Description of three-body nuclear reactions in the Faddeev formalism

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Outline

- Faddeev/AGS formalism for 3-particle scattering
  — benchmarking other methods
  — L- and π-dependent nonlocal optical potentials
  — (d,n) reactions
- core excitation (CX) in 3-body nuclear reactions
  — ^{11}Be(p,d), ^{10}Be(d,p), ^{24}Mg(d,d')
  — analyzing CX effects
- 4-particle scattering
  (p,n), (d,p) and (d,n) reactions in 4N system

$$\left.\begin{array}{l} p+{}^3\mathrm{H} \\ n+{}^3\mathrm{He} \\ d+d \end{array}\right\} \to \left\{\begin{array}{l} p+n+d \\ p+{}^3\mathrm{H} \\ n+{}^3\mathrm{He} \\ d+d \\ 2p+2n \end{array}\right.$$

Scattering: wave function vs transition operator

- Schrödinger equation

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+ impose asymptotic boundary conditions explicitly

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$$G_0 = (E + i0 - H_0)^{-1}$$

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wave function $$|\psi\rangle = |\mathbf{k}\rangle + G_0 v|\psi\rangle$$

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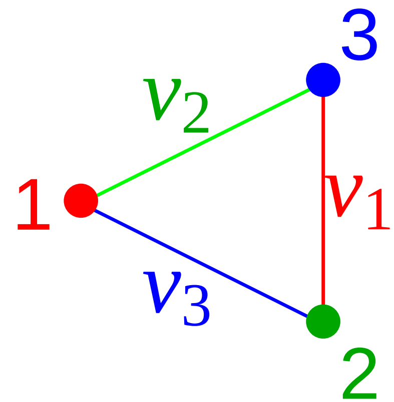
transition matrix $$T|\mathbf{k}\rangle = v|\psi\rangle$$

$$T = v + vG_0 T$$

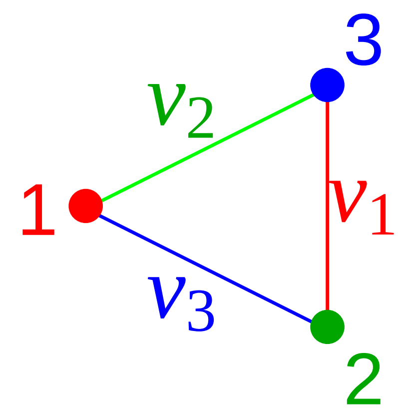
$$|\psi\rangle = |\mathbf{k}\rangle + G_0 T|\mathbf{k}\rangle$$

Three-particle system



Hamiltonian $H_0 + \sum_{\alpha} v_{\alpha}$

Three-particle system



Hamiltonian $H_0 + \sum_\alpha v_\alpha$

- Faddeev equations

$$(E - H_0 - v_\alpha)|\psi_\alpha\rangle = v_\alpha \sum_\sigma \bar{\delta}_{\alpha\sigma} |\psi_\sigma\rangle$$

$$|\Psi\rangle = \sum_\alpha |\psi_\alpha\rangle$$

Alt, Grassberger, and Sandhas equations

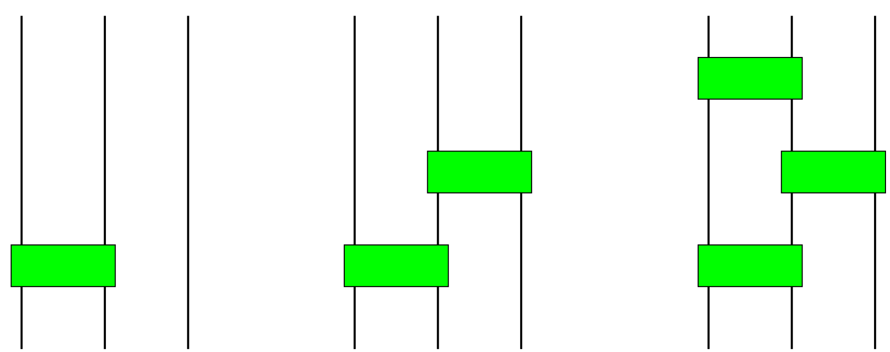
$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$

$$U_{0\alpha} = G_0^{-1} + \sum_{\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$

$$T_{\sigma} = v_{\sigma} + v_{\sigma} G_0 T_{\sigma}$$

$$G_0 = (E + i0 - H_0)^{-1}$$

channel states $\quad (E - H_0 - v_{\alpha})|\phi_{\alpha}\rangle = 0$



...

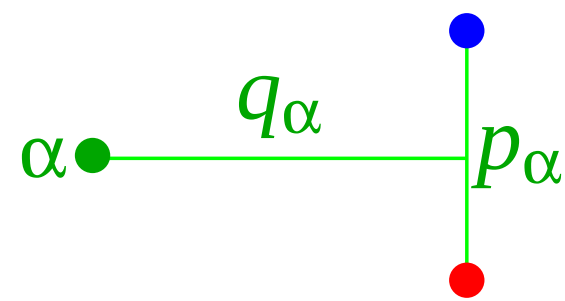
AGS equations with 3BF

$$V_{3BF} = \sum_{\alpha=1}^{3} w_\alpha$$

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_\gamma \bar{\delta}_{\beta\gamma} T_\gamma G_0 U_{\gamma\alpha} + w_\alpha + \sum_\gamma w_\gamma G_0 (1 + T_\gamma G_0) U_{\gamma\alpha}$$

AGS equations: numerical solution

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$



- 3 sets of Jacobi momenta
- momentum-space partial wave basis
- set of coupled 2-variable integral equations
- integrable singularities in kernel
- Coulomb interaction: screening and renormalization

[PRC 71, 054005; PRC 72, 054004; PRC 74, 064001]

Application to 3-body nuclear reactions

$$\left.\begin{array}{l} p+(nA) \\ d+A \end{array}\right\} \rightarrow \left\{\begin{array}{l} n+(pA) \\ p+(nA) \\ d+A \\ p+n+A \end{array}\right.$$

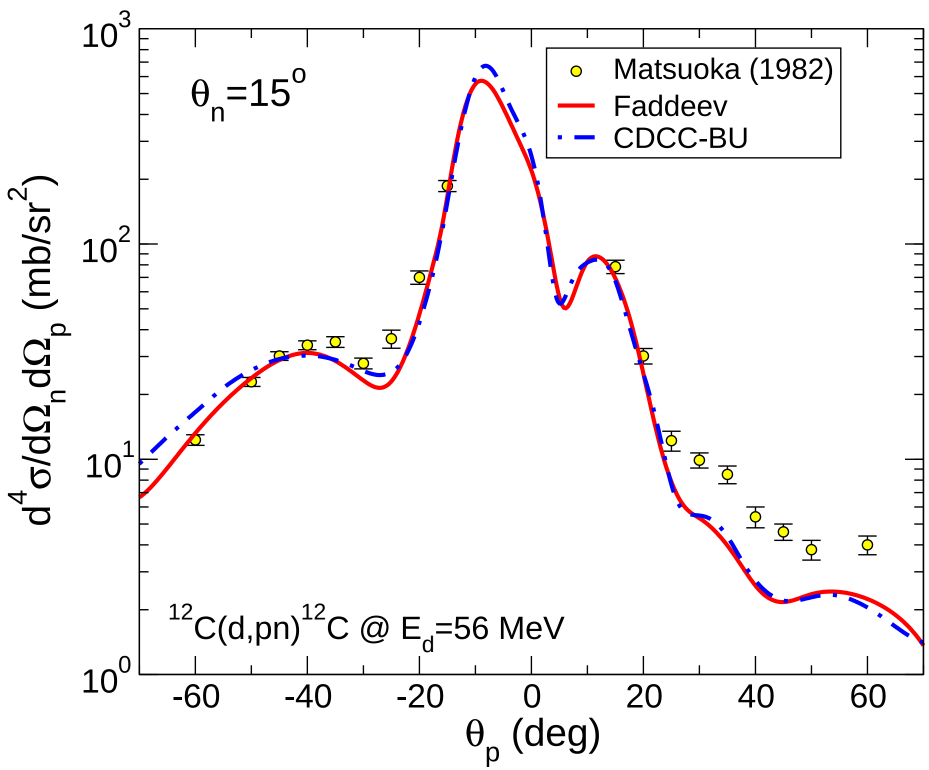
with $A = {}^4\mathrm{He},\ {}^{10}\mathrm{Be},\ {}^{12}\mathrm{C},\ {}^{14}\mathrm{C},\ {}^{16}\mathrm{O},\ {}^{28}\mathrm{Si},\ {}^{40}\mathrm{Ca},\ {}^{48}\mathrm{Ca},\ {}^{58}\mathrm{Ni},\ \ldots$

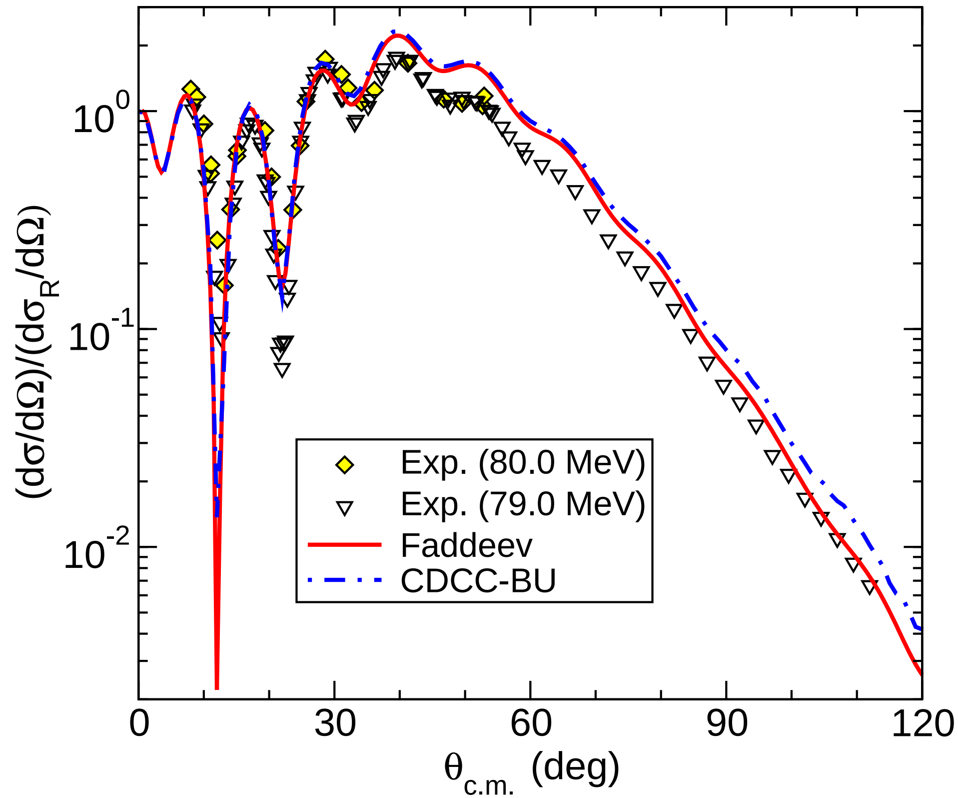
V_{np}: realistic NN potentials (CD Bonn)

V_{NA}: optical/binding NA potentials (CH89, KD, ...)

- Validity test of approximate nuclear reaction methods: DWBA, ADWA, CDCC, ...
- Novel dynamic input: nonlocal OP, L- and π-dependent OP, core excitation, ...

