Confinement in a correlated instanton-dyon ensemble

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Introduction

- Topological objects in QCD vacuum have been studied for quite some time.

- Monopoles and confinement in abelian gauge theories [Polyakov].

- Instanton models such as ILM [Schafer, Shuryak (1998)] have been successful in explaining xSB in QCD and many other hadron properties.

- Confinement mechanism is still an open puzzle and “new” (1998) topological solutions might have the answer to it.

- This is essential to the understanding of the thermodynamic properties of QCD.
CONFINEMENT IN SU(2) GAUGE THEORY
Confinement in SU(2) Gauge Theory (finite temperature)

Polyakov loop $\rightarrow L(\vec{x}) = \mathcal{P} \exp \left( i \int_0^\beta d\tau A_4(\tau, \vec{x}) \right)$

- $\frac{1}{2} \text{Tr} L$ is gauge invariant $\rightarrow$ can be parametrized

$$L(\|\vec{x}\| \rightarrow \infty) = \begin{pmatrix} e^{2\pi i \mu_1} & 0 \\ 0 & e^{2\pi i \mu_2} \end{pmatrix}$$

- The set of eigenvalues $\{\mu_i\}$ is called Holonomy with $\mu_1 + \mu_2 = 0$

  And the difference $\nu \equiv \mu_2 - \mu_1$ $\rightarrow$ Holonomy parameter

Yang-Mills is invariant under center group transformations $Z_2$ but $L(\vec{x}) \rightarrow zL(\vec{x})$

$\rightarrow \langle L_\infty \rangle$ order parameter of the breaking of center symmetry

$$L_\infty \equiv \lim_{\|\vec{x}\| \rightarrow \infty} \frac{1}{2} \text{Tr} L(\vec{x})$$
Confinement in SU(2) Gauge Theory (finite temperature)

\[
\langle L_\infty \rangle = \begin{cases} 
\cdot 0 & \text{unbroken } \mathbb{Z}_2 \text{ symmetry} \\
\cdot \text{Non-zero} & \text{broken } \mathbb{Z}_2 \text{ symmetry}
\end{cases} \rightarrow \begin{cases} 
\text{CONFINED PHASE} \\
\text{DECONFINED PHASE}
\end{cases}
\]

Holonomy \[
\begin{cases} 
\cdot \text{The set } \{\mu_i\} \text{ such that } L \in \mathbb{Z}_2 & \rightarrow \text{TRIVIAL} \\
\cdot \text{The set } \{\mu_i\} \text{ such that } L \notin \mathbb{Z}_2 & \rightarrow \text{NON-TRIVIAL}
\end{cases}
\]

Physical Interpretation \[
\langle \text{Tr} L(\vec{x}) \rangle \propto e^{-\beta F_q} = \begin{cases} 
= 0 & \rightarrow \text{CONFINED} \\
\neq 0 & \rightarrow \text{DECONFINED}
\end{cases}
\]

\[
\langle \text{Tr} L(\vec{x}) \text{ Tr} L(\vec{y}) \rangle \propto e^{-\beta F_{q\bar{q}}} \quad \text{Static-heavy quark-antiquark potentials}
\]
INSTANTONS IN YANG-MILLS THEORY
BPST-Instanton

Belavin-Polyakov-Schwarz-Tyupkin (1975) → BPST-Instanton:

(ANTI)INSTANTON = Classical (anti)self-dual \( (F_{\mu\nu}^a = \pm \tilde{F}_{\mu\nu}^a) \) solution to the Euclidean e.o.m. with finite action.

\[
A_\mu(x) = -\tilde{\eta}_{\mu\nu}^a \partial_\nu \log W(x) \quad \text{with} \quad W(x) = 1 + \frac{\rho^2}{(x - x_0)^2}
\]

- Topological charge \( Q_T = \frac{1}{32\pi^2} \int d^4x \, F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a = \pm 1 \)

- Instanton action \( S = \frac{8\pi^2}{g^2} \)
H-S Caloron (trivial holonomy)

