Three-Body Forces in Hypernuclear Mean-Field Model

Jan Pokorný[1,2], Petr Veselý[2]

[1]Faculty of Nuclear Sciences and Physical Engineering of Czech Technical University in Prague,
[2]Nuclear Physics Institue of Czech Academy of Sciences in Řež

25$^{\mathrm{th}}$ October 2017

ECT* ASTRA Workshop

Motivation

- aim – description and **spectra** of **medium** and **heavier hypernuclei** with **realistic baryon forces**
- **Hamiltonian**

$$\widehat{\mathbf{H}} = \widehat{\mathbf{T}}_{\mathbf{N}} + \widehat{\mathbf{T}}_{\mathbf{\Lambda}} + \widehat{\mathbf{V}}^{\mathbf{NN}} + \widehat{\mathbf{V}}^{\mathbf{NNN}} + \widehat{V}^{\Lambda NN} - \widehat{\mathbf{T}}_{\mathbf{CM}} \tag{1}$$

- the **hypernuclear mean field** is generated by the **HF equations** in **proton-neutron-Λ** formalism
- **HF formalism** is derived for **both three-body terms** *NNN* and *ΛNN* interactions, so far **implemented only the** ***NNN*** **term**
- we study the **effect of the** ***NNN*** **interactions** on the **structure of hypernuclei** and their **nuclear cores**

Three-body Interactions

- baryon-baryon potentials in the low-energy scale can be described by **Chiral Perturbation Theory** (ChPT)
- ChPT – effective field theory with consistent hierarchy of two-, three-, four-, ..., and many-baryon forces
- three-body interactions emerge at the **next-to-next-to leading order** of perturbation

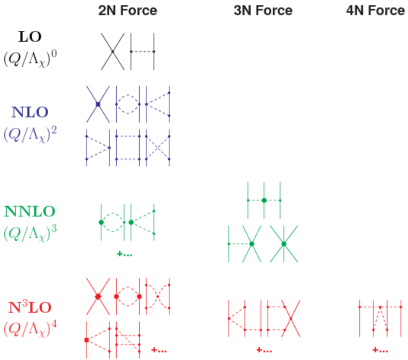

Figure: Hierarchy of nuclear forces in ChPT [R. Machleidt, arXiv:1308.0103].

Mean-Field Model

- mean field – (hyper)nuclear potential **self-consistently generated** from interactions between all constituents (nucleons or nucleons + Λ)
- protons, neutrons, and Λ are considered to be different particles placed in different potential wells
- **Hartree-Fock method** in the **p-n-Λ formalism**
- HF method traditionally based on **phenomenological** two-body NN interactions
- our approach – formalism with **realistic** $NN, NNN, \Lambda N, \Lambda NN$ interactions
- employed chiral potential $\mathrm{NNLO_{sat}}$ [A. Ekström et al., PRC 91, 051301(R) (2015)] – **realistic** NN and NNN interactions
- LO ΛN potential [H. Polinder et al., NPA 779, 244 (2006)] – **strong cutoff dependence** (linear), arbitrarily chosen cutoff $\lambda = 550$ MeV
- computations only with NN, NNN interactions (nuclei); $NN, NNN, \Lambda N$ interactions (hypernuclei)

The Hartree-Fock Method

- the starting point of this method is the hypernuclear Hamiltonian

$$\widehat{\mathbf{H}} = \widehat{\mathbf{T}}_{\mathbf{N}} + \widehat{\mathbf{T}}_{\mathbf{\Lambda}} + \widehat{\mathbf{V}}^{\mathbf{NN}} + \widehat{\mathbf{V}}^{\mathbf{\Lambda N}} + \widehat{\mathbf{V}}^{\mathbf{NNN}} + \widehat{V}^{\Lambda NN} - \widehat{\mathbf{T}}_{\mathbf{CM}}, \tag{1}$$

- $\widehat{T}_N, \widehat{T}_\Lambda$ ... kinetic terms
- $\widehat{V}^{NN}, \widehat{V}^{\Lambda N}$ ... two-body terms
- $\widehat{V}^{NNN}, \widehat{V}^{\Lambda NN}$ ... three-body terms
- $\widehat{T}_{CM}$ ... center-of-mass kinetic term

$$\widehat{T}_{CM} = \frac{1}{2M(A+0.19)} \left(\sum_{a=1}^{A} \widehat{\vec{P}}_a^2 + 2 \sum_{a<b} \widehat{\vec{P}}_a \cdot \widehat{\vec{P}}_b \right) \tag{2}$$