CDCC test: $^{12}C(d,pn)^{12}C$ & $^{58}Ni(d,d)^{58}Ni$

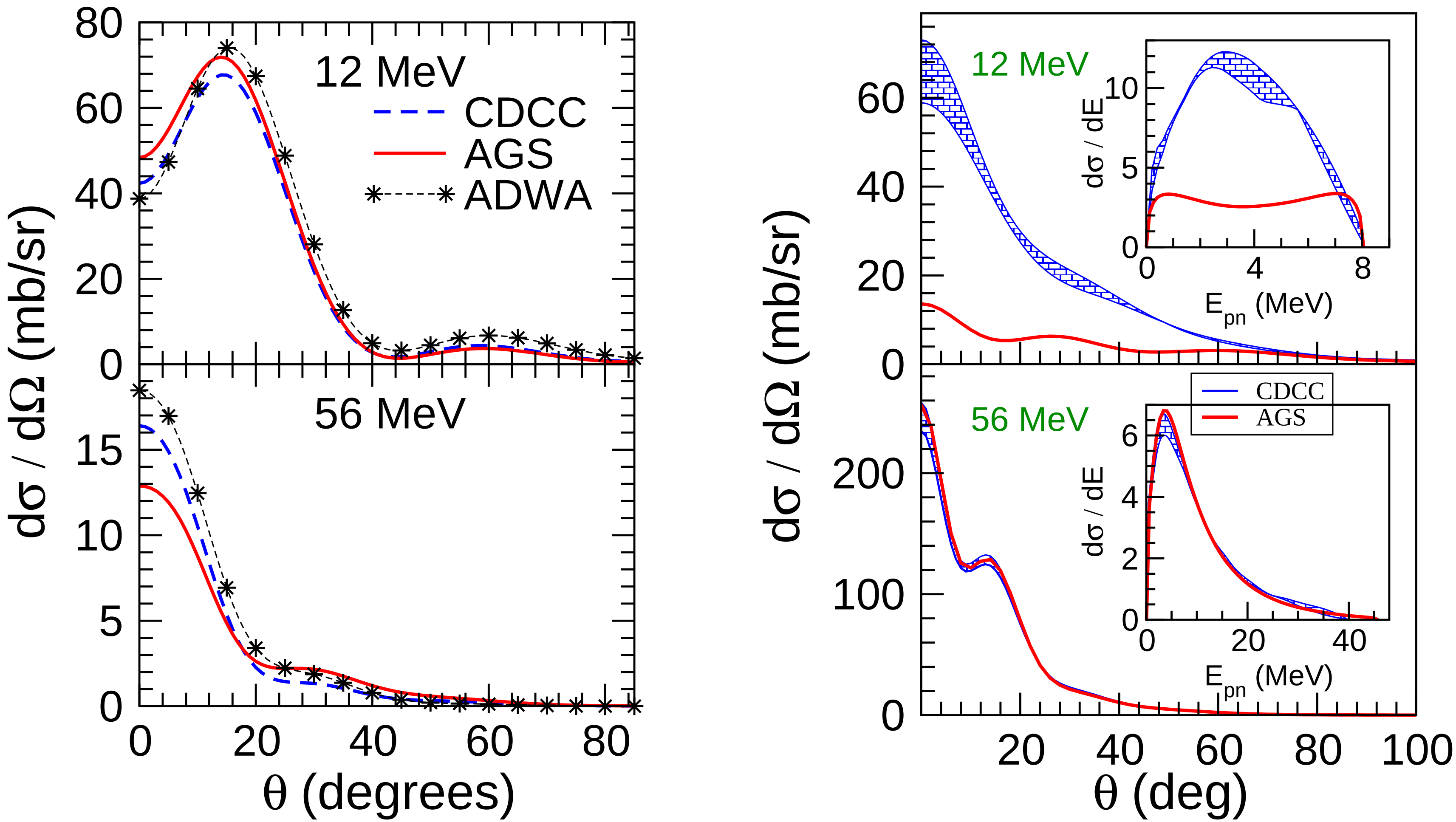




CDCC: A. M. Moro & F. M. Nunes

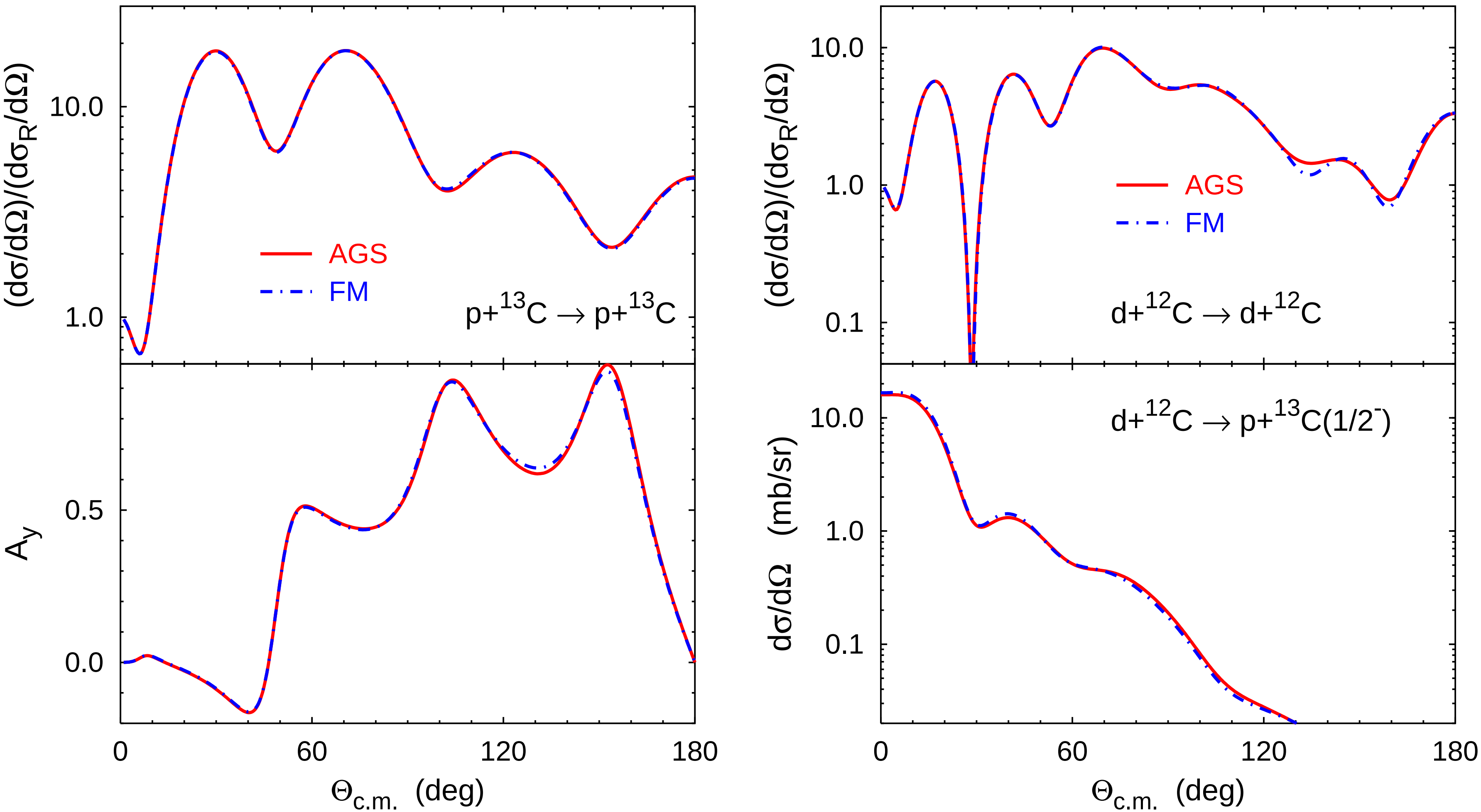
[PRC 76, 064602]

CDCC test: $^{12}C(d,p)^{13}C$ and $^{12}C(d,pn)^{12}C$



CDCC/ADWA: F. M. Nunes, N. Upadhyay [PRC 85, 054621]

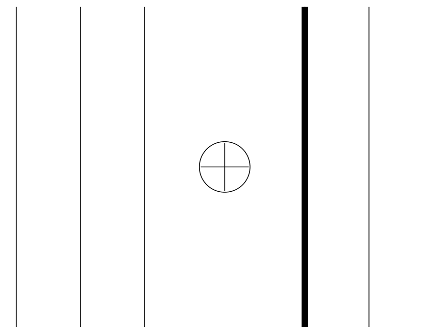
Comparison with r-space FM results



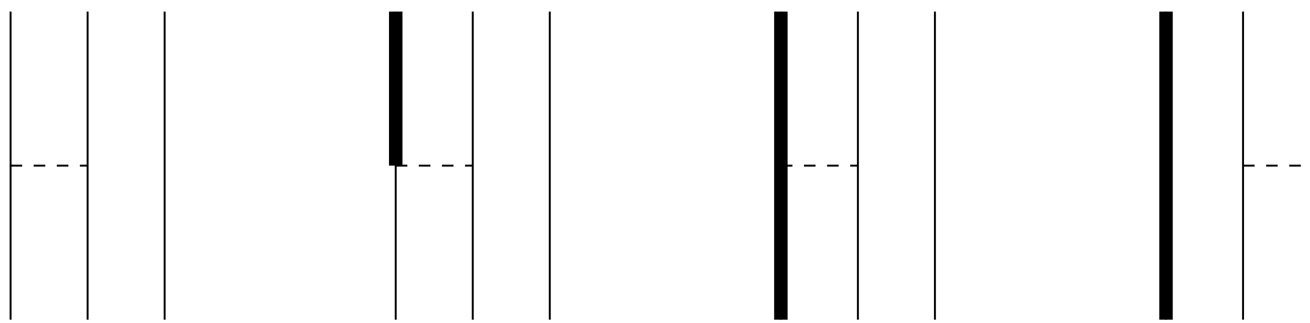
FM: R. Lazauskas [LNP 875, Clusters in Nuclei V.3]

Core excitation (CX): extended Hilbert space

$$\mathcal{H} = \mathcal{H}_g \oplus \mathcal{H}_x$$



sector coupling by interaction



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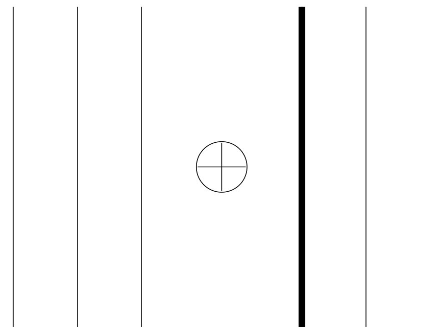

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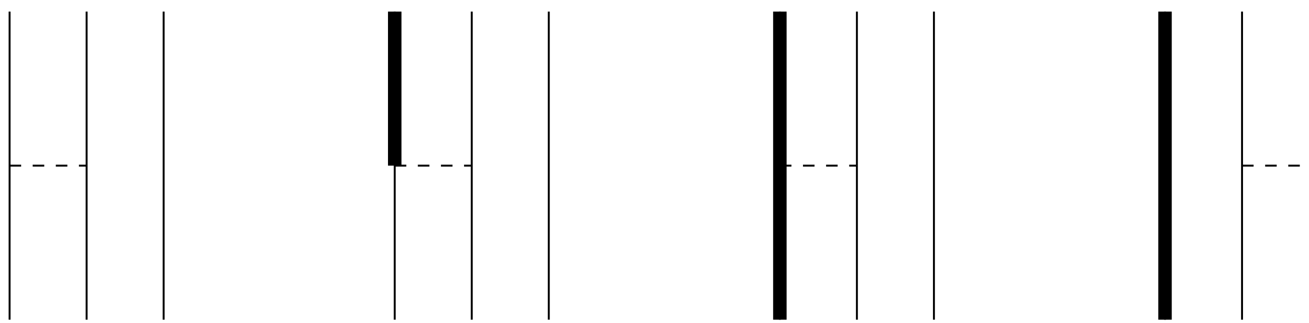


standard form of Alt-Grassberger-Sandhas (AGS)
3-body equations with $H_0 \to H_0 + h_A^{\text{int}}$

$$h_A^{\text{int}} |\mathcal{H}_a\rangle = (m_{A^*} - m_A)\delta_{ax} |\mathcal{H}_a\rangle$$

