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Low-energy limit of the eLSM from the FRG approach

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Functional methods in hadron and nuclear physics

Trento | August 24, 2017

Overview

1. Introductory part

2. Methods

    The extended Linear Sigma Model (eLSM)

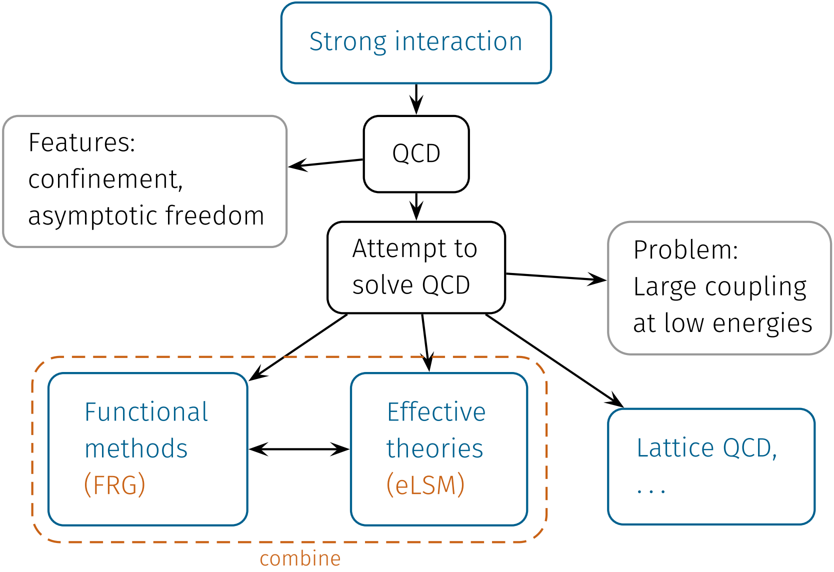
    Functional Renormalization Group (FRG)

3. Low-energy limit of the eLSM

4. Summary and outlook

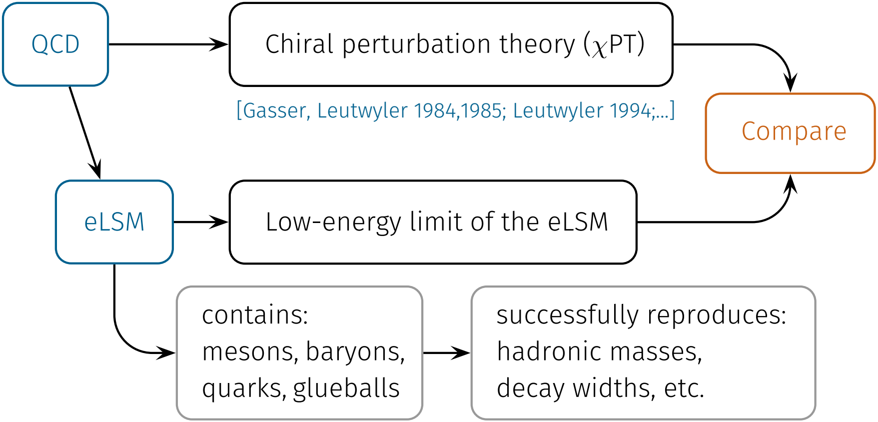
Introductory part

Quantum chromodynamics



Outline

Is the eLSM a valid low-energy description of QCD?



[Gallas et al. 2010; Parganlija et al. 2010; Janowski et al. 2011;
Parganlija et al. 2013; Eshraim et al. 2015; Olbrich et al. 2016;...]

