Tetraquarks from lattice QCD

"Functional methods in hadron and nuclear physics", Trento

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Basic idea to investigate tetraquarks

- Study tetraquarks in the following way:

  (1) **Compute potentials of two heavy quarks** (e.g. $\bar{b}\bar{b}$) **in the presence of two lighter quarks** (e.g. ud, ss, cc) **using lattice QCD.**

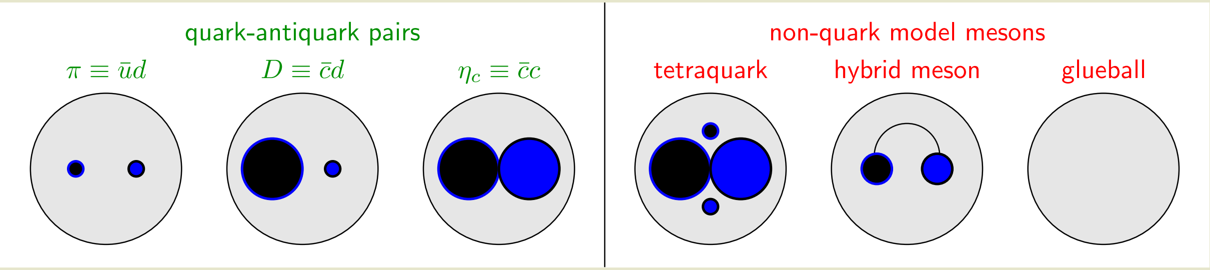
  (2) **Explore, whether these potentials are sufficiently attractive to host bound states or resonances** ($\rightarrow$ tetraquarks) **by using techniques from quantum mechanics and scattering theory.**

- This talk is a summary of
  [P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]
  [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., Phys. Rev. D **92**, 014507 (2015) [arXiv:1505.00613]]
  [P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]
  [P. Bicudo, J. Scheunert, M.W., Phys. Rev. D **95**, 034502 (2017) [arXiv:1612.02758]]
  [P. Bicudo, J. Scheunert, M.W., Phys. Rev. D **95**, 034502 (2017) [arXiv:1612.02758]]
  [P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, M.W., arXiv:1704.02383].

- For recent work from other groups using a similar approach cf. e.g.
  [W. Detmold, K. Orginos, M. J. Savage, Phys. Rev. D **76**, 114503 (2007) [arXiv:hep-lat/0703009]]
  [G. Bali, M. Hetzenegger, PoS **LATTICE2010**, 142 (2010) [arXiv:1011.0571 [hep-lat]]
  [Z. S. Brown and K. Orginos, Phys. Rev. D **86**, 114506 (2012) [arXiv:1210.1953 [hep-lat]]

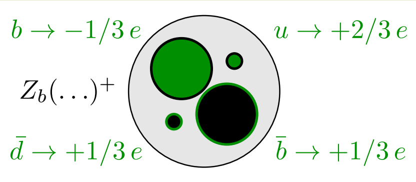
Why are such studies important? (1)

- **Meson**: system of quarks and gluons with integer total angular momentum $J = 0, 1, 2, \ldots$
- Most mesons seem to be **quark-antiquark pairs** $\bar{q}q$, e.g. $\pi \equiv \bar{u}d$, $D \equiv \bar{c}d$, $\eta_s \equiv \bar{c}c$ (quark-antiquark model calculations reproduce the majority of experimental results).
- Certain mesons are poorly understood (significant discrepancies between experimental results and quark model calculations), could have a more complicated structure, e.g.
  - **2 quarks and 2 antiquarks (tetraquark)**,
  - **a quark-antiquark pair and gluons (hybrid meson)**,
  - **only gluons (glueball)**.



Why are such studies important? (2)

- Indications for tetraquark structures:
  - Electrically charged mesons $Z_b(10610)^+$ and $Z_b(10650)^+$:
    - Mass suggests a $b\bar{b}$ pair ...
    - ... but $b\bar{b}$ is electrically neutral ...?
    - **Easy to understand, when assuming a tetraquark structure**: $Z_b(\ldots)^+ \equiv b\bar{b}u\bar{d}$ ($u \rightarrow +2/3\,e$, $\bar{d} \rightarrow -1/3\,e$).



  - Electrically charged Z_c states:
    - Similar to Z_b.
  - Mass ordering of light scalar mesons:
    - E.g. $m_\kappa > m_{a_0(980)}$ ...?

Outline

- A brief introduction to lattice QCD and the computation of hadron masses.
  - QCD: definition.
  - QCD: computation of hadron masses.
  - Lattice QCD.
- $\bar{b}\bar{b}qq$ tetraquarks.
- $\bar{b}\bar{b}qq$ / BB potentials.
- $\bar{b}\bar{b}qq$ tetraquarks (2).
- Inclusion of heavy spin effects.
- $\bar{b}\bar{b}qq$ tetraquark resonances.
- $\bar{b}b$ hybrid mesons.
- Summary and outlook.

