Ab-initio calculations of η-nuclear quasi bound states

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Introduction

Current status

- moderate attractive ηN interaction with scattering length $a_{\eta N} \sim 0.27 + \mathrm{i}0.22$ fm
  $\Rightarrow \exists$ of η nuclear bound states (starting ^{12}C)
  (Haider, Liu **PLB 172** (1986) 257, **PRC 34** (1986) 1845)

Numerous studies:

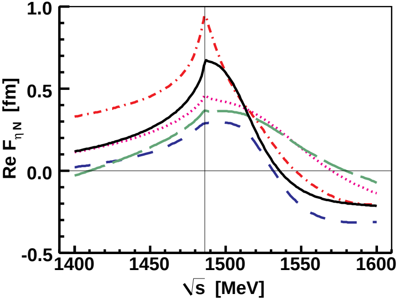
- chiral coupled channel models Re$a_{\eta N} \in (0.3\ ,\ 0.7)$ fm
- K matrix methods Re$a_{\eta N} \approx 1.0$ fm
  (fitting πN and γN reaction data in $N^\star$ (1535) resonance region)
  (bounds states already in He isotops)

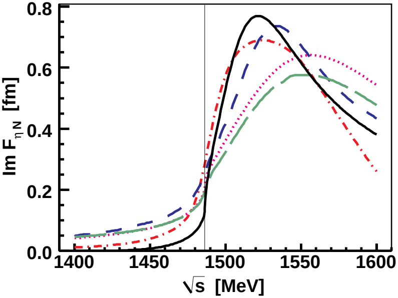
No decisive experimental evidence so far

- $^{25}_{\eta}\mathrm{Mg}$?, $B_\eta = 13.1 \pm 1.5\mathrm{MeV}$ and $\Gamma_\eta = 10.2 \pm 3.0\mathrm{MeV}$
  (COSY-GEM, **PRC 79** (2009) 012201(R))
- $^{3}_{\eta}\mathrm{He}$ not found
  (MAMI, **PLB 709** (2012) 21; Xie et al., **PRC 95** (2017) 015202)
- $^{4}_{\eta}\mathrm{He}$ not found
  (WASA@COSY, **PRC 87** (2013) 035204; **NPA 959** (2017) 102)

ηN scattering amplitudes

- ηN amplitude for various models
- strong energy dependence of the scattering amplitudes





line	$a_{\eta N}$ [fm]	model
dotted	0.46+i0.24	N. Kaiser, P.B. Siegel, W. Weise, PLB 362 (1995) 23
short-dashed	0.26+i0.25	T. Inoue, E. Oset, NPA 710 (2002) 354 (GR)
dot-dashed	0.96+i0.26	A.M. Green, S. Wycech, PRC 71 (2005) 014001 (GW)
long-dashed	0.38+i0.20	M. Mai, P.C. Bruns, U.-G. Meißner, PRD 86 (2012) 094033 (M2)
full	0.67+i0.20	A. Cieply, J. Smejkal, Nucl. Phys. A 919 (2013) 334 (CS)

η in many-body systems

(A. Cieply, E. Friedman, A. Gal, J. Mares, PLB **725** (2013) 334, NPA **925** (2014) 126)

- K.-G. equation:

  $$\left[\omega_\eta^2 + \vec{\nabla}^2 - m_\eta^2 - \Pi_\eta(\omega_\eta, \rho)\right] \phi_\eta = 0$$

  complex energy $\omega_\eta = m_\eta - B_\eta - \mathrm{i}\Gamma_\eta/2$

- $\Pi_\eta(\omega_\eta, \rho) = 2\omega_\eta V_\eta = -4\pi \frac{\sqrt{s}}{E_N} F_{\eta N}(\sqrt{s}, \rho)\rho$

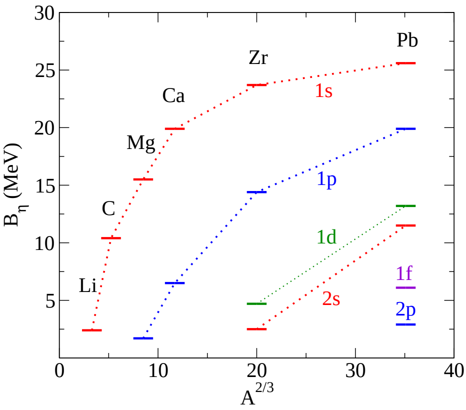
- η in a nucleus $\Rightarrow$ polarized (compressed) $\rho \longrightarrow \Pi_\eta(\rho)$