3-body AGS equations with core excitation

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$

$$U_{0\alpha} = G_0^{-1} + \sum_{\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$

$$T_{\sigma} = v_{\sigma} + v_{\sigma} G_0 T_{\sigma}$$

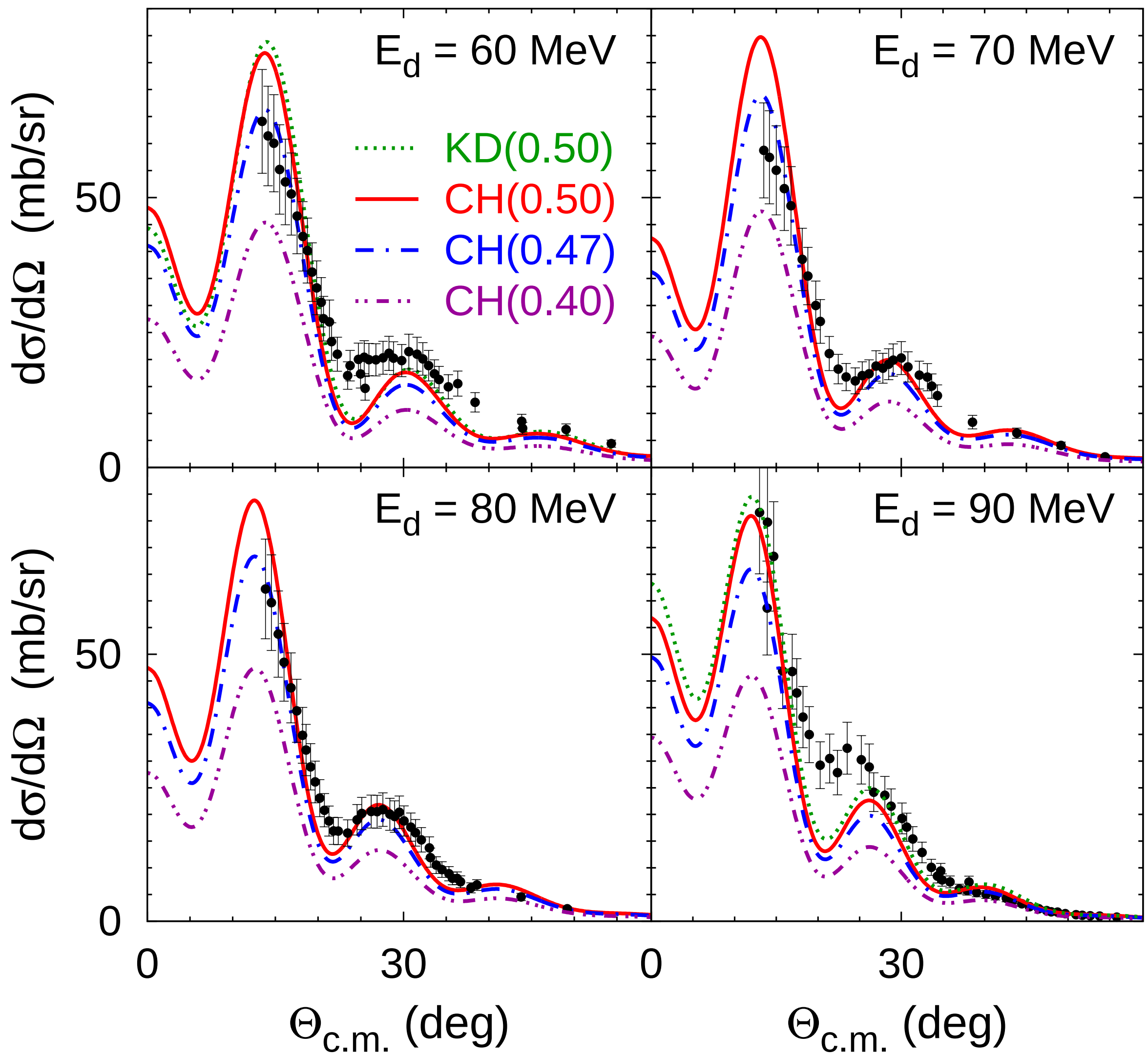
$$G_0 = (E + i0 - H_0)^{-1}$$

channel states $(E - H_0 - v_{\alpha})|\phi_{\alpha}\rangle = 0$

$$H_0|\mathbf{p}_{\alpha}\mathbf{q}_{\alpha}\rangle_a = [p_{\alpha}^2/2\mu_{\alpha} + q_{\alpha}^2/2M_{\alpha} + (m_{A^*} - m_A)\delta_{ax}]|\mathbf{p}_{\alpha}\mathbf{q}_{\alpha}\rangle_a$$

[PRC 88, 011601(R)]

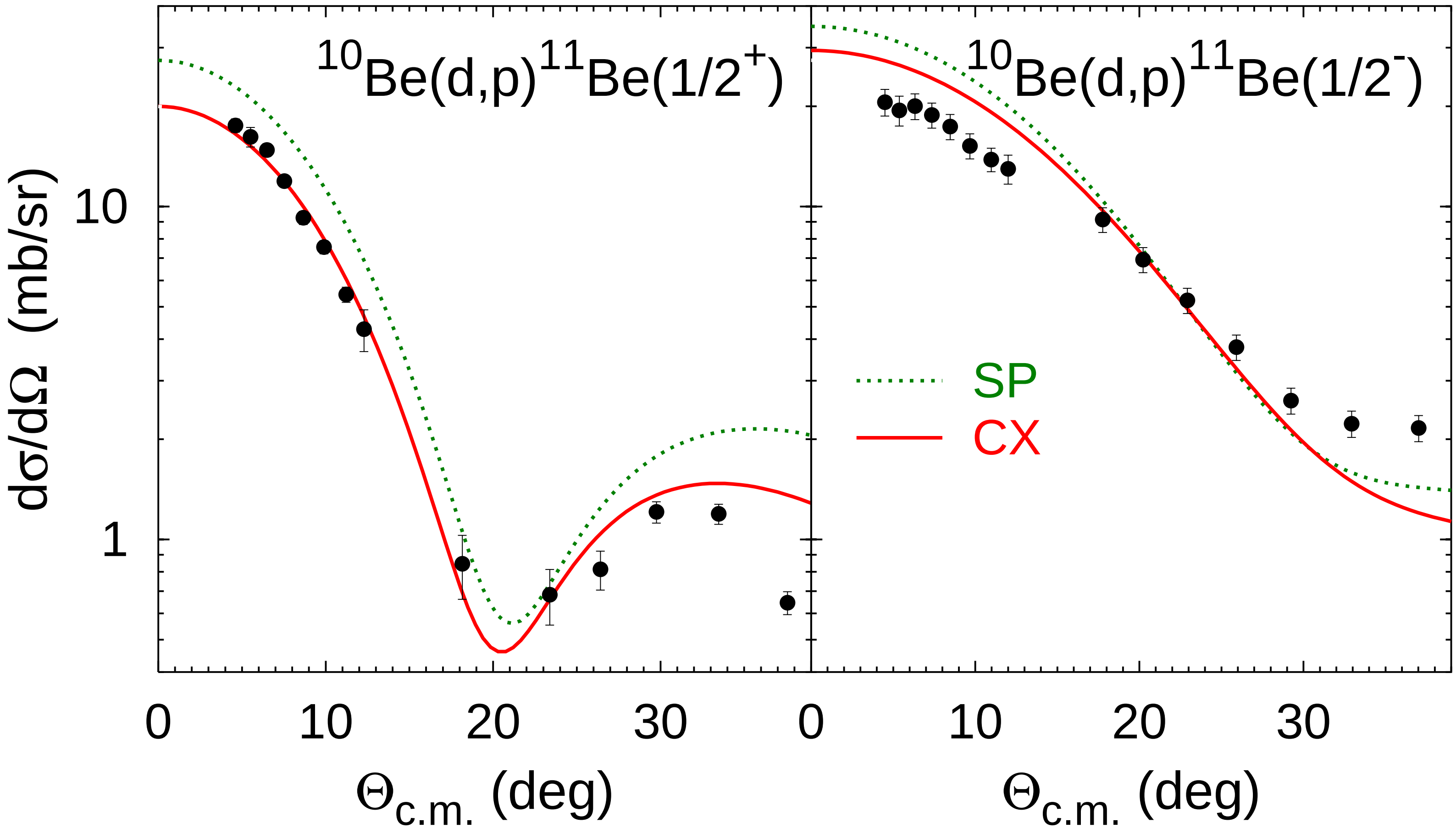
^{24}Mg(d,d')^{24}Mg(2^+) inelastic scattering



Rotational model for V_{NA} with $\beta_2 = 0.4$, 0.47, 0.5

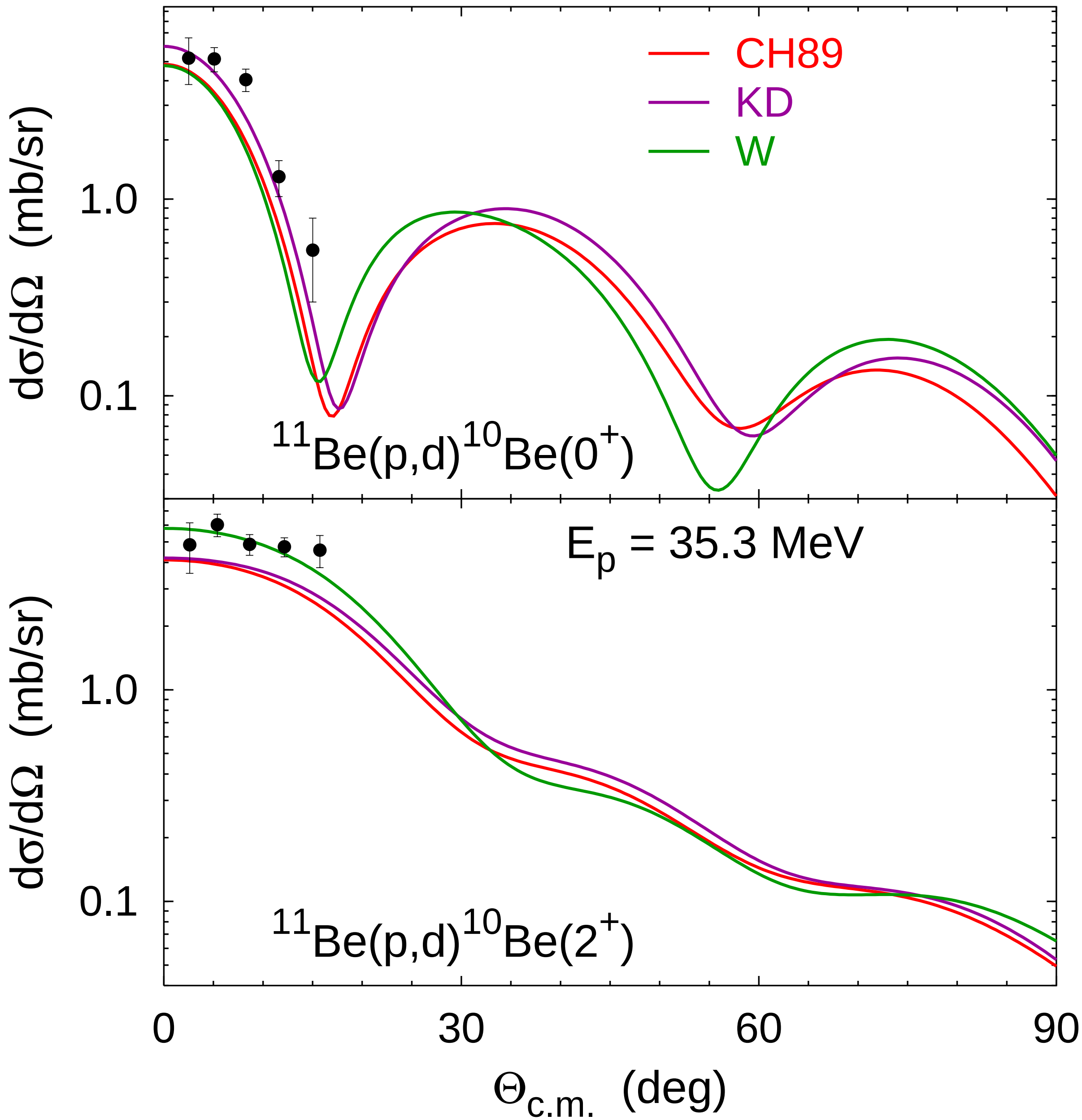
DWBA: $\beta_2 \sim 0.5\ (p,p'), \quad \beta_2 \sim 0.4\ (d,d')$

CX effect in ^{10}Be(d,p)^{11}Be at 21.4 MeV



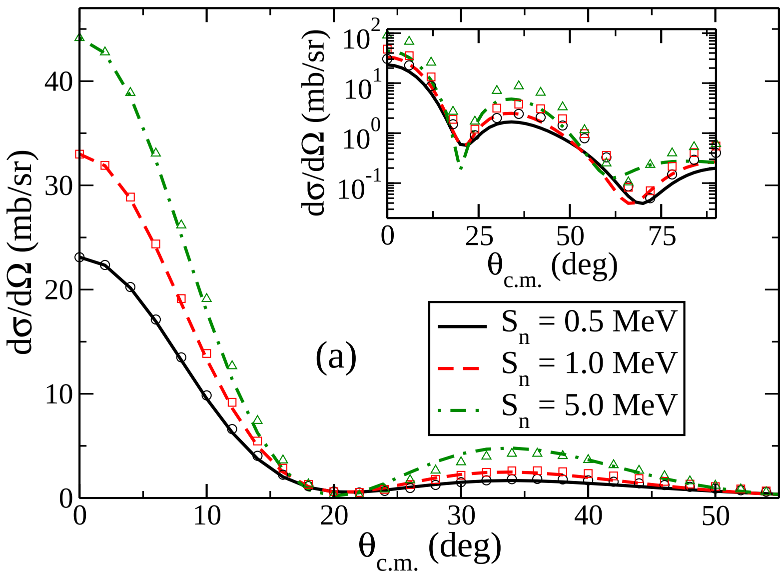
CH89, rotational model for V_{NA} with $\beta_2 = 0.67$