- $M \approx 938$ MeV ... nucleon mass, A ... baryon number, $\widehat{\vec{P}}_a$ ... a-th particle momentum
- **we derive formalism of the HF method for Hamiltonian (1)**
- **we implement only $\widehat{\mathbf{T}}_{\mathbf{N}}, \widehat{\mathbf{T}}_{\mathbf{\Lambda}}, \widehat{\mathbf{V}}^{\mathbf{NN}}, \widehat{\mathbf{V}}^{\mathbf{NNN}}, \widehat{\mathbf{V}}^{\mathbf{\Lambda N}}$ – tests on ^{40}Ca, ^{16}O, $^{41}_{\Lambda}$Ca, $^{17}_{\Lambda}$O**

The Wave Function of the Ground State

- the ground-state wave function is a product of proton and neutron **Slater determinants** (fermions, antisymmetrization) and one single-particle wave function of the Λ particle

$$|\Psi_0\rangle = |\Psi_0\rangle_{\mathrm{p}} \otimes |\Psi_0\rangle_{\mathrm{n}} \otimes |\Psi_0\rangle_{\Lambda} \tag{1}$$

- we introduce the HF method in the formalism of the **second quantization**, i.e. in terms of **creation, annihilation operators** ($a^\dagger, a$ for protons, $b^\dagger, b$ for neutrons, $c^\dagger, c$ for Λ)

$$|\Psi_0\rangle_{\mathrm{p}} = \prod_{i=1}^{Z} a_i^\dagger |0\rangle, \quad |\Psi_0\rangle_{\mathrm{n}} = \prod_{i=1}^{N} b_i^\dagger |0\rangle, \quad |\Psi_0\rangle_{\Lambda} = c_1^\dagger |0\rangle \tag{2}$$

- Z ... proton number, N ... neutron number, products run over states lowest in energy

Second quantization

- single-particle wave function $|i\rangle_{\mathrm{p}} = a_i^\dagger|0\rangle$, $|i\rangle_{\mathrm{n}} = b_i^\dagger|0\rangle$, $|i\rangle_\Lambda = c_i^\dagger|0\rangle$
- kinetic operators $t_{ij}^{\mathrm{p}} = {}_{\mathrm{p}}\langle i|\widehat{T}_{\mathrm{p}}|j\rangle_{\mathrm{p}}$, $t_{ij}^{\mathrm{n}} = {}_{\mathrm{n}}\langle i|\widehat{T}_{\mathrm{n}}|j\rangle_{\mathrm{n}}$, $t_{ij}^{\Lambda} = {}_{\Lambda}\langle i|\widehat{T}_{\Lambda}|j\rangle_{\Lambda}$
- two-body operators

$$V_{ijkl}^{\mathrm{pp}} = \langle ij|\widehat{V}^{\mathrm{pp}}|kl - lk\rangle, \tag{1a}$$

$$V_{ijkl}^{\mathrm{nn}} = \langle ij|\widehat{V}^{\mathrm{nn}}|kl - lk\rangle, \tag{1b}$$

$$V_{ijkl}^{\mathrm{pn}} = \langle ij|\widehat{V}^{\mathrm{pn}}|kl\rangle, \tag{1c}$$

$$V_{ijkl}^{\mathrm{p}\Lambda} = \langle ij|\widehat{V}^{\mathrm{p}\Lambda}|kl\rangle, \tag{1d}$$

$$V_{ijkl}^{\mathrm{n}\Lambda} = \langle ij|\widehat{V}^{\mathrm{n}\Lambda}|kl\rangle. \tag{1e}$$