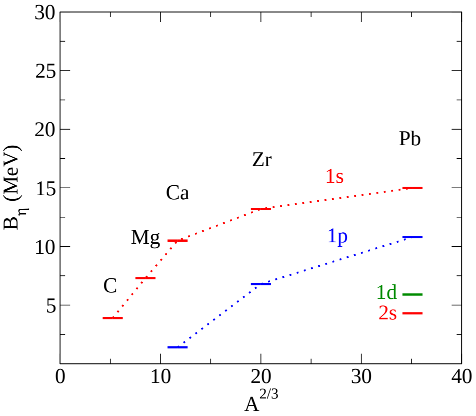
  $\Rightarrow$ selfconsistent solution
- Pauli correlations $\leftarrow$ WRW method

  (T. Wass, M. Rho, W. Weise, NPA **617** (1997) 449)

η in many-body systems

- Predictions of GW and CS models:
  all states in selected nuclei are shown; both models give small widths





η in few-body systems

Faddeev (AGS) calculations of ηNNN and $\eta NNNN$ systems

- A. Fix, H Arenhovel, Phys. Rev. **C 66** (2002) 024002
- A. Fix, O. Kolesnikov, **PLB 772** (2017) 663

Variational calculations

Hyperspherical basis

- N. Barnea, E. Friedman, A. Gal, **PLB 747** (2015) 345

Stochastic Variational Method (SVM)

- N. Barnea, E. Friedman, A. Gal, **NPA 968** (2017) 35
- N. Barnea, B. Bazak, E. Friedman, A, Gal, **PLB 771** (2017) 297

Stochastic Variational Method

(K. Varga et al., Nucl. Phys. **A 571** (1994) 447)
(K. Varga, Y. Suzuki, Phys. Rev. **C 52** (1995) 2885)

optimizes variational basis in a **random trial and error procedure**

Variational basis states

- antisymmetrized correlated Gaussians (assuming L=0)

$$\psi_{SM_S TM_T}(\vec{x}, A) = \mathcal{A}\{G_A(\vec{x})\chi_{SM_S}\eta_{TM_T}\}, \quad G_A(\vec{x}) = e^{-\frac{1}{2}\vec{x}A\vec{x}}$$

- Jacobi coordinates $\vec{x}$, symmetric positive definite matrix of variational parameters A, spin χ_{SM_S} and isospin η_{TM_T} parts
- $\frac{N(N-1)}{2}$ real parameters for one basis state
- explicit antisymmetrization → computational complexity grows with N!

$$\mathcal{A} = \sum_{i=1}^{N!} p_i \mathcal{P}_i$$

Variation procedure

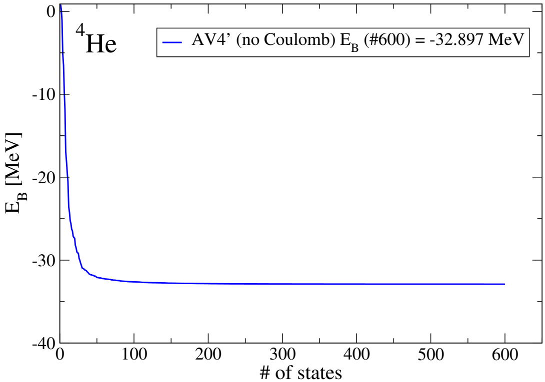
Hamiltonian and wave function

$$\hat{H} = \hat{T}_N + \hat{T}_\eta + \hat{V}_{NN} + \hat{V}_{\eta N} - \hat{T}_{\rm cm} \ , \qquad \Psi = \sum_{i=1}^{K} c_i \psi_{SM_S TM_T}(\vec{x}, A_i)$$

Step-by step optimization procedure

- starting with the 1st basis state $\psi_{1\ SM_S TM_T}(\vec{x}, A_1)$
- optimization loop over variational parameters of the 1st basis state A_1
  - random selection of variational parameters A_1
  - solution of generalized eigenvalue problem
- saving the optimized set of randomly selected variational parameters A_1 (giving the lowest binding energy E_B)
- addition of the 2nd basis state $\psi_{2\ SM_S TM_T}(\vec{x}, A_2)$
- optimization loop over variational parameters of the 2nd basis state $\psi_{1\ SM_S TM_T}(\vec{x}, A_2)$
- .....

