

SATURATION FRONTS IN RESUMMED HIGH ENERGY EVOLUTION

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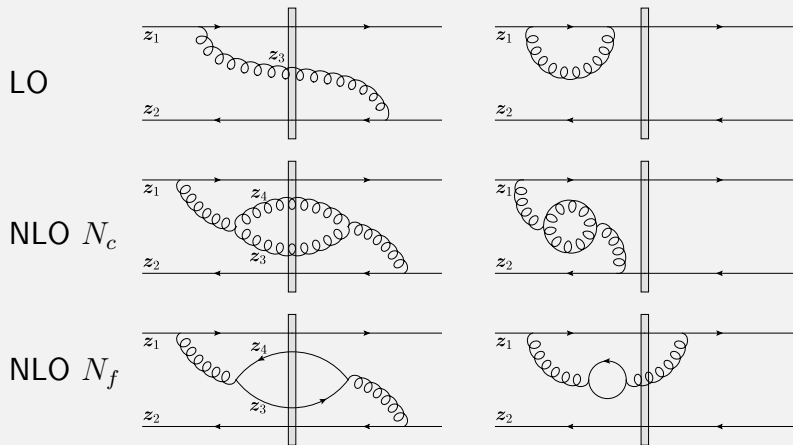
ECT*/FBK, Trento, Italy

“Probing QCD at the High Energy Frontier”, ECT*, May 2018

B. Ducloué, E. Iancu, A.H. Mueller, E. Iancu, DT, in preparation

- Large transverse logarithms in NLO BK and instabilities
- Resummation (double logs) and stability
- Issues with resummed evolution in projectile rapidity (Y)
- Comparison of saturation fronts in Y and η
- Resummed evolution in target rapidity (η)

NLO BK EVOLUTION



Evolution of right moving projectile: modes with smaller longitudinal momentum k^+ . Soft plus non-soft (in general).

NLO BK EVOLUTION

Proj: (z_1, z_2) , q^+ , $q^- = 1/2z_{12}^2q^+$ Targ: Q_0^2 , q_0^- , $q_0^+ = Q_0^2/2q_0^-$
 NLO evolution for $S_{12} \equiv S(z_1, z_2; Q_0^2; Y)$, with $Y = \ln q^+/q_0^+$:

$$\begin{aligned}
 \frac{dS_{12}}{dY} = & \frac{\bar{\alpha}_s}{2\pi} \int \frac{d^2z_3z_{12}^2}{z_{13}^2z_{32}^2} \left[1 + \underbrace{\bar{\alpha}_s \left(\bar{b} \ln z_{12}^2 \mu^2 - \bar{b} \frac{z_{13}^2 - z_{23}^2}{z_{12}^2} \ln \frac{z_{13}^2}{z_{12}^2} \right)}_{\text{RC: choose } \mu=1/z_{ij\min}} \right. \\
 & \left. - \underbrace{\frac{1}{2} \ln \frac{z_{13}^2}{z_{12}^2} \ln \frac{z_{23}^2}{z_{12}^2}}_{\text{TO in Projectile}} \right] (S_{13}S_{32} - S_{12}) \\
 & + \frac{\bar{\alpha}_s^2}{8\pi^2} \int \frac{d^2z_3d^2z_4}{z_{34}^4} \left(\underbrace{-2 + \frac{z_{13}^2z_{24}^2 + z_{14}^2z_{23}^2 - 4z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2 - z_{14}^2z_{23}^2} \ln \frac{z_{13}^2z_{24}^2}{z_{14}^2z_{23}^2}}_{\text{Proj \& Targ DGLAP} \rightsquigarrow -11/12} \right) \\
 & \times (S_{13}S_{34}S_{42} - S_{13}S_{32}) \\
 & + \mathcal{O}(N_f) + \mathcal{O}(1/N_c^2) + \text{regular terms}
 \end{aligned}$$

LARGE TRANSVERSE LOGARITHMS

Strongly ordered large perturbative dipoles (DLA)

$$z_{12} \ll z_{13} \simeq z_{23} \ll z_{14} \simeq z_{24} \simeq z_{34} \ll 1/Q_s$$