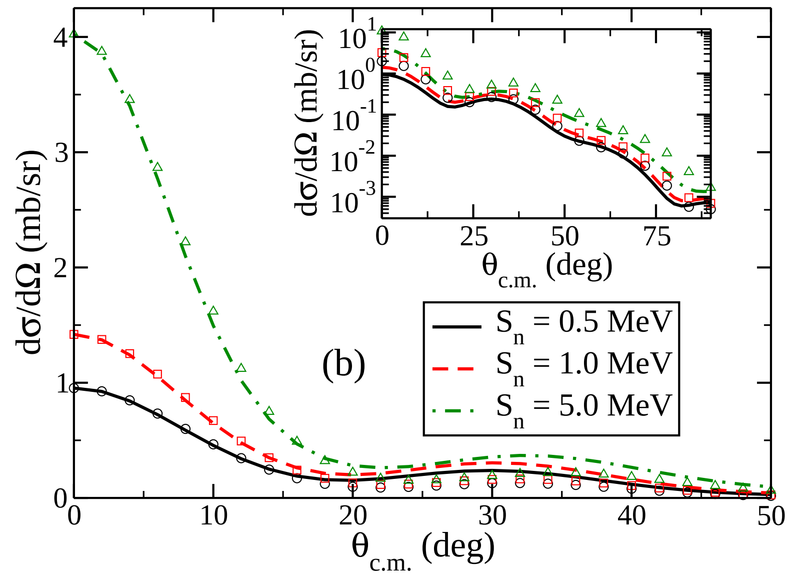
^{11}Be(p,d)^{10}Be: sensitivity to V_{NA}



CX effect: ^{10}Be(d,p)"^{11}Be" at 20 and 80 MeV

varying neutron separation energy in "^{11}Be"

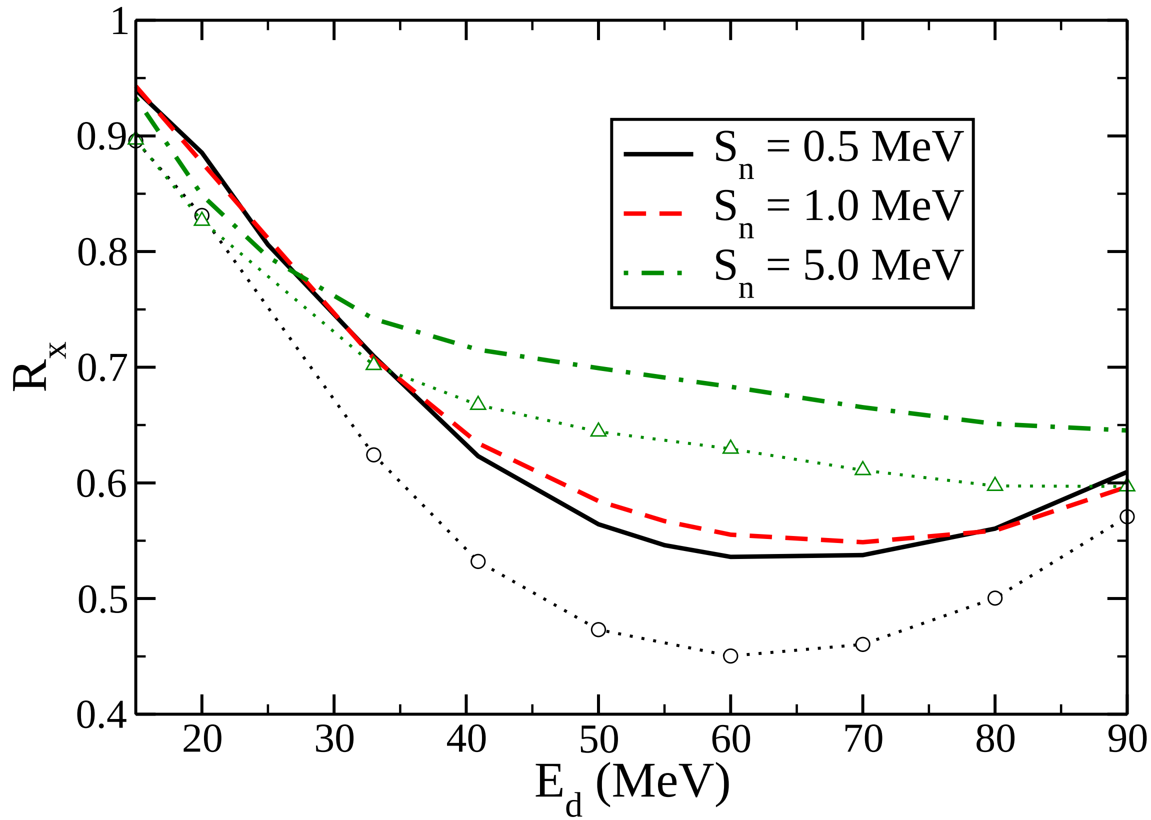




CX($\beta_2 = 0.67$): lines
SP: symbols (normalized to CX at 0° on linear scale)

[In collaboration with A. Ross, E. Norvaišas, F.M. Nunes]

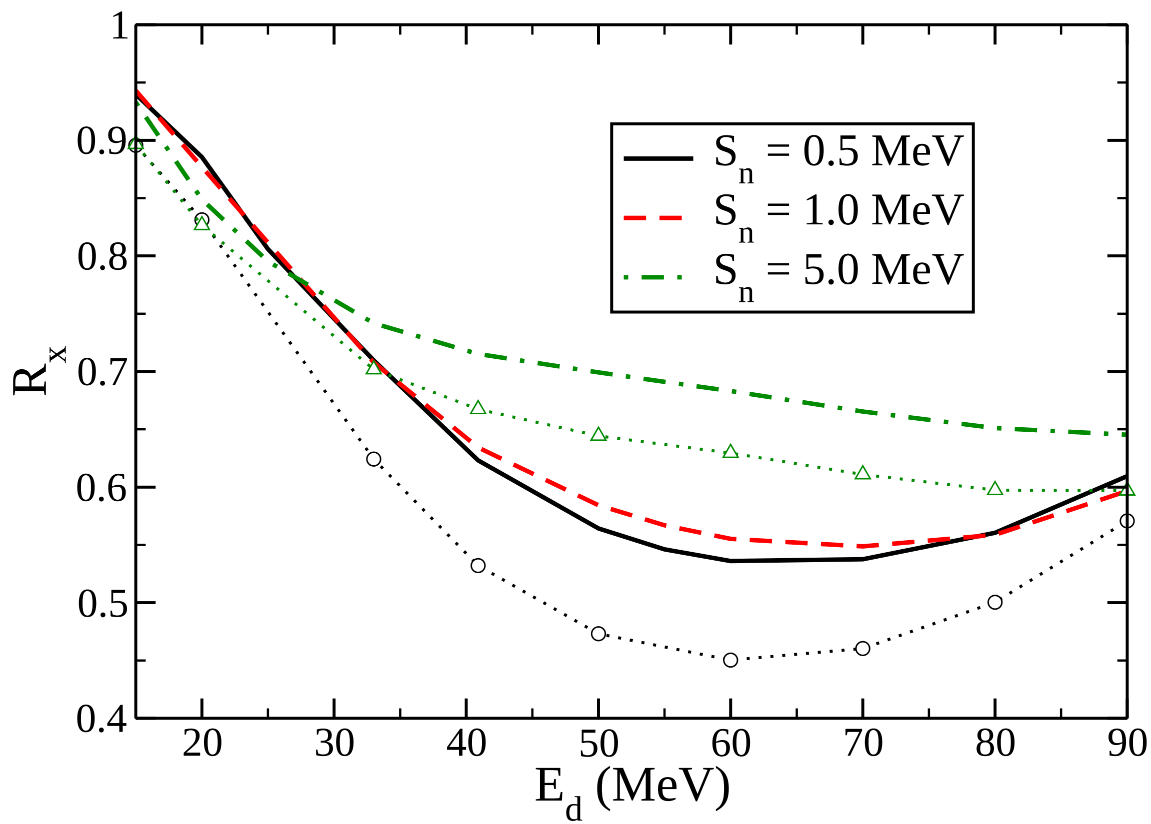
CX effect: ^{10}Be(d,p)"^{11}Be"



$E_x = 3.368$ MeV (lines) vs $E_x = 0.5$ MeV (symbols)

$$R_x = (d\sigma/d\Omega)_{\mathrm{CX}}/[SF*(d\sigma/d\Omega)_{\mathrm{SP}}]|_{\Theta=0^\circ}$$

CX effect: ^{10}Be(d,p)"^{11}Be"



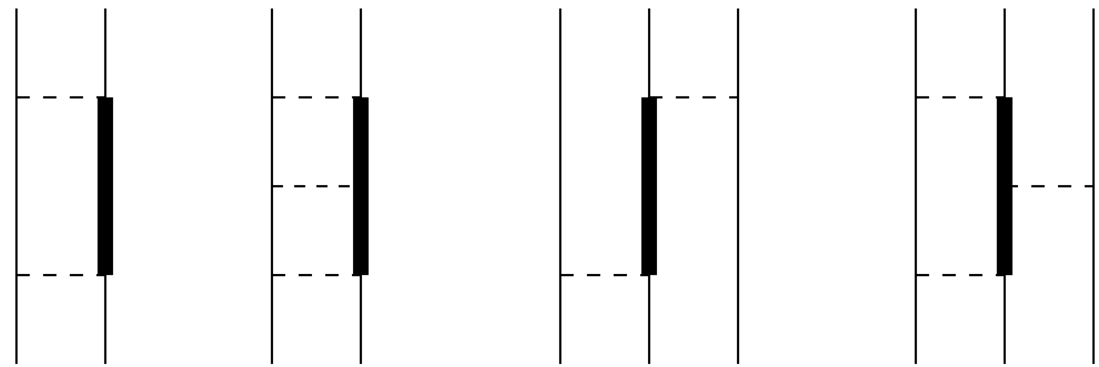
$E_x = 3.368$ MeV (lines) vs $E_x = 0.5$ MeV (symbols)

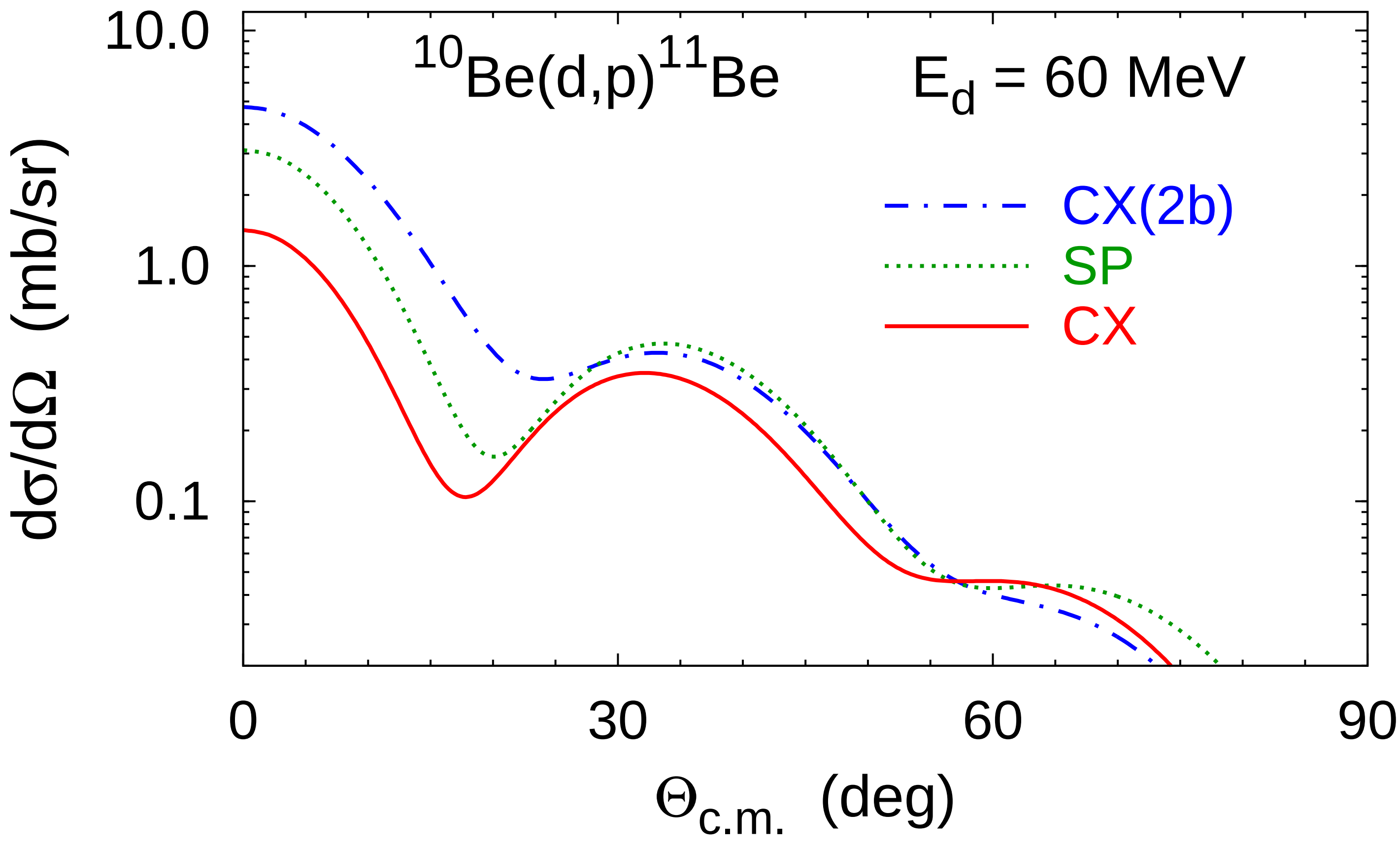
$$R_x = (d\sigma/d\Omega)_{\rm CX}/[SF * (d\sigma/d\Omega)_{\rm SP}]|_{\Theta=0^\circ}$$