Methods

Degrees of freedom

- (Pseudo-)scalar meson matrix $\Sigma = \Sigma_a t_a$, generators t_a of $U(N_f)$
- Vector and axial-vector mesons $V_\mu = V_{\mu,a} t_a$ and $A_\mu = A_{\mu,a} t_a$
- Left-/right-handed vector fields $L_\mu = L_{\mu,a} t_a$, $R_\mu = R_{\mu,a} t_a$:

$$L_\mu = V_\mu + A_\mu, \quad R_\mu = V_\mu - A_\mu.$$

- Transformations under $U(N_f)_L \times U(N_f)_R$:

$$\Sigma \to U_L \Sigma U_R^\dagger, \quad L_\mu \to U_L L_\mu U_L^\dagger, \quad R_\mu \to U_R R_\mu U_R^\dagger.$$

- "Covariant derivative":

$$D_\mu \Sigma = \partial_\mu \Sigma - i g_1 \left(L_\mu \Sigma - \Sigma R_\mu \right).$$

- Field-strength tensors:

$$L_{\mu\nu} = \partial_\mu L_\nu - \partial_\nu L_\mu, \quad R_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu.$$

Mesonic Lagrangian of the eLSM

$$
\begin{aligned}
\mathcal{L}_{\text{mesons}} = &\operatorname{tr}\left[(D_\mu\Sigma)^\dagger D_\mu\Sigma\right] + m_0^2 \operatorname{tr}\left(\Sigma^\dagger\Sigma\right) \\
&+\lambda_1 \left[\operatorname{tr}\left(\Sigma^\dagger\Sigma\right)\right]^2 + \lambda_2 \operatorname{tr}\left[\left(\Sigma^\dagger\Sigma\right)^2\right] \\
&+\frac{1}{4}\operatorname{tr}\left(L_{\mu\nu}^2 + R_{\mu\nu}^2\right) + \operatorname{tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)\left(L_\mu^2 + R_\mu^2\right)\right] \\
&- \operatorname{tr}\left[H\left(\Sigma + \Sigma^\dagger\right)\right] - c_A\left(\det\,\Sigma + \det\,\Sigma^\dagger\right) \\
&- i\frac{g_2}{2}\left(\operatorname{tr}\left\{L_{\mu\nu}\left[L_\mu, L_\nu\right]\right\} + \operatorname{tr}\left\{R_{\mu\nu}\left[R_\mu, R_\nu\right]\right\}\right) \\
&+\frac{h_1}{2}\operatorname{tr}\left(\Sigma^\dagger\Sigma\right)\operatorname{tr}\left(L_\mu^2 + R_\mu^2\right) + h_2\operatorname{tr}\left(\left|L_\mu\Sigma\right|^2 + \left|\Sigma R_\mu\right|^2\right) \\
&+2h_3\operatorname{tr}\left(\Sigma R_\mu \Sigma^\dagger L_\mu\right) - g_3\left[\operatorname{tr}\left(L_\mu L_\nu L_\mu L_\nu\right) + \operatorname{tr}\left(R_\mu R_\nu R_\mu R_\nu\right)\right] \\
&- g_4\left[\operatorname{tr}\left(L_\mu L_\mu L_\nu L_\nu\right) + \operatorname{tr}\left(R_\mu R_\mu R_\nu R_\nu\right)\right] - g_5\operatorname{tr}\left(L_\mu L_\mu\right)\operatorname{tr}\left(R_\nu R_\nu\right) \\
&- g_6\left[\operatorname{tr}\left(L_\mu L_\mu\right)\operatorname{tr}\left(L_\nu L_\nu\right) + \operatorname{tr}\left(R_\mu R_\mu\right)\operatorname{tr}\left(R_\nu R_\nu\right)\right].
\end{aligned}
$$

Yukawa coupling and $N_f = 2$

- Left-/right-handed spinors $\psi_{L/R} = \frac{1}{2}(\mathbb{1} \mp \gamma_5)\psi$
- Quark Lagrangian (flavor-symmetric quark-chemical potential μ):

$$\begin{aligned}\mathcal{L}_{\text{quarks}} &= \bar{\psi}\left(\gamma_\mu \partial_\mu + \mu\gamma_0\right)\psi + y\left(\bar{\psi}_L \Sigma \psi_R + \bar{\psi}_R \Sigma^\dagger \psi_L\right) \\ &\equiv \bar{\psi}\left(\gamma_\mu \partial_\mu + \mu\gamma_0 + y\Sigma_5\right)\psi.\end{aligned}$$

