Finite-t and target mass corrections to DVCS

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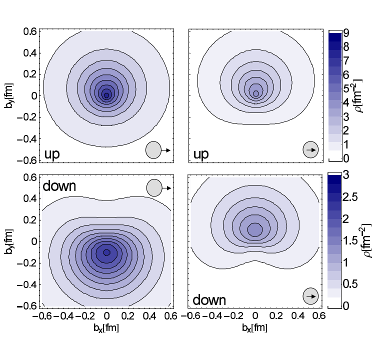
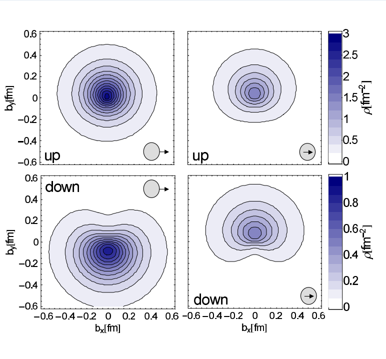
access to three-dimensional picture of the nucleon (M. Burkardt)





$\hookrightarrow$ **first two moments of transverse spin parton density**

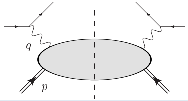
computer simulations: M. Göckeler *et al.*, Phys.Rev.Lett. 98 (2007) 222001

- **paradigm shift: finite t a "nuisance" $\longrightarrow$ important tool**

Planar vs. non-planar kinematics

- **paradigm shift: finite t a "nuisance" $\longrightarrow$ important tool**

DIS

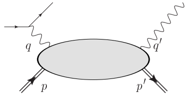


Define (p, q) as longitudinal plane:

$$p = (p_0, \vec{0}_\perp, p_z)$$
$$q = (q_0, \vec{0}_\perp, q_z)$$

$\Rightarrow$ parton fraction = Bjorken x

DVCS



Many choices possible:

$$p = (p_0, \vec{0}_\perp, p_z), \quad q = (q_0, \vec{0}_\perp, q_z)$$

or

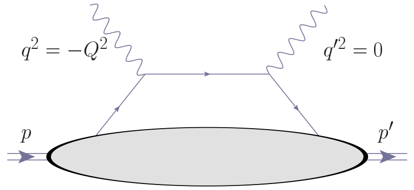
$$p + p' = (P_0, \vec{0}_\perp, P_z), \quad q = (q_0, \vec{0}_\perp, q_z)$$

etc.

$\Rightarrow$ parton fraction $2\xi = x_B[1+\mathcal{O}\left(\frac{t}{Q^2}\right)]$, redefinition of helicity amplitudes

- **Ambiguity is resolved by adding "kinematic" power corrections t/Q^2, m^2/Q^2**

Braun, Manashov, Pirnay: PRD **86** (2012) 014003



longitudinal plane (q, q')

$$n = q', \qquad \tilde{n} = -q + \frac{Q^2}{Q^2+t}\, q'$$

with this choice $\Delta = q - q'$ is longitudinal and

$$|P_\perp|^2 = -m^2 - \frac{t}{4}\frac{1-\xi^2}{\xi^2} \;\sim\; t_{\rm min} - t$$

where

$$P = \frac{1}{2}(p+p')\,, \qquad \xi_{\rm BMP} = -\frac{(\Delta\cdot q')}{2(P\cdot q')} \;=\; \frac{x_B(1+t/Q^2)}{2 - x_B(1-t/Q^2)}$$

photon polarization vectors

$$\varepsilon^0_\mu = -\left(q_\mu - q'_\mu q^2/(qq')\right)/\sqrt{-q^2}\,,$$

$$\varepsilon^\pm_\mu = (P^\perp_\mu \pm i\bar{P}^\perp_\mu)/(\sqrt{2}|P_\perp|)\,, \qquad \bar{P}^\perp_\mu = \epsilon^\perp_{\mu\nu}P^\nu$$