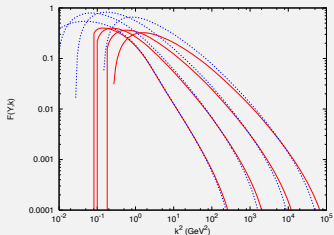
Linearize, large dipoles strong interaction, reals terms dominate

$$\frac{dT_{12}}{dY} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} dz_{13}^2 \frac{z_{12}^2}{z_{13}^4} \left(1 - \frac{\bar{\alpha}_s}{2} \ln^2 \frac{z_{13}^2}{z_{12}^2} - \bar{\alpha}_s \frac{11}{12} \ln \frac{z_{13}^2}{z_{12}^2} \right) T_{13}$$

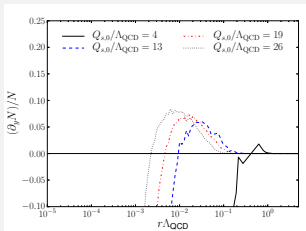
NLO > LO, unstable expansion in coupling

Even single iteration leads to negative solution

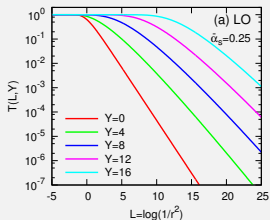
UNSTABLE NUMERICAL SOLUTIONS



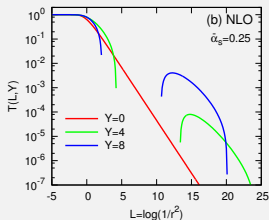
NLO BFKL + Sat Bound



NLO BK

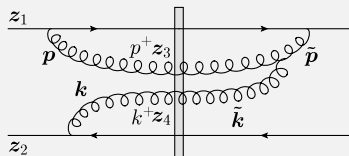


LO BK



LO BK + Double Log

TIME ORDERING AND RESUMMATION IN DLA



Hard to soft projectile evolution: $\mathbf{k} \ll \mathbf{p}$ and $k^+ \ll p^+$

Time ordering non-trivial, requires: $\tau_k \sim k^+ z_4^2 \ll \tau_p \sim p^+ z_3^2$

Resum to all orders in a non-local equation

$$\frac{dT_{12}(Y, z_{12}^2)}{dY} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} \frac{dz_{13}^2}{z_{13}^2} \frac{z_{12}^2}{z_{13}^2} \Theta \left(Y - \ln \frac{z_{13}^2}{z_{12}^2} \right) T \left(Y - \ln \frac{z_{13}^2}{z_{12}^2}, z_{13}^2 \right)$$

Mathematically equivalent to local equation

$$\frac{dT_{12}}{dY} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} \frac{dz_{13}^2}{z_{13}^2} \frac{z_{12}^2}{z_{13}^2} \frac{J_1 \left(2\sqrt{\bar{\alpha}_s \ln^2 \frac{z_{13}^2}{z_{12}^2}} \right)}{\sqrt{\bar{\alpha}_s \ln^2 \frac{z_{13}^2}{z_{12}^2}}} T_{13}$$

RESUMMATION IN BK

Match local resummed DLA equation to include BK physics

$$\frac{dS_{12}}{dY} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{d^2 z_3 z_{12}^2}{z_{13}^2 z_{32}^2} \mathcal{K}_{\text{DLA}} \left(\sqrt{\ln \frac{z_{13}^2}{z_{12}^2} \ln \frac{z_{23}^2}{z_{12}^2}} \right) (S_{13} S_{32} - S_{12})$$

