

Heavy Quarkonium Hybrids

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Rubén Oncala, JS, Phys. Rev. D 96, 014004 (2017) (arXiv:1702.03900)

Heavy Quarkonium Hybrids

- Heavy Quarkonium: contain a $Q\bar{Q}$ pair, $Q = b, c$
- Hybrids: have a non-trivial gluon content
 - ▶ May have exotic J^{PC} : 0^{--} , 0^{+-} , 1^{-+} , 2^{+-} , ...
 - ▶ If not, difficult to quantify for light hadrons, but easier for heavy ones

Pioneering works:

- Exotics: MIT bag model (Jaffe, Johnson, 76)
- Heavy Hybrids:
 - ▶ String model (Giles, Tye, 77; Horn, Mandula, 78)
 - ▶ Born-Oppenheimer approximation (Hasenfratz, Horgan, Kuti, Richard, 80)
 - ▶ Lattice potentials (Griffiths, Rakow, Michael, 83)

Outline

- 1 Motivation
- 2 Heavy Quarkonium
- 3 Heavy Hybrids
 - Spectrum
 - Mixing
 - Decay
 - Hyperfine splittings
- 4 Conclusions

Motivation

Understanding the XYZ states from QCD:

- Hidden charm (hidden bottom)
- Above open charm (bottom) threshold
- Do not fit usual potential model expectations

Tools:

- Effective Field Theories
- Lattice QCD inputs

XYZ with $c\bar{c}$ (Olsen, 15)

State	M (MeV)	Γ (MeV)	J^{PC}	Process (decay mode)
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \rightarrow K + (J/\psi \pi^+ \pi^-)$ $p\bar{p} \rightarrow (J/\psi \pi^+ \pi^-) + \dots$ $B \rightarrow K + (J/\psi \pi^+ \pi^- \pi^0)$ $B \rightarrow K + (D^0 \bar{D}^0 \pi^0)$ $B \rightarrow K + (J/\psi \gamma)$ $B \rightarrow K + (\psi' \gamma)$ $pp \rightarrow (J/\psi \pi^+ \pi^-) + \dots$
$X(3915)$	3917.4 ± 2.7	28_{-9}^{+10}	0^{++}	$B \rightarrow K + (J/\psi \omega)$ $e^+ e^- \rightarrow e^+ e^- + (J/\psi \omega)$
$X(3940)$	3942_{-8}^{+9}	37_{-17}^{+27}	$0(?)^{-(?)+}$	$e^+ e^- \rightarrow J/\psi + (D^* \bar{D})$ $e^+ e^- \rightarrow J/\psi + (\dots)$
$G(3900)$	3943 ± 21	52 ± 11	1^{--}	$e^+ e^- \rightarrow \gamma + (D \bar{D})$
$Y(4008)$	4008_{-49}^{+121}	226 ± 97	1^{--}	$e^+ e^- \rightarrow \gamma + (J/\psi \pi^+ \pi^-)$
$Y(4140)$	4144 ± 3	17 ± 9	$?^{?+}$	$B \rightarrow K + (J/\psi \phi)$
$X(4160)$	4156_{-25}^{+29}	139_{-65}^{+113}	$0(?)^{-(?)+}$	$e^+ e^- \rightarrow J/\psi + (D^* \bar{D})$

State	M (MeV)	Γ (MeV)	J^{PC}	Process (decay mode)
$Y(4260)$	4263_{-9}^{+8}	95 ± 14	1^{--}	$e^+e^- \rightarrow \gamma + (J/\psi \pi^+ \pi^-)$ $e^+e^- \rightarrow (J/\psi \pi^+ \pi^-)$ $e^+e^- \rightarrow (J/\psi \pi^0 \pi^0)$
$Y(4360)$	4361 ± 13	74 ± 18	1^{--}	$e^+e^- \rightarrow \gamma + (\psi' \pi^+ \pi^-)$
$X(4630)$	4634_{-11}^{+9}	92_{-32}^{+41}	1^{--}	$e^+e^- \rightarrow \gamma (\Lambda_c^+ \Lambda_c^-)$
$Y(4660)$	4664 ± 12	48 ± 15	1^{--}	$e^+e^- \rightarrow \gamma + (\psi' \pi^+ \pi^-)$
$Z_c^+(3900)$	3890 ± 3	33 ± 10	1^{+-}	$Y(4260) \rightarrow \pi^- + (J/\psi \pi^+)$ $Y(4260) \rightarrow \pi^- + (D\bar{D}^*)^+$
$Z_c^+(4020)$	4024 ± 2	10 ± 3	$1(?)^{+(?) -}$	$Y(4260) \rightarrow \pi^- + (h_c \pi^+)$ $Y(4260) \rightarrow \pi^- + (D^* \bar{D}^*)^+$
$Z_c^0(4020)$	4024 ± 4	10 ± 3	$1(?)^{+(?) -}$	$Y(4260) \rightarrow \pi^0 + (h_c \pi^0)$
$Z_1^+(4050)$	4051_{-43}^{+24}	82_{-55}^{+51}	$?^{?+}$	$B \rightarrow K + (\chi_{c1} \pi^+)$
$Z^+(4200)$	4196_{-32}^{+35}	370_{-149}^{+99}	1^{+-}	$B \rightarrow K + (J/\psi \pi^+)$
$Z_2^+(4250)$	4248_{-45}^{+185}	177_{-72}^{+321}	$?^{?+}$	$B \rightarrow K + (\chi_{c1} \pi^+)$
$Z^+(4430)$	4477 ± 20	181 ± 31	1^{+-}	$B \rightarrow K + (\psi' \pi^+)$ $B \rightarrow K + (J\psi \pi^+)$