- Full Lagrangian:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{mesons}} + \mathcal{L}_{\text{quarks}}.$$

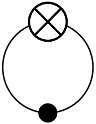
- $N_f = 2$:

$$\begin{aligned}\Sigma &= (\sigma + i\eta)t_0 + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{t}, \\ \Sigma_5 &= (\sigma + i\gamma_5\eta)t_0 + (\vec{a}_0 + i\gamma_5\vec{\pi}) \cdot \vec{t}, \\ R_\mu &= (\omega_\mu - f_{1\mu})t_0 + (\vec{\rho}_\mu - \vec{a}_{1\mu}) \cdot \vec{t}, \\ L_\mu &= (\omega_\mu + f_{1\mu})t_0 + (\vec{\rho}_\mu + \vec{a}_{1\mu}) \cdot \vec{t}.\end{aligned}$$

FRG

- Implementation of the Wilsonian RG idea
- Renormalization scale(k)-dependent effective action Γ_k
- FRG flow equation [Wetterich 1993]:

$$\partial_k \Gamma_k = \frac{1}{2} \, \mathrm{str} \left[\partial_k \mathbf{R}_k \left(\mathbf{\Gamma}_k^{(2)} + \mathbf{R}_k \right)^{-1} \right] = \frac{1}{2}$$



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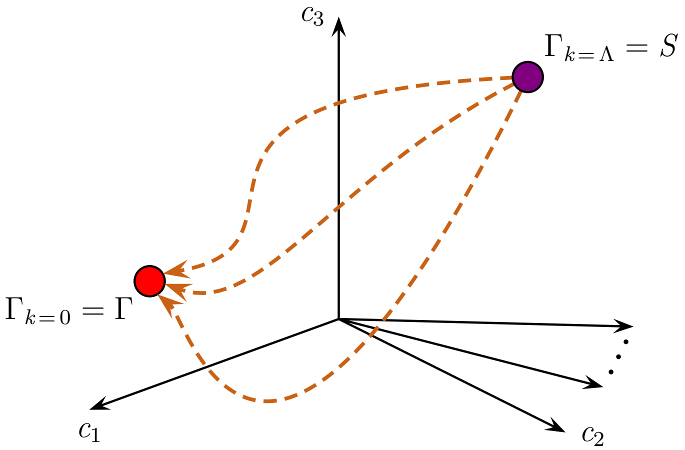
- Regulator function $\mathbf{R}_k$ provides correct integration limits