Variation procedure



Result

- wave function in a basis of optimized correlated Gaussians

$$\Psi = \sum_{i=1}^{K} c_i \psi_{SM_S TM_T}(\vec{x}, A_i)$$

V_{NN} and $V_{\eta N}$ potential input

NN two-body potentials

- Argonne AV4' potential

  (R. B. Wiringa, S. C. Pieper, Phys. Lett. 89 (2002) 182501)

- Minnesota MN potential

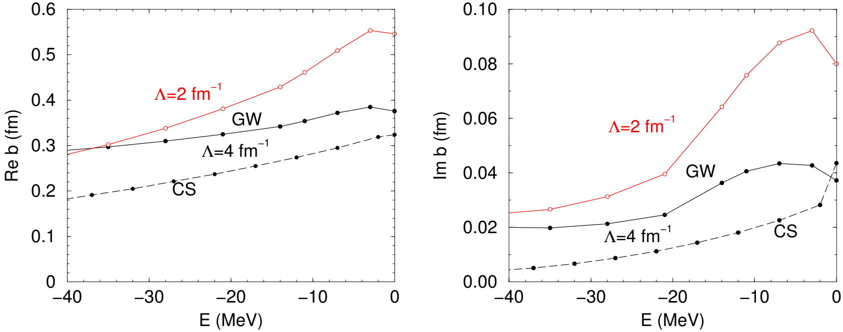
  (D. R. Thomson, M. LeMere, Y. C. Tang, Nucl. Phys **A 286** (1977) 53)

ηN two-body potential

- complex energy-dependent local potential derived from the full chiral coupled-channels model
- energy dependent $b(E)$ fitted to phase shifts δ derived from $F_{\eta N}$ in GW and CS models
- scale parameter Λ proportional to the $V_{\eta N}$ range

$$V_{\eta N}(E, r) = -\frac{4\pi}{2\mu_{\eta N}} b(E) \rho_\Lambda(r) \ , \qquad \rho_\Lambda(r) = \left(\frac{\Lambda}{2\sqrt{\pi}} \right)^3 \exp\left\{ -\frac{\Lambda^2 r^2}{4} \right\}$$

V_{NN} and $V_{\eta N}$ potential input



(N. Barnea, E. Friedman, A. Gal, **PLB 747** (2015) 345)

Perturbative estimate of converison widths Γ

$$\Gamma = -2 < \Psi_{gs} | \mathrm{Im} V_{\eta N} | \Psi_{gs} >$$

Energy dependence of η-N potential

Energy dependence of $V_{\eta N}(\sqrt{s})$

- A nucleons + η meson:

  $s = (\sqrt{s_{\rm th}} - B_\eta - B_N)^2 - (\vec{p_\eta} + \vec{p_N})^2 \leq s_{\rm th}$
  where $\sqrt{s_{\rm th}} = m_N + m_\eta$

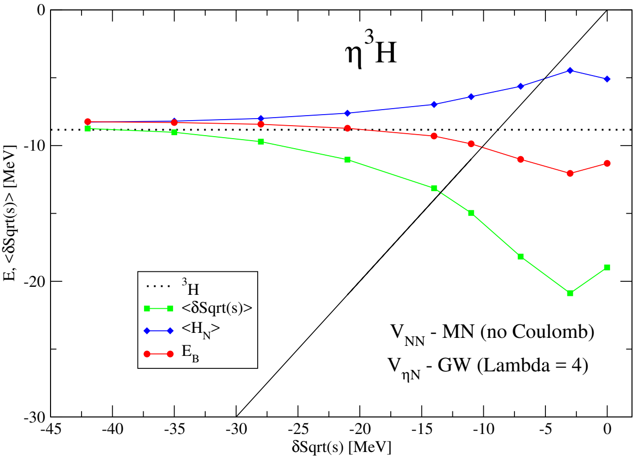
- near threshold approximated by:
  $\sqrt{s} = \sqrt{s_{\rm th}} + \delta\sqrt{s}, \quad \delta\sqrt{s} < 0!$

  $\langle\delta\sqrt{s}\rangle = -\frac{B}{A} - \frac{A-1}{A}B_\eta - \xi_N\frac{A-1}{A}\langle T_{N:N}\rangle - \xi_\eta\left(\frac{A-1}{A}\right)^2\langle T_\eta\rangle,$

  where B = total binding energy, $\xi_{N(\eta)} = m_{N(\eta)}/(m_N + m_\eta)$,
  $T_\eta = \eta$ kin. energy, $T_{N:N}$ = pairwise NN kin. energy

- $\langle\delta\sqrt{s}\rangle \Rightarrow$ selfconsistency

Energy dependence of η-N potential



Selfconsistency

We are looking for such solution where $< \delta\sqrt{s} >= \delta\sqrt{s}$.
($\delta\sqrt{s}$ enters $V_{\eta N}(\sqrt{s})$ potential)