Generalization to finite temperature (1978) → Harrington-Shepard

CALORON:

\[ A_\mu(x) = -\tilde{\eta}_{\mu\nu}^a \partial_\nu \log \Pi(x) \quad \text{with} \quad \Pi(x) = 1 + \frac{\pi \rho^2}{\beta r} \frac{\sinh \left( \frac{2\pi r}{\beta} \right)}{\cosh \left( \frac{2\pi r}{\beta} \right) - \cos \left( \frac{2\pi \tau}{\beta} \right)} \]

- Periodic in Euclidean time \( A_\mu(\vec{r}, \tau) = A_\mu(\vec{r}, \tau + \beta) \), \( \beta = \frac{1}{T} \)

- Trivial holonomy \( \lim_{r \to \infty} A_4(\vec{r}, \tau) \sim \lim_{r \to \infty} \frac{1}{r^2} = 0 \)

\[ \implies \langle \text{Tr} L(\vec{r}) \rangle \neq 0 \quad \text{NO CONFINEMENT!} \]
KvBLL Calorons (Non-trivial holonomy)

Kraan, van Baal, Lee & Lu (1998)

\[
A^KvBLL_{\mu} = \delta_{\mu4}v^{\frac{3}{2}} + \frac{\tau^3}{2}\bar{\eta}_{\mu\nu}\partial_{\nu}\log\Phi + \frac{\Phi}{2}\text{Re} \left[ (\bar{\eta}_{\mu\nu} - i\bar{\eta}_{\mu\nu}^2) (\tau^1 + i\tau^2) (\partial_{\nu} + iv\delta_{\nu4}) \tilde{\chi} \right]
\]

Non-trivial holonomy \( v = 2\pi T\nu \) with \( \nu \in [0, 1] \)

- In the limit \( \nu \to 0 \) it reduces to the H-S caloron.

- “Made of” 2 BPS-monopoles: \textit{INSTANTON-DYONS} (a.k.a. Instanton-quarks, monopoles)

\[
L_{\infty} = \cos(\pi\nu) \Rightarrow L_{\infty} = 0 \quad \text{for} \quad \nu = \frac{1}{2}
\]
KvBLL Caloron (Non-trivial holonomy)

- The solution is parametrized in terms of the dyon positions.
- In the vicinities of $\vec{r}_L$ or $\vec{r}_M$, the field becomes that of the constituent dyons:

\[
A_4^M = \frac{\tau_3}{2} \left( v \coth(vs) - \frac{1}{s} \right) \xrightarrow{s \to \infty} v \frac{\tau_3}{2}
\]

\[
A_4^L = \frac{\tau_3}{2} \left( 2\pi T - \bar{v} \coth(\bar{v}r) + \frac{1}{r} \right) \xrightarrow{r \to \infty} v \frac{\tau_3}{2}
\]

where $\bar{v} \equiv 2\pi T - v$

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<th>$M$</th>
<th>$\bar{M}$</th>
<th>$L$</th>
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<td>Magnetic charge</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
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<tr>
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<td>$\nu \frac{8\pi^2}{g^2}$</td>
<td>$\bar{\nu} \frac{8\pi^2}{g^2}$</td>
<td>$\bar{\nu} \frac{8\pi^2}{g^2}$</td>
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<tr>
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<td>$v^{-1}$</td>
<td>$v^{-1}$</td>
<td>$\bar{v}^{-1}$</td>
<td>$\bar{v}^{-1}$</td>
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</table>
THE DYON ENSEMBLE
Building the instanton-dyon ensemble

1. Semiclassical calculation of Yang-Mills partition function around the KvBLL caloron:

\[
Z = \int \mathcal{D}A_{\mu} \ e^{-S[A_{\mu}]} \quad \rightarrow \quad A_{\mu}(x) = A_{\mu}^{\text{KvBLL}}(x) + a(x)
\]

\[
Z_{\text{KvBLL}} = e^{-VP(\nu)} \int d^3r_L \ d^3r_M T^6 C 2\pi \left( \frac{8\pi^2}{g^2} \right)^4 \left( \frac{\Lambda_{PV} e^\gamma E}{4\pi T} \right)^{\frac{22}{3}} \left( \frac{1}{T r_{LM}} \right)^{\frac{5}{3}}
\]

\[
\times (1 + 2\pi T \nu \bar{\nu} r_{LM}) (1 + 2\pi T \nu r_{LM})^{\frac{8\nu}{3} - 1} (1 + 2\pi T \bar{\nu} r_{LM})^{\frac{8\bar{\nu}}{3} - 1}
\]