Flow in theory space



Low-energy limit of the eLSM

Recall the mesonic Lagrangian of the eLSM

$$
\begin{aligned}
\mathcal{L}_{\text{mesons}} = & \operatorname{tr}\left[(D_\mu \Sigma)^\dagger D_\mu \Sigma\right] + m_0^2 \operatorname{tr}\left(\Sigma^\dagger \Sigma\right) \\
& + \lambda_1 \left[\operatorname{tr}\left(\Sigma^\dagger \Sigma\right)\right]^2 + \lambda_2 \operatorname{tr}\left[\left(\Sigma^\dagger \Sigma\right)^2\right] \\
& + \frac{1}{4} \operatorname{tr}\left(L_{\mu\nu}^2 + R_{\mu\nu}^2\right) + \operatorname{tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)\left(L_\mu^2 + R_\mu^2\right)\right] \\
& - \operatorname{tr}\left[H\left(\Sigma + \Sigma^\dagger\right)\right] - c_A\left(\det\,\Sigma + \det\,\Sigma^\dagger\right) \\
& - i\frac{g_2}{2}\left(\operatorname{tr}\left\{L_{\mu\nu}\left[L_\mu, L_\nu\right]\right\} + \operatorname{tr}\left\{R_{\mu\nu}\left[R_\mu, R_\nu\right]\right\}\right) \\
& + \frac{h_1}{2} \operatorname{tr}\left(\Sigma^\dagger \Sigma\right) \operatorname{tr}\left(L_\mu^2 + R_\mu^2\right) + h_2 \operatorname{tr}\left(\left|L_\mu \Sigma\right|^2 + \left|\Sigma R_\mu\right|^2\right) \\
& + 2h_3 \operatorname{tr}\left(\Sigma R_\mu \Sigma^\dagger L_\mu\right) - g_3\left[\operatorname{tr}\left(L_\mu L_\nu L_\mu L_\nu\right) + \operatorname{tr}\left(R_\mu R_\nu R_\mu R_\nu\right)\right] \\
& - g_4\left[\operatorname{tr}\left(L_\mu L_\mu L_\nu L_\nu\right) + \operatorname{tr}\left(R_\mu R_\mu R_\nu R_\nu\right)\right] - g_5 \operatorname{tr}\left(L_\mu L_\mu\right) \operatorname{tr}\left(R_\nu R_\nu\right) \\
& - g_6\left[\operatorname{tr}\left(L_\mu L_\mu\right) \operatorname{tr}\left(L_\nu L_\nu\right) + \operatorname{tr}\left(R_\mu R_\mu\right) \operatorname{tr}\left(R_\nu R_\nu\right)\right].
\end{aligned}
$$

In more detail...

Compare low-energy couplings $C_1, \ldots, C_4$:

- Chiral expansion of the QCD generating functional (χPT):

$$\mathcal{L}_{\chi\mathrm{PT}} = \frac{1}{2}\left(\partial_\mu \vec{\pi}\right)^2 - \frac{1}{2} m_\pi^2 \vec{\pi}^{\,2} + C_{1,\chi\mathrm{PT}} \left(\vec{\pi}^{\,2}\right)^2 + C_{2,\chi\mathrm{PT}} \left(\vec{\pi} \cdot \partial_\mu \vec{\pi}\right)^2 + C_{3,\chi\mathrm{PT}} \left(\partial_\mu \vec{\pi}\right)^2 \left(\partial_\nu \vec{\pi}\right)^2 + C_{4,\chi\mathrm{PT}} \left(\partial_\mu \vec{\pi} \cdot \partial_\nu \vec{\pi}\right)^2 + \mathcal{O}\left(\pi^6, \partial^6\right).$$

- Low-energy limit of the eLSM at tree level:

$$\mathcal{L}_{\mathrm{eLSM}} = \frac{1}{2}\left(\partial_\mu \vec{\pi}\right)^2 - \frac{1}{2} m_\pi^2 \vec{\pi}^{\,2} + C_{1,\mathrm{eLSM}} \left(\vec{\pi}^{\,2}\right)^2 + C_{2,\mathrm{eLSM}} \left(\vec{\pi} \cdot \partial_\mu \vec{\pi}\right)^2 + C_{3,\mathrm{eLSM}} \left(\partial_\mu \vec{\pi}\right)^2 \left(\partial_\nu \vec{\pi}\right)^2 + C_{4,\mathrm{eLSM}} \left(\partial_\mu \vec{\pi} \cdot \partial_\nu \vec{\pi}\right)^2 + \mathcal{O}\left(\pi^6, \partial^6\right).$$

Numerical results at tree level

Table 1: Comparison of the low-energy couplings at tree level, cf. [Divotgey 2016; Bijnens, Ecker 2014].

Coupling	χPT	eLSM
C_1	-0.29 ± 0.34	-0.268 ± 0.021
$C_2 \times 10^5\ \mathrm{MeV}^2$	5.882 ± 0.013	5.399 ± 0.081
$C_3 \times 10^{11}\ \mathrm{MeV}^4$	-5.57 ± 0.88	$-9.30 \pm 0.59 + \mathrm{corr.}$
$C_4 \times 10^{11}\ \mathrm{MeV}^4$	2.58 ± 0.29	$9.45 \pm 0.59 + \mathrm{corr.}$

- How do these values change if we include loop corrections?
- Can we produce similar values within the FRG formalism?