$R_x \neq 1 \Longrightarrow$ SF$= \sigma_{\rm exp}/\sigma_{\rm SP}$ unreliable !

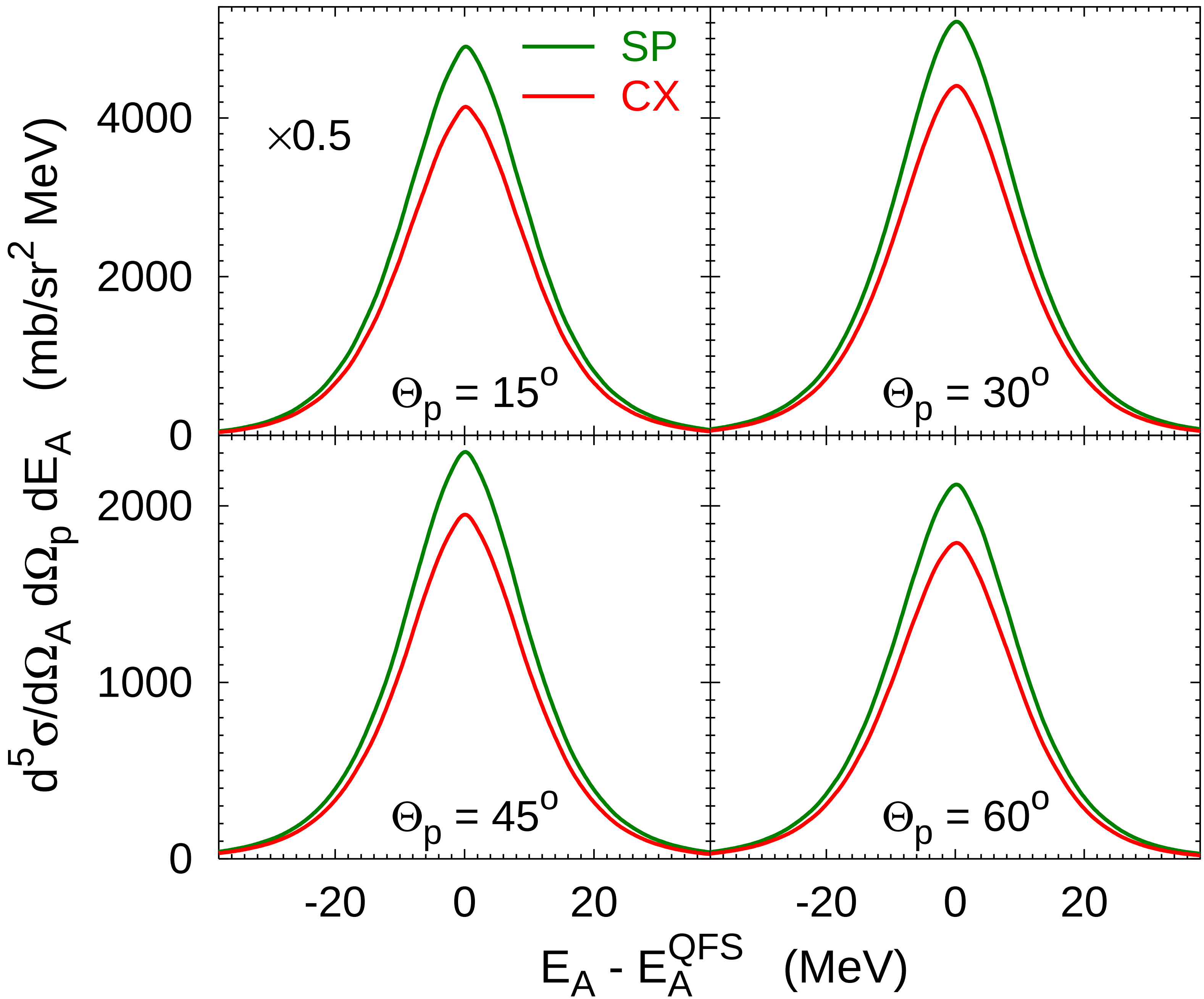
Faddeev/AGS: (V_{NA} - SF - data) compatibility check

CX effects of 2- and 3-body nature

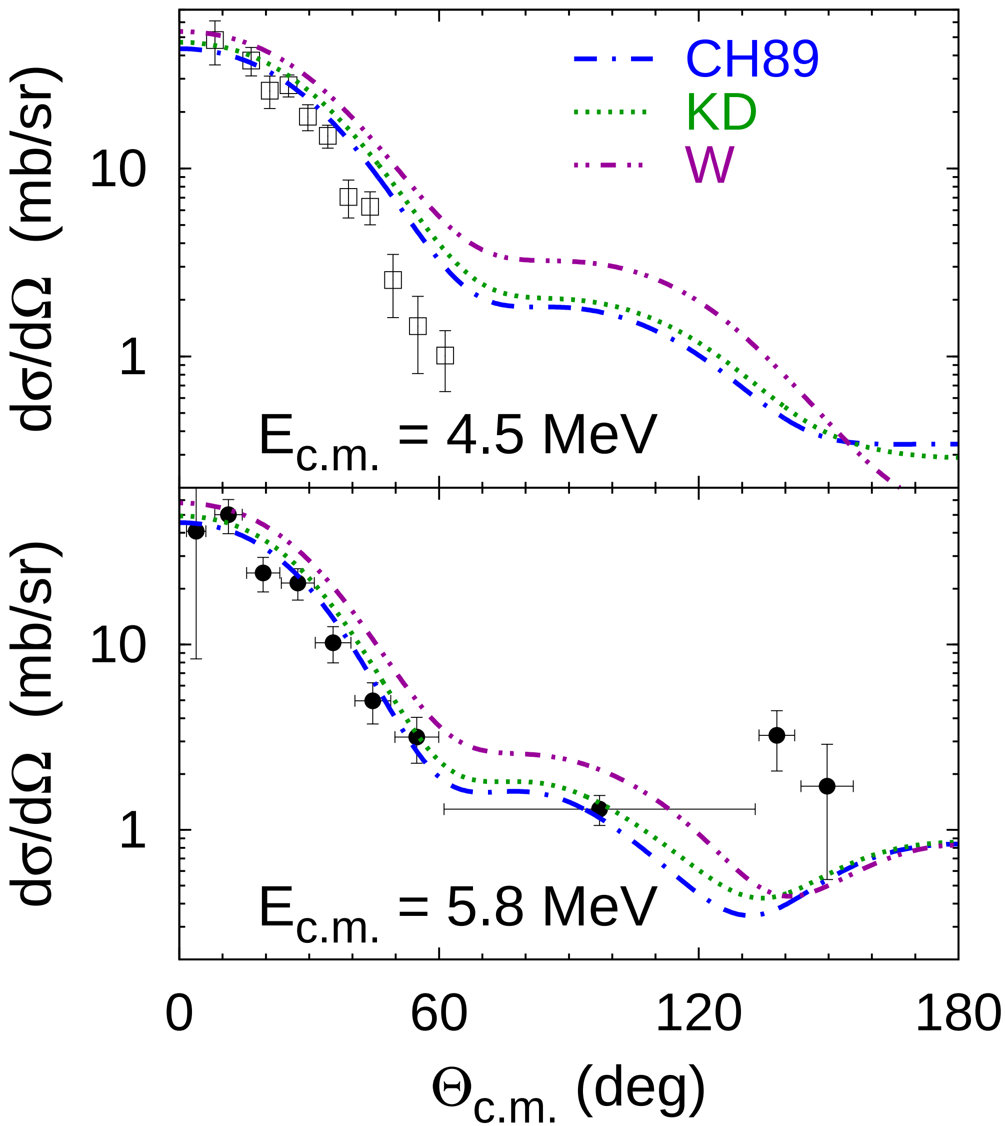




^{11}Be(p,pn)^{10}Be at 200 MeV/u near np QFS ($\Theta_A = 0^\circ$)

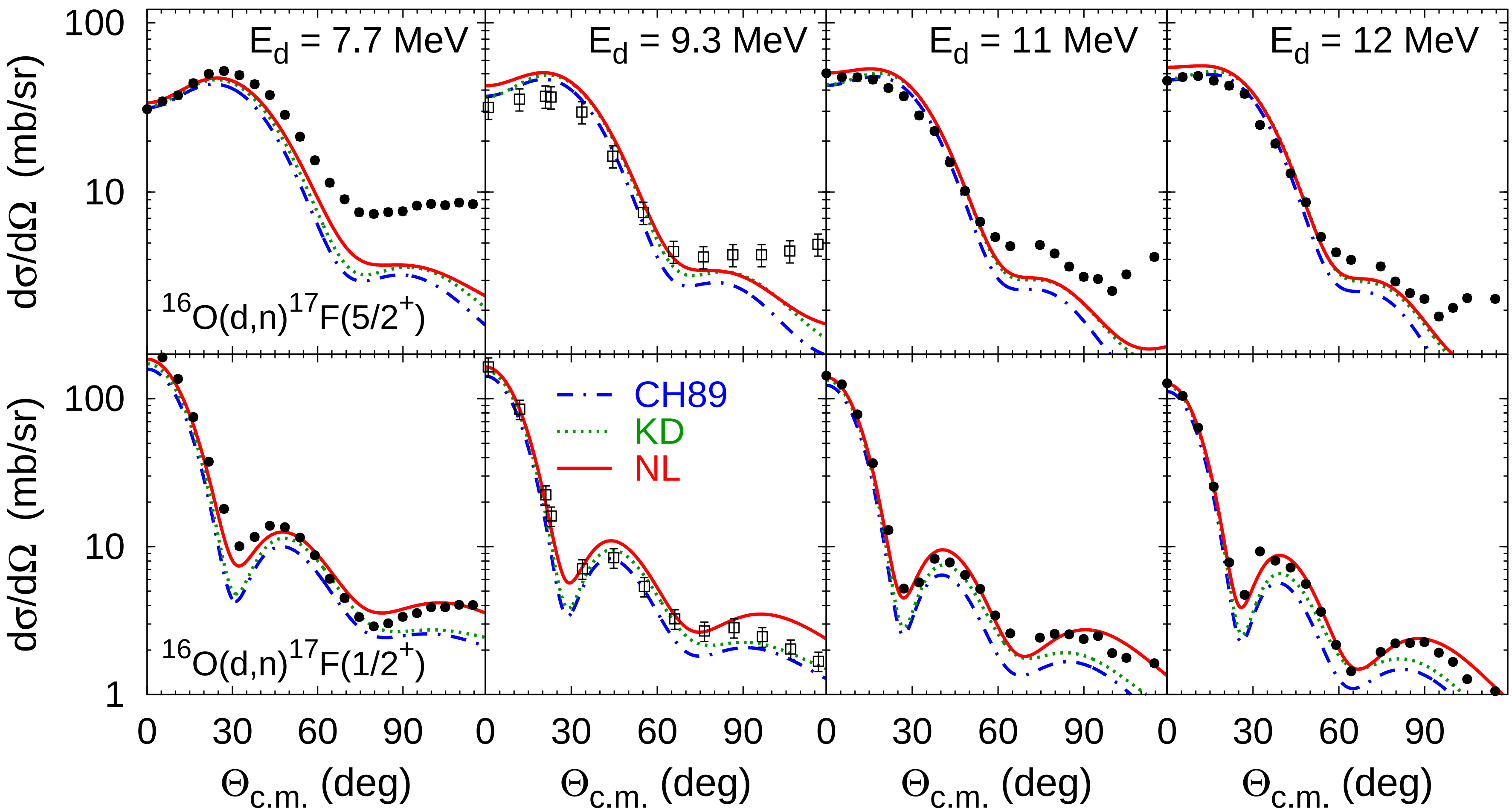


$^{7}Be(d,n)^{8}B$



CH89
KD
W
10
1
dσ/dΩ (mb/sr)
E c.m. = 4.5 MeV
10
1
dσ/dΩ (mb/sr)
E c.m. = 5.8 MeV
0
60
120
180
Θ c.m. (deg)