XYZ with $b\bar{b}$ (Olsen, 15)

State	M (MeV)	Γ (MeV)	J^{PC}	Process (decay mode)
$Y_b(10890)$	10888.4 ± 3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \rightarrow (\Upsilon(nS) \pi^+ \pi^-)$
$Z_b^+(10610)$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	$\Upsilon(5S) \rightarrow \pi^- + (\Upsilon(1, 2, 3S) \pi^+)$ $\Upsilon(5S) \rightarrow \pi^- + (h_b(1, 2P) \pi^+)$ $\Upsilon(5S) \rightarrow \pi^- + (B\bar{B}^*)^+$
$Z_b^0(10610)$	10609 ± 6		1^{+-}	$\Upsilon(5S) \rightarrow \pi^0 + (\Upsilon(1, 2, 3S) \pi^0)$
$Z_b^+(10650)$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$\Upsilon(5S) \rightarrow \pi^- + (\Upsilon(1, 2, 3S) \pi^+)$ $\Upsilon(5S) \rightarrow \pi^- + (h_b(1, 2P) \pi^+)$ $\Upsilon(5S) \rightarrow \pi^- + (B^* \bar{B}^*)^+$

Heavy Quarkonium

$Q\bar{Q}$ bound state , $m_Q \gg \Lambda_{QCD}$, $\alpha_s(m_Q) \ll 1$

Heavy Quarkonium

$Q\bar{Q}$ bound state , $m_Q \gg \Lambda_{QCD}$, $\alpha_s(m_Q) \ll 1$

- Heavy quarks move slowly $v \ll 1$

Heavy Quarkonium

$Q\bar{Q}$ bound state , $m_Q \gg \Lambda_{QCD}$, $\alpha_s(m_Q) \ll 1$

- Heavy quarks move slowly $v \ll 1$
- Non-relativistic system \rightarrow multiscale problem
 - ▶ $m_Q \gg m_Q v$ (Relative momentum)
 - ▶ $m_Q v \gg m_Q v^2$ (Binding energy)
 - ▶ $m_Q \gg \Lambda_{QCD}$

Heavy Quarkonium

$Q\bar{Q}$ bound state , $m_Q \gg \Lambda_{QCD}$, $\alpha_s(m_Q) \ll 1$

- Heavy quarks move slowly $v \ll 1$
- Non-relativistic system \rightarrow multiscale problem
 - ▶ $m_Q \gg m_{Qv}$ (Relative momentum)
 - ▶ $m_Q v \gg m_{Qv}^2$ (Binding energy)
 - ▶ $m_Q \gg \Lambda_{QCD}$
- EFTs are useful (N. Brambilla, A. Pineda, JS and A. Vairo, Rev. Mod. Phys. 77, 1423 (2005))
 - ▶ NRQCD: $m_Q \gg m_{Qv}, m_{Qv}^2, \Lambda_{QCD}$ (W.E. Caswell and G.P. Lepage, Phys. Lett. **167B**, 437 (1986))
 - ▶ pNRQCD (weak coupling): $m_{Qv} \gg m_{Qv}^2, \Lambda_{QCD}$ (A. Pineda, JS, Nucl.Phys.Proc.Suppl.64:428-432,1998)
 - ▶ pNRQCD (strong coupling): $m_{Qv}, \Lambda_{QCD} \gg m_{Qv}^2$ (N. Brambilla, A. Pineda, JS, A. Vairo, Nucl.Phys.B566:275,2000)

pNRQCD strong coupling regime at LO

- Matching to NRQCD in the static limit $\Rightarrow V_s^{(0)}$ is the ground state energy of two static color sources separated at a distance r
- Can be extracted from lattice calculations of the Wilson loop

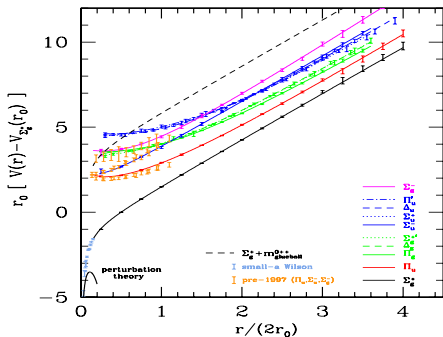


Figure: Meyer, Swanson, 2015

- Well fitted by the Cornell potential

$$V_s^{(0)} = V_{\Sigma_g^+}(r) \approx -\frac{k_g}{r} + \kappa r + E_g^{Q\bar{Q}} \quad , \quad k_g = 0.489 \quad , \quad \kappa = 0.187 \text{ GeV}^2$$

- The symmetry group of a diatomic molecule (two equal atoms separated at a distance r)
- The generators are
 - ▶ Rotations around the z-axis, labeled by $|L| = 0, 1, 2, \dots \equiv \Sigma, \Pi, \Delta, \dots$
 - ▶ Reflections about the xz plain, labeled by \pm (only important for Σ states)
 - ▶ Parity, labeled by g (positive) and u (negative). In the case of a $Q\bar{Q}$ pair is replaced by CP.
- When $r \rightarrow 0$ reduces to $O(3)$ (plus C in the case of $Q\bar{Q}$)
 - ▶ Implies short distance degeneracies

$$(\Sigma_u^-, \Pi_u) \quad , \quad (\Sigma_g^-, \Pi_g, \Delta_g) \quad , \quad (\Sigma_g^{+'}, \Pi_g') \quad , \quad (\Sigma_u^+, \Pi_u', \Delta_u) \quad , \quad \dots$$