The KvBLL (L-M pair) 1-loop partition function [Diakonov et al (2004)]

where \( P(\nu) = \frac{4\pi^2}{3} T^3 \nu^2 \bar{\nu}^2 \) [Gross, Pisarski, Yaffe (1981)] and

\[
\text{Instanton action} \quad \rightarrow \quad S(T) = \frac{8\pi^2}{g^2(T)} = \frac{22}{3} \log \left( \frac{4\pi T}{\Lambda_{PV} e^\gamma E} \right)
\]
Building the instanton-dyon ensemble

2. The moduli space metric for arbitrary number of dyons

\[ \sqrt{\det(g)} \approx \det(G) \]

[Diakonov & Petrov (2007)] of dimension \((N_L + N_M) \times (N_L + N_M)\)

\[
G_{m,i,n,j} = \delta_{mn} \delta_{i,j} \left( 4\pi \nu - \sum_{k \neq i} \frac{2}{T|\vec{r}_{m,i} - \vec{r}_{m,k}|} + \sum_{k} \frac{2}{T|\vec{r}_{m,i} - \vec{r}_{l_k}|} \right) \\
\quad + \frac{2\delta_{mn}}{T|\vec{r}_{m,i} - \vec{r}_{n,j}|} \bigg|_{i \neq j} - \frac{2}{T|\vec{r}_{m,i} - \vec{r}_{n,j}|} \bigg|_{m \neq n}
\]

For a single L-M pair:

\[
G = \begin{pmatrix}
4\pi \nu + \frac{1}{Tr_{LM}} & -\frac{1}{Tr_{LM}} \\
-\frac{1}{Tr_{LM}} & 4\pi \bar{\nu} + \frac{1}{Tr_{LM}}
\end{pmatrix}
\]

\[ r_{LM} \gg \frac{1}{2\pi T\nu}, \frac{1}{2\pi T\bar{\nu}} \]

\[ Z_{KvBLL} = e^{-VP(\nu)} \int d^3 r_L d^3 r_M \det(G) T^6 \Gamma^2 S^4 e^{-S \nu^{8/3} - 1 - \bar{\nu}^{8/3} - 1} \]
3. To include antidyons in the ensemble: 
\[ \text{det}(G) \text{det}(\tilde{G}) e^{-V_{D\bar{D}}} \]

**Uncorrelated**

**Correlated**

\[ V_{D\bar{D}} \rightarrow \text{Action of Dyon-Antidyon interactions} \]

- Of the same kind: [Larsen & Shuryak, (2015,2016)]

\[
\begin{align*}
V_{LL} &= -2\bar{\nu}S \left( \frac{1}{\zeta_L} - 1.632e^{-0.704\zeta_L} \right) \\
V_{MM} &= -2\nu S \left( \frac{1}{\zeta_M} - 1.632e^{-0.704\zeta_M} \right)
\end{align*}
\]

where
\[ \zeta_j = 2\pi\nu_j \text{T}r_{jj} \]
\[ r_{jj} = |\vec{r}_j - \vec{r}_{\bar{j}}| \]

- Repulsive core potential:

\[ V_{jj}^C = \frac{\nu_j V_c}{1 + e^{(\zeta_j - \zeta_j^0)}} \]

- Long range interaction (Abelian Electric/Magnetic):

\[ V_{ij} = \frac{S}{2\pi T r_{ij}} (e_i e_j + m_i m_j - 2h_i h_j) \]
4. Screening in all Coulomb terms $\frac{1}{r} \rightarrow \frac{e^{-MD r}}{r}$, including those in $\det(G)$