Relating CFFs in the laboratory and photon reference frame

$$\mathcal{F}^{\text{lab}}_{++} = \mathcal{F}^{\text{phot}}_{++} + \frac{\varkappa}{2}\left[\mathcal{F}^{\text{phot}}_{++} + \mathcal{F}^{\text{phot}}_{-+}\right] - \varkappa_0\, \mathcal{F}^{\text{phot}}_{0+},$$
$$\mathcal{F}^{\text{lab}}_{0+} = -(1+\varkappa)\,\mathcal{F}^{\text{phot}}_{0+} + \varkappa_0\left[\mathcal{F}^{\text{phot}}_{++} + \mathcal{F}^{\text{phot}}_{-+}\right]$$

$$\mathcal{F} \in \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$$

where

$$\varkappa_0 \sim \sqrt{(t_{\min} - t)/Q^2}\,, \qquad \varkappa \sim (t_{\min} - t)/Q^2$$

and different skewedness parameter

$$\xi^{\text{lab}} \simeq \frac{x_B}{2 - x_B} \quad \text{vs.} \quad \xi^{\text{phot}} = \frac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)}$$

Kumerički-Müller convention (KM)

$$\mathrm{LT_{KM}}: \begin{cases} \mathcal{F}^{\mathsf{lab}}_{++} = T_0 \otimes F, & \mathcal{F}^{\mathsf{lab}}_{0+} = 0, \\ \mathcal{F}^{\mathsf{lab}}_{-+} = 0, & \xi_{\mathrm{KM}} = \xi^{\mathsf{lab}} \end{cases}$$

Braun-Manashov-Pirnay convention (BMP)

$$\mathrm{LT_{BMP}}: \begin{cases} \mathcal{F}^{\mathsf{phot}}_{++} = T_0 \otimes F, & \mathcal{F}^{\mathsf{phot}}_{0+} = 0, \\ \mathcal{F}^{\mathsf{phot}}_{-+} = 0, & \xi_{\mathrm{BMP}} = \xi^{\mathsf{phot}} \end{cases}$$

$$\Downarrow$$

$$\mathrm{LT_{BMP}}: \begin{cases} \mathcal{F}^{\mathsf{lab}}_{++} = \left(1+\frac{\varkappa}{2}\right) T_0 \otimes F, & \mathcal{F}_{0+} = \varkappa_0 \, T_0 \otimes F \\ \mathcal{F}^{\mathsf{lab}}_{-+} = \frac{\varkappa}{2} T_0 \otimes F, & \xi = \xi_{\mathrm{BMP}}, \end{cases}$$

- **Changing frame of reference results in**
  - **Different skewedness parameter for a given x_B**
  - **Numerically significant excitation of helicity-flip CFFs**
- **Different results for experimental observables**

- **For many observables, “photon frame” LT calculation is very close to full twist-4**

Braun, Manashov, Müller, Pirnay: PRD89 (2014) 074022

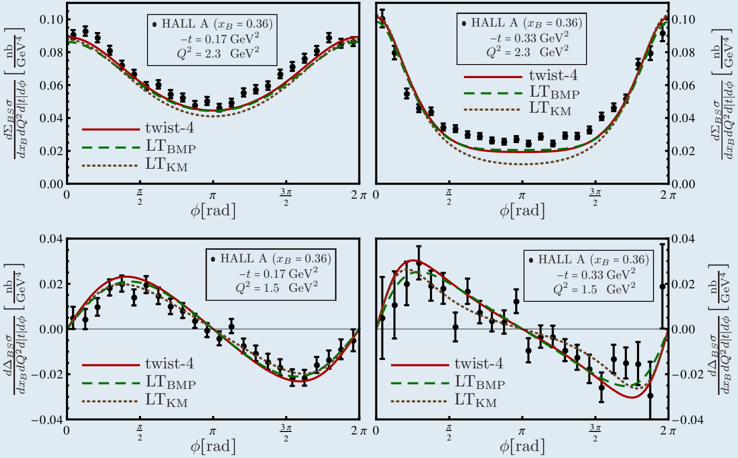
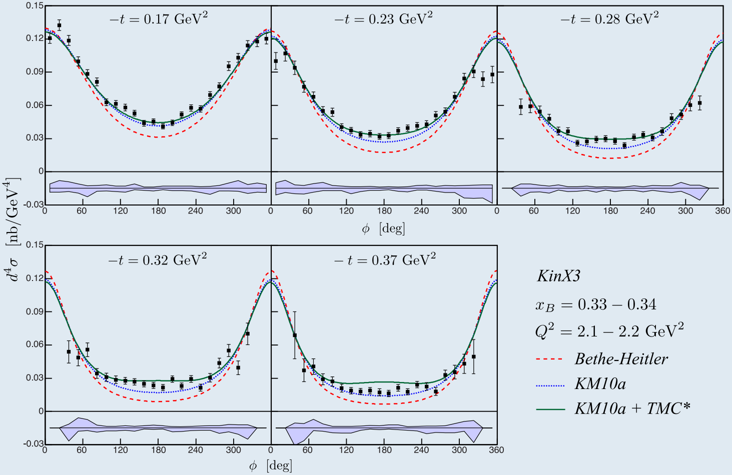