Truncation

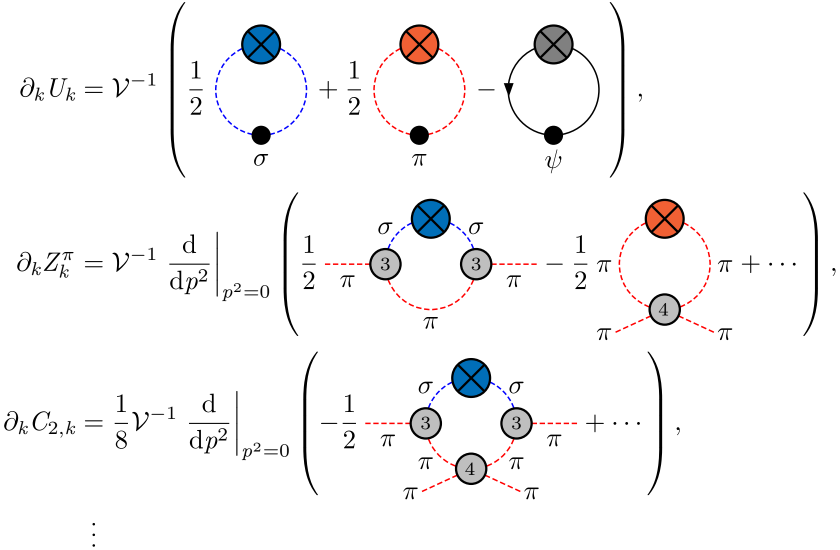
- Restrict analysis to an $O(4)$ model with quarks (σ, π, ψ)
- Ansatz for the effective action:

$$\Gamma_k = \int_x \Big[\frac{1}{2} Z_k^\sigma \left(\partial_\mu \sigma\right) \partial_\mu \sigma + \frac{1}{2} Z_k^\pi \left(\partial_\mu \vec{\pi}\right) \cdot \partial_\mu \vec{\pi} + U_k(\rho) - h_{\mathrm{ESB}} \sigma$$
$$+ C_{2,k} \left(\vec{\pi} \cdot \partial_\mu \vec{\pi}\right)^2 - C_{3,k} \left(\partial_\mu \vec{\pi}\right)^2 \left(\partial_\nu \vec{\pi}\right)^2$$
$$+ \bar{\psi} \left(Z_k^\psi \gamma_\mu \partial_\mu + y \Sigma_5 \right) \psi \Big],$$
$$\rho = \sigma^2 + \vec{\pi}^2.$$

- Assume scale-independent Yukawa coupling y
- $C_{1,k}$ is part of the effective potential U_k
- $C_{4,k}$ remains zero in the absence of vector mesons

Goal: Oppose IR values from the FRG to an one-loop approximation and to a recent tree-level study [Divotgey 2016]

Flow equations



Symmetry breaking and renormalized masses

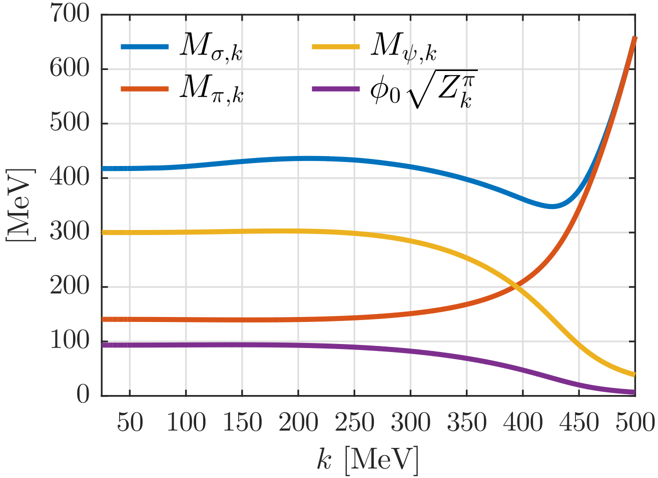


Figure 1: Scale evolution of the renormalized meson/quark masses and the vacuum expectation value $\phi_0\sqrt{Z_k^\pi}$. $k_{\mathrm{IR}} = 25$ MeV.