The total Dyon-Antidyon interactions look like

$$V_{D\bar{D}} = \left\{ \begin{array}{ll}
\sum_{j \neq \bar{j}} 2S \left( \frac{1}{\zeta_j} - 1.632 e^{-0.704 \zeta_j} \right) e^{-MD r_{j\bar{j}}} & \text{if } \zeta_j > \zeta_j^0, \text{ for } L\bar{L}, M\bar{M} \\
\sum_{i \neq j} V_{i\bar{j}}^C & \text{if } \zeta_j < \zeta_j^0, \text{ for } LL, \bar{L}\bar{L}, MM, \bar{M}\bar{M}, L\bar{L}, M\bar{M} \\
\sum_{i,\bar{j}} \frac{S}{\pi T_{r_{i\bar{j}}}} e^{-MD r_{i\bar{j}}} & \text{for } \bar{M}L, \bar{L}M \\
0 & \text{for } LM, \bar{L}\bar{M}.
\end{array} \right.$$
The Partition Function of the dyon ensemble

\[ Z = e^{-VP(\nu)} \sum_{N_M, N_L, N_{\bar{L}}, N_{\bar{M}}} \frac{1}{N_L!N_M!N_{\bar{L}}!N_{\bar{M}}!} \int \prod_{l=1}^{N_L} f_L T^3 \, d^3 r_{L_l} \prod_{m=1}^{N_M} f_{\bar{M}} T^3 \, d^3 r_{\bar{M}_m} \det(G_D) \det(G_{\bar{D}}) e^{-V_D\bar{D}} \]

\[ \times \prod_{\bar{l}=1}^{N_{\bar{L}}} f_{\bar{L}} T^3 \, d^3 r_{\bar{L}_{\bar{l}}} \prod_{\bar{m}=1}^{N_{\bar{M}}} f_{\bar{M}} T^3 \, d^3 r_{\bar{M}_{\bar{m}}} \]

where \( f_M = f_{\bar{M}} = \Gamma S^2 e^{-\nu S} \nu^{8/3} \nu^{-1} \) and \( f_L = f_{\bar{L}} = \Gamma S^2 e^{-\bar{\nu} S} \bar{\nu}^{8/3} \bar{\nu}^{-1} \)

Dyon “fugacities”

Integration over dyon positions

\[ d^3 L_l \, d^3 M_m \, d^3 r_{\bar{L}_{\bar{l}}} \, d^3 r_{\bar{M}_{\bar{m}}} \]
The ensemble settings

- Dimensionless units scaled by temperature
  
  \[ rT \rightarrow r \quad VT^3 \rightarrow V \quad \frac{F}{T} \rightarrow F \quad \frac{M_D}{T} \rightarrow M_D \]

- Temperature dependence is parametrized by the instanton action
  
  \[ S(T) = \frac{8\pi^2}{g^2(T)} = \frac{22}{3} \log \left( \frac{T}{\Lambda} \right) \]

  At \( T = T_c \), the scale parameter is defined
  \[ \Rightarrow \frac{\Lambda}{T_c} = \exp \left[ -\frac{3}{22} S(T_c) \right] \]

- The simulation was set on a box of length \( L \approx 3.5/T \) with p.b.c.

- \( N_D = N_{\bar{D}} \) in the range \( N_D \in [0, 22] \) or \( n_D \in [0, 0.5] \)

- Around 3000 MC configurations generated for each set of parameters:

  \[ \{ \nu, S, N_L, N_M, M_D, V_c, s_c, V \} \]
The phase transition was found for \( S = 7 \Rightarrow \Lambda = 0.385T_c \)

For \( 0.761 \leq T/T_c \leq 1 \) the minimum of free energy density was still at \( \nu = 0.5 \)

For \( S > 7, \nu < 0.5 \) → Deconfined phase

For \( 5 \leq S \leq 7 \) → Maximal non-trivial holonomy → Confined phase
The order parameter and universality

- Second order phase transition
- From Svetitsky-Yaffe conjecture:

It belongs to same universality class as 3D-Ising Model.

\[ \langle L_\infty \rangle \sim (T/T_c - 1)^\beta [1 + (T/T_c - 1)^\omega] \]