ηNN, η^3He and η^4He systems

ηNN

- unbound

(N. Barnea, E. Friedman, A. Gal, PLB 747 (2015) 345)

η^3He

$V_{\eta N}$	V_{NN}	$\delta\sqrt{s_s c}$	B_η	Γ_{gs}
GW, $\Lambda = 2$	MN	-9.385	0.099	1.144
	AV4'	-11.478	-0.028	0.769
GW, $\Lambda = 4$	MN	-13.392	0.990	3.252
	AV4'	-14.881	0.686	2.438
CS, $\Lambda = 2$	MN	-8.388	-0.217	0.057
CS, $\Lambda = 4$	MN	-8.712	-0.161	0.227

(N. Barnea, E. Friedman, A. Gal, NPA 968 (2017) 35)

ηNN, η^3He and η^4He systems

ηNN

- unbound

(N. Barnea, E. Friedman, A. Gal, PLB 747 (2015) 345)

		η^3He			η^4He		
$V_{\eta N}$	V_{NN}	$\delta\sqrt{s_s c}$	B_η	Γ_{gs}	$\delta\sqrt{s_s c}$	B_η	Γ_{gs}
GW, $\Lambda = 2$	MN	-9.385	0.099	1.144	-19.48	0.96	1.98
	AV4'	-11.478	-0.028	0.769	-23.65	0.38	1.21
GW, $\Lambda = 4$	MN	-13.392	0.990	3.252	-29.75	4.69	4.50
	AV4'	-14.881	0.686	2.438	-32.41	3.51	3.62
CS, $\Lambda = 2$	MN	-8.388	-0.217	0.057	-16.70	-0.16	0.13
CS, $\Lambda = 4$	MN	-8.712	-0.161	0.227	-19.25	0.47	0.90

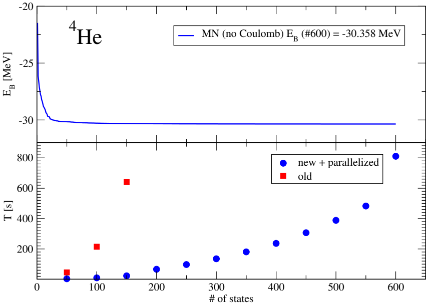
(N. Barnea, E. Friedman, A. Gal, NPA 968 (2017) 35)

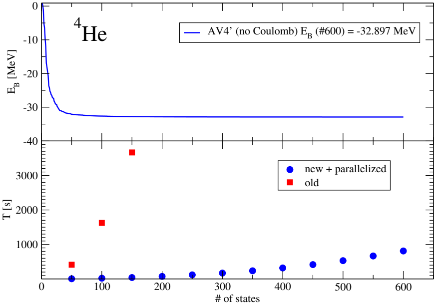
(N. Barnea, B. Bazak, E. Friedman, A. Gal, PLB 771 (2017) 297)

Optimization

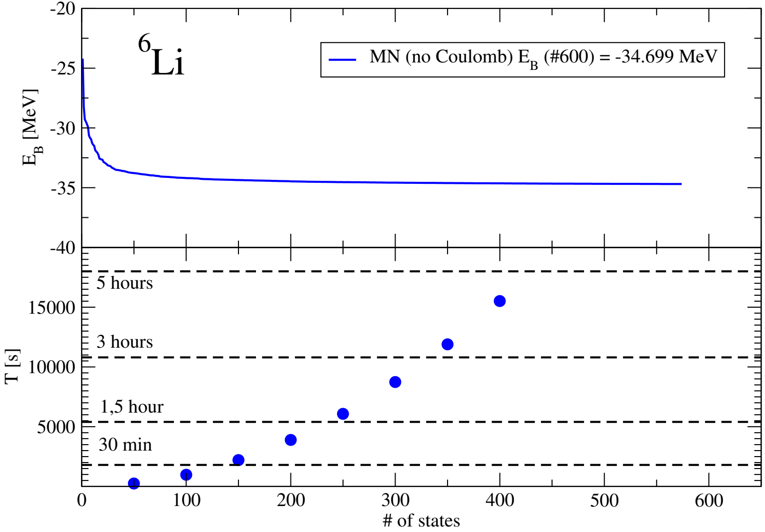
Problems

- computationl complexity increases with N! (antisymmetrization)
- convergence becomes more expensive for heavier systems or hard-core potentials (more basis states)

