Multichannel scattering method for medium-light nuclei

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Three-body systems in reactions with rare isotopes

MCAS COLLABORATION

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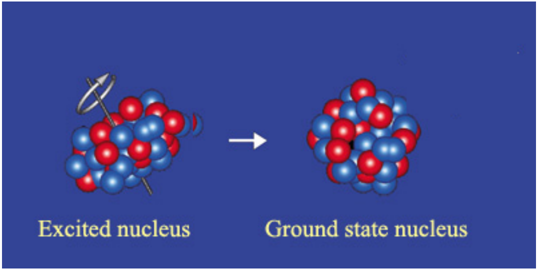
INTRODUCTION

The existing radioactive ion beam (RIB) facilities, and the construction of new ones, allow the formation and investigation of new species, particularly at or beyond the drip lines.

This talk is a review of the efforts by the MCAS collaboration to investigate exotic and non-exotic systems via a coupled-channel method, in the low-energy domain (scattering, bound states, resonances).

Treating scattering on light-medium nuclei

COUPLED-CHANNEL dynamics: including the collective low-energy excitations of the core.



C13 (n-C12), N13 (p-C12), C15 (n-C14), F15 (p-O14), He7, B7, Be7, Li7, Be9, B9, C17 (n-C16), Na-17 (p-Ne16) C-19 (n-C18), $^{9}{}_{\Lambda}Be$, $^{13}{}_{\Lambda}C$,
and also ... Ne23 (n-Ne22), Mn23, Na23 (p-Ne22), Al23, O17 (n-O16), F17 (p-O16), O19 ... O16 (alpha-C12), Be10 (alpha-He6)

Model of nuclear interaction

Current description: nucleon-nucleus scattering (light-medium nuclei with 0^+ g.s.) including first core excitations of collective nature (quadrupole, octupole, etc).

$$V_{cc'}(r) = \sum_{n=C,LS,LL,SI} V_n < (\ell s)jI; J^\pi | \mathcal{O}_n f_n(r, R, \theta_{\mathbf{r},\mathbf{R}}) | (\ell' s)j'I'; J^\pi >$$

For all operators, the functional forms are expanded to second order in the core-deformation parameter ($R = R_0(1 + \beta_2 P_2(\theta))$)

$$f_n(r, R, \theta) = f_n^{(0)}(r) - \beta_2 R_0 P_2(\theta) \frac{d}{dr} f_n^{(0)}(r)$$
$$+ \frac{\beta_2^2 R_0^2}{2\sqrt{\pi}} \left(P_0 - \frac{2\sqrt{5}}{7} P_2(\theta) + \frac{2}{7} P_4(\theta) \right) \frac{d^2}{dr^2} f_n^{(0)}(r)$$

MCAS: A low-energy scattering tool

we want to determine S-matrices to evaluate:
Total elastic scattering cross section

$$\sigma_{EL} = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} \left\{ (\ell + 1) \left| S_\ell^+(k) - 1 \right|^2 + \ell \left| S_\ell^-(k) - 1 \right|^2 \right\}$$

Total reaction cross section

$$\sigma_R = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} \left\{ (\ell + 1) \left(1 - |S_\ell^+(k)|^2 \right) + \ell \left(1 - |S_\ell^-(k)|^2 \right) \right\}$$

Solution of Lippmann-Schwinger coupled-channel equation

$$T^{J^\pi}_{cc'}(p,q;E) \;=\; V^{J^\pi}_{cc'}(p,q) + \mu \sum_{c''=1}^{\text{closed}} \int_0^\infty V^{J^\pi}_{cc''}(p,x) \frac{x^2}{h^2_{c''}+x^2} T^{J^\pi}_{c''c'}(x,q;E)\;dx$$
$$-\mu \sum_{c''=1}^{\text{open}} \int_0^\infty V^{J^\pi}_{cc''}(p,x) \frac{x^2}{k^2_{c''}-x^2+i\epsilon} T^{J^\pi}_{c''c'}(x,q;E)\;dx$$

Finte-Rank expansion of realistic CC interaction

$$V_{cc'}(p,q) \sim V^N_{cc'}(p,q) = \sum_{n=1}^{N} \hat{\chi}_{cn}(p)\;\eta_n^{-1}\;\hat{\chi}_{c'n}(q)\;.$$

