Phases of cold nuclear matter and the equation of state of neutron stars

Gordon Baym
University of Illinois, Urbana

New perspectives on Neutron Star Interiors
ECT*, Trento
10 October 2017
Better understanding of dense nuclear matter:

Driven by:

- observations of two heavy neutron stars with $M \approx 2 \, M_\odot$

- ongoing observational simultaneous determinations of neutron star masses and radii in low mass x-ray binaries, and by NICER on International Space Station

- emerging understanding in QCD of how nuclear matter can turn into deconfined quark matter in the interior

- creation of quark gluon plasmas in ultrarelativistic heavy ion collisions

- creation of Bose and BCS superfluids in ultracold atoms.

New sources of gravitational radiation events from merging of neutron stars with black holes or neutron stars. New window on dense matter.

16 Oct. 2017
High mass neutron star, PSR J1614-2230 -- in neutron star-white dwarf binary


Spin period = 3.15 ms; orbital period = 8.7 day
Inclination = 89:17° ± 0:02°: edge on

\[ M_{\text{neutron star}} = 1.928 \pm 0.017 M_\odot; \quad M_{\text{white dwarf}} = 0.500 \pm 0.006 M_\odot \]

(Gravitational) Shapiro delay of light from pulsar when passing the companion white dwarf
Second high mass neutron star, PSR J0348+0432 -- in neutron star-white dwarf binary

Spin period = 39 ms; orbital period = 2.46 hours
Inclination = 40.2°

$M_{\text{neutron star}} = 2.01 \pm 0.04 M_\odot$; $M_{\text{white dwarf}} = 0.172 \pm 0.003 M_\odot$

Significant gravitational radiation

$\frac{\dot{P}}{\dot{P}_{GR}} = 1.05 \pm 0.18$

400 Myr to coalescence!  AAAALigo
Possible high mass neutron stars (< 2.7\(M_{\odot}\)) in extreme "black widow pulsars"

J1311-3430, B1957+20, J2215+5135
(neutron star - He star binaries)

van Kerkwijk, Breton, & Kulkarni (B1957+20),” Ap.J. 728, 95 (2011),

\[ M_{\text{ns-J1311}} \sim 1.8 - 2.7 \, M_{\odot} \quad ; \quad M_{\text{companion}} \sim 0.01M_{\odot} \]

Uncertainties arise from incomplete modeling of heating of the companions by the neutron stars.
Neutron star interior

- Mass $\sim 1.4-2 \, M_{\text{sun}}$
- Radius $\sim 10-12 \, \text{km}$
- Temperature $\sim 10^6-10^9 \, \text{K}$
- Surface gravity $\sim 10^{14}$ that of Earth
- Surface binding $\sim 1/10 \, m c^2$
Nuclei before neutron drip

e^{-} + p \rightarrow n + \nu : \text{makes nuclei neutron rich}
as electron Fermi energy increases with depth

n \rightarrow p + e^{-} + \bar{\nu} : \text{not allowed if } e^{-} \text{ state already occupied}

Beta equilibrium: \[ \mu_n = \mu_p + \mu_e \]

Fermi seas

Shell structure (spin-orbit forces) for very neutron rich nuclei?
Do N=50, 82 remain neutron magic numbers? Proton shell structure?
Being explored at rare isotope accelerators: RIKEN Rare Ion Beam Facility, and later GSI (MINOS), FRIB, RAON (KoRIA)
No shell effect for Mg(Z=12), Si(14), S(16), Ar(18) at N=20 and 28 
Bastin. et al. PRL (2007, ...)

Oxygen has new shell closure at N=16 
Otsuka et al PRL (2005)

Calcium has new shell closure at N=34 

Spin-orbit forces and hence shell structure modified by tensor and 3-body forces in neutron rich nuclei
The liquid interior

Neutrons (likely superfluid) $\sim 95\%$  Non-relativistic
Protons (likely superconducting) $\sim 5\%$  Non-relativistic
Electrons (normal, $T_c \sim T_f e^{-137}$) $\sim 5\%$  Fully relativistic

Eventually muons, hyperons??, quark matter and possible exotica:
- pion condensation
- kaon condensation
- quark droplets

Phase transition from crust to liquid at $n_b \sim 0.7$  $n_0 \sim 0.09$  fm$^{-3}$
(mass density $\sim 2 \times 10^{14}$  g/cm$^3$).  10% uncertainty!