(Brambilla, Pineda, JS, Vairo, 99)

pNRQCD strong coupling regime at LO

Charmonium spin averages

$$m_c = 1.47 \text{ GeV} \quad , \quad E_g^{c\bar{c}} = -0.242 \text{ GeV}$$

nL	$M_{c\bar{c}}$	$M_{c\bar{c}EXP}$	$S = 0$	$S = 1$
1S	3068 (0)	3068	0^{-+}	1^{--}
1P	3494 (-31)	3525	1^{+-}	$(0, 1, 2)^{++}$
2S	3678 (4)	3674	0^{-+}	1^{--}
1D	3793 (20)	3773	2^{-+}	$(1, 2, 3)^{--}$
2P	3968 (41)	3927	1^{+-}	$(0, 1, 2)^{++}$
3S	4130 (91)	4039	0^{-+}	1^{--}
2D	4210 (57)	4153	2^{-+}	$(1, 2, 3)^{--}$
4S	4517 (96)	4421	0^{-+}	1^{--}

pNRQCD strong coupling regime at LO

Bottomonium spin averages

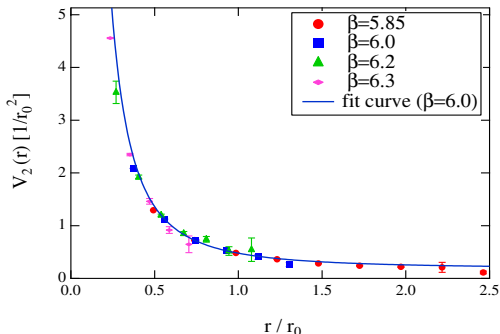
$$m_b = 4.88 \text{ GeV} \quad , \quad E_g^{b\bar{b}} = -0.228 \text{ GeV}$$

nL	$M_{b\bar{b}}$	$M_{b\bar{b}EXP}$	$S = 0$	$S = 1$
1S	9442 (-3)	9445	0^{-+}	1^{--}
1P	9908 (8)	10017	0^{-+}	1^{--}
1D	10155 (-9)	10164	2^{-+}	$(1, 2, 3)^{--}$
2P	10265 (5)	10260	1^{+-}	$(0, 1, 2)^{++}$
3S	10356 (1)	10355	0^{-+}	1^{--}
4S	10638 (59)	10579	0^{-+}	1^{--}
5S	10885 (9)	10876	0^{-+}	1^{--}
6S	11110 (91)	11019	0^{-+}	1^{--}

pNRQCD strong coupling regime beyond LO

An example at $\mathcal{O}(1/m_Q^2)$: the $V_{L_2 S_1}^{(1,1)}$ spin-orbit potential

$$V_{L_2 S_1}^{(1,1)}(r) = -\frac{c_F^{(1)}}{r^2} \mathbf{ir} \cdot \lim_{T \rightarrow \infty} \int_0^T dt t \ll g \mathbf{B}(t, \mathbf{r}/2) \times g \mathbf{E}(0, -\mathbf{r}/2) \gg$$



$$V_2 = V_{L_2 S_1}^{(1,1)} / c_F^{(1)}, \text{ Koma, Koma, 09}$$

pNRQCD strong coupling regime beyond LO

An example at $\mathcal{O}(1/m_Q^2)$: the $V_{L_2 S_1}^{(1,1)}$ spin-orbit potential

- Short distance constraint: it must coincide with the perturbative evaluation,

$$V_{L_2 S_1}^{(1,1)}(r) \sim c_F^{(1)} \frac{C_F \alpha_s}{r^3} \quad , \quad r \rightarrow 0$$

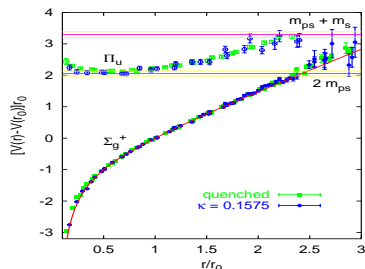
(Gupta, Radford, 81)

- Long distance constraint: it must coincide with the QCD effective string theory result

$$V_{L_2 S_1}^{(1,1)}(r) = -\frac{c_F^{(1)} g^2 \Lambda^2 \Lambda'}{\kappa r^2} \quad , \quad r \rightarrow \infty$$

(Perez-Nadal, JS, 08)

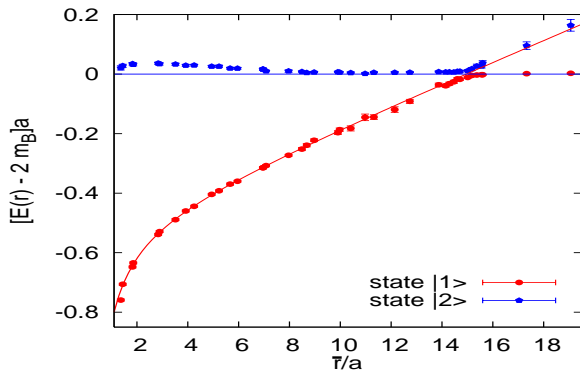
The Static Limit



G.S. Bali et al. (TXL Collaboration), Phys. Rev. **D62**, (2000):054503

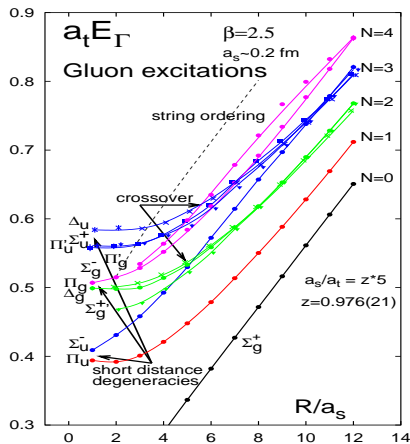
- Heavy Quarkonium: states built on the Σ_g^+ potential (color singlet)
- Heavy Hybrids: states built on the Π_u, \dots potentials (color octet)
- Heavy Tetraquarks, Pentaquarks, \dots : analogous to Hybrids but using operators that involve light quarks
- Molecular states: states built on $D\bar{D}$ or $B\bar{B}$
- \dots