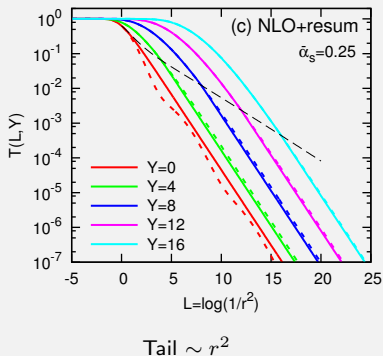
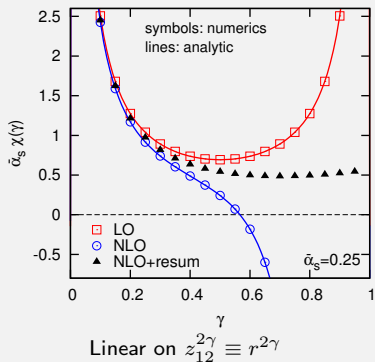
with

$$\mathcal{K}_{\text{DLA}}(\rho) = \frac{J_1\left(2\sqrt{\bar{\alpha}_s \rho^2}\right)}{\sqrt{\bar{\alpha}_s \rho^2}} = 1 - \frac{\bar{\alpha}_s \rho^2}{2} + \frac{(\bar{\alpha}_s \rho^2)^2}{12} + \dots$$

Resums double logarithms to all orders (and nothing more)

LO BK + NLO double log when truncated to $\bar{\alpha}_s^2$

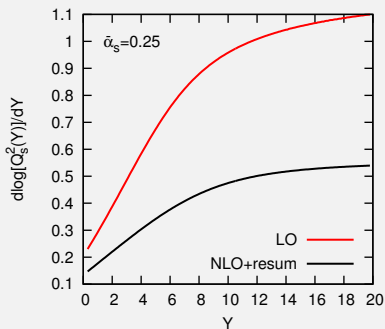
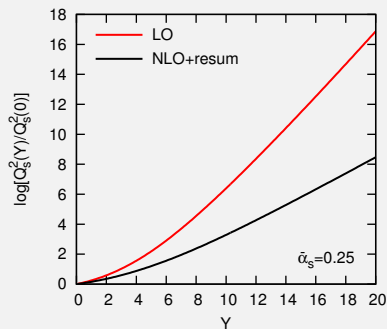
NUMERICAL SOLUTION



$$\omega_{\text{LO}} = \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s}{1-\gamma} + \text{regular} \quad \omega_{\text{NLO}} = \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s}{1-\gamma} - \frac{\bar{\alpha}_s^2}{(1-\gamma)^3} + \text{regular}$$

$$\omega = \frac{\bar{\alpha}_s}{\gamma} + \frac{1}{2} \left[-(1-\gamma) + \sqrt{(1-\gamma)^2 + 4\bar{\alpha}_s} \right] + \text{regular}$$

NUMERICAL SOLUTION



Considerable speed reduction, perhaps too much?

TARGET RAPIDITY η

Target rapidity determines the kinematics

$$\begin{aligned}\eta &\equiv \ln \frac{1}{x_{\text{Bj}}} = \ln \frac{s}{Q^2} = \ln \frac{2q^+q_0^-}{2q^+q^-} = \ln \frac{q_0^-}{q^-} \\ &= \ln \frac{Q_0^2}{2q_0^+} \frac{2q^+}{Q^2} \rightarrow Y - \ln \frac{1}{r^2 Q_0^2} \equiv Y - \rho\end{aligned}$$

NB:

Could have started from target DGLAP in η

Change variables from (η, ρ) to (Y, ρ)

Get large logarithms in the BK kernel

- Initial condition at $Y = 0$ or boundary at $\eta = 0 \Leftrightarrow Y = \rho$
- Erroneous use of MV or GBW type IC leads to unphysical pushed front
- $\gamma_s \sim 1$, where is BFKL dynamics? λ_s seems too small (compare to DT 03)
- For DIS express final result in terms of $\eta = Y - \rho$ and ρ
Saturation: target property, need $Q_s^2(\eta)$
- Fronts in Y and η very different for relevant $\bar{\alpha}_s$ values

STARTING THE EVOLUTION

- Physical initial condition given at $\eta = 0$
- Construct initial condition at unphysical value $Y = 0$
Can do at level of DLA, e.g. for GBW physical IC

$$z_{12}^2 Q_0^2 \rightarrow z_{12}^2 Q_0^2 J_0 \left(2\sqrt{\bar{\alpha}_s \ln^2 z_{12}^2 Q_0^2} \right) \quad \text{for } z_{12}^2 Q_0^2 \ll 1$$