- three-body operators

$$V_{ijklmn}^{\mathrm{ppp}} = \langle ijk|\widehat{V}^{\mathrm{ppp}}|lmn - lnm + nlm - nml + mnl - mln\rangle, \tag{2a}$$

$$V_{ijklmn}^{\mathrm{nnn}} = \langle ijk|\widehat{V}^{\mathrm{nnn}}|lmn - lnm + nlm - nml + mnl - mln\rangle, \tag{2b}$$

$$V_{ijklmn}^{\mathrm{ppn}} = \langle ijk|\widehat{V}^{\mathrm{ppn}}|lmn - mln\rangle, \tag{2c}$$

$$V_{ijklmn}^{\mathrm{pnn}} = \langle ijk|\widehat{V}^{\mathrm{pnn}}|lmn - lnm\rangle, \tag{2d}$$

$$V_{ijklmn}^{\mathrm{pp}\Lambda} = \langle ijk|\widehat{V}^{\mathrm{pp}\Lambda}|lmn - mln\rangle, \tag{2e}$$

$$V_{ijklmn}^{\mathrm{nn}\Lambda} = \langle ijk|\widehat{V}^{\mathrm{nn}\Lambda}|lmn - mln\rangle, \tag{2f}$$

$$V_{ijklmn}^{\mathrm{pn}\Lambda} = \langle ijk|\widehat{V}^{\mathrm{pn}\Lambda}|lmn\rangle. \tag{2g}$$

Hypernuclear Hamiltonian in the Second Quantization

- hypernuclear Hamiltonian in the second quantization

$$
\begin{aligned}
\widehat{H} &= \sum_{ij} t^{\mathrm{p}}_{ij} a^\dagger_i a_j + \sum_{ij} t^{\mathrm{n}}_{ij} b^\dagger_i b_j + \sum_{ij} t^{\Lambda}_{ij} c^\dagger_i c_j \\
&+ \frac{1}{4}\sum_{ijkl} V^{\mathrm{pp}}_{ijkl} a^\dagger_i a^\dagger_j a_l a_k + \frac{1}{4}\sum_{ijkl} V^{\mathrm{nn}}_{ijkl} b^\dagger_i b^\dagger_j b_l b_k + \sum_{ijkl} V^{\mathrm{pn}}_{ijkl} a^\dagger_i b^\dagger_j b_l a_k \\
&+ \sum_{ijkl} V^{\mathrm{p}\Lambda}_{ijkl} a^\dagger_i c^\dagger_j c_l a_k + \sum_{ijkl} V^{\mathrm{n}\Lambda}_{ijkl} b^\dagger_i c^\dagger_j c_l b_k \\
&+ \frac{1}{36}\sum_{ijklmn} V^{\mathrm{ppp}}_{ijklmn} a^\dagger_i a^\dagger_j a^\dagger_k a_n a_m a_l + \frac{1}{36}\sum_{ijklmn} V^{\mathrm{nnn}}_{ijklmn} b^\dagger_i b^\dagger_j b^\dagger_k b_n b_m b_l \\
&+ \frac{1}{4}\sum_{ijklmn} V^{\mathrm{ppn}}_{ijklmn} a^\dagger_i a^\dagger_j b^\dagger_k b_n a_m a_l + \frac{1}{4}\sum_{ijklmn} V^{\mathrm{pnn}}_{ijklmn} a^\dagger_i b^\dagger_j b^\dagger_k b_n b_m a_l \\
&+ \frac{1}{4}\sum_{ijklmn} V^{\mathrm{pp}\Lambda}_{ijklmn} a^\dagger_i a^\dagger_j c^\dagger_k c_n a_m a_l + \frac{1}{4}\sum_{ijklmn} V^{\mathrm{nn}\Lambda}_{ijklmn} b^\dagger_i b^\dagger_j c^\dagger_k c_n b_m b_l \\
&+ \sum_{ijklmn} V^{\mathrm{pn}\Lambda}_{ijklmn} a^\dagger_i b^\dagger_j c^\dagger_k c_n b_m a_l . \qquad (1)
\end{aligned}
$$

Variational Equation

- protons, neutrons, Λ in **separate** HO bases
- **unitary** transformations A, B, C between **HO basis** and **self-consistent basis**

$$a_i'^\dagger = \sum_{ij} A_{ij} a_j^\dagger ; \quad a_i' = \sum_{ij} a_j A_{ij}^* , \tag{1a}$$

$$b_i'^\dagger = \sum_{ij} B_{ij} b_j^\dagger ; \quad b_i' = \sum_{ij} b_j B_{ij}^* , \tag{1b}$$

$$c_i'^\dagger = \sum_{ij} C_{ij} c_j^\dagger ; \quad c_i' = \sum_{ij} c_j C_{ij}^* . \tag{1c}$$