String breaking



Bali, Neff, Duessel, Lippert, Schilling, 2005

The Static Limit



Juge, Kuti, Morningstar (2002)

Heavy Hybrids

- The Hybrid potentials have a minimum at $r \sim 1/\Lambda_{QCD}$
- The energy fluctuations about the minimum
$$E \sim \sqrt{\Lambda_{QCD}^3/m_Q} \ll \Lambda_{QCD}$$
- The energy scale Λ_{QCD} can be integrated out
- An EFT can be built for the short distance multiplets, in a way analogous to the strong coupling regime of pNRQCD
- We focus on the lowest lying doublet
 - ▶ At short distances it is described by $\text{tr}(\mathbf{OB})$ in weak coupling pNRQCD
 - ▶ We introduce a wave function field \mathbf{H} with the same symmetry properties as $\text{tr}(\mathbf{OB})$
 - ▶ The potentials for \mathbf{H} are obtained from fitting the static energies of [Juge, Kuti, Morningstar \(2002\)](#), and imposing:
 - ★ Weak coupling pNRQCD constraints at short distances
 - ★ QCD string constraints at long distances

$$\mathcal{L} = H^{i\dagger} (\delta_{ij} i \partial_0 - h_{Hij}) H^j$$

$$h_{Hij} = \left(-\frac{\nabla^2}{m_Q} + V_{\Sigma_u^-}(r) \right) \delta_{ij} + (\delta_{ij} - \hat{r}_i \hat{r}_j) V_q(r)$$

$$V_q(r) = V_{\Pi_u}(r) - V_{\Sigma_u^-}(r)$$

$$\left[-\frac{1}{m_Q} \frac{\partial^2}{\partial r^2} + \begin{pmatrix} \frac{(J-1)J}{m_Q r^2} + V_q(r) \frac{J+1}{2J+1} & V_q(r) \frac{\sqrt{(J+1)J}}{2J+1} \\ V_q(r) \frac{\sqrt{(J+1)J}}{2J+1} & \frac{(J+1)(J+2)}{m_Q r^2} + V_q(r) \frac{J}{2J+1} \end{pmatrix} + V_{\Sigma_u^-}(r) \right] \begin{pmatrix} P_J^-(r) \\ P_J^+(r) \end{pmatrix} = E \begin{pmatrix} P_J^-(r) \\ P_J^+(r) \end{pmatrix}$$

$$\left(-\frac{1}{m_Q} \frac{\partial^2}{\partial r^2} + \frac{J(J+1)}{m_Q r^2} + V_{\Pi_u}(r) \right) P_J^0(r) = E P_J^0(r)$$

- $\mathbf{J} = \mathbf{L} + \mathbf{J}_g$
- \mathbf{L} = orbital angular momentum of the heavy quarks
- \mathbf{J}_g = total angular momentum of the gluons, $|\mathbf{J}_g| = 1$
- Heavy quark spin independence

Results for charm

NL_J	w-f	$c\bar{c}$	Hybrid	$\mathcal{J}^{PC} (S=0)$	$\mathcal{J}^{PC} (S=1)$	Λ_{η}^c
1s	S	3068		0^{-+}	1^{--}	Σ_u^+
2s	S	3678		0^{-+}	1^{--}	Σ_u^+
3s	S	4131		0^{-+}	1^{--}	Σ_u^+
1p ₀	P ⁺		4486	0^{++}	1^{+-}	Σ_u^-
4s	S	4512		0^{-+}	1^{--}	Σ_u^+
2p ₀	P ⁺		4920	0^{++}	1^{+-}	Σ_u^-
3p ₀	P ⁺		5299	0^{++}	1^{+-}	Σ_u^-
4p ₀	P ⁺		5642	0^{++}	1^{+-}	Σ_u^-
1p	S	3494		1^{+-}	$(0, 1, 2)^{++}$	Σ_u^+
2p	S	3968		1^{+-}	$(0, 1, 2)^{++}$	Σ_u^+
1(s/d) ₁	P [±]		4011	1^{--}	$(0, 1, 2)^{+-}$	$\Pi_u \Sigma_u^-$
1p ₁	P ⁰		4145	1^{++}	$(0, 1, 2)^{+-}$	Π_u
2(s/d) ₁	P [±]		4355	1^{--}	$(0, 1, 2)^{+-}$	$\Pi_u \Sigma_u^-$
3p	S	4369		1^{+-}	$(0, 1, 2)^{++}$	Σ_u^+
2p ₁	P ⁰		4511	1^{++}	$(0, 1, 2)^{+-}$	Π_u
3(s/d) ₁	P [±]		4692	1^{--}	$(0, 1, 2)^{+-}$	$\Pi_u \Sigma_u^-$
4(s/d) ₁	P [±]		4718	1^{--}	$(0, 1, 2)^{+-}$	$\Pi_u \Sigma_u^-$
4p	S	4727		1^{+-}	$(0, 1, 2)^{++}$	Σ_u^+
3p ₁	P ⁰		4863	1^{++}	$(0, 1, 2)^{+-}$	Π_u
5(s/d) ₁	P [±]		5043	1^{--}	$(0, 1, 2)^{+-}$	$\Pi_u \Sigma_u^-$
5p	S	5055		1^{+-}	$(0, 1, 2)^{++}$	Σ_u^+
1d	S	3793		2^{+-}	$(1, 2, 3)^{--}$	Σ_u^+
2d	S	4210		2^{+-}	$(1, 2, 3)^{--}$	Σ_u^+
1(p/f) ₂	P [±]		4231	2^{++}	$(1, 2, 3)^{+-}$	$\Pi_u \Sigma_u^-$
1d ₂	P ⁰		4334	2^{--}	$(1, 2, 3)^{+-}$	Π_u
2(p/f) ₂	P [±]		4563	2^{++}	$(1, 2, 3)^{+-}$	$\Pi_u \Sigma_u^-$
3d	S	4579		2^{+-}	$(1, 2, 3)^{--}$	Σ_u^+
2d ₂	P ⁰		4693	2^{--}	$(1, 2, 3)^{+-}$	Π_u
3(p/f) ₂	P [±]		4886	2^{++}	$(1, 2, 3)^{+-}$	$\Pi_u \Sigma_u^-$
4d	S	4916		2^{+-}	$(1, 2, 3)^{--}$	Σ_u^+
4(p/f) ₂	P [±]		4923	2^{++}	$(1, 2, 3)^{+-}$	$\Pi_u \Sigma_u^-$
3d ₂	P ⁰		5036	2^{--}	$(1, 2, 3)^{+-}$	Π_u