QCD: definition

- Definition of QCD rather simple:

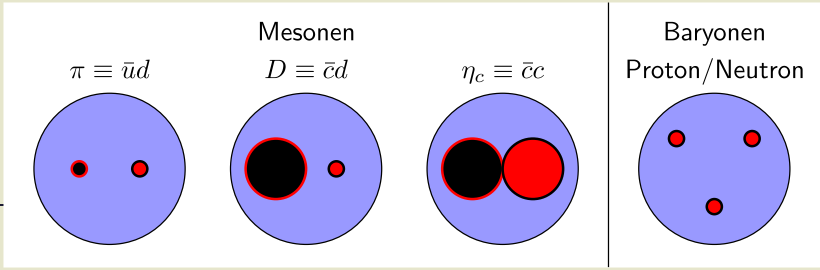
$$S = \int d^4x \left(\sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}^{(f)} \Big(\gamma_\mu \Big(\partial_\mu - iA_\mu\Big) + m^{(f)}\Big) \psi^{(f)} + \frac{1}{2g^2} \mathrm{Tr}\Big(F_{\mu\nu} F_{\mu\nu}\Big)\right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

  - $\psi^{(f)}(\mathbf{r}, t)$, $\bar{\psi}^{(f)}(\mathbf{r}, t)$: **quark fields**.
  - $A_\mu(\mathbf{r}, t)$: **gluon field**.

- No analytical solutions for e.g. meson or baryon masses available, because
  - field equations non-linear,
  - no small parameter (coupling constant), i.e., perturbation theory in general not applicable.

- Numerical method necessary $\rightarrow$ **lattice QCD**.



QCD: computation of hadron masses (1)

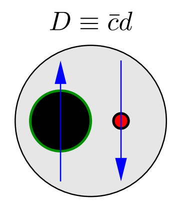
- Lattice QCD computation of a hadron mass in three steps:

Step (1): define a suitable hadron creation operator O

- A hadron creation operator is essentially a combination of quark field operators $\psi^{(f)}(\mathbf{r}) \equiv u(\mathbf{r}), d(\mathbf{r}), s(\mathbf{r}), c(\mathbf{r}), b(\mathbf{r}), t(\mathbf{r})$ and gluon field operators $A_\mu(\mathbf{r})$.
- The quark field operator $u(\mathbf{r})$ creates a u quark at position $\mathbf{r}$, $d(\mathbf{r})$ creates a d quark, ...
- A **suitable hadron creation operator** O generates in **crude approximation** the hadron of interest:
  - **Details are irrelevant, the final result for the hadron mass does not depend on these details.**
  - **Example:** D **meson** ... essentially a quark-antiquark pair $\bar{c}d$ with total angular momentum $J = 0$ and parity $P = -$; a possible D meson creation operator is

$$O \equiv \int d^3r \, \bar{c}(\mathbf{r})\gamma_5 d(\mathbf{r})$$

($\gamma_5 \to J^P = 0^-$, $\int d^3r \to \mathbf{p} = 0$).



QCD: computation of hadron masses (2)

Step (2): Compute the temporal correlation function $C(t)$ of the hadron creation operator O using lattice QCD

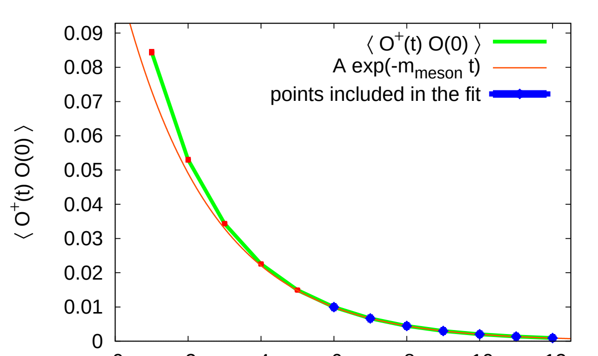
- **Correlation function**: $C(t) \equiv \langle\Omega|O^\dagger(t)O(0)|\Omega\rangle$ $(|\Omega\rangle$ = QCD ground state = vacuum).
- Lattice QCD is very technical:
  - Sophisticated codes have to be developed ...
  - ... which run on high performance computers several weeks or months ...
  - ... a few details on the next slide.

Step (3): extract the hadron mass from the exponential decay of the correlation function $C(t)$

- Using elementary quantum mechanics one can show

$$C(t) \quad = \quad \langle\Omega|O^\dagger(t)O(0)|\Omega\rangle \quad \overset{t\to\infty}{\propto} \quad e^{-m_D t}.$$

- Fit of $Ae^{-m_D t}$ to the lattice QCD results for $C(t)$ yields the D meson mass m_D.



Lattice QCD

- **Goal**: numerical calculation of QCD observables, e.g. a temporal correlation function (and from that correlation function a hadron hadron mass).

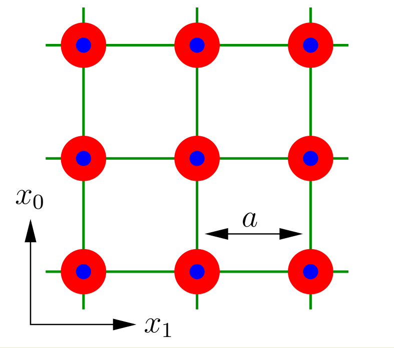
- Starting point is the **path integral formulation**,

$$C(t) \quad = \quad \langle \Omega | O^\dagger(t) O(0) | \Omega \rangle \quad = \quad \prod_{x_\nu \in \text{Gitter}} \bigg(\prod_f d\psi^{(f)}(x_\nu) \, d\bar{\psi}^{(f)}(x_\nu) \bigg) dA_\mu(x_\nu) \, \ldots ,$$

  - on each spacetime point x_ν (there are infinitely many) one has to solve an "ordinary integral" over the field variables $\psi^{(f)}(x_\nu)$ and $A_\mu(x_\nu)$,
  - i.e., an infinite-dimensional integral.