Optimal representations of $\hat{\chi}_{cn}(p)$ in terms of Sturmians $|\Phi_{cn}\rangle$

$$\sum_{c'} G^0_c V_{cc'} |\Phi_{c'n}\rangle \;\;=\;\; -\eta_n |\Phi_{cn}\rangle$$

with $\hat{\chi}_{cn}(p)$ defined as $|\hat{\chi}_{cn}\rangle = V_{cc'}|\Phi_{c'n}\rangle$

Multichannel S matrices

Sturmian expansion of $V_{cc'}$ leads to algebraic form for S

$$\begin{aligned} S_{cc'} &= \delta_{cc'} \; - \; i\pi\mu\sqrt{k_c k_{c'}} \; T_{cc'} \\ &= \delta_{cc'} \; - \; i^{\phi}\pi\mu \sum_{n,n'=1}^{N} \sqrt{k_c}\, \hat{\chi}_{cn}(k_c) \; \left([\boldsymbol{\eta} - \mathbf{G}_0]^{-1} \right)_{nn'} \; \hat{\chi}_{c'n'}(k_{c'}) \sqrt{k_{c'}} \; , \end{aligned}$$

with matrix elements

$$\begin{aligned} [\mathbf{G}_0]_{nn'} = \; \mu \Bigg[& \sum_{c=1}^{\text{open}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{k_c^2 - x^2 + i\epsilon} \hat{\chi}_{cn'}(x) \; dx \\ & - \sum_{c=1}^{\text{closed}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{h_c^2 + x^2} \hat{\chi}_{cn'}(x) \; dx \Bigg] \end{aligned}$$

$$[\boldsymbol{\eta}\,]_{nn'} = \eta_n \; \delta_{nn'} \; .$$

(Energy dependent) Sturmians

Sturmians (*aka* Weinberg states): a *different* way to QM.
Consider a two-body like Hamiltonian:

$$(E - H_o)\Psi_E = V\Psi_E \,, \tag{1}$$

where E is the spectral variable, and Ψ_E is the eigenstate.
Sturmians are the eigensolutions of:

$$(E - H_o)\Phi_i(E) = \frac{V}{\eta_i(E)}\Phi_i(E) \,, \tag{2}$$

where E is a parameter. The eigenvalue η_i is the potential scale.
SPECTRUM: all the potential rescalings that give solution to that equation, for given energy E, and with well-defined boundary conditions.

(Energy dependent) Sturmians

Then, the single-channel S-matrix can be written as

$$S(E) = \frac{\Pi_i(1-\eta_i(E^{(-)}))}{\Pi_i(1-\eta_i(E^{(+)}))} \tag{3}$$

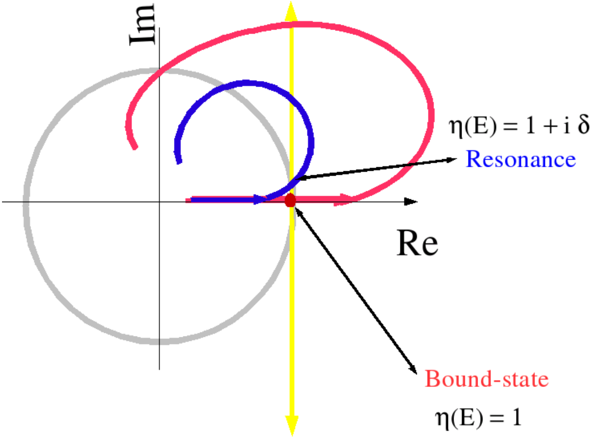
Alternatively, in CC dynamics, introducing the form factor in momentum space

$$\hat{\chi}_{ci}(E^{(+)};k) =< k, c|V|\Phi_i(E) >, \tag{4}$$

the CC S-matrix can be rewritten also as

$$S_{cc'}(E) = \delta_{cc'} - i\pi\sqrt{k_c k'_c}\sum_i \hat{\chi}_{ci}(E^{(+)};k_c)\frac{1}{1-\eta_i(E^{(+)})}\hat{\chi}_{c'i}(E^{(+)};k_{c'}) \tag{5}$$

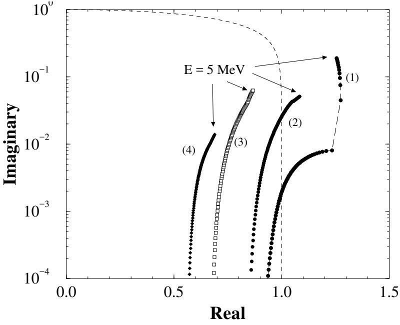
How resonances and bound states are found in Sturmian theory.