- **self-consistent** basis is defined as

$$|\mathrm{HF}\rangle = \prod_{i=1}^{Z} a_i'^\dagger |0\rangle \otimes \prod_{i=1}^{N} b_i'^\dagger |0\rangle \otimes c_1'^\dagger |0\rangle \tag{2}$$

- the HF method is a **variational method**, it **minimizes** the energy functional

$$\frac{\delta}{\delta A, B, C} \left[\langle \mathrm{HF} | \widehat{H} | \mathrm{HF} \rangle - \varepsilon_i^{p,n,\Lambda} \langle \mathrm{HF} | \mathrm{HF} \rangle \right] = 0 \tag{3}$$

The HF Equations for Protons, Neutrons, and the Λ Hyperon

$$
\begin{aligned}
t^{\mathrm{p}}_{ij} + \sum_{kl} V^{\mathrm{pp}}_{ikjl}\rho^{\mathrm{p}}_{lk} + \sum_{kl} V^{\mathrm{pn}}_{ikjl}\rho^{\mathrm{n}}_{lk} + \sum_{kl} V^{\mathrm{p}\Lambda}_{ikjl}\rho^{\Lambda}_{lk} + \frac{1}{2}\sum_{klmn} V^{\mathrm{ppp}}_{ikljmn}\rho^{\mathrm{p}}_{mk}\rho^{\mathrm{p}}_{nl} \\
+ \frac{1}{2}\sum_{klmn} V^{\mathrm{pnn}}_{ikljmn}\rho^{\mathrm{n}}_{mk}\rho^{\mathrm{n}}_{nl} + \sum_{klmn} V^{\mathrm{ppn}}_{ikljmn}\rho^{\mathrm{p}}_{mk}\rho^{\mathrm{n}}_{nl} + \sum_{klmn} V^{\mathrm{pp}\Lambda}_{ikljmn}\rho^{\mathrm{p}}_{mk}\rho^{\Lambda}_{nl} \\
+ \sum_{klmn} V^{\mathrm{pn}\Lambda}_{ijklmn}\rho^{\mathrm{n}}_{mk}\rho^{\Lambda}_{nl} = \underline{\underline{\varepsilon^{\mathrm{p}}_{i}}}\delta_{ij}. \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
t^{\mathrm{n}}_{ij} + \sum_{kl} V^{\mathrm{nn}}_{ikjl}\rho^{\mathrm{n}}_{lk} + \sum_{kl} V^{\mathrm{pn}}_{kilj}\rho^{\mathrm{p}}_{lk} + \sum_{kl} V^{\mathrm{n}\Lambda}_{ikjl}\rho^{\Lambda}_{lk} + \frac{1}{2}\sum_{klmn} V^{\mathrm{nnn}}_{ikljmn}\rho^{\mathrm{n}}_{mk}\rho^{\mathrm{n}}_{nl} \\
+ \frac{1}{2}\sum_{klmn} V^{\mathrm{ppn}}_{klimnj}\rho^{\mathrm{p}}_{mk}\rho^{\mathrm{p}}_{nl} + \sum_{klmn} V^{\mathrm{pnn}}_{klimnj}\rho^{\mathrm{p}}_{mk}\rho^{\mathrm{n}}_{nl} + \sum_{klmn} V^{\mathrm{nn}\Lambda}_{ikljmn}\rho^{\mathrm{n}}_{mk}\rho^{\Lambda}_{nl} \\
+ \sum_{klmn} V^{\mathrm{pn}\Lambda}_{klimnj}\rho^{\mathrm{p}}_{mk}\rho^{\Lambda}_{nl} = \underline{\underline{\varepsilon^{\mathrm{n}}_{i}}}\delta_{ij}. \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
t^{\Lambda}_{ij} + \sum_{kl} V^{\mathrm{p}\Lambda}_{kilj}\rho^{\mathrm{p}}_{lk} + \sum_{kl} V^{\mathrm{n}\Lambda}_{kilj}\rho^{\mathrm{n}}_{lk} + \frac{1}{2}\sum_{klmn} V^{\mathrm{pp}\Lambda}_{klimnj}\rho^{\mathrm{p}}_{mk}\rho^{\mathrm{p}}_{nl} + \frac{1}{2}\sum_{klmn} V^{\mathrm{nn}\Lambda}_{klimnj}\rho^{\mathrm{n}}_{mk}\rho^{\mathrm{n}}_{nl} \\
+ \sum_{klmn} V^{\mathrm{pn}\Lambda}_{klimnj}\rho^{\mathrm{p}}_{mk}\rho^{\mathrm{n}}_{nl} = \underline{\underline{\varepsilon^{\Lambda}_{i}}}\delta_{ij}. \qquad (3)
\end{aligned}
$$

