D-term in various (space@time-like) processes

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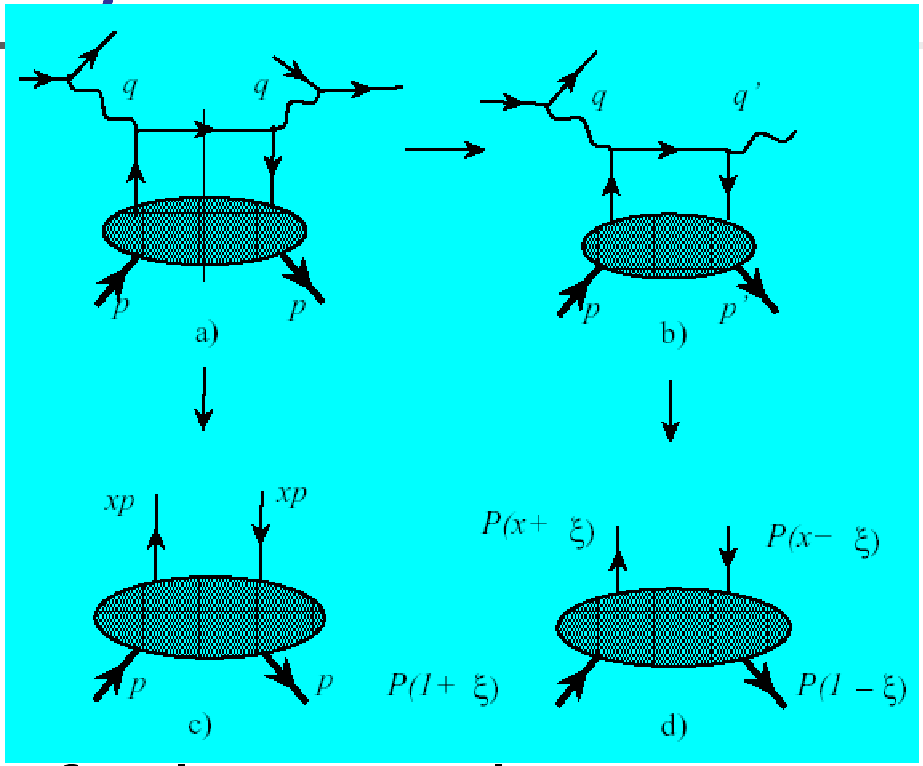
Outline

- GPDs, analyticity and D-term
- Specifics of time-like processes: s and Q2 cuts
- DDCVS for space@timelike cases
- D-term in 2 hadron processes
- Interference between hadronic exclusive lepton pair production mechanisms

QCD Factorization for DIS and DVCS/DVMP



- Manifestly spectral

$$\mathcal{H}(x_B) = \int_{-1}^{1} dx \frac{H(x)}{x - x_B + i\epsilon}$$

- Extra dependence on ξ

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x - \xi + i\epsilon},$$

Unphysical regions

- DIS : Analytical function – if $1 \leq |x_B|$ polynomial in $1/x_B$

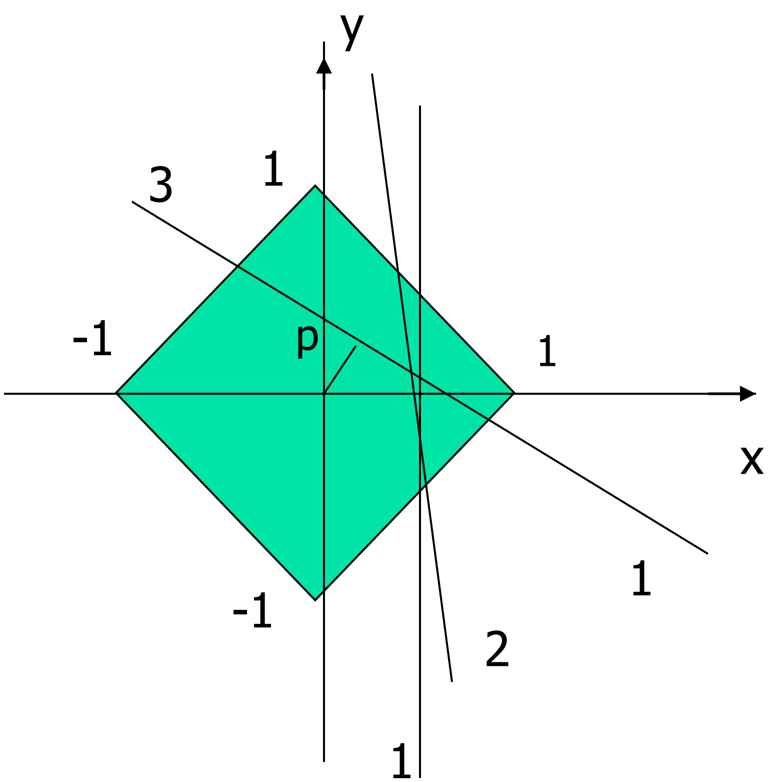
$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- DVCS – additional problem of analytical continuation of $H(x,\xi)$
- Solved by using of Double Distributions Radon transform (talks P. Kroll, K. Semenov-Tian-Shansky