- Numerical realization:

  - Discretize spacetime by introducing a hypercubic lattice with sufficiently small lattice spacing $a \approx 0.05\,\text{fm} \ldots 0.10\,\text{fm}$.
  - Consider only a limited region of spacetime with extent $L \approx 2.0\,\text{fm} \ldots 4.0\,\text{fm}$.

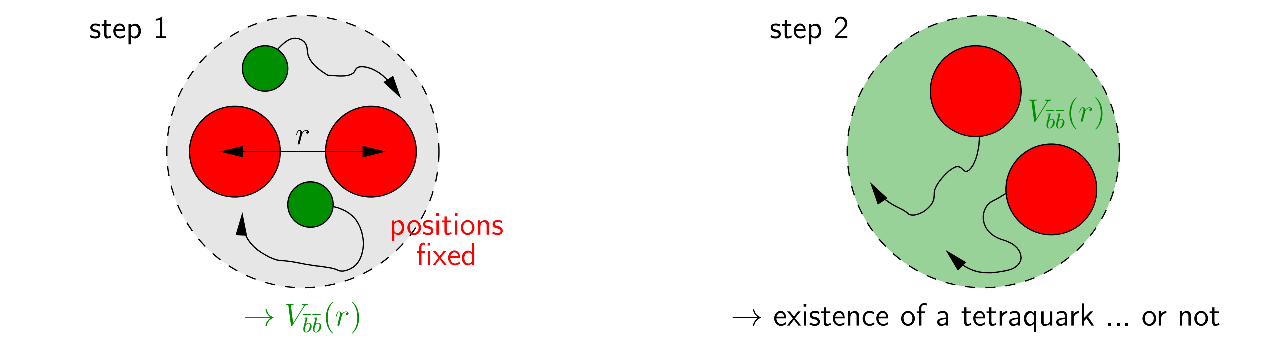


$\rightarrow$ Path integral reduced to a finite-dimensional integral, but $\mathcal{O}(10^8)$ **integration variables**.
$\rightarrow$ Specifically developed stochastic algorithms are necessary.
$\rightarrow$ High performance computer systems are needed.

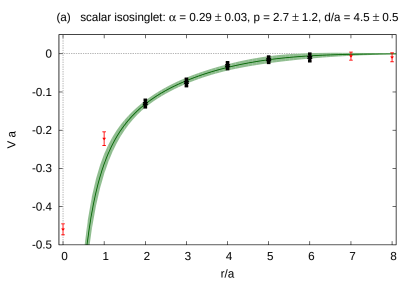
$\bar{b}\bar{b}qq$ tetraquarks (1)

- **Basic idea**: Investigate existence of heavy tetraquarks $\bar{b}\bar{b}qq$ in two steps.
  (1) **Compute potentials of two static antiquarks ($\bar{b}\bar{b}$) in the presence of two lighter quarks ($qq \in \{ud, ss, cc\}$) using lattice QCD.**
  (2) **Check, whether these potentials are sufficiently attractive, to host a bound state by solving a corresponding Schrödinger equation.**
  ($\rightarrow$ This would indicate a stable $\bar{b}\bar{b}qq$ tetraquark.)

  ((1) + (2) $\rightarrow$ Born-Oppenheimer approximation).



$\bar{b}\bar{b}qq$ tetraquarks (2)



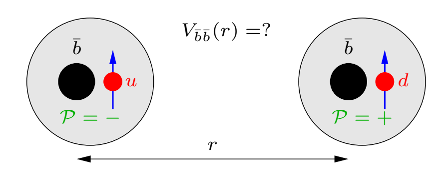
Born-Oppenheimer approximation, step (1)

- Lattice QCD computation of $\bar{b}\bar{b}$ potentials $V_{\bar{b}\bar{b}}(r)$ (2 flavor ETMC gauge link configurations).

  (1) Use $\bar{b}\bar{b}qq$ creation operators

$$O_{\bar{b}\bar{b}qq} \equiv (\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{b}_C(-\mathbf{r}/2)q_A^{(1)}(-\mathbf{r}/2)\Big)\Big(\bar{b}_D(+\mathbf{r}/2)q_B^{(2)}(+\mathbf{r}/2)\Big).$$

  - Different light quark flavors $qq \in \{ud, ss, cc\}$.
  - Different light quark spin/parity.
  - Different heavy quark spin/parity (no effect on $V_{\bar{b}\bar{b}}(r)$).

  → Many different channels
  ... some attractive, some repulsive
  ... some correspond for large $\bar{b}\bar{b}$ separations to pairs of ground state mesons (B and/or B^*), some to excited mesons (one or two B_0^* and/or B_1^*).