Figure: Unpolarized cross section [upper panels] and electron helicity dependent cross section difference [lower panels] from HALL A (old data) compared to the GK12 GPD model

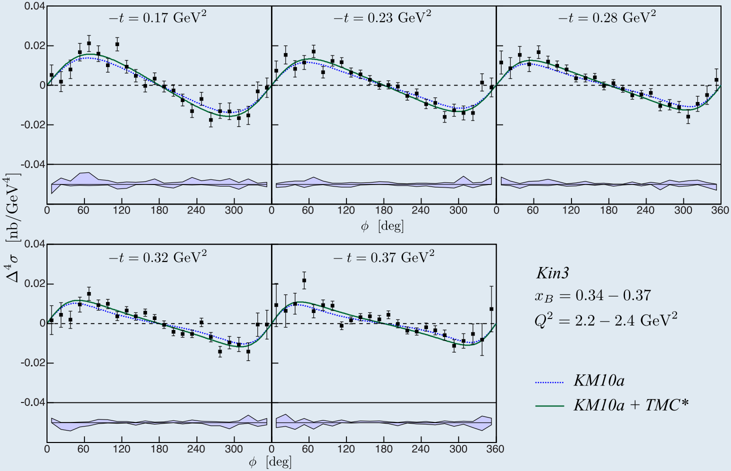
M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



- **TMC* refers to the calculation that includes full kinematic twist-4 corrections**

GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



- **TMC* refers to the calculation that includes full kinematic twist-4 corrections**

GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

- **Early work:**

— Extension of Nachtmann's approach to target mass corrections in DIS
— Spin-rotation (Wandzura-Wilczek)

Blümlein, Robaschik: NPB581 (2000) 449
Radyushkin, Weiss: PRD63 (2001) 114012
Belitsky, Müller: NPB589 (2000) 611
...

— Results not gauge invariant
— Results not translation invariant

— A related discussion in a different context:
Ball, Braun: NPB543 (1999) 201

- **A conceptual problem?**

Contributions of different twist are intertwined by symmetries:

- Conservation of the electromagnetic current and translation invariance

$$\partial^\mu T\left\{j_\mu^{em}(x) j_\nu^{em}(0)\right\} = 0$$
$$T\left\{j_\mu^{em}(2x) j_\nu^{em}(0)\right\} = e^{-i\mathbf{P}\cdot x} T\left\{j_\mu^{em}(x) j_\nu^{em}(-x)\right\} e^{i\mathbf{P}\cdot x}$$

  are valid in the sum of all twists but not for each twist separately

- Higher-twist contributions that restore gauge/translation invariance are due to descendants of leading-twist operators obtained by adding total derivatives

$$T\left\{j_\mu^{em}(x) j_\nu^{em}(0)\right\} = \underbrace{\sum_N a_N \mathcal{O}_N}_{\text{leading-twist}} + \sum_N \left(b_N \partial^2 \mathcal{O}_N + c_N (\partial\mathcal{O})_N\right) + \text{ other operators}$$

- These operators contribute to finite-t and target mass corrections

Task: Find the contributions to the OPE of all descendants of leading-twist operators

Invisible operator?