Wave-function renormalization

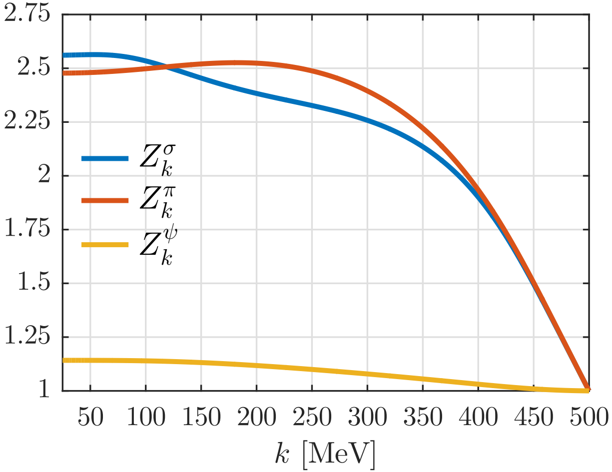


Figure 2: Scale evolution of the wave-function renormalization factors Z_k^σ, Z_k^π, and Z_k^ψ. $k_{\mathrm{IR}} = 25$ MeV.

Higher couplings

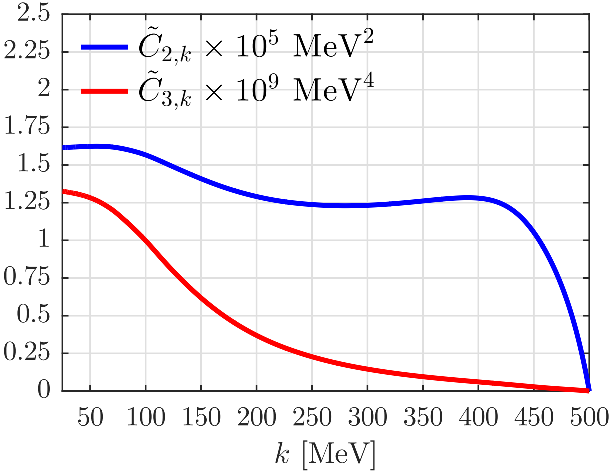


Figure 3: Scale evolution of the renormalized couplings $\tilde{C}_{2,k}$ and $\tilde{C}_{3,k}$. $k_{\mathrm{IR}} = 25$ MeV.

Symmetry breaking and renormalized masses (1 loop)

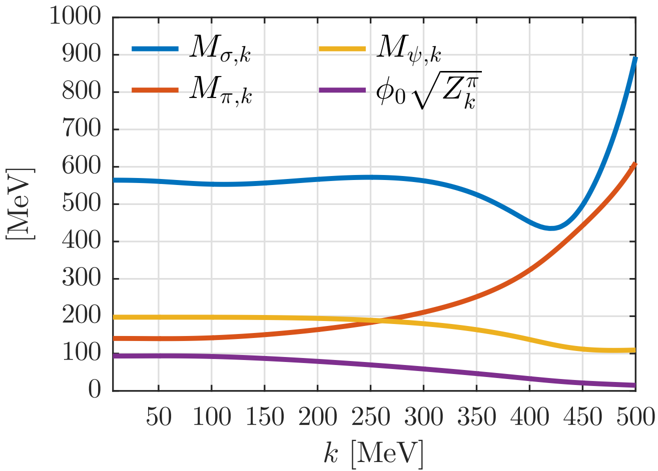


Figure 4: Scale evolution of the renormalized meson/quark masses and the vacuum expectation value in the one-loop approximation. $k_{\mathrm{IR}} = 7\ \mathrm{MeV}$.