Numerical Implementation

- we implement the **extension of the HF+TDA code** which has been used in the study of multipole response in neutron-rich nuclei [D. Bianco, F.Knapp, N. Lo Iudice, P. Vesely, F. Andreozzi, G. De Gregorio, A. Porrino, JPG 41, 025109 (2014).]
- we employ **chiral potential NNLO**$_{\mathrm{sat}}$ [A. Ekström et al., PRC 91, 051301(R) (2015)]
  - includes NN and NNN interactions
- **chiral LO ΛN interaction** [H. Polinder et al., NPA 779, 244 (2006).]
  - explicit linear dependence on the cut-off parameter
  - this interaction was derived without counter terms
  - we implement cut-off 550 MeV – closest to realistic results
- the terms with ΛNN **interactions are not taken into account**

Truncation of the Configuration Space

- the HF method is implemented into to the computer code
- **truncation** of the **single-particle configuration space** $N_{\max}$

$$\{|i\rangle : 2n_i + l_i = N_i \leq N_{\max}\} \tag{1}$$

- 2body elements **– products of two single-particle states** $|ij\rangle = |i\rangle|j\rangle$, truncation $N_{\max}^{(12)}$

$$\{|ij\rangle : 2n_i + l_i + 2n_j + l_j = N_i + N_j \leq N_{\max}^{(12)} = 2N_{\max}\} \tag{2}$$

- 3body elements **– products of three single-particle states** $|ijk\rangle = |i\rangle|j\rangle|k\rangle$, truncation $N_{\max}^{(123)}$

$$\{|ijk\rangle : 2n_i + l_i + 2n_j + l_j + 2n_k + l_k = N_i + N_j + N_k \leq N_{\max}^{(123)} = 3N_{\max}\} \tag{3}$$

- testing calculations done in **small config. space** ($N_{\max} = 4, N_{\max}^{(12)} = 8, N_{\max}^{(123)} = 12$)
- **qualitative effect** of the *NNN* interactions is present in this small space
- in future: **convergence, systematics, cutoff** $N_{\max} = N_{\max}^{(12)} = N_{\max}^{(123)}$

Results

- studied nuclei: ^{16}O, ^{40}Ca – doubly magic and spherically symmetric
- studied hypernuclei: $^{17}_{\Lambda}$O, $^{41}_{\Lambda}$Ca
- small configuration space – truncation: $N_{\mathrm{max}} = 4$, $N^{(12)}_{\mathrm{max}} = 8$, $N^{(123)}_{\mathrm{max}} = 12$
- documentation of the **qualitative effect of the *NNN* interactions**
  - **radial density distributions**
  - **charge radii**
  - **neutron single particle spectra** (the same effect applies to proton s.p. spectra)
  - Λ **s.p. spectra**
- comparison of calculations done purely with the *NN* interactions to the ones done with the *NN* and the *NNN* interactions
  - **(2B)** vs. **(2B + 3B)**