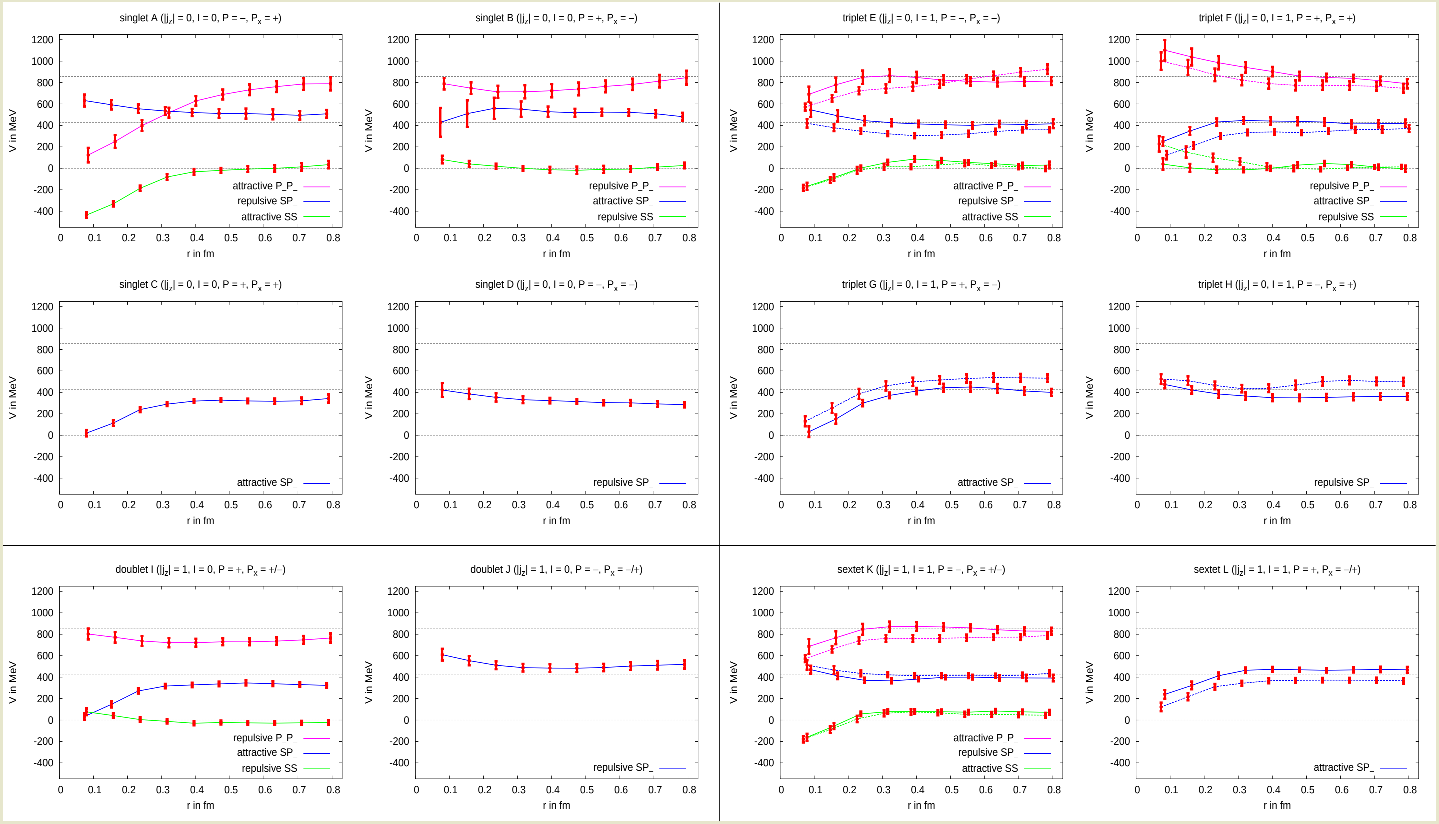
  (2) Compute temporal correlation functions.

  (3) Determine $V_{\bar{b}\bar{b}}(r)$ from the exponential decays of the correlation functions.

- **First principles QCD computation of forces between hadrons.**

$\bar{b}\bar{b}qq$ / BB potentials (1)

- $I = 0$ (left) and $I = 1$ (right); $|j_z| = 0$ (top) and $|j_z| = 1$ (bottom).



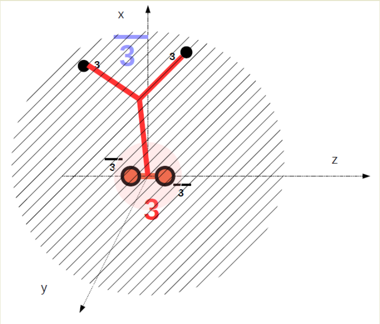
$\bar{b}\bar{b}qq$ / BB potentials (2)

Why are certain channels attractive and others repulsive? (1)

- Fermionic wave function must be antisymmetric (Pauli principle); in quantum field theory/QCD automatically realized on the level of states.
- qq isospin: $I = 0$ antisymmetric, $I = 1$ symmetric.
- qq angular momentum/spin: $j = 0$ antisymmetric, $j = 1$ symmetric.
- qq color:
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$: must be antisymmetric, i.e., a triplet $\bar{3}$.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$: must be symmetric, i.e., a sextet 6.
- The four quarks $\bar{b}\bar{b}qq$ must form a color singlet:
  - qq in a color triplet $\bar{3}$ $\rightarrow$ static quarks $\bar{b}\bar{b}$ also in a triplet 3.
  - qq in a color sextet 6 $\rightarrow$ static quarks $\bar{b}\bar{b}$ also in a sextet $\bar{6}$.

$\bar{b}\bar{b}qq$ / BB potentials (3)

Why are certain channels attractive and others repulsive? (2)



- Assumption: attractive/repulsive behavior of $\bar{b}\bar{b}$ at small separations r is mainly due to 1-gluon exchange,
  - color triplet 3 is attractive, $V_{\bar{b}\bar{b}}(r) = -2\alpha_s/3r$,
  - color sextet $\bar{6}$ is repulsive, $V_{\bar{b}\bar{b}}(r) = +\alpha_s/3r$

  (easy to calculate in LO perturbation theory).