Wave-function renormalization (1 loop)

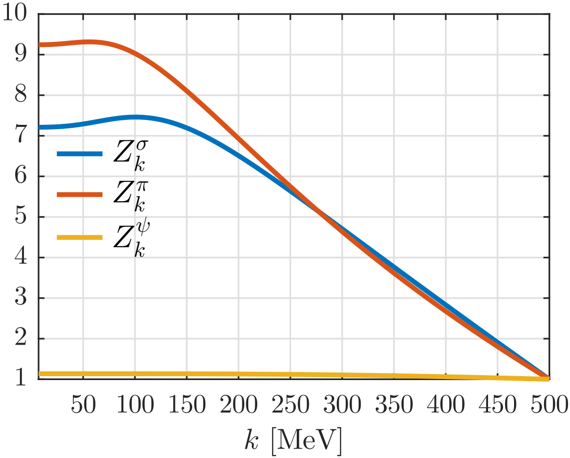


Figure 5: Scale evolution of the wave-function renormalization in the one-loop approximation. $k_{\mathrm{IR}} = 7$ MeV.

Higher couplings (1 loop)

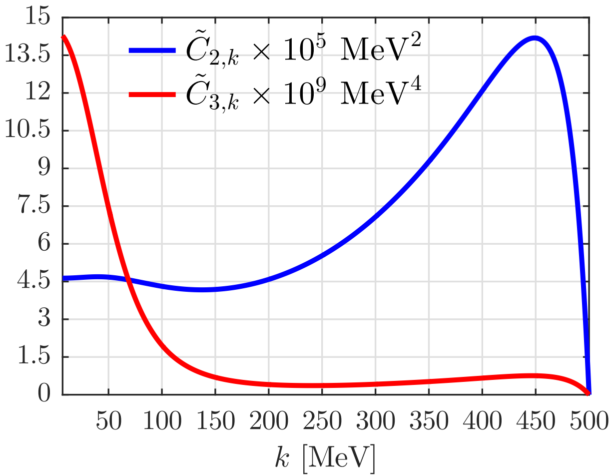


Figure 6: Scale evolution of the renormalized couplings $\tilde{C}_{2,k}$ and $\tilde{C}_{3,k}$ in the one-loop approximation. $k_{\mathrm{IR}} = 7$ MeV.

Tree-level estimation

- Tree-level potential:

$$U = \frac{1}{2} m_0^2 \left(\sigma^2 + \vec{\pi}^2\right) + \frac{\lambda}{4} \left(\sigma^2 + \vec{\pi}^2\right)^2 .$$

- Tree-level masses ($\phi = \langle \sigma \rangle$):

$$m_\sigma^2 = m_0^2 + 3\lambda\phi^2,$$
$$m_\pi^2 = m_0^2 + \lambda\phi^2.$$

- Low-energy couplings:

$$C_2 = \frac{(m_\sigma^2 - m_\pi^2)^2}{m_\sigma^4 \phi^2} \left(\frac{1}{2} + 2\frac{m_\pi^2}{m_\sigma^2}\right),$$
$$C_3 = \frac{(m_\sigma^2 - m_\pi^2)^2}{2 m_\sigma^6 \phi^2}.$$

IR values (1)

Table 2: Low-energy couplings of the $O(4)$ theory.

Coupling	Tree level	1 loop	FRG
$C_2 \times 10^5\ \mathrm{MeV}^2$	6.57	4.63	1.62
$C_3 \times 10^9\ \mathrm{MeV}^4$	0.26	14.28	1.33

Remark: The values in the second and third column correspond to the renormalized couplings $\tilde{C}_{2,k}$ and $\tilde{C}_{3,k}$ in the IR limit.