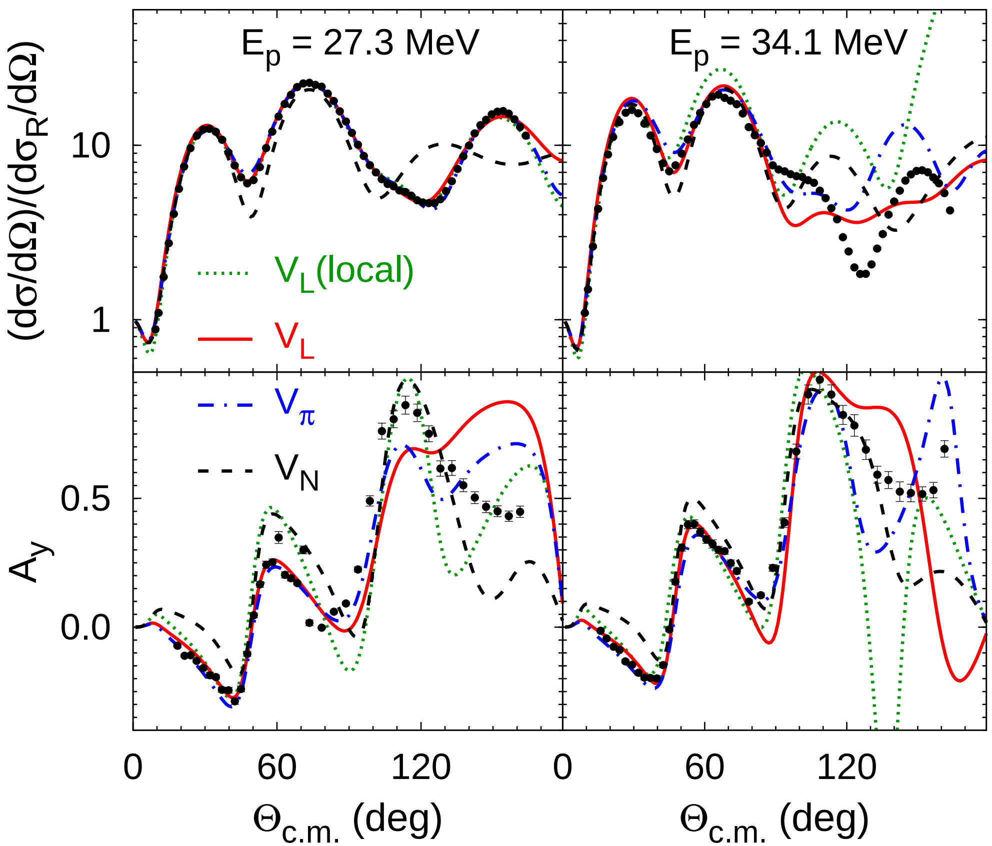
$^{16}O(d,n)^{17}F$



E_d = 7.7 MeV
E_d = 9.3 MeV
E_d = 11 MeV
E_d = 12 MeV
16O(d,n)17F(5/2+)
16O(d,n)17F(1/2+)
CH89
KD
NL
dσ/dΩ (mb/sr)
Θc.m. (deg)
100
10
1
0
30
60
90

L- and π-dependent nonlocal OP: ^{16}O(p,p)^{16}O

$$V_L(\mathbf{r}',\mathbf{r}) = -H_c(x)[V_V\, f_V(y) + iW_V\, f_W(y) + iW_S\, g_S(y)]$$
$$-H_s(x)V_s\frac{2}{y}\frac{d\,f_s(y)}{dy}\,\boldsymbol{\sigma}\cdot\mathbf{L}$$
$$-H_c(x)[\tilde{V}\, g_{\tilde{V}}(y) + i\tilde{W}\, g_{\tilde{W}}(y)]\, f_L(L^2)$$



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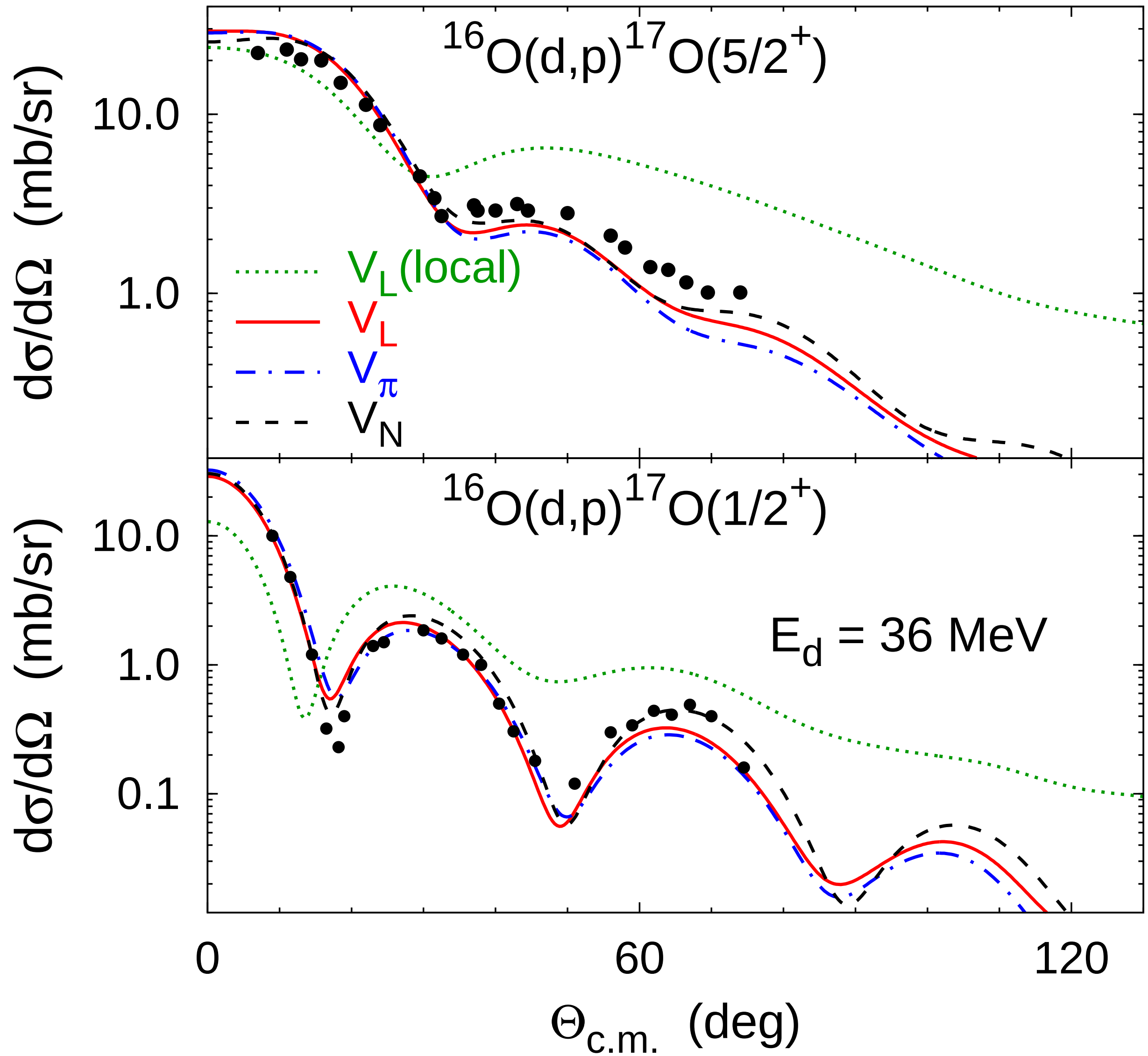
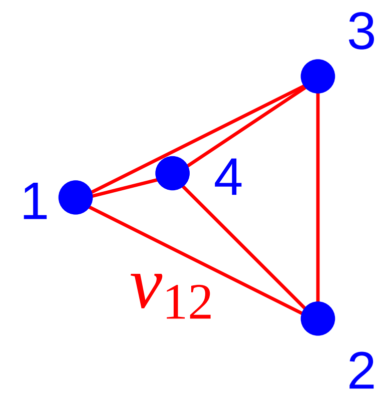


[In collaboration with D. Jurčiukonis]

4N scattering



Hamiltonian $$H_0 + \sum_{i>j} v_{ij}$$

- Wave function:
  Schrödinger equation (HH + Kohn VP, r-space)
  [M. Viviani, A. Kievsky, L. E. Marcucci, S. Rosati, L. Girlanda]
- Wave function components:
  Faddeev-Yakubovsky equations (r-space)
  [R. Lazauskas, J. Carbonell]
- Transition operators:
  Alt-Grassberger-Sandhas equations (p-space)
  [AD, A. C. Fonseca]

4-body scattering: AGS equations

4-body transition operators

$$t_i = v_i + v_i G_0 t_i$$

$$G_0 = (E + i0 - H_0)^{-1}$$

$$U_\gamma^{jk} = G_0^{-1} \bar{\delta}_{jk} + \sum_i \bar{\delta}_{ji} t_i G_0 U_\gamma^{ik}$$

$$\mathcal{U}_{\beta\alpha}^{ji} = (G_0 t_i G_0)^{-1} \bar{\delta}_{\beta\alpha} \delta_{ji} + \sum_{\gamma k} \bar{\delta}_{\beta\gamma} U_\gamma^{jk} G_0 t_k G_0 \, \mathcal{U}_{\gamma\alpha}^{ki}$$

i, j, k: pairs ($\equiv$ three-cluster (2+1+1) partitions)
α, β, γ: two-cluster (1+3 or 2+2) partitions

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i, j, k: pairs ($\equiv$ three-cluster (2+1+1) partitions)
α, β, γ: two-cluster (1+3 or 2+2) partitions

wave function

$$|\Psi_\alpha\rangle = |\Phi_\alpha\rangle + \sum_{\gamma jki} G_0 t_j G_0 U_\gamma^{jk} G_0 t_k G_0 \mathcal{U}_{\gamma\alpha}^{ki} |\phi_\alpha^i\rangle$$

$$|\Phi_\alpha\rangle = \sum_i |\phi_\alpha^i\rangle, \qquad |\phi_\alpha^i\rangle = G_0 \sum_j \bar{\delta}_{ij} t_j |\phi_\alpha^j\rangle$$

4-body scattering amplitudes

two-cluster reactions:

$$\langle \Phi_\beta | T_{\beta\alpha} | \Phi_\alpha \rangle = \sum_{ji} \langle \phi_\beta^j | \mathcal{U}_{\beta\alpha}^{ji} | \phi_\alpha^i \rangle$$

three-cluster breakup:

$$\langle \Phi^j | T_\alpha^j | \Phi_\alpha \rangle = \sum_{\beta k i} \langle \Phi^j | U_\beta^{jk} G_0 \, t_k \, G_0 \, \mathcal{U}_{\beta\alpha}^{ki} | \phi_\alpha^i \rangle$$

four-cluster breakup:

$$\langle \Phi_0 | T_{0\alpha} | \Phi_\alpha \rangle = \sum_{\beta jki} \langle \Phi_0 | t_j \, G_0 \, U_\beta^{jk} G_0 \, t_k \, G_0 \, \mathcal{U}_{\beta\alpha}^{ki} | \phi_\alpha^i \rangle$$