- **The problem:**
  S. Ferrara, A. F. Grillo, G. Parisi and R. Gatto, Phys. Lett. **B38**, 333 (1972):

— matrix elements of $\partial^\mu \mathcal{O}_{\mu\mu_1\ldots\mu_N}$ over free quarks vanish

  - ? Go over to NLO
  - ? More complicated quark-gluon matrix elements
  - ? Off-shell OPE

— main difficulty is to disentangle the contribution of interest from "genuine" twist-4 operators

- Using EOM $\partial^\mu \mathcal{O}_{\mu\mu_1\ldots\mu_N}$ can be expressed in terms of quark-gluon operators. e.g.:

Kolesnichenko '84

$$\partial^\mu O_{\mu\nu} = 2i\bar{q}gF_{\nu\mu}\gamma^\mu q\,,$$

where

$$O_{\mu\nu} = (1/2)[\bar{q}\gamma_\mu \overset{\leftrightarrow}{D}_\nu q + (\mu \leftrightarrow \nu)]$$

- **One cannot ignore $\bar{q}Fq$ operators**

— Is it possible at all to define the separation between "kinematic" and "genuine" twist four?

Guidung principle:

Braun, Manashov, PRL 107 (2011) 202001

— "kinematic" approximation amounts to the assumption that genuine twist-four matrix elements are zero

— for consistency, they must remain zero at all scales

— they must not reappear at higher scales due to mixing with "kinematic" operators

- **"Kinematic" and "Dynamic" contributions must have autonomous scale-dependence**

Indeed, otherwise:

$$\Big(\langle\bar q F q\rangle + c\langle(\partial\mathcal{O})\rangle\Big)^{\mu^2} = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_4/\beta_0}\Big(\langle\bar q F q\rangle + c\langle(\partial\mathcal{O})\rangle\Big)^{\mu_0^2}$$

$$\Longrightarrow$$

$$\langle\bar q F q\rangle^{\mu^2} = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_4/\beta_0}\langle\bar q F q\rangle^{\mu_0^2} + c\left[\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_4/\beta_0} - \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_2/\beta_0}\right]\langle(\partial\mathcal{O})\rangle^{\mu_0^2}$$

[The "kinematic" approximation corresponds to taking into account *all* operators with the same anomalous dimensions as the leading twist operators]

- Explicit diagonalization of the mixing matrix for twist-4 operators not feasible

- In a conformal theory

$$\Big(\mu\partial_\mu + \cancel{\beta(\alpha)\partial_\alpha} + \mathbb{H}\Big)O_j = 0 \quad \Longrightarrow \quad \langle T\{O_{j_1}(x)O_{j_2}(0)\}\rangle \sim \delta_{j_1 j_2}$$

therefore

$$T\Big\{j(x)j(0)\Big\} = \sum_N C_N(x,\partial)\mathcal{O}_N + \ldots,$$

$$C(x,\partial)\mathcal{O}_N = a_N\mathcal{O}_N + b_N\partial^2\mathcal{O}_N + c_N(\partial\mathcal{O})_N + \ldots$$

$\Longrightarrow$

$$\langle T\Big\{j(x)j(0)\mathcal{O}_N(y)\Big\}\rangle = C_N(x,\partial)\langle T\Big\{\mathcal{O}_N(0)\mathcal{O}_N(y)\Big\}\rangle + 0$$

— might work in QCD

- Orthogonality of eigenoperators suggests that $\mathbb{H}$ is a hermitian operator w.r.t. a certain scalar product

  Braun, Manashov, Rohrwild, Nucl. Phys. **B807** (2009) 89; Nucl. Phys. **B826** (2010) 235.

Summary of this part:

- **noncomplanarity makes separation of collinear directions ambiguous**

  — hence “leading twist approximation” ambiguous

  — related to violation of translation invariance and EM Ward identities

- **Complete results available to t/Q^2, m^2/Q^2 accuracy**

  — translation and gauge invariance restored

  — for many observables, complete results close to LT in “photon frame”