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

Double distributions and their integration

- Slope of the integration line-skewness
- Kinematics of DIS: $\xi = 0$
  ("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$
  - line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography

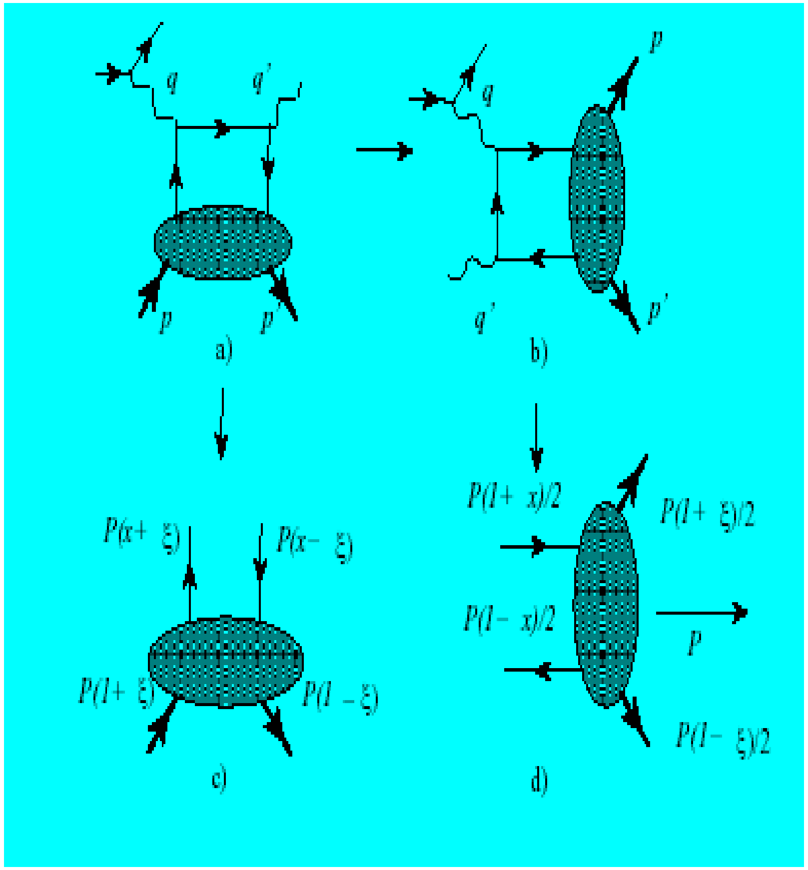


$$f(x,y) = -\frac{1}{2\pi^2}\int_0^\infty \frac{dp}{p^2}\int_0^{2\pi} d\phi|cos\phi|(H(p/cos\phi + x + ytg\phi, tg\phi) - H(x + ytg\phi, tg\phi)) =$$

$$= -\frac{1}{2\pi^2}\int_{-\infty}^\infty \frac{dz}{z^2}\int_{-\infty}^\infty d\xi(H(z + x + y\xi, \xi) - H(x + y\xi, \xi))$$

Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes



GDA -> back to unphysical regions for DIS and DVCS

- Recall DIS

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- Non-positive powers of x_B

- DVCS

$$H(\xi) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

- Polynomiality (general property of Radon transforms): moments - integrals in x weighted with x^n - are polynomials in $1/\xi$ of power $n+1$
- As a result, analyticity is preserved: only non-positive powers of ξ appear

