Dynamical Origin of the Proton Mass

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Why is the proton (or hadron) mass interesting?

- The quark masses contribute only a small fraction of the proton mass.

(How to define this small contribution needs a careful discussion)

- Thus most of the mass comes from dynamic origin.
- The proton mass is an interesting probe of QCD dynamics!

Massless QCD

- To focus on dynamics, we consider a limit of QCD in which quark masses do no appear:
- We let heavy quark masses go to infinity (charm, beauty, top)
- Let light quark masses go to zero (up and down)
- The strange quark is a tough call, as its mass is not so small compared to the proton mass, nor so large.
- We can consider massless QCD or QCD lite in two versions: n_f=2 or 3

QCD Lite

- Simple lagrangian

  $$L = -\frac{1}{4}F^2 + \sum \ \ \bar{\psi}_i iD\psi_i$$

which does not contain any dimensionful parameter.

- Classical conformal symmetry!

  O(5,1) which includes the Poincare symmetry.

- The symmetry is broken to O(4,1) in quantum theory: quantum anomaly: trace anomaly.

Trace anomaly

- At classical level, the energy-momentum tensor $T^{\mu\nu}$ in QCD lite has a zero trace (scalar)

$$T^{\mu}_{\mu} = 0$$

[which would imply all masses are zero!]

- However, once the conformal symmetry is broken in quantum mechanically, it acquires a trace

$$T^{\mu}_{\mu} = \frac{\beta(g)}{2g} F^2$$

which is scale invariant (? Only after subtractions).

- To understand origin of mass, we need understand what the anomaly does.

Perturbation theory

- Consider perturbatice QCD vacuum $|0\rangle$, the trace anomaly in this state is non-zero

$$\langle 0 | T_\mu^\mu | 0 \rangle \sim (c_1 \alpha_s + c_2 \alpha_s^2 + \cdots) \int d^4 k$$

each gluon modes contribute equally to the M.E.

and the total result is proportional to the phase-space volume!

This is similar to UV divergence in black-body radiation formula (UV catastrophe), where T acts as cut-off scale.

Here we cut off manually as it is non-physical (except in gravity), $\int d^4 k \sim \frac{1}{a^4}$, on lattice

Non-perturbative vacuum

- The physical or true QCD vacuum $|\Phi\rangle$ is different from the perturbative vacuum in that the IR modes are strongly modified by dynamics, while the UV modes are the same due to asymptotic freedom.
- Thus, we can write

$$\langle\Phi|T_\mu^\mu|\Phi\rangle = \langle 0|T_\mu^\mu|0\rangle + \langle\delta T\rangle$$

- $\langle\delta T\rangle$ is a quasi-observable, truly renormalization scale independent! It becomes a dimensionaful parameter of QCD theory!
- $\langle\delta T\rangle = \#\,\Lambda_{QCD}^4$ where Λ_{QCD} is defined by the running coupling, probed at high energy.

Dynamics in trace anomaly

- One can imagine,

$$\langle \delta T \rangle = \Lambda_{QCD}^{4} \int \phi\left(\frac{k^2}{\Lambda_{QCD}^2}\right) d\left(\frac{k^2}{\Lambda_{QCD}^2}\right)$$

where the scale-invariance, but scheme-dependent function $\phi(x)$ encodes the non-perturbative dynamics, how gluon modes are modified in the non-perturbative vacuum

- This modification sets up the stage for hadron mass from the standpoint of trace anomaly.

Comparing with other arguments on scale generation

- Confinement

As exemplified by the MIT bag model where the bag radius is determined through the bag constant B, the energy difference between pert and non-pert vacuum.

$$B \sim \langle \delta T \rangle$$

- Chiral symmetry breaking

Chiral condensate generates the hadron physics scale

Modification of the low-energy Dirac spectrum (density of near zero modes) due to non-perturbative dynamics.

Hadron masses

- In QCD lite, hadron masses arise entirely from QCD dynamics.
- From the Einstein’ s famous equation

$$E = Mc^2$$

mass becomes a probe of the dynamical energy sources

$$M = E/c^2$$

- Although relativity allows one to look at the mass in different frames, this involves additional boost dynamics (most protons in the universe are “at rest”)

Subtraction

- In terms of QCD Hamiltonian

$$m_p = \langle p|H|p\rangle - \langle \Phi|H|\Phi\rangle$$

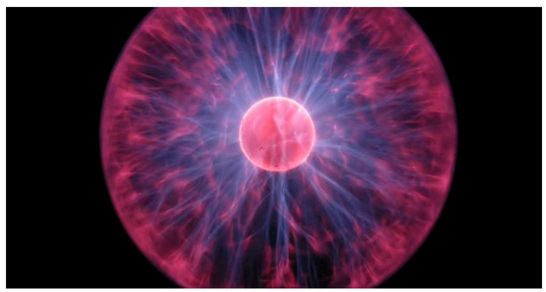
where for simplicity |p> is a hadron(proton) at rest

thus in a way, it does not directly probe the vacuum structure itself.

- This subtraction is implicit in path integral formulation.

Mass and vacuum response

- Hadron mass arises from the response of the QCD vacuum after inserting a certain combination of massless quarks which have color!



- Reflects dynamics of the real QCD vacuum when perturbed by color sources

Hadron mass scale

- The hadron mass can be measured through QCD scale arising in vacuum dynamics,

$$m_p = c_p \Lambda_{QCD}$$

which is not known until the parameter Λ_{QCD} is decided.