- Summary:
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$ $\rightarrow$ attractive $\bar{b}\bar{b}$ potential $V_{\bar{b}\bar{b}}(r)$.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$ $\rightarrow$ repulsive $\bar{b}\bar{b}$ potential $V_{\bar{b}\bar{b}}(r)$.

  Expectation is consistent with the obtained lattice results (for ground state potentials $[B,\ B^*]$; can be extended to excitations $[B_0^*,\ B_1^*]$).

$\bar{b}\bar{b}qq$ / BB potentials (4)

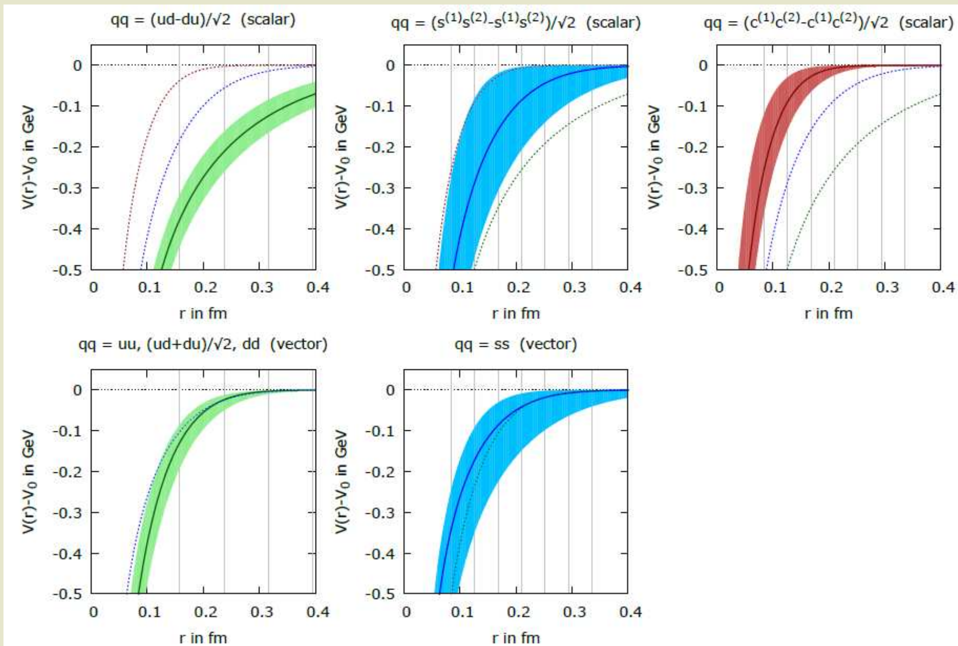
- Focus on the two attractive channels between B and B^*:
  - Scalar isosinglet ($(I = 0, j = 0)$, more attractive): $qq = (ud - du)/\sqrt{2}$, $\Gamma = (1 + \gamma_0)\gamma_5$.
  - Vector isotriplet ($(I = 1, j = 1)$, less attractive): $qq \in \{uu, (ud + du)/\sqrt{2}, dd\}$, $\Gamma = (1 + \gamma_0)\gamma_j$.
- Computations for $qq = ll, ss, cc$ ($l \in \{u, d\}$) to study the mass dependence.
- Parameterize lattice potential results by contiuous functions obtained by χ^2 minimizing fits of

$$V_{\bar{b}\bar{b}}(r) \quad = \quad -\frac{\alpha}{r}\exp\left(-\left(\frac{r}{d}\right)^p\right) + V_0 :$$

  - $1/r$: 1-gluon exchange at small $\bar{b}\bar{b}$ separations.
  - $\exp(-(r/d)^p)$: color screening at large $\bar{b}\bar{b}$ separations due to meson formation.
  - Fit parameters α, d and V_0; $p = 2$ from quark models.

$\bar{b}\bar{b}qq$ / BB potentials (5)

- Potentials for $qq = ll$, $l \in \{u, d\}$ are wider and deeper than potentials for $qq = ss, cc$.
  $\rightarrow$ **Good candidates to find tetraquarks are systems of two very heavy and two very light quarks, i.e.,** $\bar{b}\bar{b}ll$.



$\bar{b}\bar{b}qq$ tetraquarks (3)

Born-Oppenheimer approximation, step (2)

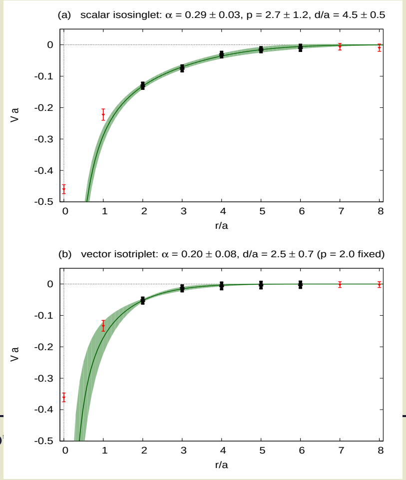
- Solve the Schrödinger equation for the relative coordinate of the heavy quarks $\bar{b}\bar{b}$,

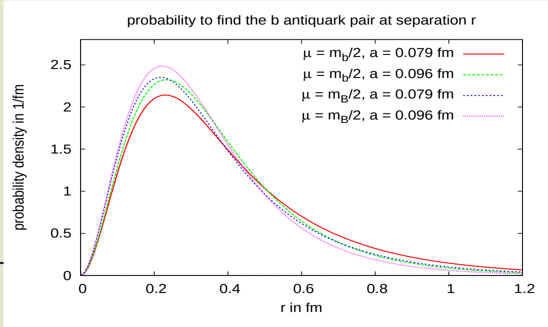
$$\left(-\frac{1}{2\mu}\triangle + V_{\bar{b}\bar{b}}(r)\right)\psi(\mathbf{r}) \quad = \quad E\psi(\mathbf{r}) \quad , \quad \mu \quad = \quad m_b/2;$$

  possibly existing bound states, i.e., $E < 0$, indicate $\bar{b}\bar{b}qq$ tetraquarks.