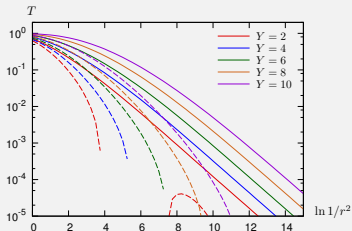
Analytically continued backwards DLA evolution to

$$\eta = -\rho \Leftrightarrow Y = 0$$

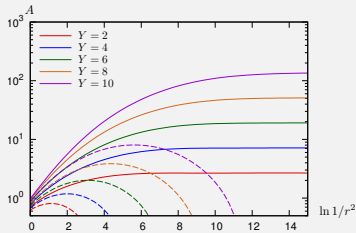
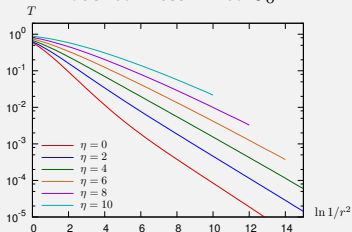
Exponentiate to unitarize

- Evolution will not likely reproduce physical IC at $\eta = 0$

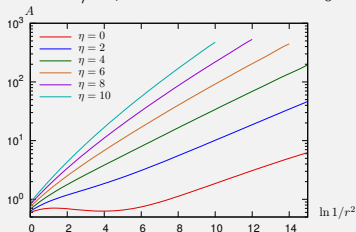
(A LITTLE) UGLY SOLUTION?



dashed: resummed J_0



$A = T/r^2$, dashed: resummed J_0

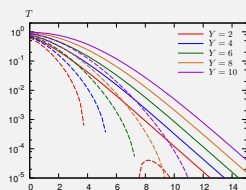


Difficult to trust solution, not able to solve BC problem

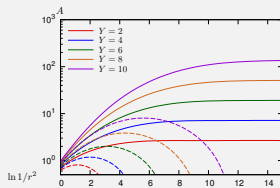
MORE BEAUTIFUL BUT MOSTLY WRONG SOLUTION

Try GBW IC at $Y = 0$?

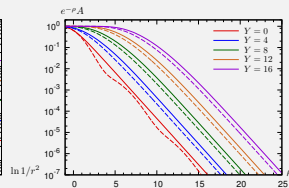
A mixture of GBW and resummed IC?



solid: GBW



$A = T/r^2$, solid: GBW



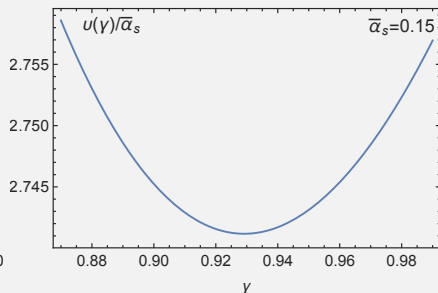
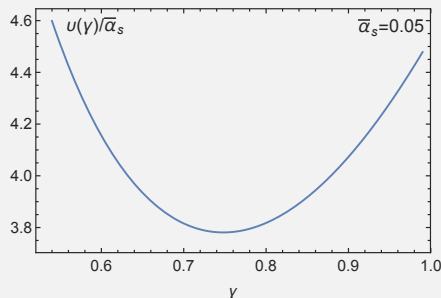
$[(1 + J_0)/2]$ GBW, GBW

Fronts look nice.

They are unphysical.

Why $\gamma_s = 1$?