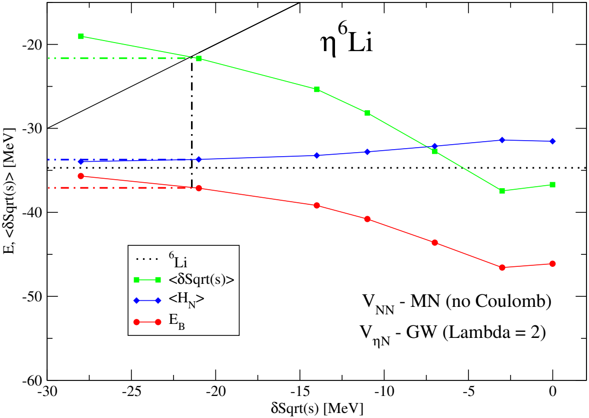




^{6}Li - preliminary

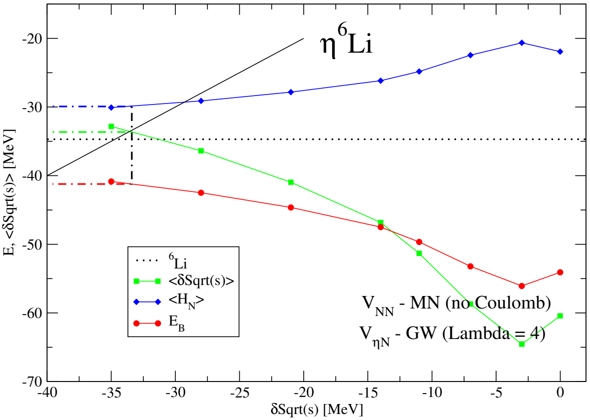


η^6Li - preliminary



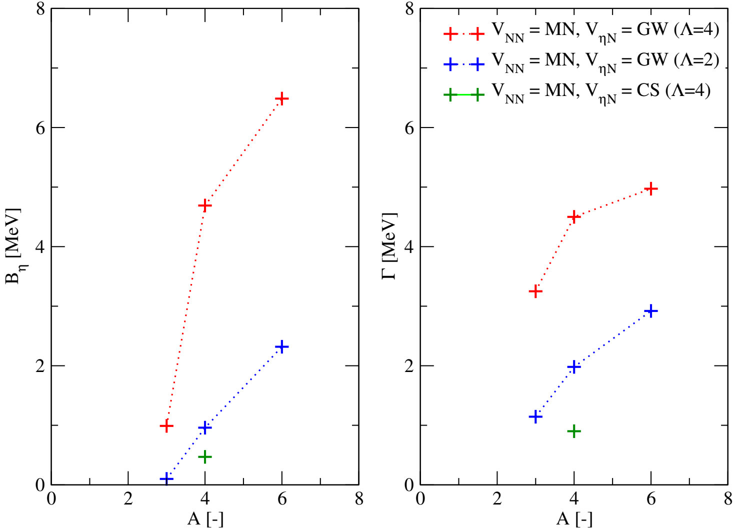
$V_{\eta N}$	V_{NN}	$\delta\sqrt{s_s c}$	B_η	Γ_{gs}
GW, $\Lambda = 2$	MN	-21.49	2.14	2.92

η^6Li - preliminary



$V_{\eta N}$	V_{NN}	$\delta\sqrt{s_s c}$	B_η	Γ_{gs}
GW, $\Lambda = 2$	MN	-21.49	2.14	2.92
GW, $\Lambda = 4$	MN	-33.55	6.49	4.97

Overall results



Summary

- ab-inition selfconsistent calculations of ηNN, ηNNN, $\eta NNNN$ systems
  - $\eta\mathrm{d}$ unbound
  - $\eta^3\mathrm{He}$ quasi-bound state
  - $\eta^4\mathrm{He}$ quasi-bound state
- developed new optimized SVM version
  - selfconsistent ab-initio calculations of $\eta^6\mathrm{Li}$ system (preliminary)
    - $\mathrm{B}_\eta = 2.14$ MeV , $\Gamma_\eta = 2.92$ MeV (MN, GW $\Lambda = 2$)
    - $\mathrm{B}_\eta = 6.49$ MeV , $\Gamma_\eta = 4.97$ MeV (MN, GW $\Lambda = 4$)
- consistency with RMF calculations of η-nuclear many-body systems ($\mathrm{B}_\eta \approx 2.5$ MeV , $\Gamma_\eta \approx 4$ MeV)

Next steps :

- calculation of few-body systems with $L > 0 \rightarrow \eta^7\mathrm{Li}$
- realistic interactions (AV6, AV8)
- reaching heavier few-body systems (9, 10-body systems)