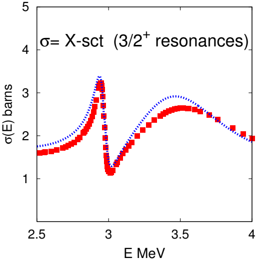


The case of neutron-^{12}C in the $3/2^+$ channel

A realistic case: low-energy resonances in $3/2^+$ n-^{12}C system.

Sturmian patterns (left) and $3/2^+$ resonant X-sect (right).





CC dynamics and the Pauli principle

First application $n - C12/p - C12$ aborted: Why?

Bound states

^{13}C four observed → 12 computed

^{13}N one observed → 8 computed

The deep forbidden states contaminate the physical solution due to Coupled-Channel dynamics. Problems in CC formalisms (but not only)...

CC dynamics and the Pauli principle

The solution: Elimination of these forbidden states in the definition of the Hamiltonian.

- (OCM) constrained hamiltonian. (Saito '69)
- The OPP technique highly nonlocal. (Kukulin et al. '74)
- (susyQM) mainly local potentials. (Witten '81/ Baye '87)

C.V.Sukumar, D.M. Brink (2004) examined connections with inverse scattering approaches (J. Phys. A **37**).

The OPP approach (Kukulin, Pomerantsev et al.) eliminates the deep bound states introducing a new term in the nuclear potential.

The OPP "potential"

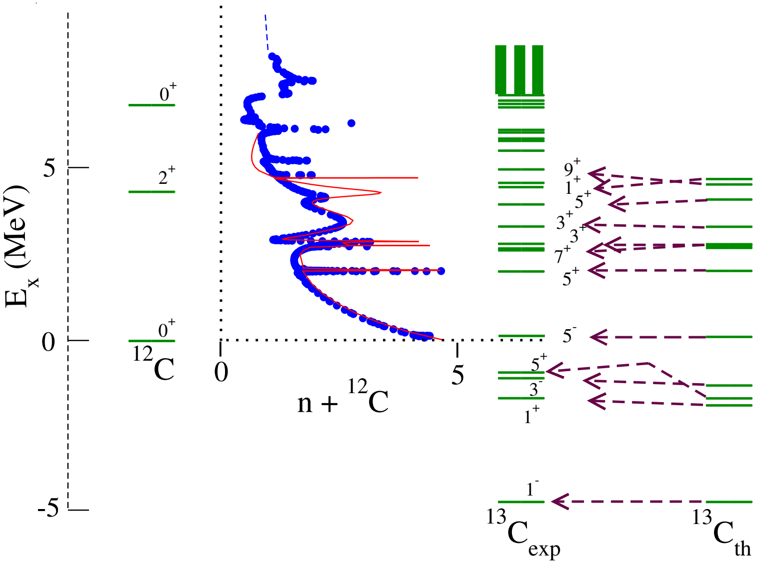
The full nuclear potential $\mathcal{V}_{cc}(r)$ is not the local potential $n - C12$:
The "complete" potential is (in partial-wave decomposition)

$$\mathcal{V}_{cc'}(r, r') = V_{cc'}(r)\delta(r - r')$$

$$+\delta_{cc'}\lambda_c A_c(r)A_c(r')(\delta_{c=s\frac{1}{2}^+}) + \delta_{cc'}\lambda_c A_c(r)A_c(r')(\delta_{c=p\frac{3}{2}^-})$$

$A_c(r)$ are the Pauli-forbidden deep (CC-uncoupled) bound states.

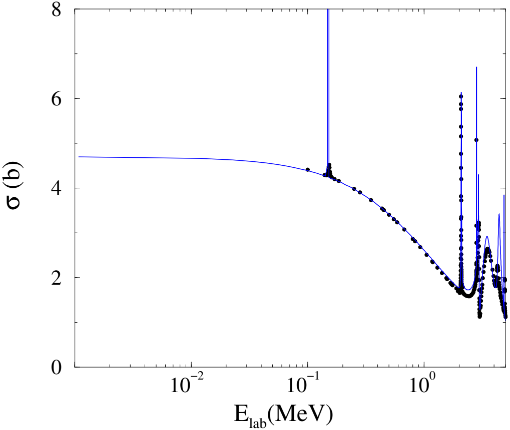
A state in the OPP approach is:
forbidden in the limit $\lambda \to +\infty$
allowed when $\lambda \to 0$