Radial nuclear density distributions of the ^{40}Ca and the ^{16}O

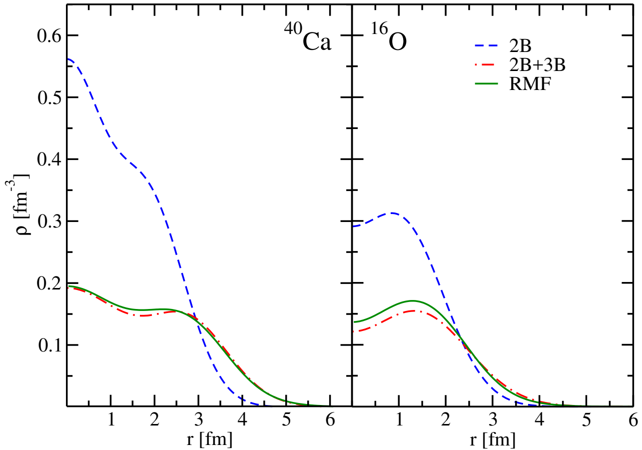

Figure: Radial density distribution of ^{40}Ca and ^{16}O calculated only with *NN* interactions (2B – dashed blue line), radial density distribution of ^{40}Ca and ^{16}O calculated with *NN* and *NNN* interactions (2B + 3B – dash-dotted red line), realistic radial density distribution of ^{40}Ca and ^{16}O calculated with RMF model (RMF – full green line).

- **calculations with *NNN* interactions (2B+3B) are closer to the realistic density distributions**

The Charge Radii of the ^{40}Ca and the ^{16}O

Table: The charge radii r_{ch} of the ^{40}Ca and the ^{16}O calculated with the *NN* interactions (2B) and the charge radii of the ^{40}Ca and the ^{16}O calculated with the $NN + NNN$ interactions (2B+3B) compared to the experimental data (exp) taken from [I. Angeli, ADNDT 87, 185 (2004)].

	r_{ch} [fm]		
^{A}X	2B	2B+3B	exp
^{40}Ca	2.58	**3.18**	**3.48**
^{16}O	2.23	**2.67**	**2.70**

- **charge radii calculated with the *NNN* interactions (2B+3B) are realistic**
- **configuration space too small** for ^{40}Ca - we expect results closer to experiment in bigger space

The Neutron Single-Particle Spectra in the ^{40}Ca and in the ^{16}O

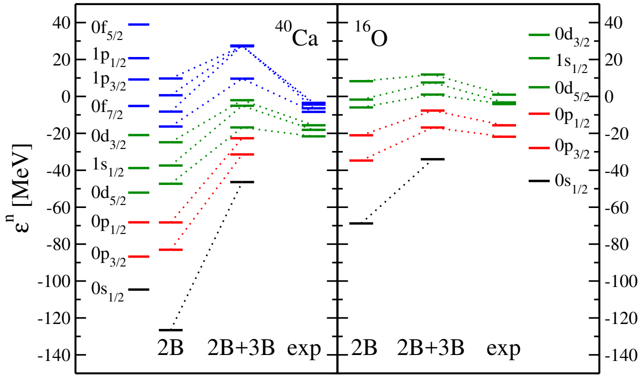

Figure: The neutron single-particle energies ε^n in ^{40}Ca and ^{16}O calculated with only two-body *NN* interactions (2B) and with two-body *NN* plus three-body *NNN* interactions (2B+3B) compared to the empirical values (exp) [V.I. Isakov et al., PJA 14, 29-36 (2002)].

- **the *NNN* interactions (2B+3B) quenche the gaps between major shells and the gaps within the shells**
- the results for the sd- shells in ^{40}Ca are not realistic – small config. space

The Λ Single-Particle Spectra in the ${}^{41}_{\Lambda}$Ca and in the ${}^{17}_{\Lambda}$O

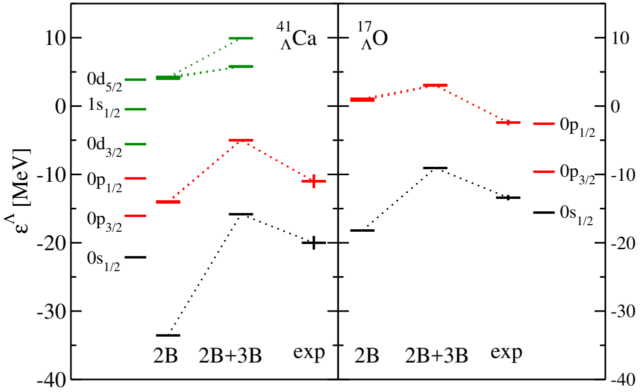

Figure: The Λ single-particle energies ε^{Λ} in ${}^{41}_{\Lambda}$Ca and ${}^{17}_{\Lambda}$O calculated with only two-body *NN* interactions (2B) and with two-body *NN* plus three-body *NNN* interactions (2B+3B) compared to the experimental data (exp) [M. Agnello et al., PLB 698, 219 (2011), R. E. Chrien, NPA 478, 705c (1998)].