Using the Ising model C.E. values \( \beta \approx 0.3265 \) and \( \omega \approx 0.84 \)
Dyon densities

Dyon size:

\[ L \sim \frac{1}{2\pi T \bar{\nu}} \quad M \sim \frac{1}{2\pi T \nu} \]

As \( \nu \to 0 \) the action of the \( M \) type dyons is much smaller than that of \( L \) ones.
Static quark-antiquark potentials

- Another important criterion of confinement

\[ e^{-F_{qq}} \sim \langle \text{Tr} L^\dagger(\bar{x}) \text{Tr} L(\bar{y}) \rangle \quad \longrightarrow \quad F_{qq}|_{|\bar{x}-\bar{y}| \to \infty} \approx \sigma |\bar{x} - \bar{y}| \]

- The total \( A_4(x) \) field in the ensemble is needed

**NOTICE:** Far away from their cores, the dyon fields become:

\[ A_4^{L,\bar{L}} \approx \frac{\tau^3}{2} \left( \nu + \frac{1}{|\bar{x}|} \right) \quad \text{and} \quad A_4^{M,\bar{M}} \approx \frac{\tau^3}{2} \left( \nu - \frac{1}{|\bar{x}|} \right) \]

\[ \Rightarrow A_4(\bar{x}) = \frac{\tau^3}{2} \left[ 2\pi \nu + \sum_{l,m}^{N_L,N_M} \left( \frac{1}{|\bar{x} - \bar{r}_{L_l}|} - \frac{1}{|\bar{x} - \bar{r}_{M_m}|} + \frac{1}{|\bar{x} - \bar{r}_{L_l}'|} - \frac{1}{|\bar{x} - \bar{r}_{M_m}'|} \right) \right] \]
Static quark-antiquark potentials

- For color sources (quarks) in the **fundamental representation**: 
  \[ \frac{1}{2} \text{Tr} \ L^f(\vec{x}) = \cos \left( \pi \nu + \frac{1}{2} l(\vec{x}) \right) \]

- Quark-Antiquark interact via a **singlet** and **triplet** channels \[ 2 \otimes \bar{2} = 1 \oplus 3 \]

\[ e^{-F^\text{avg}_{q\bar{q}}} \equiv \frac{1}{4} \left\langle \text{Tr} \ L^f(\vec{x}) \text{Tr} \ L^f(\vec{y}) \right\rangle \implies e^{F^\text{avg}_{q\bar{q}}} = \frac{1}{4} e^{-F^1_{q\bar{q}}} + \frac{3}{4} e^{-F^3_{q\bar{q}}}, \quad e^{-F^1_{q\bar{q}}} \equiv \frac{1}{2} \left\langle \text{Tr} \ [L^f(\vec{x}) L^f(\vec{y})] \right\rangle \]

**String tension** at “intermediate” distances to avoid finite volume effects

\[ F_{q\bar{q}} \sim \sigma |\vec{x} - \vec{y}| \]
For quarks in the *adjoint representation*:  
\[ \text{Tr} \, L^a(x) = |\text{Tr} \, L^f(x)|^2 - 1 \]

So the quark-antiquark potential is defined as
\[ e^{-F_{qq}^a} = \frac{\langle \text{Tr} \, L^+a(x) \text{Tr} \, L^a(y) \rangle}{\langle |\text{Tr} \, L^a(0)|^2 \rangle} \]

At large distances the plateau in the potential indicates "string breaking" due to screening effects, as expected.
Spatial Wilson Loop

- At finite $T$, the spatial Wilson loop does not provide a good measure for confinement since it has Area Law even in the deconfined phase.

- At $T \to 0$ Lorentz symmetry (O(4) Euclidean) is restored and the “electric” and “magnetic” string tension should coincide.