Results for bottom

N_{L_j}	w -f	$b\bar{b}$	Hybrid	$\mathcal{J}^{PC} (S = 0)$	$\mathcal{J}^{PC} (S = 1)$	Λ_{η}^{\prime}
1s	S	9442		0^{-+}	1^{--}	Σ_g^+
2s	S	10009		0^{-+}	1^{--}	Σ_g^+
3s	S	10356		0^{-+}	1^{--}	Σ_g^+
4s	S	10638		0^{-+}	1^{--}	Σ_g^+
1p ₀	P ⁺		11011	0^{++}	1^{+-}	Σ_u^-
2p ₀	P ⁺		11299	0^{++}	1^{+-}	Σ_u^-
3p ₀	P ⁺		11551	0^{++}	1^{+-}	Σ_u^-
4p ₀	P ⁺		11779	0^{++}	1^{+-}	Σ_u^-
1p	S	9908		1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+
2p	S	10265		1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+
3p	S	10553		1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+
1(s/d) ₁	P [±]		10690	1^{--}	$(0, 1, 2)^{+-}$	$\Pi_u \Sigma_u^-$
1p ₁	P ⁰		10761	1^{++}	$(0, 1, 2)^{+-}$	Π_u
4p	S	10806		1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+
2(s/d) ₁	P [±]		10885	1^{--}	$(0, 1, 2)^{+-}$	$\Pi_u \Sigma_u^-$
2p ₁	P ⁰		10970	1^{++}	$(0, 1, 2)^{+-}$	Π_u
5p	S	11035		1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+
3(s/d) ₁	P [±]		11084	1^{--}	$(0, 1, 2)^{+-}$	$\Pi_u \Sigma_u^-$
4(s/d) ₁	P [±]		11156	1^{--}	$(0, 1, 2)^{+-}$	$\Pi_u \Sigma_u^-$
3p ₁	P ⁰		11175	1^{++}	$(0, 1, 2)^{+-}$	Π_u
6p	S	11247		1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+
5(s/d) ₁	P [±]		11284	1^{--}	$(0, 1, 2)^{+-}$	$\Pi_u \Sigma_u^-$
1d	S	10155		2^{-+}	$(1, 2, 3)^{--}$	Σ_g^+
2d	S	10454		2^{-+}	$(1, 2, 3)^{--}$	Σ_g^+
3d	S	10712		2^{-+}	$(1, 2, 3)^{--}$	Σ_g^+
1(p/f) ₂	P [±]		10819	2^{++}	$(1, 2, 3)^{+-}$	$\Pi_u \Sigma_u^-$
1d ₂	P ⁰		10870	2^{--}	$(1, 2, 3)^{+-}$	Π_u
4d	S	10947		2^{-+}	$(1, 2, 3)^{--}$	Σ_g^+
2(p/f) ₂	P [±]		11005	2^{++}	$(1, 2, 3)^{+-}$	$\Pi_u \Sigma_u^-$
2d ₂	P ⁰		11074	2^{--}	$(1, 2, 3)^{+-}$	Π_u
5d	S	11163		2^{-+}	$(1, 2, 3)^{--}$	Σ_g^+
3(p/f) ₂	P [±]		11197	2^{++}	$(1, 2, 3)^{+-}$	$\Pi_u \Sigma_u^-$
3d ₂	P ⁰		11275	2^{--}	$(1, 2, 3)^{+-}$	Π_u
4(p/f) ₂	P [±]		11291	2^{++}	$(1, 2, 3)^{+-}$	$\Pi_u \Sigma_u^-$

Comments

- Only inputs:
 - ▶ Lattice potential
 - ▶ Heavy Quarkonium spectrum
 - ▶ $m_c = 1.47$ GeV, $m_b = 4.88$ GeV
- Braaten, Langmack, Hudson Smith, 2014
 - ▶ Neglect $\Pi_u - \Sigma_u^-$ coupled channel
 - ▶ Fix the lowest lying state for each potential to the Hadron Spectrum Collaboration, 2012 (HSC)
- Berwin, Brambilla, Tarrús Castellà, Vairo, 2015
 - ▶ Based on the weak coupling regime of pNRQCD
 - ▶ Equivalent equations, different potentials and normalization
 - ▶ Our results are consistent with the lower end of their error bars
- The hierarchy of the lower lying hybrid multiplets agrees with the lattice HSC 2016 but our numbers are about 380-150 MeV lower
- Marginal agreement with the QCD sum rule results of Chen, Kleiv, Steele, Bulthuis, Harnett, Ho, Richards, Zhu, 2013

Hybrids in XYZ?