Holographic property (OT'05)

- Factorization Formula ->

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x-\xi+i\epsilon}$$

- Analyticity ("dynamical") -> Imaginary part -> Dispersion relation:

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,x)}{x-\xi+i\epsilon}$$

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x,x)-H(x,\xi)}{x-\xi+i\epsilon}$$

- "Holographic" equation (DVCS AND VM)

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} \int_{-1}^{1} H(x,\xi) dx (x-\xi)^{n-1} = const$$

Holographic property - II

- Directly follows from double distributions

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

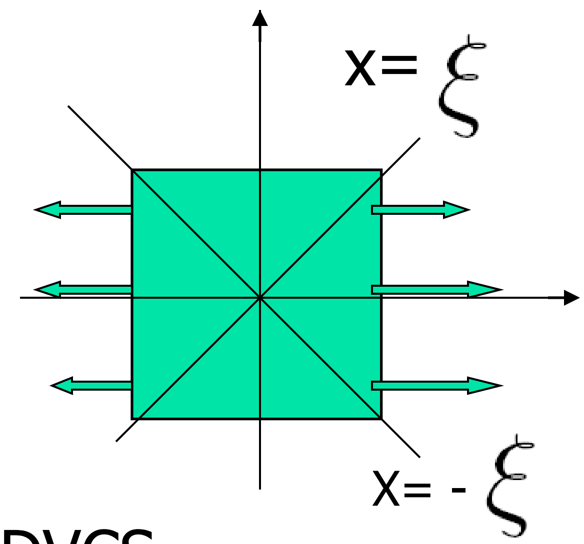
- Constant is the SUBTRACTION one - due to the (generalized) D-term G(x,y)

$$\Delta\mathcal{H}(\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy \frac{G(x,y)}{1-y}$$

$$= \int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z-1} = const$$

Holographic property - III

- 2-dimensional space -> 1-dimensional section!
- Momentum space: any relation to holography in coordinate space ?!



- ERBL ⟶ "GDA" region
- Strategy (now adopted – talk of K. Kumericky) of GPD's

studies: start at diagonals

(through SSA due to imaginary part of DVCS amplitude) and restore by making use of dispersion relations + subtraction constants

Angular distribution in hadron pairs production

- Back to GDA region
- Moments of H(x,x) - define the coefficients of powers of cosine!– 1/ ξ
- Higher powers of cosine in t-channel – threshold in s - channel
- Larger for pion than for nucleon pairs because of less fast decrease at x ->1
- Continuation of D-term from t to s channel – dispersion relation in t (Pasquini, Polyakov, Vanderhaegen)

$$\mathscr{H}(\xi) = -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,\xi)\frac{x^n}{\xi^{n+1}}$$

$$= -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,x)\frac{x^n}{\xi^{n+1}} + \Delta\mathscr{H}.$$

Analyticity of Compton amplitudes in energy plane (Anikin,OT'07)

- Finite subtraction implied

$$\mathrm{Re}\mathcal{A}(\nu, Q^2) = \frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu', Q^2)}{(\nu'^2-\nu^2)} + \Delta \qquad \Delta = 2\int_{-1}^{1} d\beta \frac{D(\beta)}{\beta - 1}$$

$$\Delta^p_{\mathrm{CQM}}(2) = \Delta^n_{\mathrm{CQM}}(2) \approx 4.4, \qquad \Delta^p_{\mathrm{latt}} \approx \Delta^n_{\mathrm{latt}} \approx 1.1$$

- Numerically close to Thomson term for REAL proton (but NOT neutron) Compton Scattering!
- Duality (sum of squares vs square of sum; proton: 4/9+4/9+1/9=1)?!
- Stability of subtraction against NPQCD?

2-photons scattering

- Real photons limit

$$\mathrm{Re}\mathcal{A}(\nu, Q^2) = \frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu', Q^2)}{(\nu'^2-\nu^2)} + \Delta$$

- v=(s-u)/4M -> (t-u)/4M
- Scattering at 90^0 in c.m. is defined by subtractiion constant
- Dominance of Thomson term (better for proton-antiproton – sum of charges squared argument)

Is D-term independent?