$\quad n_0 \sim 0.16$  fm$^{-3}$

Uncertainties in nuclear matter liquid: interpolations between pure neutron matter and symmetric nuclear matter.
Standard construction of neutron star models

1) Compute energy per nucleon in neutron matter (pure or in beta equilibrium: $\mu_n = \mu_p + \mu_e$). Include 2 and 3 body forces between nucleons.

Neutron star models using static interactions between nucleons

\[ E = \text{energy density} = \rho \, c^2 \]
\[ n_b = \text{baryon density} \]
\[ P(\rho) = \text{pressure} = n_b^2 \frac{\partial (E/n_b)}{\partial n_b} \]

TOV equation

\[
\frac{\partial P(r)}{\partial r} = -\frac{G}{r^2} \frac{(\rho(r) + P(r)/c^2)}{1 - 2m(r)G/rc^2} \left( m(r) + 4\pi P(r)r^3/c^2 \right)
\]

Mass vs. central density

Akmal, Pandharipande and Ravenhall, 1998

APR equation of state
Neutron stars: cold quark matter

Fundamental limitations of eq. of state based on NN interactions alone

Accurate for \( n \approx n_0 \). But for \( n >> n_0 \):

- Can forces be described with static few-body potentials?

- Force range \( \approx 1/2m_\pi \Rightarrow \) relative importance of 3 (and higher) body forces \( \approx n/(2m_\pi)^3 \approx 0.4n_{fm^{-3}} \).

- No well defined expansion in terms of 2,3,4,...body forces.

- Can one even describe system in terms of well-defined \``asymptotic'' laboratory particles? Early percolation of nucleonic volumes!

Role of strangeness – hyperons (very model dependent!)

How can quark matter give stiff eq. of state, to explain large masses?
Learning about dense matter from neutron star observations

Masses and radii of neutron stars
Binary systems: stiff e.o.s
Thermonuclear bursts in X-ray binaries => Mass vs. Radius, strongly constrains eq.of state.
NICER to measure M and R directly

Glitches: probe n,p superfluidity and crust

Cooling of n-stars: search for exotica
Measuring equation of state in crust
(Andrew Cumming’s talk)
The equation of state is very stiff

Softer equation of state => lower maximum mass and higher central density

Binary neutron stars \( \sim 1.4 \, M_\odot \): consistent with soft eq. of state

PSR J1614-2230: \( M_{\text{neutron star}} = 1.93 \pm 0.02 M_\odot \)
PSR J0348+0432: \( M_{\text{neutron star}} = 2.01 \pm 0.04 M_\odot \)

require very stiff equation of state! How possible?
Neutron star masses


PSR J1614-2230:
\[ M_{\text{nstar}} = 1.928 \pm 0.017 M_\odot \]

PSR J0348+0432:
\[ M_{\text{nstar}} = 2.01 \pm 0.04 M_\odot \]

Galactic black hole masses

Wiktorowicz & Belcynski
Measuring masses and radii of neutron stars in thermonuclear bursts in X-ray binaries


Measurements of apparent surface area, flux at Eddington limit (radiation pressure = gravity), combined with distance to star, constrains M and R.
Mass vs. radius determination of neutron stars in burst sources (low mass x-ray binaries)

PHASE DIAGRAM OF NUCLEAR MATTER.
Modern phase diagram

Deconfined quarks and gluons

Asakawa-Yazaki critical point (1989)

States of color superconductivity – diquark BCS pairing

2SC / Color flavor locked (Alford, Rajagopal, Wilczek, ...)

Quarks confined
Crossover at zero net density: see no evidence of phase transition in pressure, entropy, or energy density.

Wuppertal-Budapest lattice collaboration
WB: S. Borsanyi et al., PLB (2014)
HotQCD: A. Bazavov et al., PRD (2014)

Lattice gauge theory not yet well implemented for finite baryon density!! Fermion sign problem
Crossover at zero net baryon density

QCD lattice gauge theory -- for finite light quark masses -- predicts crossover from confined phase at lower T to deconfined phase at higher T.

Do quarks roam freely in the deconfined phase? If so, they must also roam freely at lower T.

Are there really quarks running about freely in this room?
No free quarks even above the crossover!

In confined region quarks are inside hadrons. Also have quarks and antiquarks in the QCD forces between hadrons. With higher density or temperature, form larger clusters, which percolate at the crossover. In deconfined regime clusters extending across all of space.

