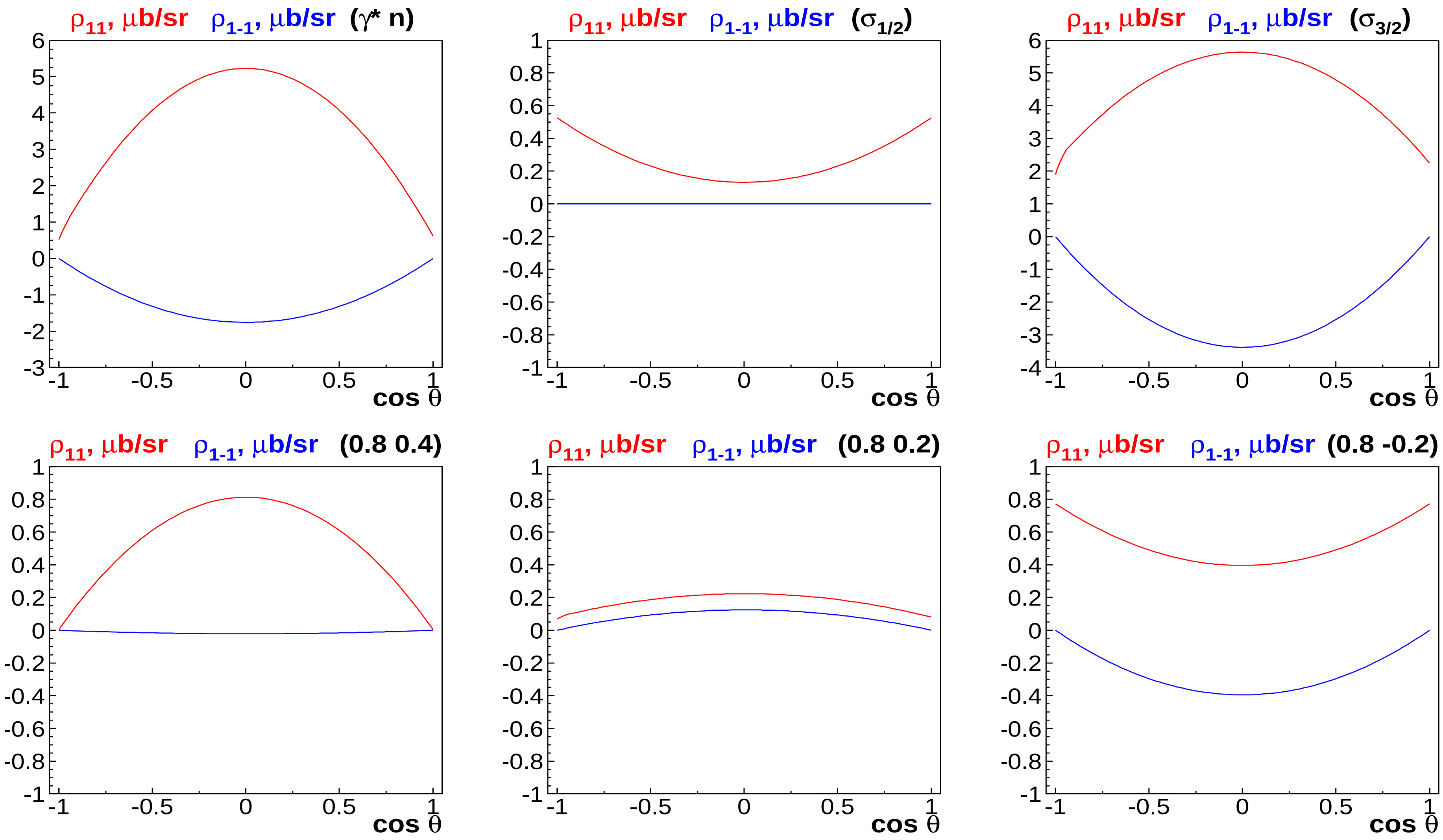
Density matrix elements:

$$N\rho_{11}^{(H)} = \sum_{ij} \left(H_{xx}^{ij} + H_{yy}^{ij}\right) F_i(Q^2)F_j(Q^2)$$

$$2\mathbf{Re}\rho_{1-1}^{(H)} = \sum_{ij} \left(H_{yy}^{ij} - H_{xx}^{ij}\right) F_i(Q^2)F_j(Q^2)$$

The matrix elements $\rho_{ij}^{(H)} = \rho_{ij}^{(H)}(W^2, Q^2, z_{\pi\gamma})$ **where** $z = \cos(\Theta)$ **is the angle between initial pion and** γ**-particle in c.m.s. of the reaction.**

Prediction for the density matrix elements $2\rho_{11}$ and $2\mathrm{Re}\rho_{1-1}$ from $\pi^- p \to \gamma^* n$



The analysis of the simulated data (photoproduction and the case with a longitudinal couplings)