State	M	J^{PC}	XYZ	M_{exp}	Γ_{exp}	J_{exp}^{PC}
$1(s/d)_1$	4011	$1^{--}, (0, 1, 2)^{-+}$	Y(4008)	4008^{+121}_{-49}	226 ± 97	1^{--}
$1p_1$	4145	$1^{++}, (0, 1, 2)^{+-}$	Y(4140) X(4160)	$4144, 5 \pm 2, 6$ 4156^{+29}_{-25}	15^{+11}_{-7} 139^{+113}_{-65}	1^{++} $?^{?+}$
$2(s/d)_1$	4355	$1^{--}, (0, 1, 2)^{-+}$	X(4320) X(4350) Y(4360) Y(4390)	4320 ± 17 4351 ± 5 4361 ± 13 4391 ± 6	101 ± 30 13^{+18}_{-10} 74 ± 18 139 ± 16	1^{--} $?^{?+}$ 1^{--} 1^{--}
$1p_0$	4486	$0^{++}, 1^{+-}$	X(4500)	4506^{+16}_{-19}	92^{+30}_{-29}	0^{++}
$3(s/d)_1$	4692	$1^{--}, (0, 1, 2)^{-+}$	Y(4660) X(4630)	4664 ± 12 4634^{+9}_{-11}	48 ± 15 92^{+41}_{-32}	1^{--} 1^{--}
$2(s/d)_1$	10885	$1^{--}, (0, 1, 2)^{-+}$	$Y_b(10890)$	$10888, 4 \pm 3$	$30, 7^{+8,9}_{-7,7}$	1^{--}

- C-parity implies that only spin zero hybrids would have been observed, except for X(4350).
- Decays to vector heavy quarkonium states have been observed for all spin zero 1^{--} states above, except for X(4630) and Y(4390), which disfavors the hybrid interpretation due to spin symmetry.

Mixing with Heavy Quarkonium

- States with the same quantum numbers may mix
 - Mixing is a $1/m_Q$ suppressed effect
 - If the energy gap $\Delta E \lesssim \Lambda_{QCD}^2/m_Q$ it becomes next to leading order
 - For $\Lambda_{QCD} = 400$ MeV: $\Delta E|_c \sim 100$ MeV, $\Delta E|_b \sim 30$ MeV

NL_J	w-f	$c\bar{c}$	Hybrid	$\mathcal{J}^{PC}(S=0)$	$\mathcal{J}^{PC}(S=1)$	Λ_η^ϵ
3s	S	4131		0 ⁻⁺	1 ⁻⁻	Σ_g^+
1p ₀	P ⁺		4486	0 ⁺⁺	1 ^{+−}	Σ_u^-
2p	S	3968		1 ^{+−}	(0, 1, 2) ⁺⁺	Σ_g^+
1(s/d) ₁	P [±]		4011	1 ⁻⁻	(0, 1, 2) ^{+−}	$\Pi_u \Sigma_u^-$
1p ₁	H ⁰		4145	1 ⁺⁺	(0, 1, 2) ^{+−}	Π_u
2(s/d) ₁	P [±]		4355	1 ⁻⁻	(0, 1, 2) ^{+−}	$\Pi_u \Sigma_u^-$
3p	S	4369		1 ^{+−}	(0, 1, 2) ⁺⁺	Σ_g^+
2d	S	4210		2 ^{+−}	(1, 2, 3) ⁻⁻	Σ_g^+
1(p/f) ₂	P [±]		4231	2 ⁺⁺	(1, 2, 3) ^{+−}	$\Pi_u \Sigma_u^-$

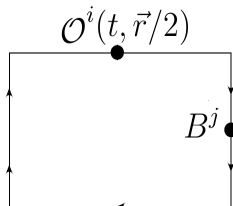
NL_J	w-f	$b\bar{b}$	Hybrid	$\mathcal{J}^{PC}(S=0)$	$\mathcal{J}^{PC}(S=1)$	Λ_η^ϵ
2(s/d) ₁	P [±]		10885	1 ⁻⁻	(0, 1, 2) ^{+−}	$\Pi_u \Sigma_u^-$
4d	S	10947		2 ^{+−}	(1, 2, 3) ⁻⁻	Σ_g^+

Mixing with Heavy Quarkonium

- Symmetries imply

$$\mathcal{L}_{\text{mixing}} = \text{tr} \left[S^\dagger V_S^{ij} \{ \sigma^i, H^j \} + \text{h.c.} \right]$$

- Matching to NRQCD at $O(1/m_Q)$ implies $V_S^{ij}(\mathbf{r}) \sim \frac{1}{m_Q} \times$



NRQCD

- $\mathcal{O}^i \sim B^i$

This term mixes $S = 0$ Hybrids with $S = 1$ Quarkonium, and may explain spin symmetry violating decays of the 1^{--} Y states.