Percolation of clusters along the density axis, at zero temperature.

Quarks can still be bound even if deconfined.

\[ n_{\text{perc}} \approx 0.34 \left( \frac{3}{4\pi} r_n^3 \right) \text{ fm}^{-3} \]

\[ r_n = \text{nucleon radius} \]

\[ n_0 = \text{density of matter inside large nucleus} \]
Critical points similar to those in liquid-gas phase diagram ($\text{H}_2\text{O}$)

Can go continuously from A to B around the upper critical point. Liquid-gas phase transition.

In lower shaded region have BCS pairing of nucleons, of quarks, and possibly other states (meson condensates). Different symmetry structure than at higher $T$. 
Phase diagram of ultracold atomic fermion gases: in T and strength of the particle interactions

Unitary regime (Feshbach resonance) – BEC-BCS crossover. No phase transition through crossover.
Phase diagram of ultracold atomic fermion gases: in $T$ and strength of the particle interactions

- Free fermions + di-fermion molecules
- $T_c \sim 0.22 \, E_f$
- $T_c \sim e^{-\pi/2k_f a}$
- $T_c < 0$
- $T_c > 0$
- BEC of di-fermion molecules
- BCS

BEC - diatomic molecules - strongly interacting pairs - Cooper pairs
Similarly, as nuclear matter becomes denser can one expect “continuous” evolution from hadrons (nucleons) to quark pairs (diquarks)?

Smooth evolution of states in atomic clouds and nuclear matter


Evolution of Fermi atoms with weakening attraction between atoms:

Similarly, as nuclear matter becomes denser can one expect “continuous” evolution from hadrons (nucleons) to quark pairs (diquarks)?

Quark hadron continuity
$T$ 

$\sim 150$ MeV 

quark-gluon plasma 

hadrons $\rightarrow$ quarks 

hadron resonance gas 

nuclear 

color superconductivity 

$M_N$ 

$\mu_B$
Have good idea of equation of state at nuclear densities and at high densities. Look at pressure vs. baryon chemical potential.

Quarks in Nambu-Jona-Lasinio (NJL) model with universal repulsive short-range qq coupling (Kunihiro)

\[ \mathcal{L}_{V}^{(4)} = -g_{V} (\bar{q} \gamma_{\mu} q)^{2} \]

APR = Akmal, Pandharipande, Ravenhall nucleonic equation of state with nucleonic potentials (2 and 3 body) fit to NN scattering and light nuclei.
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Quark matter cores in neutron stars

Canonical picture: compare calculations of eqs. of state of hadronic matter and quark matter.

*GB & S.A. Chin (1976)*

Crossing of thermodynamic potentials => first order phase transition. ex. nuclear matter using 2 & 3 body interactions, vs. perturbative expansion or bag models.

Assumes hadronic state at high densities – not possible when hadrons substantially overlap

Allows only quark equations of state lying under hadronic at high density. Soft only and therefore can’t support two solar mass stars.

Typically conclude transition at \( n \sim 10n_{\text{nm}} \) -- would not be reached even in high mass neutron stars => at most small quark matter cores.
How can QCD give large mass neutron stars?

Pressure $P$ is a continuous function of baryon chemical potential $\mu$.

Stiff equation of state has high pressure for given mass (or energy) density or equivalently low energy density for given pressure.

Stiffer equations of state given more massive neutron stars, with lower central densities.

Green equation of state is stiffer than red. Has larger pressure for given mass density $\rho$, and has smaller $\rho$ for given pressure $P$. 
How can QCD give large mass neutron stars?

Energy or mass density \( \varepsilon = \rho c^2 = \mu n - P \)

slope:

\[ n_* = \left. \frac{\partial P}{\partial \mu} \right|_{\mu_*} \]

smaller for stiffer equation of state
Hybrid eqs. of state are intrinsically softer

Phase with larger \( P \) at given \( \mu \) thermodynamically preferred

Assumes hadronic state at high densities – not possible when hadrons substantially overlap

Continuous eqs. of state can be much stiffer

Hadrons only at low density and quark matter at high density. In between???
Model calculations of neutron star matter within NJL model