- sd- shells in ${}^{41}_{\Lambda}$Ca not realistic – not taken into account
- **the *NNN* interactions shrink the gaps between the s- and p- shells**
- spectra shifted upwards in energy – **cut-off dependence of the Λ*N* interaction**
- spin-orbit splitting in p- levels

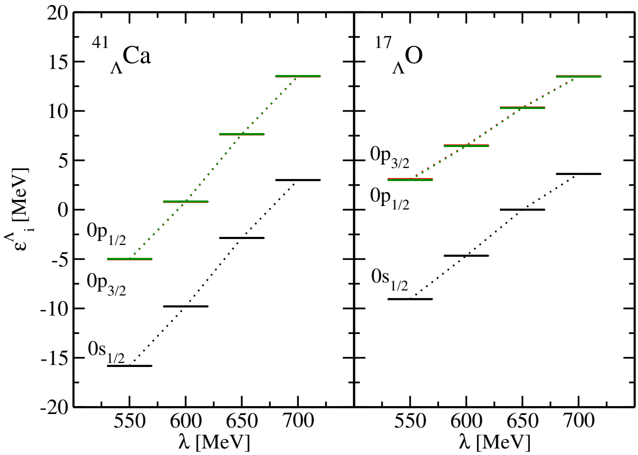

Figure: Single-particle energies of the Λ particle in the $^{41}_{\Lambda}$Ca and $^{17}_{\Lambda}$O as a function of the cutoff parameter λ.

- **spectra** depends **linearly** on cutoff
- **the gaps remain constant!**

The Λ single-particle spectra in the $^{41}_{\Lambda}$Ca and in the $^{17}_{\Lambda}$O II

Table: Single-particle energies of the Λ hyperon in $^{41}_{\Lambda}$Ca and $^{17}_{\Lambda}$O calculated with *NN* interactions (2B) and with *NN* + *NNN* interactions (2B+3B) compared to the experimental data (exp) – $^{41}_{\Lambda}$Ca [R.E. Chrien, NPA 478, 705c (1998)], $^{17}_{\Lambda}$O [M. Agnello et al., PLB 698, 219 (2011)].

	ε^{Λ} [MeV]					
		$^{41}_{\Lambda}$Ca			$^{17}_{\Lambda}$O	
s.-p. level	2B	2B+3B	exp	2B	2B+3B	exp
$0s_{1/2}$	-33.561	-15.820	-20.0 $\pm$ 1.0	-18.203	-9.055	-13.5 $\pm$ 0.4
$0p_{3/2}$	-14.095	-5.016	-11.0 $\pm$ 1.0	1.076	3.090	-2.4 $\pm$ 0.4
$0p_{1/2}$	-13.958	-4.987	-11.0 $\pm$ 1.0	0.805	3.005	-2.4 $\pm$ 0.4

- **the *NNN* interactions give realistic gaps between s- and p- shells**
- standard ordering of levels in the p- shell: $0p_{3/2}$, $0p_{1/2}$
- **good results for $^{41}_{\Lambda}$Ca, opposite ordering in the p- shell in the $^{17}_{\Lambda}$O**
- **property of the employed ΛN interaction, not the model!!**
    - weak tensor term

Conclusions

- **derivation of the formalism of the HF method** which includes NNN and ΛNN potentials
- **implementation of the** NN **and** NNN **interactions** in the chiral potential $\mathrm{NNLO_{sat}}$ **in the HF code**
- test calculations – the NNN interactions **flatten the density distributions, extend the charge radii, shrink gaps between the major shells in neutron s.p. spectra, shrink gaps between the major shells in** Λ **s.p. spectra**
- **employed** ΛN **interaction is strongly cut-off dependent**
  - opposite ordering of p- levels in the $^{17}_{\Lambda}\mathrm{O}$ – possibly weak tensor term

Outlook

- **systematics** – dependence of the spectra (proton, neutron, Λ) on the size of the configuration space
- **$\boldsymbol{\Lambda} - \boldsymbol{\Sigma}$ mixing!!!**
- **implementation of the ΛNN interaction**
- **coupling** of **$\boldsymbol{\Lambda}$** with **core exctitations** (**beyond mean-field approach**)