Mixing with Heavy Quarkonium

- $V_S^{ij} = (\delta^{ij} - \hat{r}^i \hat{r}^j) V_S^{\Pi_u} + \hat{r}^i \hat{r}^j V_S^{\Sigma_u^-}$
- No lattice evaluation of $V_S^{\Pi_u}$, $V_S^{\Sigma_u^-}$, so far
- Short distance constraints: pNRQCD at weak coupling implies for $r \rightarrow 0$

$$V_S^{\Pi_u}(r) \sim V_S^{\Sigma_u^-}(r) \rightarrow \pm \frac{\lambda^2}{m_Q} = \text{const.}$$

- Long distance constraints: QCD string (EST) implies for $r \rightarrow \infty$

$$V_S^{\Sigma_u^-}(r) \rightarrow -\frac{2\pi^2 \Lambda''' }{m_Q \kappa r^3}, \quad V_S^{\Pi_u}(r) \rightarrow \sqrt{\frac{\pi^3}{\kappa}} \frac{\Lambda'}{m_Q r^2}$$

- ▶ κ , Λ' and Λ''' also appear in the spin dependent potentials (Perez-Nadal, JS, 2011; Brambilla, Groher, Martinez, Vairo, 2014)
- ▶ Can be extracted from available lattice results of the long distance potentials (Koma, Koma, 2007; 2010)

$$\kappa \sim 0.187 \text{GeV}^2, \quad \Lambda' \sim -59 \text{MeV}, \quad \Lambda''' \sim \pm 230 \text{MeV}$$

Modeling the mixing potential

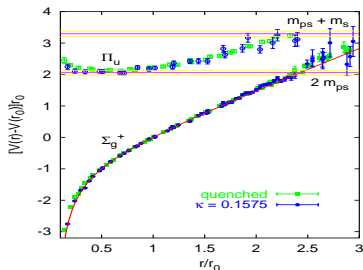
$$V_S^\Pi[\pm-](r) = \frac{\lambda^2}{m_Q} \left(\frac{\pm 1 - \left(\frac{r}{r_\Pi}\right)^2}{1 + \left(\frac{r}{r_\Pi}\right)^4} \right), \quad r_\Pi = \left(\frac{|g\Lambda'| \pi^{\frac{3}{2}}}{2\lambda^2 \kappa^{\frac{1}{2}}} \right)^{\frac{1}{2}}$$
$$V_S^\Sigma[\pm\pm](r) = \frac{\lambda^2}{m_Q} \left(\frac{\pm 1 \pm \left(\frac{r}{r_\Sigma}\right)^2}{1 + \left(\frac{r}{r_\Sigma}\right)^5} \right), \quad r_\Sigma = \left(\frac{|g\Lambda''| \pi^2}{\lambda^2 \kappa} \right)^{\frac{1}{3}}$$

- Simple interpolation that allows for a sign flip between short and long distance behavior
- We focus on charm and scan $\lambda = 100, 300, 600$ MeV
- $V_S^\Pi[+-]$ with $V_S^\Sigma[++]$ and $\lambda = 600$ MeV produce the maximum mixing
- The spin symmetry violating decays of $Y(4008)$ (29% $S=1$), $Y(4360)$ (35% $S=1$) and $Y(4660)$ (17% $S=1$) are qualitatively explained
- It would be important that lattice calculations confirm the signs and size of the mixing potentials

Hybrid-Quarkonium coupled equations

- The $S = 0$ case:
 - ▶ $P_{0\mathcal{J}\mathcal{M}}^L(r)$, $J = \mathcal{J}$, $L = J \pm 1 (\pm)$, $J(0)$
 - ▶ $S_{1\mathcal{J}\mathcal{M}}^L(r)$, $L = \mathcal{J} \pm 1 (\pm)$, $\mathcal{J}(0)$
 - ▶ 2 coupled equations for $S_{1\mathcal{J}\mathcal{M}}^0(r)$, $P_{0\mathcal{J}\mathcal{M}}^0(r)$
 - ▶ 4 coupled equations for $S_{1\mathcal{J}\mathcal{M}}^+(r)$, $S_{1\mathcal{J}\mathcal{M}}^-(r)$, $P_{0\mathcal{J}\mathcal{M}}^+(r)$, $P_{0\mathcal{J}\mathcal{M}}^-(r)$
- The $S = 1$ case:
 - ▶ $P_{1\mathcal{J}\mathcal{M}}^{LJ}(r)$, $J = \mathcal{J} \pm 1 (\pm)$, $\mathcal{J}(0)$, $L = J \pm 1 (\pm)$, $J(0)$
 - ▶ $S_{0\mathcal{J}\mathcal{M}}(r)$, $L = \mathcal{J}$
 - ▶ 2 coupled equations $P_{1\mathcal{J}\mathcal{M}}^{+0}(r)$, $P_{1\mathcal{J}\mathcal{M}}^{-0}(r)$
 - ▶ 6 coupled equations
 $S_{0\mathcal{J}\mathcal{M}}(r)$, $P_{1\mathcal{J}\mathcal{M}}^{++}(r)$, $P_{1\mathcal{J}\mathcal{M}}^{-+}(r)$, $P_{1\mathcal{J}\mathcal{M}}^{+-}(r)$, $P_{1\mathcal{J}\mathcal{M}}^{--}(r)$, $P_{1\mathcal{J}\mathcal{M}}^{00}(r)$