E_x (MeV)
5
0
-5
0+
2+
0+
12C
0
5
n + 12C
9+
1+
5+
3+
3+
7+
5+
5-
5+
3-
1+
1-
13C exp
13C th

n-^{12} C: Low energy details

MCAS calculation



$\frac{5}{2}^-$ resonance centroid very sensitive to Pauli blocking

CC dynamics and the Pauli principle

BUT THERE IS MORE, in many applications we had to loosen the OPP constraints

[Eric.Schmid, 1978] "The states can be Pauli-forbidden, Pauli-allowed, or Pauli-suppressed" Trieste, IAEA Few-Body Conference.

[Langanke-Friedrich, 1986] "The role of partially redundant states" Adv in Nucl. Phys.

RGM theory:

Forbidden - Eigenvalue in the Norm Kernel $e = 0$
Allowed - Eigenvalue in the Norm Kernel $e = 1$
Quasi-Rendundant - Eigenvalue in the Norm Kernel $0 < e < 1$

OPP Approach:

Forbidden - Strength of $\lambda = \infty$
Allowed - Strength of $\lambda = 0$
Hindered - Strength of $0 < \lambda < \infty$

Analysis of ^{15}F (vs. ^{15}C mirror partner)

- It started in 2006, we were triggered by recent data by:
  V.Z. Goldberg *et al.* PRC **69** ('04)
  F.Q. Guo *et al.* PRC **72** ('05).

Analysis of ^{15}F (vs. ^{15}C mirror partner)

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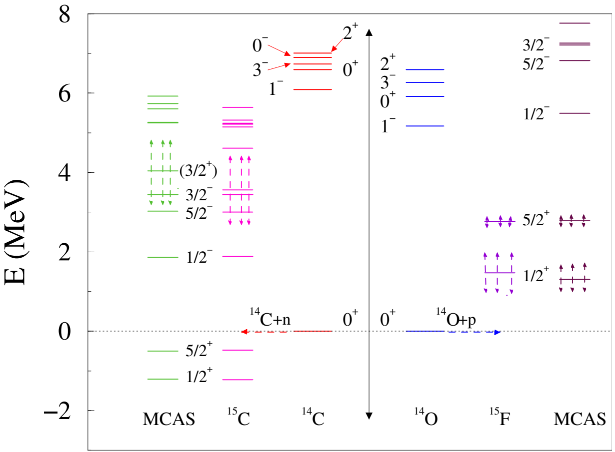
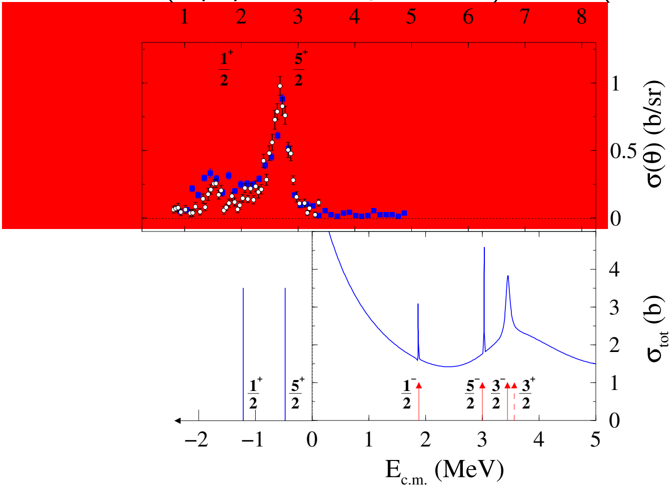
  V.Z. Goldberg *et al.* PRC **69** ('04)
  F.Q. Guo *et al.* PRC **72** ('05).
- **OUR STUDY** L.Canton, J.Svenne, K.Amos,*et al.*:
  PRL **96** ('06) used ^{15}C in fit-analysis
  to reproduce the resonant GS and first excited state in ^{15}F