- The mass scale is a parameter. QCD itself does not determine what is the mass the proton. The proton mass can be 940 MeV, or 9.4 GeV, or 94 GeV, all allowed!

What decides QCD scale?

- QCD itself leaves the mass scale as a parameter.
- To determine this parameter, we have to embed the theory into a large theory, such as a GUT.
- In such a theory, the QCD scale is determined by, for example, running coupling unification at a very large scale (10^{16} GeV)

“Dynamical origin” of mass

- The mass is the result of the equilibrium reached through dynamical processes.
- Virial theorem is a statement that in equilibrium, how does the energy of the different sources balancing out.
  - Harmonic oscillator, T=V
  - Coulomb system, T= -V/2
- Thus one of the most important insight on QCD dynamics from mass is about the balancing contributions of the different parts.

QCD predictions: ratios as pure numbers

- However, what QCD lite can predict is the ratios of dimensionful quantities,

$$\frac{m_p}{f_\pi} = \text{a numerical number depends only on}$$

the number of light flavors.

- These numerical numbers reflect the underlying theory in a deep way and can be computed in lattice gauge theory, but they are mostly not very illuminating.

Splitting the QCD energy source

- In X. Ji, Phys.Rev.Lett. 74 (1995) 1071-1074

$$H_{\rm QCD} = H_q + H_m + H_g + H_a .$$

$$H_q = \int d^3\vec{x}\ \bar{\psi}(-i\mathbf{D}\cdot\alpha)\psi,$$ ← Quark energy

$$H_m = \int d^3\vec{x}\ \bar{\psi}m\psi,$$ ← Quark mass

$$H_g = \int d^3\vec{x}\ \frac{1}{2}(\mathbf{E}^2+\mathbf{B}^2),$$ ← Gluon kinetic energy

$$H_a = \int d^3\vec{x}\ \frac{9\alpha_s}{16\pi}(\mathbf{E}^2-\mathbf{B}^2).$$ ← Trace anomaly

- Critics say this is not scale invariant….

Trace anomaly contribution

- It is easy to show, like a virial theorem, that the trace anomaly contributes ¼ of the proton mass.

$$\langle \Delta T \rangle = \frac{\mathrm{M}}{4} = \left(\langle \mathrm{p} | T_\mu^\mu | \mathrm{p} \rangle - \langle \Phi | T_\mu^\mu | \Phi \rangle \right) / 4$$

- This contribution is very much like to bag constant contribution to the mass of the proton in the MIT bag model.
- It suggests that the bag constant actually corresponds to the difference of trace anomaly in the proton state and true vacuum.

$$\mathrm{BV} = \left(\langle \mathrm{p} | T_\mu^\mu | \mathrm{p} \rangle - \langle \Phi | T_\mu^\mu | \Phi \rangle \right) / 4$$
$$= \Lambda_{QCD} \int \Delta\phi(x) dx$$

Equilibrium in MIT bag model

- The quarks have kinetic energy, which according to the uncertainty principle, is large when the moving space is small.
- However, the space in which quarks move has a vacuum energy, which increases as the volume gets large.
- The hadron size and mass are the result of balance between the two energies. (quark dynamical pressure balances the vacuum pressure).

Dynamical evolution

- Consider three quarks moving in a small volume V, in which the quark kinetic energy is large, which generates an expansion pressure.
- Thus the volume will increase as time goes.
- However, as the volume gets too big, the quark kinetics energy get too small. The volume will shrink due to the pressure of the true vacuum (or negative pressure of the false vacuum).
- This process of oscillations will continue as the energy of the system is conserved.

Reaching equilibrium through Euclidean time

- To set up the equilibrium corresponding to a proton, one can switch the dynamics to imaginary time （Euclidean dynamics).
- In this case, one has a dissipative system, and the extra energy will dissipate away as time goes.
- The finally equilibrium reached is a minimal energy bound state, in which the quark kinetic energy just balances the vacuum pressure.

Lattice study of mass dynamics

- One can mimic this dynamics through a mass calculation on lattice.
- Step1: create a point like source and sink on a lattice with different spacing in the time direction.
- Step2: measure the contributions to the quark and gluon kinetic energy and the trace anomaly.
- Step3: construct the picture that how does various energy changes as the time separation increases.

Physics interpretation

- At small (Euclidean) time, the point-like proton source has essentially no color charge, thus has little perturbation in the QCD vacuum.
- However, the kinetic energy of the quarks must be large due to quantum mechanical uncertainty principle. The gluon energy and trace anomaly will have small M. E.
- As time evolves, the quarks will spread to reduce the kinetic energy. In doing so, it will generate color multiples which will interact strongly with gluon modes in the QCD vacuum.

Physics interpretation

- The gluons in the vacuum will respond and generate positive contributions through its kinetic energy and trace anomaly.
- If the quarks spread too much, the gluon energy will grow too big and this will not happen in Euclidean time.
- The minimal energy is reached when quarks spread to the proton size, and the gluon energy and trace anomaly reach certain values, as determined by phenomenology.

Conclusions

- By studying the mass content of the proton, one can learn quite bit about the quark and gluon dynamics.
- Lattice studies can help to understand how does the QCD vacuum generates a mass scale through anomaly.
- And help to determine how do the quark energy and other energy contributions to mass reach their equilibrium values through studying the vacuum response to a quark source.