STEEPNESS OF INITIAL CONDITION



Velocity function: $v(\gamma) = \omega(\gamma)/\gamma$

Saturation saddle point: $v'(\gamma_s) = 0$

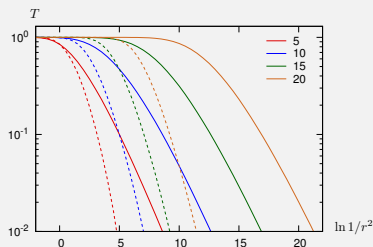
Front speed: $\lambda_s = d \ln Q_s^2 / dY = v(\gamma_s) + \mathcal{O}(1/Y)$

For $\bar{\alpha}_s < \bar{\alpha}_s^* \simeq 0.21$, $\gamma_s < \gamma_{ic} = 1$: pulled front ✓

For $\bar{\alpha}_s > \bar{\alpha}_s^* \simeq 0.21$, $\gamma_s > \gamma_{ic} = 1$: pushed front ✗

J_0 makes IC steep enough for all (relevant) values of $\bar{\alpha}_s$

SLOPE AND SPEED FOR THE η -FRONT



Front in $Y \Leftrightarrow$ Front in η , physical
Scaling in Y : $-\ln T = \tilde{\gamma}_s(\rho - \tilde{\lambda}_s Y)$
Change variable $Y = \eta + \rho$
Scaling in η : $-\ln T = \gamma_s(\rho - \lambda_s \eta)$
(valid inside diffusion radius)

$$\gamma_s = \tilde{\gamma}_s(1 - \tilde{\lambda}_s)$$

and

$$\lambda_s = \frac{\tilde{\lambda}_s}{1 - \tilde{\lambda}_s}$$

$\tilde{\lambda}_s = \mathcal{O}(\bar{\alpha}_s)$, difference is NLO. In practice it is large.

Physical η -front less steep and faster

CONSTRAINTS IN EVOLUTION: FROM Y TO η

TO in hard to soft Y -evolution, local or non-local, forbids emission of large daughter dipoles $|\mathbf{x}-\mathbf{z}|, |\mathbf{y}-\mathbf{z}| \gtrsim r = |\mathbf{x}-\mathbf{y}|$

$$\boxed{\ln \frac{q^+}{k^+} > \ln \frac{r_{>}^2}{r^2}} \quad \text{with} \quad r_{>} = \max\{|\mathbf{x}-\mathbf{z}|, |\mathbf{y}-\mathbf{z}|\}$$

Make change of variables

$$q^+ = \frac{1}{2r^2 q^-}, \quad k^+ = \frac{1}{2r_{<}^2 k^-}$$

CONSTRAINTS IN EVOLUTION: FROM Y TO η

TO constraints becomes

$$\ln \frac{k^-}{q^-} > \ln \frac{r_{>}^2}{r^2} - \ln \frac{r_{<}^2}{r^2} \Rightarrow \boxed{\ln \frac{k^-}{q^-} > \ln \frac{r_{>}^2}{r_{<}^2}}$$

TO in η -evolution forbids emission of small daughter dipoles
(or equivalently forbids disparate daughter dipole sizes)

Relevant when scattering small dipole off dense hadron?

Yes, BFKL diffusion \rightsquigarrow not uni-directional evolution

NON-LOCAL AND LOCAL EQUATION IN η

- Non-local equation [NB: Real-virtual cancellation in UV]:

$$\frac{dS_{\mathbf{x}\mathbf{y}}(\eta)}{d\eta} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{d^2z (\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-z)^2(z-\mathbf{y})^2} \Theta(\eta-\Delta_1)\Theta(\eta-\Delta_2) [S_{\mathbf{x}z}(\eta-\Delta_1)S_{z\mathbf{y}}(\eta-\Delta_2) - S_{\mathbf{x}\mathbf{y}}(\eta)]$$

$$\Delta_1 = \ln r_{>}^2/|\mathbf{x}-z|^2, \quad \Delta_2 = \ln r_{>}^2/|z-\mathbf{y}|^2$$

- Equivalent (to order of accuracy) local equation:

$$\frac{dS_{\mathbf{x}\mathbf{y}}}{d\eta} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{d^2z (\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-z)^2(z-\mathbf{y})^2} \mathcal{K}_{\text{DLA}} \left(\ln \frac{(\mathbf{x}-z)^2}{(z-\mathbf{y})^2} \right) (S_{\mathbf{x}z}S_{z\mathbf{y}} - S_{\mathbf{x}\mathbf{y}})$$