$n - {}^{15}C$ PARAMETERS

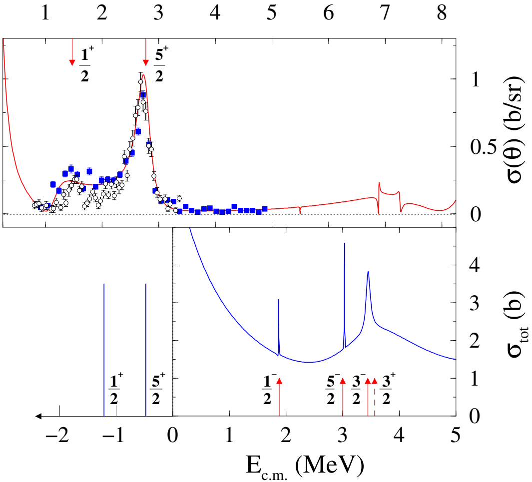
$V_0^{(\pm)} = -45.0$ MeV	$V_{ll}^{(\pm)} = 0.42$ MeV
$V_{ls}^{(\pm)} = 7.0$ MeV	$V_{ss}^{(\pm)} = --$
$R_0[a_0] = 3.1[0.65]$ fm	$\beta_2 = -0.50$



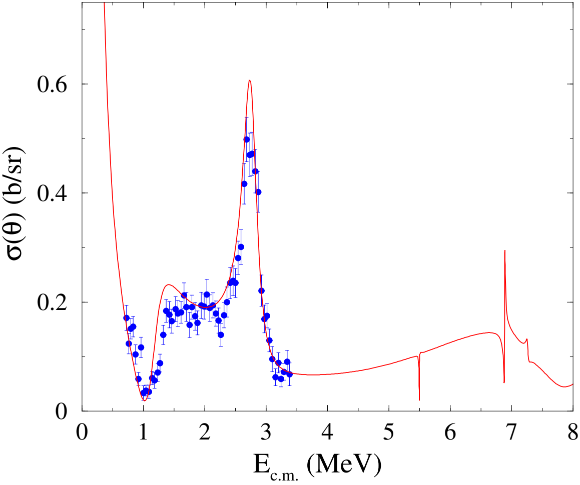
^{15}F (top, $p-^{14}O$ $\theta_{cm}=180^o$) & ^{15}C (bottom)



^{15}F (top, $p-^{14}O$ $\theta_{cm} = 180^o$) & ^{15}C (bottom)



$^{15}F \ (p -^{14} O \ \ \theta_{cm} = 147^o)$



The concept of Pauli hindered states:
Pauli Forbidden ($\lambda \simeq 1 GeV$):

$$0s_{1/2}$$
$$0p_{3/2}$$

Pauli Hindered ($\lambda \simeq 1 \div 50 MeV$)

$$0p_{1/2}$$

Pauli Allowed ($\lambda \simeq 0 MeV$)

$$1s_{1/2}$$
$$0d_{5/2}$$
$$0d_{3/2}$$
$$\ldots \quad \ldots \quad \ldots$$

Voices from theory ...

^{15}F resonant states

J^π	Theory $E, (\frac{1}{2}\Gamma)$	Experiment $E, (\frac{1}{2}\Gamma)$
$\frac{1}{2}^+$	1.31 (0.8)	1.47 (1.00)
$\frac{5}{2}^+$	2.78 (0.3)	2.77 (0.24)
$\frac{1}{2}^-$	5.49 (0.005)	
$\frac{5}{2}^-$	6.88 (0.01)	
$\frac{3}{2}^-$	7.25 (0.04)	
$\frac{1}{2}^+$	7.21 (1.2)	
$\frac{5}{2}^+$	7.75 (0.4)	
$\frac{3}{2}^+$	7.99 (3.6)	

Table: See publication PRL **96** 072502 (2006)

Voices from experiments ...

^{15} F resonant states

J^{π}	Theory $E, (\frac{1}{2}\Gamma)$	Experiment $E, (\frac{1}{2}\Gamma)$
$\frac{1}{2}^{+}$	1.31 (0.8)	1.47 (1.00)
$\frac{5}{2}^{+}$	2.78 (0.3)	2.77 (0.24)
$\frac{1}{2}^{-}$	5.49 (0.005)	4.9 ($<$0.2)
$\frac{5}{2}^{-}$	6.88 (0.01)	6.4 ($<$0.2)
$\frac{3}{2}^{-}$	7.25 (0.04)	
$\frac{1}{2}^{+}$	7.21 (1.2)	
$\frac{5}{2}^{+}$	7.75 (0.4)	7.8 (0.4) ?
$\frac{3}{2}^{+}$	7.99 (3.6)	?