- The spatial components of the dyons in the large distance limit is also Abelian:

$$A^j_\phi(x) = m_j \frac{\tan \theta}{2} \frac{\tau^3}{r}$$

**Dirac string at** $\theta = \pi$

$$B^j_r = \frac{m_j}{r^2} \frac{\tau^3}{2}$$

The total “magnetic” field in the ensemble

$$\implies B_i(x) = \frac{\tau^3}{2} \sum_{l,m}^{N_L,N_M} \left[ \frac{(x - \vec{r}_{L_l})_i}{|x - \vec{r}_{L_l}|^3} - \frac{(x - \vec{r}_{M_m})_i}{|x - \vec{r}_{M_m}|^3} - \frac{(x - \vec{r}_{L_l})_i}{|x - \vec{r}_{L_l}|^3} + \frac{(x - \vec{r}_{M_m})_i}{|x - \vec{r}_{M_m}|^3} \right]$$

Thus, we compute the traced spatial Wilson loop as

$$W_C \equiv \frac{1}{2} \text{Tr } \mathcal{P} \exp \left[ i \int_C dx_i A_i(x) \right] = \frac{1}{2} \text{Tr } \mathcal{P} \exp \left[ i \int_{AC} da_i B_i(x) \right]$$
Spatial Wilson Loop

- The interest is to show there is an area law even in the deconfined phase. \( \langle W_C \rangle \sim e^{-\sigma_s A_C} \)
- Even though \( \sigma_s \) increases with temperature, \( \sigma_s/T^2 \) actually decreases.
- Due to our choice of units, it is not possible to make a correspondence with the electric string tension.

For the adjoint representation, the screening effect of string breaking is also present.
SOME DISCUSSIONS
Influence of the core potential (Short range correlations)

- The repulsive core $V_{j}^{C} = \frac{\nu_{j}V_{c}}{1 + e^{(\zeta_{j} - \zeta_{j}^{0})}}$ was found to be essential for the confinement mechanism.

- The parameters $V_{c}$ and $\zeta_{j}^{0}$ have to be tuned properly to achieve confinement.
The screening mass $M_D$

![Graphs showing F/V vs ν for different values of $M_D$ and $T/T_c$.](image-url)
Finite volume effects

- Looking for volume effects in the probability distribution of the ensemble.
- Small effect in some observables.

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Finite volume effects
SUMMARY AND FUTURE WORK
Summary and Future Work

• For S=7 a second order phase transition was observed allowing to fix the scale parameter to $\Lambda = 0.385T_c$ and define the temperature dependence.

• The role of $\langle L_\infty \rangle$ as the order parameter of the deconfinement phase transition was established and in agreement with universality properties.

• The expected behavior of static quark-antiquark potentials and the spatial Wilson loop was observed, providing more evidence of confining properties.

• One of the key ingredients in the model is the repulsive core. Tuning of its parameters define the confined phase.

• Finite volume effects are present, but they do not affect results in a significant way.

• Our results are reported on arXiv:1611.02539 [hep-ph]

• Future work: Include dynamical fermions to explore chiral symmetry breaking/restoration and (inverse) magnetic catalysis in presence of magnetic field.
Thanks!
Static quark-antiquark potentials

• For color sources (quarks) in the **fundamental representation:** \( \frac{1}{2} \text{Tr} L^f(x) = \cos \left( \pi \nu + \frac{1}{2} l(x) \right) \)

• Quark-Antiquark interact via a **singlet** and **triplet** channels \( 2 \otimes \bar{2} = 1 \oplus 3 \)

\[
e^{-F_{qq}^{\text{avg}}} \equiv \frac{1}{4} \left\langle \text{Tr} \left[ L^f(x) \right] \text{Tr} \left[ L^f(y) \right] \right\rangle \implies e^{F_{qq}^{\text{avg}}} = \frac{1}{4} e^{-F_{qq}^{1}} + \frac{3}{4} e^{-F_{qq}^{3}}, \quad e^{-F_{qq}^{1}} \equiv \frac{1}{2} \left\langle \text{Tr} \left[ L^f(x) L^f(y) \right] \right\rangle
\]

\( T/T_c = 0.761 \quad \text{and} \quad T/T_c = 1.725 \)
The screening mass $M_D$

Self-consistent calculation of Debye mass

$$M_D^2 = \frac{2S}{V} \frac{\partial^2 F}{\partial \nu^2}$$