[PRC 75, 014005; PRA 85, 012708]

Symmetrized AGS equations

$$t = v + vG_0 t$$

$$G_0 = (E + i\varepsilon - H_0)^{-1}$$

$$U_j = P_j G_0^{-1} + P_j t G_0 U_j$$

$$3+1: \quad P_1 = P_{12} P_{23} + P_{13} P_{23}$$

$$2+2: \quad P_2 = P_{13} P_{24}$$

$$\mathcal{U}_{11} = (G_0 t G_0)^{-1} \zeta P_{34} + \zeta P_{34} U_1 G_0 t G_0 \, \mathcal{U}_{11} + U_2 G_0 t G_0 \, \mathcal{U}_{21}$$

$$\mathcal{U}_{21} = (G_0 t G_0)^{-1} (1 + \zeta P_{34}) + (1 + \zeta P_{34}) U_1 G_0 t G_0 \, \mathcal{U}_{11}$$

$$\mathcal{U}_{12} = (G_0 t G_0)^{-1} + \zeta P_{34} U_1 G_0 t G_0 \, \mathcal{U}_{12} + U_2 G_0 t G_0 \, \mathcal{U}_{22}$$

$$\mathcal{U}_{22} = (1 + \zeta P_{34}) U_1 G_0 t G_0 \, \mathcal{U}_{12}$$

$\zeta = -1 \ (+1)$ for fermions (bosons)

basis states partially symmetrized

Scattering amplitudes: $E + i\varepsilon \to E + i0$

2-cluster reactions:

$$T_{fi} = s_{fi} \langle \phi_f | \mathcal{U}_{fi} | \phi_i \rangle$$

$$|\phi_j\rangle = G_0 t P_j |\phi_j\rangle$$

$$|\Phi_j\rangle = (1 + P_j)|\phi_j\rangle$$

3-cluster breakup/recombination:

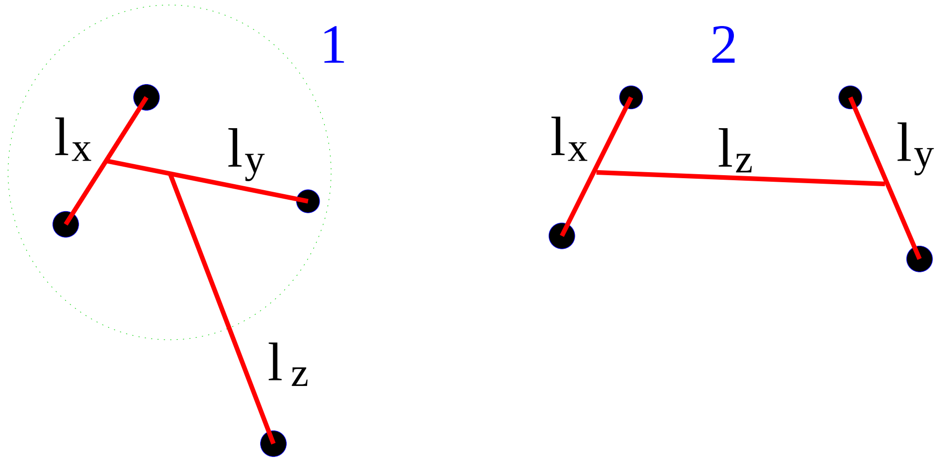
$$T_{3i} = s_{3i} \langle \phi_3 | [(1 + \zeta P_{34}) U_1 G_0 t G_0 \, \mathcal{U}_{1i} + U_2 G_0 t G_0 \, \mathcal{U}_{2i}] | \phi_i \rangle$$

4-cluster breakup/recombination:

$$T_{4i} = s_{4i} \{ \langle \phi_4 | [1 + (1 + P_1)\zeta P_{34}](1 + P_1) t G_0 U_1 G_0 t G_0 \, \mathcal{U}_{1i} | \phi_i \rangle + \langle \phi_4 | (1 + P_1)(1 + P_2) t G_0 U_2 G_0 t G_0 \, \mathcal{U}_{2i} | \phi_i \rangle \}$$

Solution of 4N AGS equations

$$\mathcal{U}_{12}|\phi_2\rangle = G_0^{-1}P_2|\phi_2\rangle - P_{34}U_1 G_0\, t\, G_0\, \mathcal{U}_{12}|\phi_2\rangle + U_2 G_0\, t\, G_0\, \mathcal{U}_{22}|\phi_2\rangle$$



- momentum-space partial-wave basis

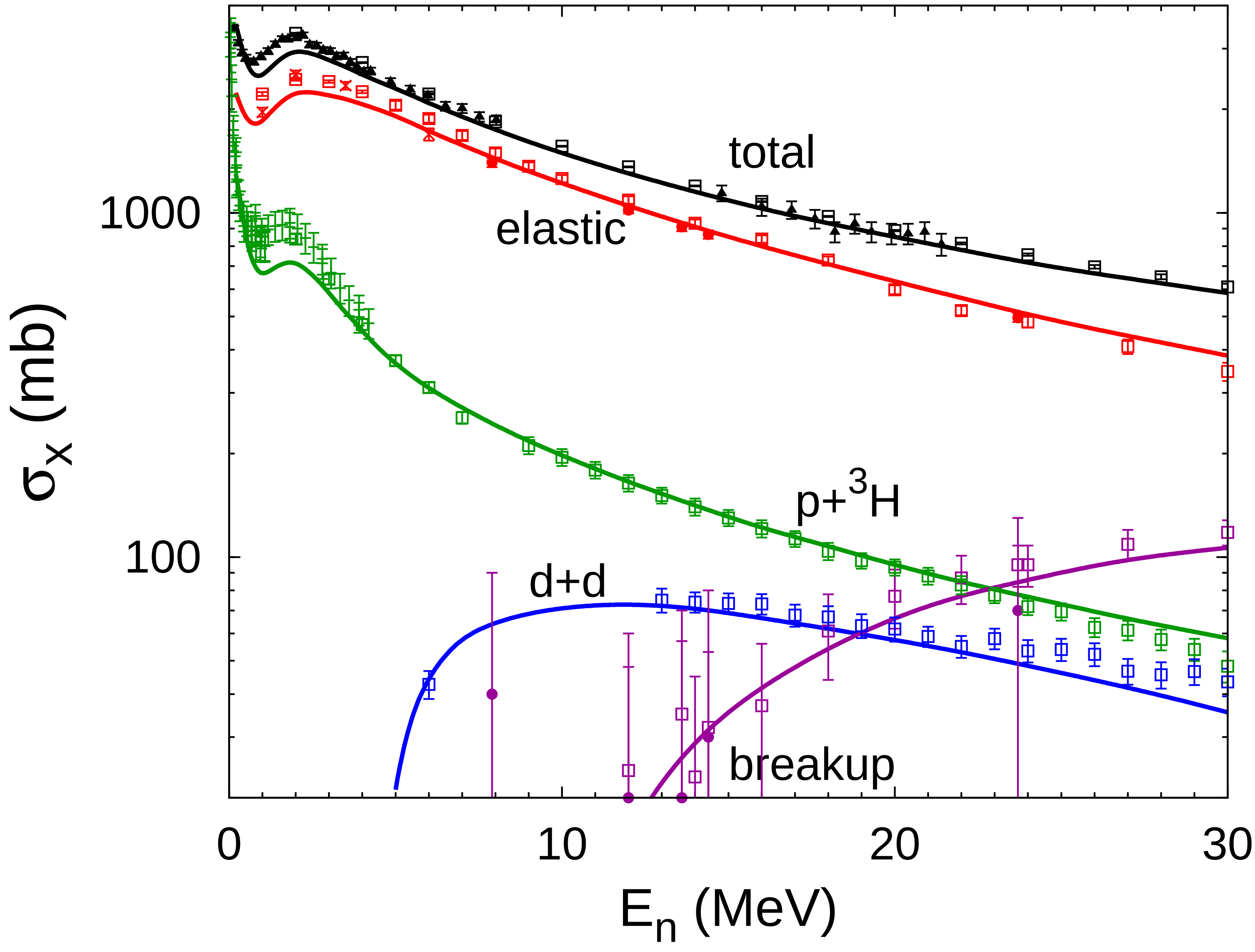
  $|k_x k_y k_z [l_z(\{l_y[(l_x S_x) j_x s_y] S_y\} J_y s_z) S_z] JM, [(T_x t_y) T_y t_z] T M_T\rangle_1$

  $|k_x k_y k_z [l_z\{(l_x S_x) j_x [l_y (s_y s_z) S_y] j_y\} S_z] JM, [T_x (t_y t_z) T_z] T M_T\rangle_2$

- large system (up to 30000) of coupled 3-variable integral equations with integrable singularities

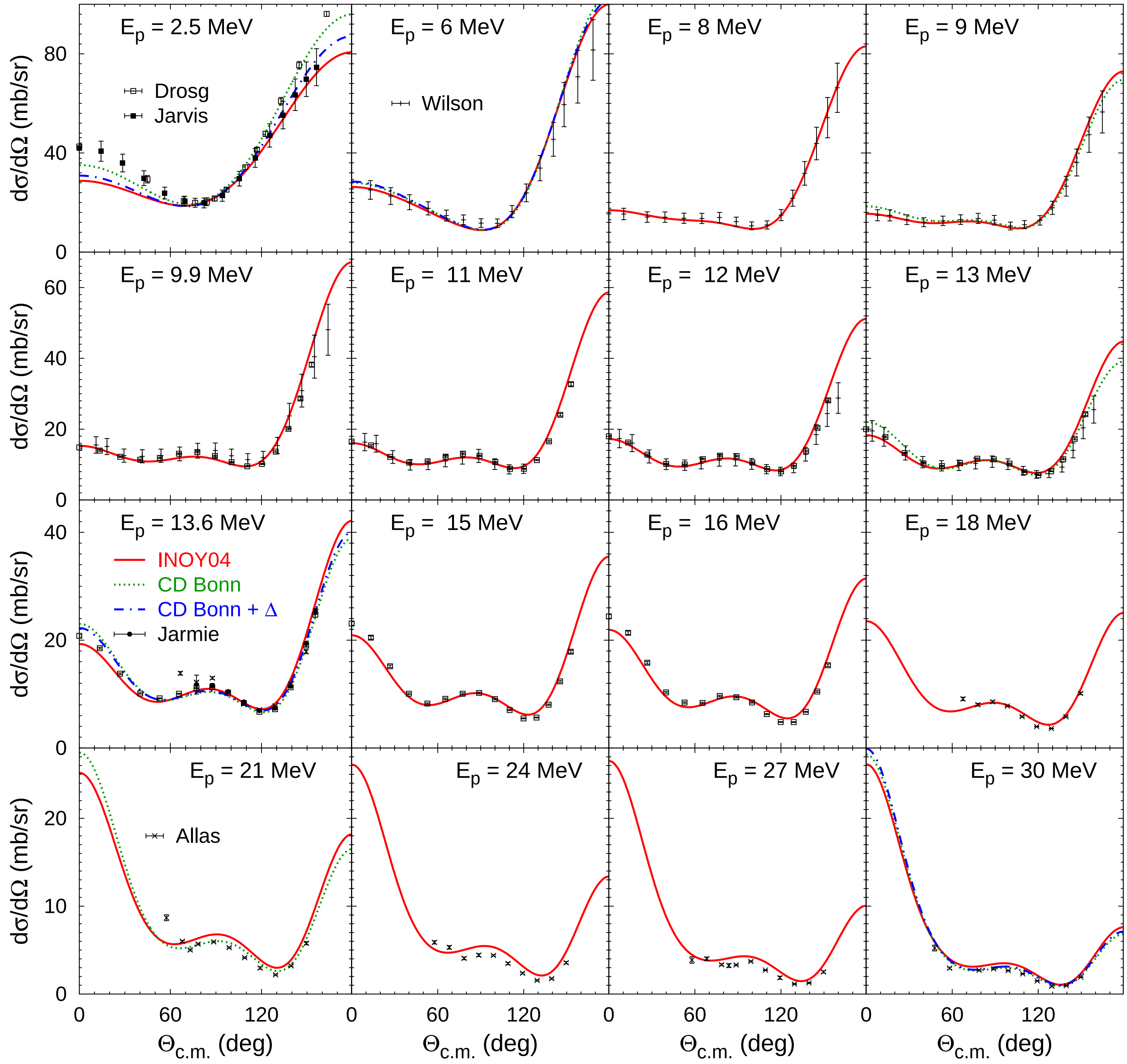
- Coulomb interaction: screening and renormalization [PRC 75, 014005, PRL 98, 162502]