Table: See publication @GSI Darmstadt Mukha et al. PRC **79** 061301 (2009)

Ten years later... Physics Letters B 758 (2016) @Ganil

An above-barrier narrow resonance in ^{15}F

F. de Grancey [a], A. Mercenne [a], F. de Oliveira Santos [a,*], T. Davinson [b], O. Sorlin [a], J.C. Angélique [c], M. Assié [a,d], E. Berthoumieux [e], R. Borcea [h], A. Buta [h], I. Celikovic [g], V. Chudoba [i], J.M. Daugas [f], G. Dumitru [h], M. Fadil [a], S. Grévy [a,j], J. Kiener [k], A. Lefebvre-Schuhl [k], N. Michel [a], J. Mrazek [i], F. Negoita [h], J. Okołowicz [l], D. Pantelica [h], M.G. Pellegriti [a], L. Perrot [a,d], M. Płoszajczak [a], G. Randisi [a], I. Ray [a], O. Roig [f], F. Rotaru [h], M.G. Saint Laurent [a], N. Smirnova [j], M. Stanoiu [h], I. Stefan [a,d], C. Stodel [a], K. Subotic [g], V. Tatischeff [k], J.C. Thomas [a], P. Ujić [g], R. Wolski [l]

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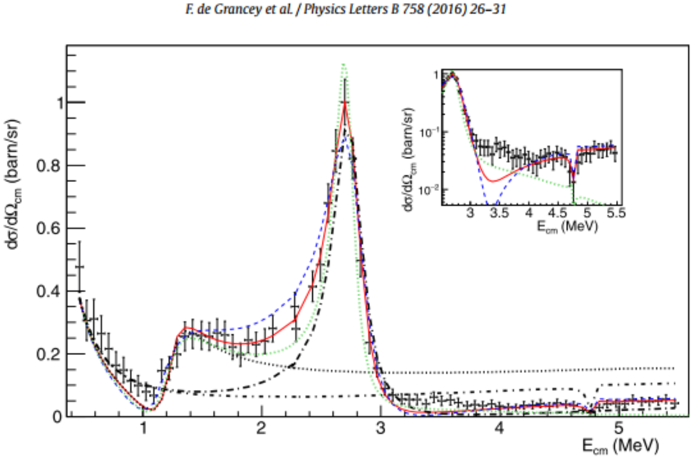
[l] Institute of Nuclear Physics, PAS, Radzikowskiego 152, PL-31342 Kraków, Poland

FOUND in proton-^{14}O scattering in inverse kinematics the first narrow state



With determined spin-parity $\frac{1}{2}^{(-)}$

Table 1
Resonance energy, width and spin measured and theoretical predictions for the second excited state of ^{15}F.

	Ref.	Second excited state		
		E_R (MeV)	Γ (keV)	J^π
Measured	[16]	4.800(100)	150(100)	–
	[31]	4.900(200)	200(200)	–
	Present	4.757(16)	36(19)	$\frac{1}{2}^-$
Predicted	[27]	5.49	5	$\frac{1}{2}^-$

Connection to ^{13}N-(2p) threshold!

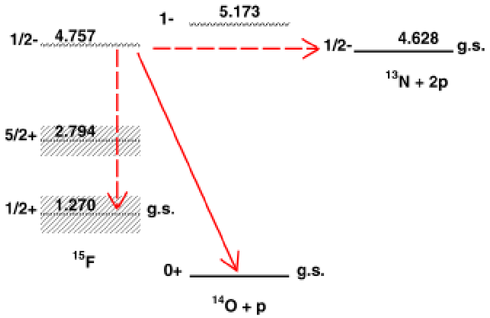


Fig. 2. (Color online.) Level scheme of ^{15}F. The possible decay channels from the $J^{\pi} = 1/2^{-}_{1}$ resonance are: the one proton emission (red arrow), gamma transition and two proton emission (red dashed arrow). The hatched areas correspond to the width of the resonances.

Extrapolating IKEDA's rule, close to that threshold, the system (and therefore that $\frac{1}{2}^{(-)}$ state) becomes a cluster ^{13}N-(2p), which explains why it has little overlap with ^{14}O-(p), and becomes a narrow resonance.

Conclusion

In this beautiful Workshop
three-body systems in reactions with rare isotopes
(thanks Organizers!)
I have presented the case of the rare isotope ^{15}F.
It is so rare that it exists only in the continuum;
it has in "isomeric" state, the second excited state,
which looks like a reef in the continuum sea.
That narrow resonance structure suggests a sudden transition:
the two bodies [^{14}O-p] become a three-body system [^{13}N-(2p)].