IR values (2)

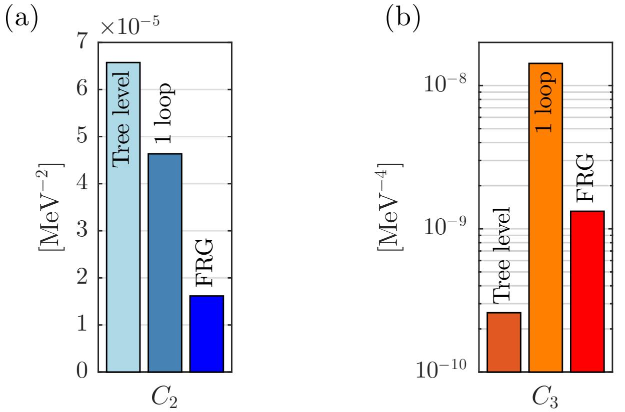


Figure 7: Comparison of the derivative couplings from the tree-level estimation, the one-loop study, and the FRG calculation. (a) C_2 and (b) C_3. The bar heights correspond to the numerical values quoted in Tab. 2.

Truncation robustness (1)

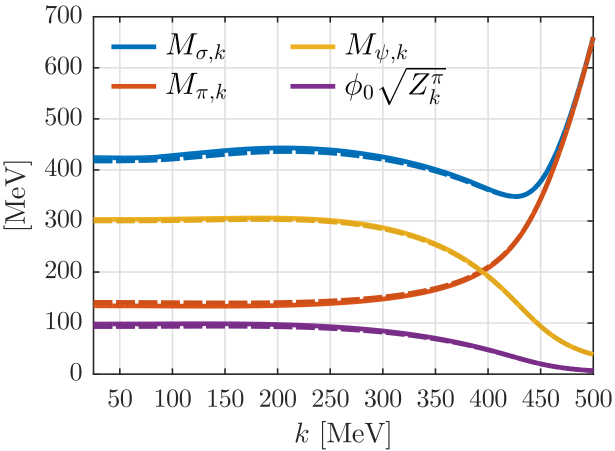


Figure 8: Scale evolution of the renormalized meson/quark masses within the LPA' truncation. The dashed lines show the related results from Fig. 1. $k_{\mathrm{IR}} = 25$ MeV.

Truncation robustness (2)

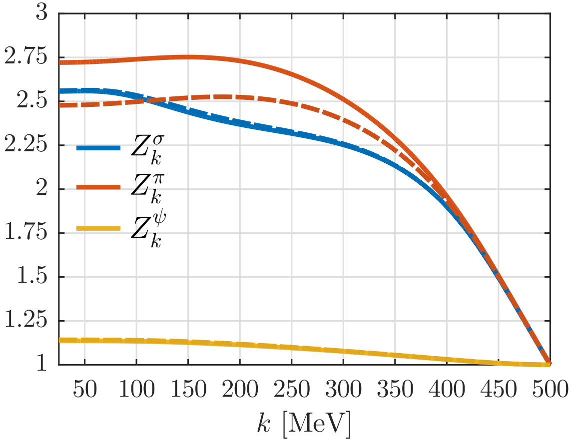


Figure 9: Scale evolution of the wave-function renormalization factors Z_k^σ, Z_k^π, and Z_k^ψ within the LPA' truncation. The dashed lines show the related results from Fig. 2. $k_{\mathrm{IR}} = 25$ MeV.

Summary and outlook

Summary and outlook

Summary:

- Low-energy couplings of the eLSM at tree level in good agreement with χPT, cf. [Divotgey 2016]
- Comparison of the $O(4)$ low-energy couplings obtained from a tree-level estimation, an one-loop approximation and a FRG study presented
- Discrepancy between these approaches observed

Outlook:

- Extend analysis to the full $O(4)$-symmetric scenario
- Include vector mesons
- Study regulator dependences
- Include scale-dependent Yukawa coupling