- **Must be taken into account in all studies aiming at 3D proton structure**

- **Outlook:**

  — all twists in LO, exact translation and gauge invariance

  — small x, matching to the BFKL formalism

  — NLO

Braun, Manashov, Pirnay: PRD **86** (2012) 014003

$$\begin{aligned}\mathcal{A}_{\mu\nu}(q,q',p) &= i\int d^4x\, e^{-i(z_1 q - z_2 q')x}\langle p',s'|T\{J_\mu(z_1x)J_\nu(z_2x)\}|p,s\rangle \\ &= \varepsilon^+_\mu\varepsilon^-_\nu\mathcal{A}^{++} + \varepsilon^-_\mu\varepsilon^+_\nu\mathcal{A}^{--} + \varepsilon^0_\mu\varepsilon^-_\nu\mathcal{A}^{0+} \\ &+ \varepsilon^0_\mu\varepsilon^+_\nu\mathcal{A}^{0-} + \varepsilon^+_\mu\varepsilon^+_\nu\mathcal{A}^{+-} + \varepsilon^-_\mu\varepsilon^-_\nu\mathcal{A}^{-+} + q'_\nu\mathcal{A}^{(3)}_\mu\end{aligned}$$

for the calculation to the twist-4 accuracy one needs

- $\mathcal{A}^{++},\mathcal{A}^{--}$: $1+\frac{1}{Q^2}$
- $\mathcal{A}^{0+},\mathcal{A}^{0-}$: $\frac{1}{Q}$ $\longleftarrow$ agree with existing results
- $\mathcal{A}^{-+},\mathcal{A}^{+-}$: $\frac{1}{Q^2}$ $\longleftarrow$ straightforward

- Photon helicity amplitudes can be expanded in a given set of spinor bilinears

$$\mathcal{A}_q^{a\pm} = \mathbb{H}^q_{a\pm} h + \mathbb{E}^q_{a\pm} e \mp \widetilde{\mathbb{H}}^q_{a\pm} \tilde{h} \mp \widetilde{\mathbb{E}}^q_{a\pm} \tilde{e}$$

with, e.g. Belitsky, Müller, Ji: NPB **878** (2014) 214

$$h = \frac{\bar{u}(p')\,(\not{q} + \not{q}')u(p)}{P\cdot(\not{q} + \not{q}')} \qquad \ldots$$

- The results read Braun, Manashov, Pirnay: PRL109 (2012) 242001

$$\mathbb{H}_{++} = T_0 \circledast H + \frac{t}{Q^2}\Big[-\frac{1}{2}T_0 + T_1 + 2\xi \mathbf{D}_\xi\, T_2\Big] \circledast H + \frac{2t}{Q^2}\xi^2\partial_\xi \xi T_2 \circledast (H+E)$$

$$\mathbb{H}_{0+} = -\frac{4|\xi P_\perp|}{\sqrt{2}Q}\Big[\xi\partial_\xi T_1 \circledast H + \frac{t}{Q^2}\partial_\xi \xi\, T_1 \circledast (H+E)\Big] - \frac{t}{\sqrt{2}Q|\xi P_\perp|}\xi T_1 \circledast \Big[\xi\,(H+E) - \widetilde{H}\Big]$$

$$\mathbb{H}_{-+} = \frac{4|\xi P_\perp|^2}{Q^2}\Big[\xi\partial^2_\xi \xi\, T_1^{(+)} \circledast H + \frac{t}{Q^2}\partial^2_\xi \xi^2\, T_1^{(+)} \circledast (H+E)\Big]$$
$$+ \frac{2t}{Q^2}\xi\Big[\xi\partial_\xi \xi\, T_1^{(+)} \circledast (H+E) + \partial_\xi\, \xi\, T_1 \circledast \widetilde{H}\Big]$$

- Photon helicity amplitudes can be expanded in a given set of spinor bilinears

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with, e.g. Belitsky, Müller, Ji: NPB **878** (2014) 214

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- The results read Braun, Manashov, Pirnay: PRL109 (2012) 242001

$$\mathbb{E}_{++} = T_0 \circledast E + \frac{t}{Q^2}\left[-\frac{1}{2}T_0 + T_1 + 2\xi\mathbf{D}_\xi\, T_2\right] \circledast E - \frac{8m^2}{Q^2}\xi^2\partial_\xi\xi\, T_2 \circledast (H+E)$$

$$\mathbb{E}_{0+} = -\frac{4|\xi P_\perp|}{\sqrt{2}Q}\left[\xi\partial_\xi T_1 \circledast E\right] + \frac{4m^2}{\sqrt{2}Q|\xi P_\perp|}\xi T_1 \circledast \left[\xi\,(H+E) - \widetilde{H}\right]$$

$$\mathbb{E}_{-+} = \frac{4|\xi P_\perp|^2}{Q^2}\left[\xi\partial_\xi^2\xi\, T_1^{(+)} \circledast E\right] - \frac{8m^2}{Q^2}\xi\left[\xi\partial_\xi\xi\, T_1^{(+)} \circledast (H+E) + \partial_\xi\,\xi\, T_1 \circledast \widetilde{H}\right]$$

etc.