- Both are initial value problems at $\eta = 0$

CHARACTERISTIC FUNCTION

Truncation: NLO BFKL triple pole with $s_0 = Q^2$ ✓

$$\omega_{\text{NLO}} = \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s}{1-\gamma} - \frac{\bar{\alpha}_s^2}{\gamma^3} + \text{regular}$$

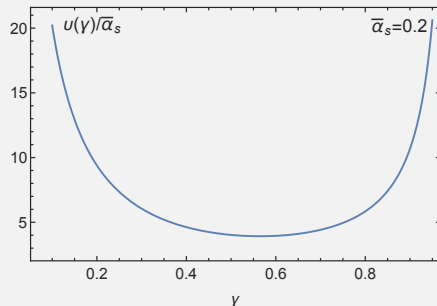
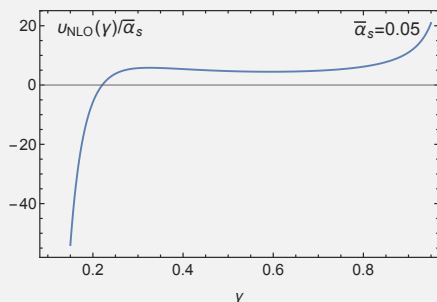
Resummed: Finite at $\gamma = 0$

$$\omega = \frac{\bar{\alpha}_s}{1-\gamma} + \underbrace{\frac{1}{2} \left[-\gamma + \sqrt{\gamma^2 + 4\bar{\alpha}_s} \right]}_{\omega_{\text{DLA}}} + \text{regular}$$

More precisely, matched to LO BFKL

$$\omega = \omega_{\text{DLA}} + \bar{\alpha}_s \left(\chi_0 - \frac{1}{\gamma} \right) + \bar{\alpha}_s^2 \left(\frac{4\zeta_3 - \chi_0^3 + 3\chi_0\chi_0' - \chi_0''}{6} + \frac{1}{\gamma^3} \right)$$

VELOCITY FUNCTION AND SADDLE POINT



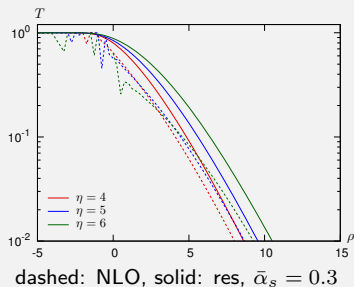
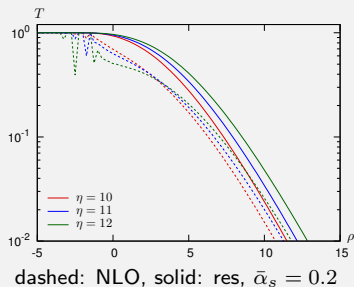
No real saddle point when truncating \rightsquigarrow expect oscillations

Well defined saddle point with resummation

$\omega_{\mathbb{P}} = 1/2 < \gamma_s < \gamma_0 = 0.628$: pulled front with MV-GBW IC

$\delta\lambda_s/\lambda_0 \sim -\#\bar{\alpha}_s$ with $\# \simeq 1$

NUMERICAL SOLUTION



Oscillations milder than in Y -evolution. But still there.

Start to appear where expected: need large r (but smaller than $1/Q_s$) to allow for logarithmic integration in $z \ll r$

Resummation: forbids such log contribution \rightsquigarrow well-defined answer

OTHER CORRECTIONS

- Resum single logarithms for projectile and target DGLAP
- Running coupling
- Include remaining regular NLO corrections, estimate:
$$\delta\omega = \pm\bar{\alpha}_s^2, \text{ find } \delta\lambda_s/\lambda_0 \sim \pm\#\bar{\alpha}_s \text{ with } \# \simeq 0.35$$

Still, what done here is a valid step

- Resummed unstable correction to render it $\mathcal{O}(\bar{\alpha}_s)$
- Definite sign (negative)
- Numerically larger than regular NLO corrections

CONCLUSIONS

- Very difficult (and not necessary?) to construct front in projectile rapidity Y
- Physical front is in terms of target rapidity η
- Front in η is easy to obtain (LO plus resummed)
- Front in η is faster and less steep