Decay to lowest lying Heavy Quarkonium



- If the energy gap ΔE to lowest lying heavy quarkonium is $\Delta E \gtrsim 1\text{GeV} \Rightarrow$ a perturbative estimate makes sense.
- If $\Delta E \langle H|r|Q\bar{Q} \rangle \ll 1 \Rightarrow$ weak coupling pNRQCD can be used

$$\Gamma(H_m \rightarrow S_n) = \frac{4\alpha_s T_F}{3 N_c} \langle H_m|r^i|S_n \rangle \langle S_n|r^i|H_m \rangle (\Delta E_n)^3$$

Results

- Hybrids with $L = J$ do not decay to Heavy Quarkonium

Charm

$NL_J \rightarrow nL$	ΔE (MeV)	$\langle r \rangle_{nm}$ (GeV^{-1})	$ \Delta E \langle r \rangle_{nm} $	Γ (MeV)
$1p_0 \rightarrow 2s$	808	0.40	0.32	6.1
$2(s/d)_1 \rightarrow 1p$	861	0.63	0.54	19

Bottom

$NL_J \rightarrow nL$	ΔE (MeV)	$\langle r \rangle_{nm}$ (GeV^{-1})	$ \Delta E \langle r \rangle_{nm} $	Γ (MeV)
$1p_0 \rightarrow 1s$	1569	-0.416	0.65	31
$1p_0 \rightarrow 2s$	1002	0.432	0.43	8.7
$2p_0 \rightarrow 1s$	1857	-0.422	0.78	53
$2p_0 \rightarrow 2s$	1290	-0.137	0.18	1.9
$2p_0 \rightarrow 3s$	943	0.462	0.44	8.3
$2(s/d)_1 \rightarrow 1p$	977	0.470	0.46	9.6

Hyperfine Splittings

Pere Solé, JS

- They appear at $\mathcal{O}(1/m_Q)$ ($\mathcal{O}(1/m_Q^2)$) in hybrids (quarkonium)
- They are controlled by a single operator

$$i\epsilon^{ijk} V^S(r) \text{tr} \left(H^{i\dagger} \left[\sigma^k, H^j \right] \right)$$

- It leads to the following mass formulae

$$\frac{M_{1J+1} - M_{0J}}{M_{1J} - M_{0J}} = -J \quad \frac{M_{1J-1} - M_{0J}}{M_{1J} - M_{0J}} = J + 1$$

$$(s/d)_1 : M_{2-+} + M_{0-+} = M_{1-+} + M_{1--}$$

$$p_1 : M_{2+-} + M_{0+-} = M_{1+-} + M_{1++}$$

$$(p/f)_2 : M_{3+-} + M_{1+-} = M_{2+-} + M_{2++}$$

$$d_2 : M_{3+-} + M_{1+-} = M_{2+-} + M_{2--}$$

- Consistent with the values of the lattice **HSC**
- Induces mixing between different hybrid states

Results

Resonance	J^{PC}	Assignment	Mass (MeV)	Observations
X(3823)	2^{--}	$1d$	3792	
X(3860)	0 or 2^{++}	$2p$	3968	
X(3872)	1^{++}	$2p$	3967	
X(3915)	0 or 2^{++}	$2p$	3968	
X(3940)	$??$	$2p$	3968	
Y(4008)	1^{--}	$1s/d_1$	4004	mixing
X(4140)	1^{++}	$??$	$??$	$1p_1$ does not decay to quarkonium
X(4160)	$??$	$1p_1$	4146	
Y(4220)	1^{--}	$2d$	4180	Y(4260) \rightarrow Y(4220), mixing
X(4230)	1^{--}	$2d$	4180	X(4230) = Y(4220), mixing
X(4350)	$?^{?+}$	$2(s/d)_1$ or $3p$	4355 or 4369	
Y(4320)	1^{--}	$2(s/d)_1$	4366	mixing
Y(4360)	1^{--}	$2(s/d)_1$	4366	Y(4360) = Y(4320)?
X(4390)	1^{--}	$2(s/d)_1$	4366	Y(4390) = Y(4360)?
X(4500)	0^{++}	$1p_0$	4566	not enough mixing
Y(4630)	1^{--}	$3d$	4559	
Y(4660)	1^{--}	$3(s/d)_1$	4711	mixing
X(4700)	0^{++}	$4p$	4703	
Υ (10860)	1^{--}	$5s$	10881	mixing
Y_b (10890)	1^{--}	$2(s/d)_1$	10890	mixing
Υ (11020)	1^{--}	$4d$	10942	

Conclusions

- Heavy Hybrids containing $c\bar{c}$ or $b\bar{b}$ have been studied from QCD in a largely model independent way (EFT+lattice inputs)
 - ▶ The lower lying states have been calculated at LO in the $1/m_Q$ expansion of the potentials, including the $\Sigma_u^- - \Pi_u$ mixing.
 - ▶ The mixing with Heavy Quarkonium states has been addressed
 - ★ It is an important source of spin symmetry violations that explains the decays of certain spin zero Hybrids to spin one Quarkonia.
 - ▶ The decay width to lower lying Heavy Quarkonium states has been estimated
 - ★ The decays of $J = L$ states are forbidden
 - ▶ Spin and velocity dependent terms in the hybrid potentials enter at $O(1/m_Q)$
 - ★ We have produced model independent formulas for the hyperfine splittings
 - ▶ A number of them can be identified with XYZ states
($Y(4008)$, $X(4160)$, $X(4350)$, $Y(4320)/Y(4360)/Y(4390)$,
 $Y(4660)$, $Y_b(10890)$)