where $F = H, E, \widetilde{H}, \widetilde{E}$ **are** C**-even GPDs**

$$T \circledast F = \sum_q e_q^2 \int_{-1}^{1} \frac{dx}{2\xi}\, T\left(\frac{\xi + x - i\epsilon}{2(\xi - i\epsilon)}\right) F(x,\xi,t)$$

the coefficient functions $T_k^{\pm}$ **are given by the following expressions:**

$$T_0(u) = \frac{1}{1-u}$$

$$T_1(u) \equiv T_1^{(-)}(u) = -\frac{\ln(1-u)}{u}$$

$$T_1^{(+)}(u) = \frac{(1-2u)\ln(1-u)}{u}$$

$$T_2(u) = \frac{\mathrm{Li}_2(1) - \mathrm{Li}_2(u)}{1-u} + \frac{\ln(1-u)}{2u}$$

and

$$\mathbf{D}_\xi = \partial_\xi + 2\frac{|\xi P_\perp|^2}{t}\partial_\xi^2 \xi \;=\; \partial_\xi - \frac{t - t_{\min}}{2t}(1-\xi^2)\partial_\xi^2 \xi$$

Main features:

- Two expansion parameters

$$\frac{t}{Q^2}; \qquad \frac{t-t_{\rm min}}{Q^2} \sim \frac{|\xi P_\perp|^2}{Q^2}$$

- All mass corrections for scalar targets absorbed in $t_{\rm min} = -4m^2\xi^2/(1-\xi^2)$; always overcompensated by finite-t corrections in the physical region
- Some extra m^2/Q^2 corrections for nucleon due to spinor algebra; disappear in certain CFF combinations
- Factorization checked to $1/Q^2$ accuracy
- Gauge and translation invariance checked to $1/Q^2$ accuracy
- Correct threshold behavior $t \to t_{\rm min}$, $\xi \to 1$
- Dispersion representation possible and worked out

Summary and Outlook

- **noncomplanarity makes separation of collinear directions ambiguous**

  — hence "leading twist approximation" ambiguous

  — related to violation of translation invariance and EM Ward identities

- **Target mass and finite-t corrections to DVCS are known to twist-4 accuracy**

  — Gauge and translation invariance of Compton tensor is restored to $1/Q^2$ accuracy
  — Convention-dependence of the common leading-twist calculations is removed
  — For many observables, complete results close to LT in "photon frame"
  — Theoretically motivated limits $-t/Q^2 \lesssim 1/4$

- **Must be taken into account in all studies aiming at 3D proton structure**

- **Outlook:**

  — time-like DVCS
  — all twists in LO, exact translation and gauge invariance

  — small x, matching to the BFKL formalism

  — NLO

Backup slides

Braun, Manashov, Müller, Pirnay: arXiv:1401.7621

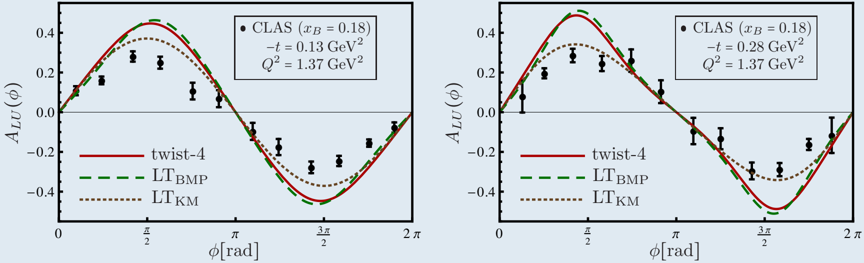

Figure: Single electron beam spin asymmetry by CLAS collaboration

GPD model: GK12 (Kroll, Moutarde, Sabatie, Eur.Phys.J. C73, 2278)