NJL Lagrangian

\[ \mathcal{L} = \bar{q} (i \gamma_\mu \partial^\mu - m_q + \mu \gamma_0) q + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} \]

\[ \mathcal{L}^{(4)}_\chi = G \sum_{a=0}^{8} [(\bar{q} \tau_a q)^2 + (\bar{q} i \gamma_5 \tau_a q)^2] \]

\[ \mathcal{L}^{(4)}_d = H \sum_{A,A'=2,5,7} [(\bar{q} i \gamma_5 \tau_A \lambda_A C \bar{q}^T)(q^T C i \gamma_5 \tau_A \lambda_A q)] \]

\[ \mathcal{L}^{(6)} = \text{Kobayashi-Maskawa-'t Hooft six quark axial anomaly} \]

chiral interactions

BCS pairing interactions

plus universal repulsive quark-quark vector coupling

\[ \mathcal{L}^{(4)}_V = -g_V (\bar{q} \gamma^\mu q)^2 \]

T. Kunihiro

Include u, d, and s quarks


GB, T. Kojo, T. Hatsuda, T. Takatsuka, & Y. Song
arXiv:1607.04966 - ROPP

\[ g_V = G \]

\[ \frac{g_V}{G} = 5 \]

pressure

mass density

baryon density

1.5

1

0
Soft quark equation of state does not allow high mass neutron stars.
Shift of pressure in quark phase towards higher $\mu$

Vector interaction stiffens eq. of state

$nuclear\ APR$

$P$

$\mu$

$g_v=0$

interpolate

increase $g_v$

stiffens eq. of state
Vector interaction stiffens eq. of state

\[ \frac{\partial^2 P}{\partial \mu^2} = \frac{\partial n}{\partial \mu} < 0 \]

unstable region

nuclear APR

Larger $g_V$ leads to unphysical thermodynamic instability
Increased BCS pairing (onset of stronger 2-body correlations) as quark matter comes nearer to becoming confined
Sample “unified” equation of state: HQC17 (= hadron-quark crossover)


Consistent with eq. of state inferred from M vs. R observations

Quark eqs. of state can be stiffer than previously thought: allow for n.s. masses > 2 $M_\odot$, and with substantial quark cores in neutron stars!!!
Masses and radii of neutron stars vs. central mass density from integrating TOV equation with HQC17


Include stronger corelations between quarks by increasing the effective pairing interaction $H$ beyond standard NJL: $H \sim 1.5 \, G$

Increased vector repulsion between quarks: $g_V \sim 0.5-1.0 \, G$
Summary

For $2n_0 < n_B < 7-8n_0$ matter is intermediate between purely hadronic and purely quark.

Quark model eqs. of state can be stiffer than previously thought, allowing for neutron star masses $> 2M_\odot$.

Interaction parameters of order vacuum values $H \sim g_v \sim G_s^{\text{vac}}$.

Much more to do:

Uncertainties in nuclear matter equation of state (APR, etc.)

Uncertainties in interpolating from nuclear matter to quark matter lead to errors in maximum neutron star masses and radii.

Uncertainties in the vector coupling and pairing forces;

Going beyond the NJL model -- running $g_v$ (Fukushima-Kojo).

Need to produce finite temperature equation of state ($\leq 50$ MeV) for modelling neutron star -- neutron star (or black hole) mergers as sources of gravitational radiation. *Masuda et al. PTEPr. 2016, 021D01*

Cooling and transport properties???
Calculating neutron star tidal deformability with HQC17

Metric tensor with neutron star of mass \( M \) and quadrupole moment \( Q_{ij} \):

\[
g_{00} \sim 1 - G \left( \frac{M}{r} - \frac{3Q_{ij}}{2r^3} \hat{n}_i \hat{n}_j \ldots \right) + \frac{E_{ij}}{2} \hat{n}_i \hat{n}_j
\]

\( E_{ij} \) = external tidal force

Tidal deformability \( \lambda \) = response of \( Q \) to \( E \):

\[
Q_{ij} = -\lambda E_{ij}
\]

Dimensionless deformability \( \Lambda \):

\[
\lambda \equiv \frac{32G}{r_5^5} \Lambda
\]

\[
r_s = \frac{2MG}{c^2}
\]

= neutron star Schwarzschild radius

Stiffer equation of state => larger star => larger \( \lambda \)

C. Markakis and M. O’Boyle (UIUC)
Lower $\lambda (~ 0.9 \times 10^{36})$ for HQC17 => smaller effect of tidal deformation on rise of chirp frequency

Chris Pankow’s talk

Hinderer, Lackey, Lang, & Read, PR D 81, 1 (2010)