- There is a bound state for $qq = (ud - du)/\sqrt{2}$ (i.e., the scalar isosinglet potential) and orbital angular momentum $l = 0$ of $\bar{b}\bar{b}$, binding energy $E = -90^{+43}_{-36}$ MeV with respect to the $B + B^*$ threshold, confidence level $\approx 2\,\sigma$.

- No further bound states, in particular not for $qq = ss, cc$ (i.e., $B_s B_s$, $B_c B_c$).



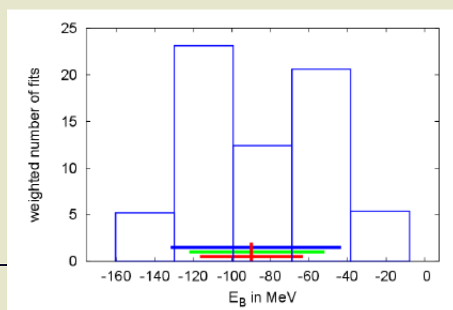


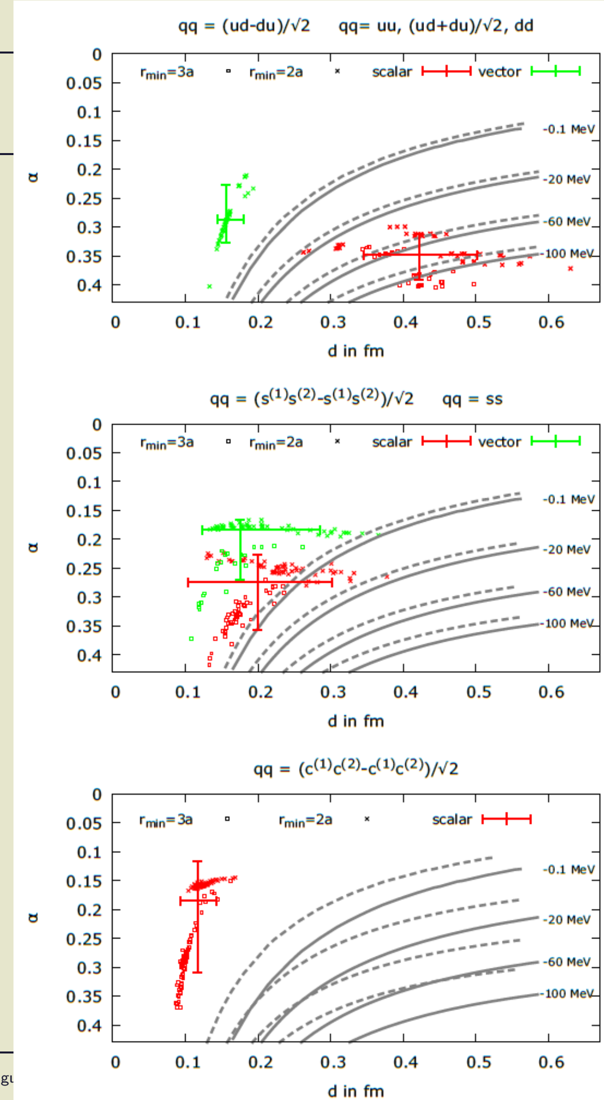
$\bar{b}\bar{b}qq$ tetraquarks (4)

- Estimate the systematic error by varying input parameters:
  - the t fitting range to extract the potential from effective masses,
  - the r fitting range for

$$V_{\bar{b}\bar{b}}(r) \quad = \quad -\frac{\alpha}{r}\exp\left(-\left(\frac{r}{d}\right)^p\right) + V_0.$$

- Left: isoline plots of the binding energy E for $l = 0$.
- Bottom: histogram for the binding energy E for $qq = (ud - du)/\sqrt{2}$ and $l = 0$.





$\bar{b}\bar{b}qq$ tetraquarks (5)

- To quantify "no binding", we list for each channel the factor, by which the reduced mass μ in the Schrödinger equation has to be multiplied, to obtain a tiny but negative energy E (again for $l = 0$).

qq	spin	factor
$(ud - du)/\sqrt{2}$	scalar	0.46
uu, $(ud + du)/\sqrt{2}$, dd	vector	1.49
$(s^{(1)}s^{(2)} - s^{(2)}s^{(1)})/\sqrt{2}$	scalar	1.20
ss	vector	2.01
$(c^{(1)}c^{(2)} - c^{(2)}c^{(1)})/\sqrt{2}$	scalar	2.57

  - Factors $\ll 1$ indicate strongly bound states, while for values $\gg 1$ bound states are essentially excluded.

- Light quarks (u/d) are unphysically heavy (correspond to $m_\pi \approx 340$ MeV); physically light u/d quarks yield similar results.