Braun, Manashov, Müller, Pirnay: arXiv:1401.7621

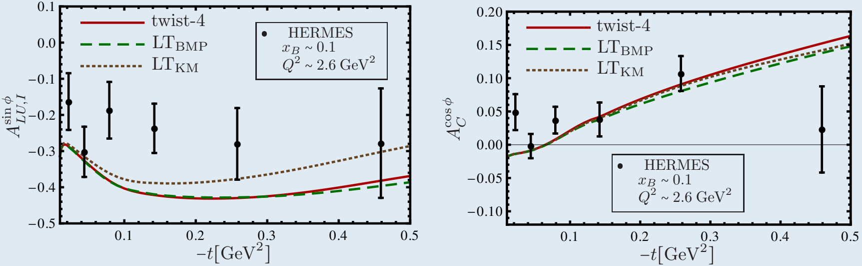

Figure: The single electron beam spin asymmetry [left panel] in the charge-odd sector and the unpolarized beam charge asymmetry [right panel] measured by the HERMES collaboration

GPD model: GK12 (Kroll, Moutarde, Sabatie, Eur.Phys.J. C73, 2278)

Braun, Manashov, Müller, Pirnay: arXiv:1401.7621

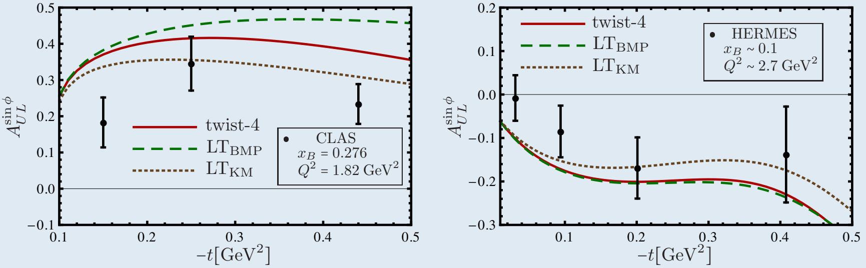

Figure: Longitudinal proton spin asymmetry from CLAS [left panel], measured with an electron beam, and HERMES [right panel], measured with a positron beam

GPD model: GK12 (Kroll, Moutarde, Sabatie, Eur.Phys.J. C73, 2278)

Braun, Manashov, Müller, Pirnay: arXiv:1401.7621

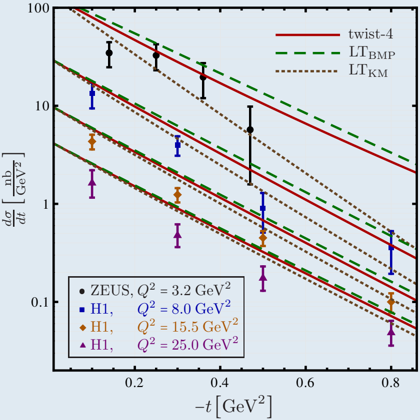

Figure: The DVCS cross section from H1 (squares, diamonds, triangles) and ZEUS (circles)

GPD model: GK12 (Kroll, Moutarde, Sabatie, Eur.Phys.J. C73, 2278)

B. Pirnay: PhD Thesis

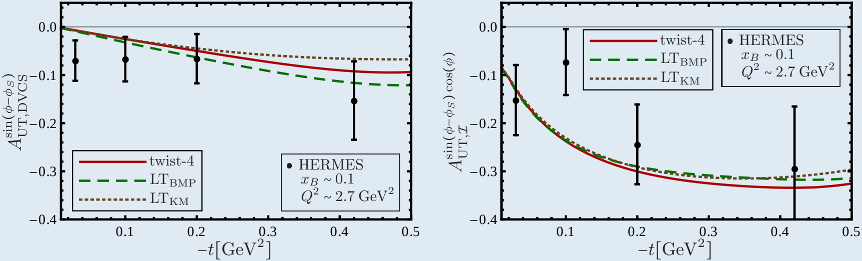

Figure: Transverse target spin asymmetries by HERMES collaboration

GPD model: GK12 (Kroll, Moutarde, Sabatie, Eur.Phys.J. C73, 2278)