n+^{3}He total and partial cross sections

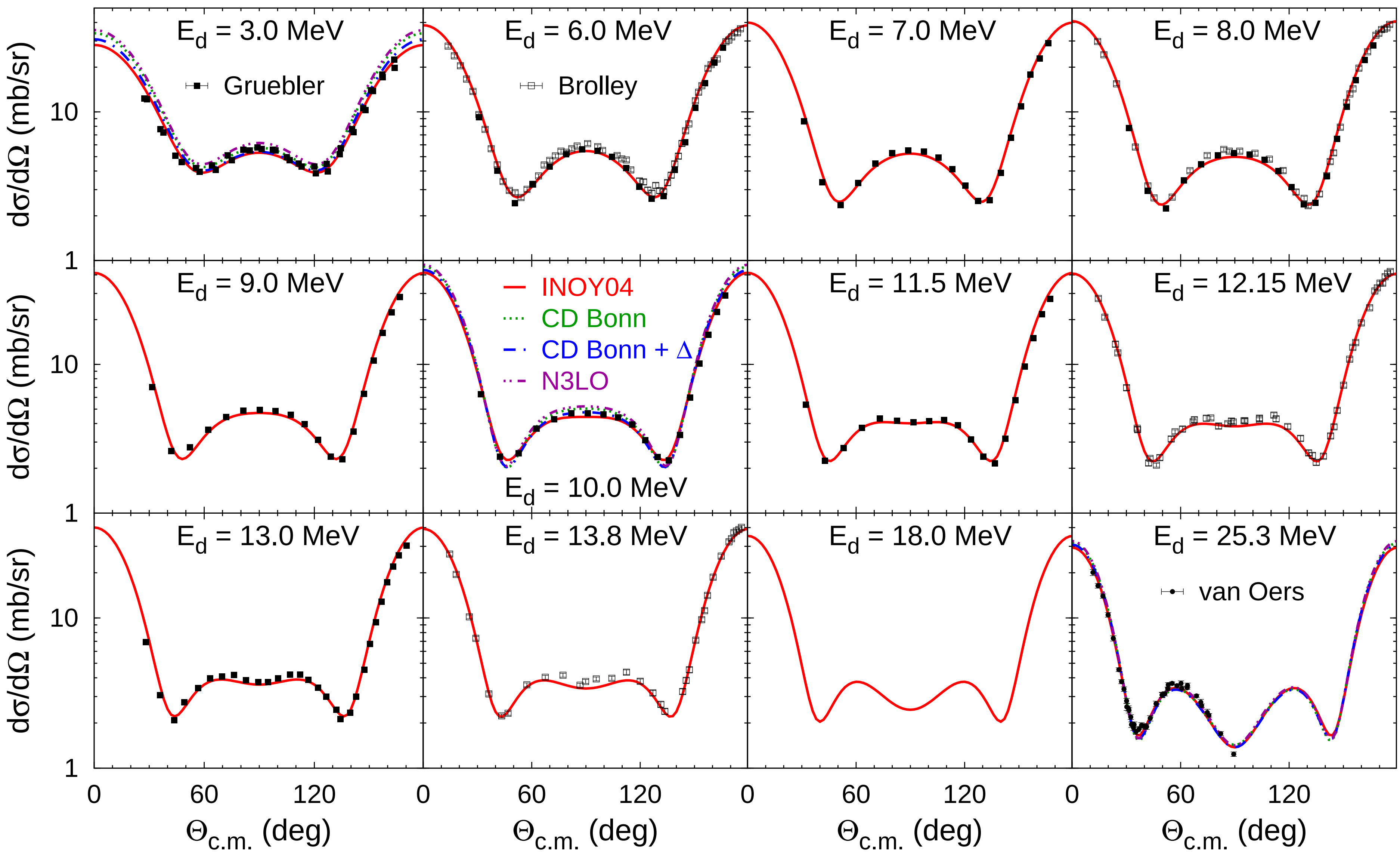


[PRL 113, 102502; PRC 90, 044002]

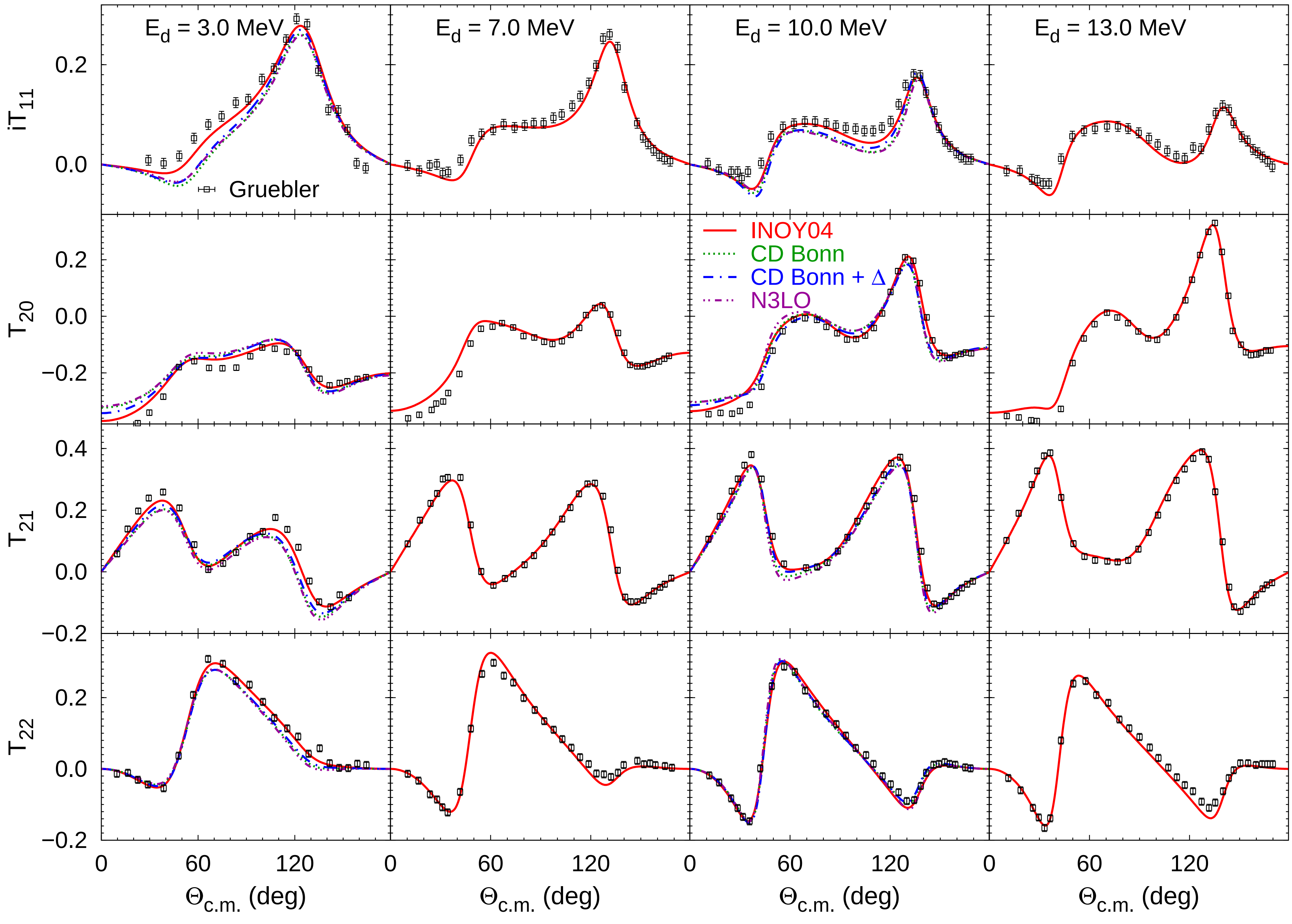
Charge exchange reaction $^3H(p,n)^3He$



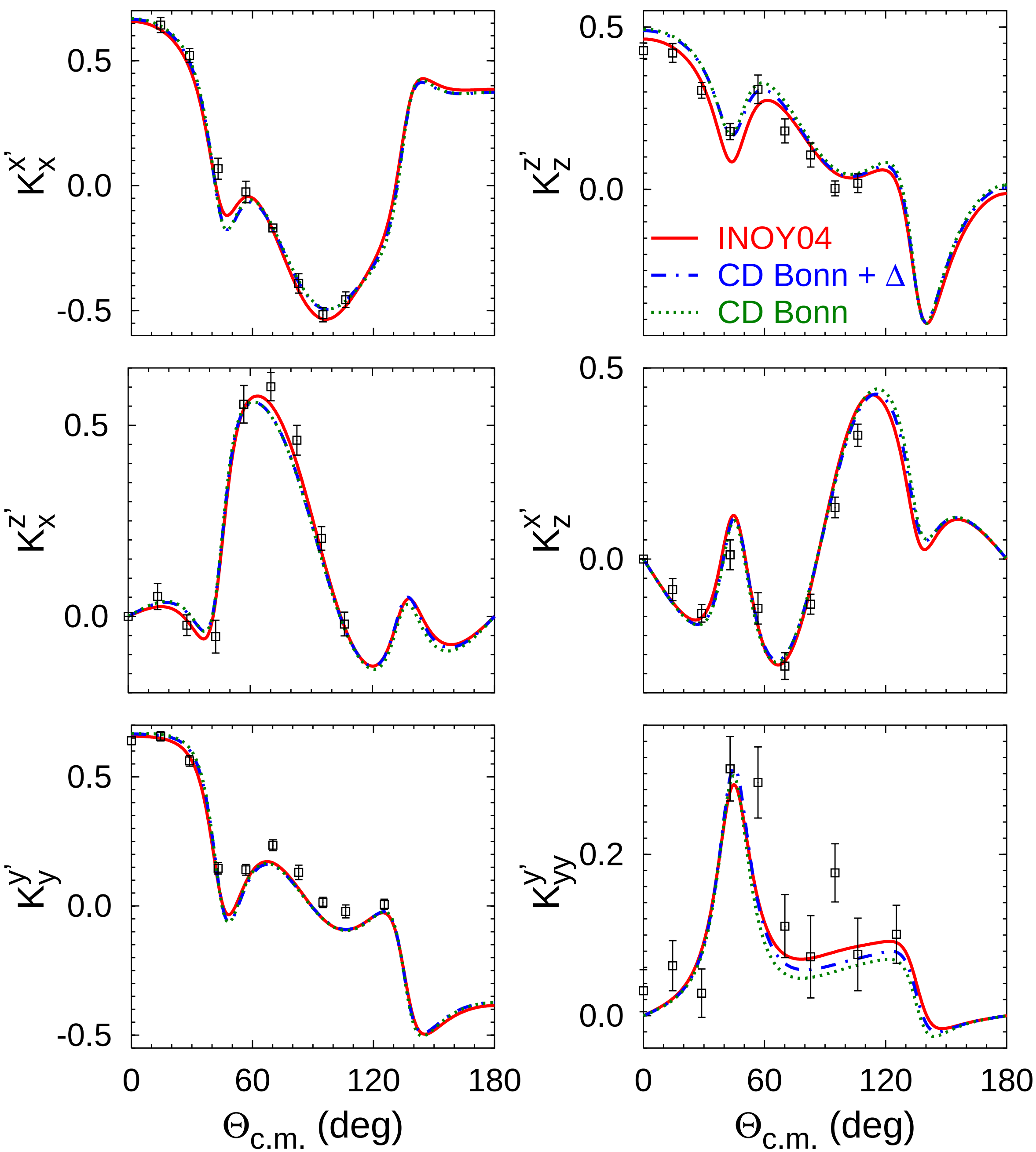
Transfer reaction $^{2}\mathrm{H}(d,p)^{3}\mathrm{H}$



Transfer reaction $^2\mathrm{H}(d,p)^3\mathrm{H}$: analyzing powers



Spin transfer in $^{2}\mathrm{H}(d,n)^{3}\mathrm{He}$ at 10 MeV



Few-body nuclear reactions

- 3-body AGS equations
  — CDCC and ADWA benchmark
  — L- and π-dependent nonlocal optical potentials
  — (d,n) reactions
- AGS extension including core excitation
- complicated CX effects in transfer reactions,
  no simple relation to SF
- CX effects of 2- and 3-body nature
- 4-body AGS equations
  — (p,n), (d,p) and (d,n) reactions in 4N system