- Mass splitting $m(B^*) - m(B) \approx 50$ MeV, neglected at the moment, is expected to weaken binding (will be discussed below).

$\bar{b}\bar{b}qq$ tetraquarks (6)

What are the quantum numbers of the predicted $\bar{b}\bar{b}qq$ tetraquark?

- $I(J^P) = 0(1^+)$.
  - Light scalar isosinglet: $qq = (ud - du)/\sqrt{2}$, $I = 0$, $j = 0$ in a color $\bar{3}$, $\bar{b}\bar{b}$ in a color 3 (antisymmetric) ... as discussed above.
  - Wave function of $\bar{b}\bar{b}$ must also be antisymmetric (Pauli principle).
    - $\bar{b}\bar{b}$ is flavor symmetric.
    - $\bar{b}\bar{b}$ spin must also be symmetric, i.e., $j_b = 1$.

  $\rightarrow$ **The predicted $\bar{b}\bar{b}qq$ tetraquark has isospin $I = 0$, spin $J = 1$.**

  - We study a state, which correspond for large $\bar{b}\bar{b}$ separations to a pairs of $B^{(*)}$ mesons in a spatially symmetric s-wave.

  $\rightarrow$ **The predicted $\bar{b}\bar{b}qq$ tetraquark has parity $P = +$** (the product of the parity quantum numbers of the two mesons, which are both negative)**.**

Inclusion of heavy spin effects

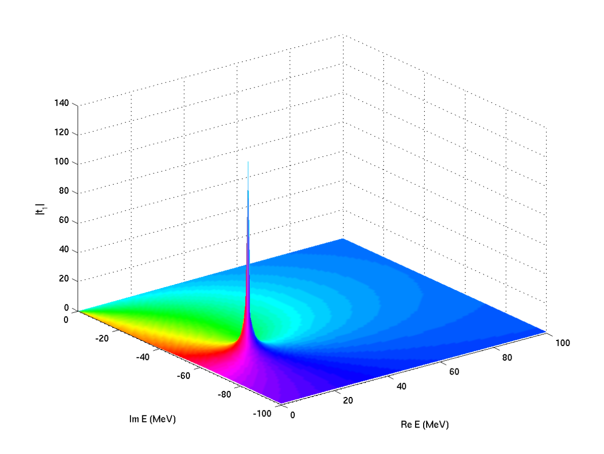
- Heavy spin effects have been neglected so far, e.g. mass splitting $m_{B^*} - m_B \approx 46\,\text{MeV}$.
- Mass splitting $m_{B^*} - m_B$ is, however, of the same order of magnitude as the previously obtained binding energy $E = -90^{+43}_{-36}\,\text{MeV}$.
- Moreover, two competing effects:
  - The attractive $\bar{b}\bar{b}ud$ channel corresponds to a linear combination of BB^* and/or B^*B^*.
  - The BB^* interaction is a superposition of attractive and repulsive $\bar{b}\bar{b}ud$ potentials.
- **Will there still be a bound state, when heavy spin effects are taken into account?**
  - Yes.
  - We include heavy spin effects by solving a coupled channel Schrödinger equation.
    [P. Bicudo, J. Scheunert, M.W., Phys. Rev. D **95**, 034502 (2017) [arXiv:1612.02758]]
  - Binding energy $E = -59^{+38}_{-30}\,\text{MeV}$.
  - Tetraquark is approximately a $50\%/50\%$ superposition of BB^* and B^*B^* (strong attraction more important than light constituents).

$\bar{b}\bar{b}qq$ tetraquark resonances (1)

- Most hadrons are unstable, i.e., resonances.
- If a $\bar{b}\bar{b}qq$ potential $V_{\bar{b}\bar{b}}(r)$ is not sufficiently attractive to host a bound state, there could still be a clear resonance.
- Comparatively easy to investigate within our approach (since we have potentials $V_{\bar{b}\bar{b}}(r)$, no Lüscher method etc. necessary).
- Use standard methods from scattering theory:
  - Solve Schrödinger equation with potential $V_{\bar{b}\bar{b}}(r)$ and appropriate boundary conditions (incident plane wave, outgoing spherical wave)
    $\rightarrow$ partial wave amplitudes $f_l(E)$.
  - Use partial wave amplitudes $f_l(E)$ to ...
    - ... determine phase shifts and contributions of partial waves to total cross section
      $\rightarrow$ peak indicates resonance mass.
    - ... determine poles of the S or the T matrix in the complex energy plane (correspond to poles of $f_l(E)$)
      $\rightarrow$ real part of a pole $\equiv$ resonance mass
      $\rightarrow$ imaginary part of a pole $\equiv$ resonance width.

$\bar{b}\bar{b}qq$ tetraquark resonances (2)

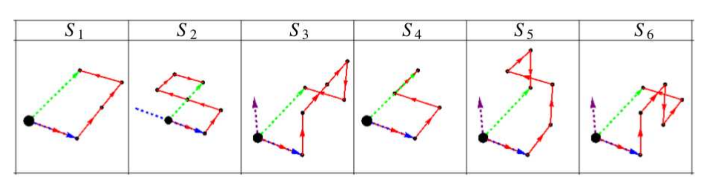
- Exploratory study mostly for $qq = (ud - du)/\sqrt{2}$ (i.e., the scalar isosinglet potential) and orbital angular momentum $l = 1$ of $\bar{b}\bar{b}$:
- There is a resonance for $qq = (ud - du)/\sqrt{2}$ and $l = 1$:
  - Resonance mass $E = +17^{+4}_{-4}$ MeV above the BB threshold.
  - Decay width $\Gamma_{\rightarrow B+B} = 112^{+90}_{-103}$ MeV.
  - Quantum numbers $I(J^P) = 0(1^-)$.
- There do not seem to be resonances in other channels ($l > 1$, vector isotriplet potential, heavier quarks qq).



$\bar{b}b$ hybrid mesons (1)

- The same two-step Born-Oppenheimer approach can also be used to study heavy hybrid mesons ($\bar{b}b$ + gluons).

Born-Oppenheimer approximation, step (1)

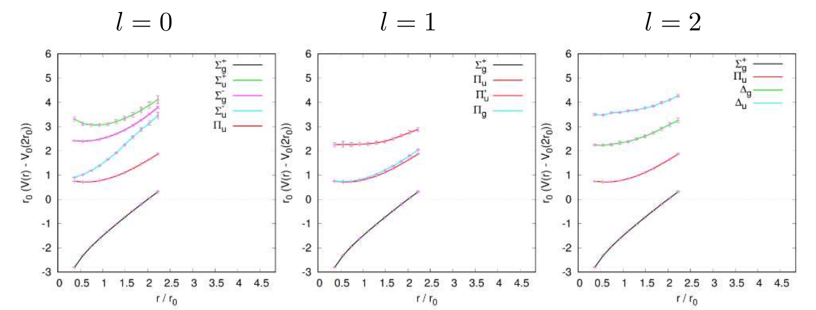


- Lattice QCD computation of $\bar{b}b$ potentials $V_{\bar{b}b}(r)$ (currently SU(3) Yang-Mills).
  - Use $\bar{b}b$ + gluons creation operators

$$O_{\bar{b}b+\text{gluons}} \equiv \Gamma_{AB}\Big(\bar{b}_A(-\mathbf{r}/2)U(-\mathbf{r}/2,+\mathbf{r}/2)b_B(+\mathbf{r}/2)\Big).$$

  - Different non-straight path of link variables (representing gluons, which contribute to the quantum numbers in a non-trivial and different way).
  - Different heavy quark spin/parity (no effect on $V_{\bar{b}b}(r)$).

[C. Reisinger, S. Capitani, O. Philipsen and M.W., arXiv:1708.05562 [hep-lat]]



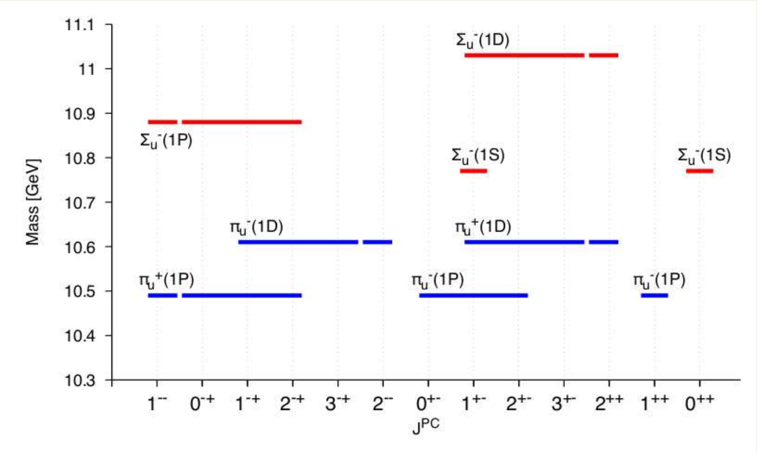
$\bar{b}b$ hybrid mesons (2)

Born-Oppenheimer approximation, step (2)

- Solve the Schrödinger equation for the relative coordinate of the heavy quarks $\bar{b}b$,

$$\left(-\frac{1}{2\mu}\triangle + V_{\bar{b}b}(r)\right)\psi(\mathbf{r}) \quad = \quad E\psi(\mathbf{r}) \quad , \quad \mu \quad = \quad m_b/2;$$

low lying states indicate $\bar{b}b$ hybrid mesons.
[C. Riehl, Bachelor of Physics thesis, Goethe University Frankfurt am Main (2017)]



Summary and outlook

- Prediction of a stable $\bar{b}\bar{b}qq$, $qq = (ud - du)/\sqrt{2}$ tetraquark.
  - Quantum numbers $I(J^P) = 0(1^+)$.
  - Binding energy $E = -59^{+38}_{-30}$ MeV with respect to the $B + B^*$ threshold.
- Prediction of a $\bar{b}\bar{b}qq$, $qq = (ud - du)/\sqrt{2}$ tetraquark resonance.
  - Quantum numbers $I(J^P) = 0(1^-)$.
  - Resonance mass $E = +17^{+4}_{-4}$ MeV above the $B + B$ threshold.
  - Decay width $\Gamma_{\rightarrow B+B} = 112^{+90}_{-103}$ MeV.
- Future plans:
  - Explore $\bar{b}\bar{b}qq$ tetraquark resonances in more detail.
  - Investigate the structure of the predicted $I(J^P) = 0(1^+)$ tetraquark ... is it a mesonic molecule or rather a diaquark-antidiquark?
  - Study $\bar{b}b\bar{q}q$ / BB, which is experimentally more relevant ($Z_b(10610)^+$, $Z_b(10650)^+$, ...), but theoretically much harder.
  - Explore $\bar{b}b$ hybrid mesons in more detail.