- Fast enough decrease at large energy

$$\mathrm{Re}\,\mathcal{A}(\nu) = \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2 - \nu^2} + \mathrm{C}_0$$

$$\mathrm{C}_0 = \Delta - \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2}$$

$$= \Delta + \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,x)}{x}.$$

- FORWARD limit of Holographic equation

$$\Delta = \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,0) - H^{(+)}(x,x)}{x}$$

$$= 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0) - H(x,x)}{x},$$

$$\mathrm{C}_0(t) = 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0,t)}{x}$$

“D – term” 30 years before…

- Cf Brodsky, Close, Gunion’72
- D-term – a sort of renormalization constant
- Recover through special regularization procedure (D. Mueller, K. Semenov-Tyan-Shansky)?
- Cf mass-shell (‘physical”) and MS renormalizations
- j-analyticity spoiled by distributions growth?

Time-like amplitudes and analyticity

- Extra cut in Q^2 appears
- Scaling: $F(Q^2/s) = F(-Q^2/-s)$: cuts cancellation (cf inclusive electron-positron annihilation)
- $F(Q^2/u) = F(-Q^2/-u)$ – cuts in Q^2 has a form of cuts in u (opposite pole prescription in TCS!) – diagonals on holographic plot interchanged
- DR in Q^2 – explains why for the opposite pole prescription one have the same holographic SR!

2 scales - DDVCS

- Different variables in coefficient function and GPD = GPD modification
- Simplest case (Diehl, Ivanov'07) – two spacelike variables (S.Brodsky-positronium beam) – only s and u cuts – ʒ independence of subtraction proved
- Easy to supplement that by calculation of subtraction:
- <G(x,y)/(y-1)> -> <G(x,y)/(y-r)>
- $r=(q^2 + q'^2)/(q^2 - q'^2)$->$(Q^2 - M^2)/(Q^2 + M^2)$
- DDVCS : $r > 1$ -> $r<1$ (pole in integration region)
- How to treat?

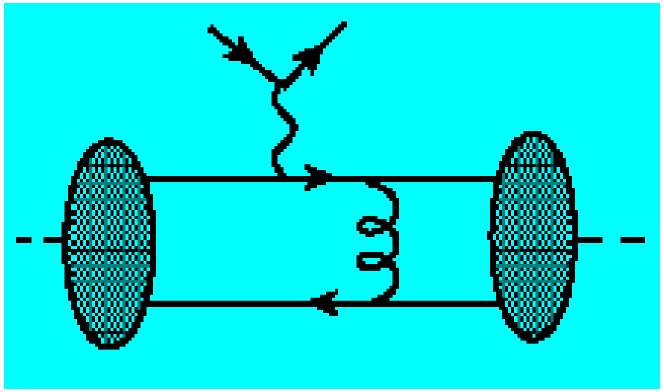
D-term in DDVCS: factorization instead of DR

- DVCS: constant reproduced by direct calculation
- DDVCS with reduced to D-term by gauge transformation: $\langle D(z)/(z-r+i\varepsilon)\rangle$
- Imaginary part due to D-term
- Im M(ʒ,r) ~ D(r)
- Energy-independent contribution to (numerator of) beam SSA

D-term in processes with 2hadrons

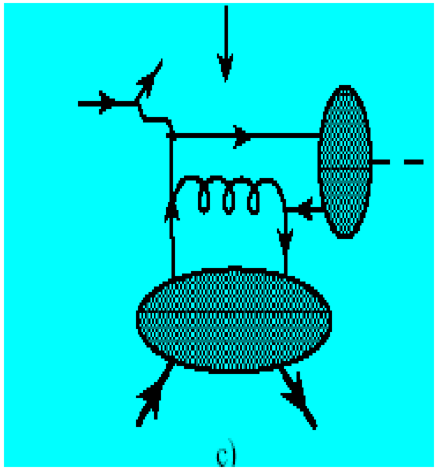
- Starting from (Pion) form factor- 2 DA's –no cuts

$$F \sim (\int dx \frac{\phi(x)}{1-x})^2$$



- 1 DA -> GPD :Exclusive mesons production: Factorization = DR + D-subtraction
- (DVMP/DY) - +/-

$$M \sim \int dx \frac{\phi(x)}{1-x} \int dx \frac{H(x,\xi)}{x-\xi+i\varepsilon}$$

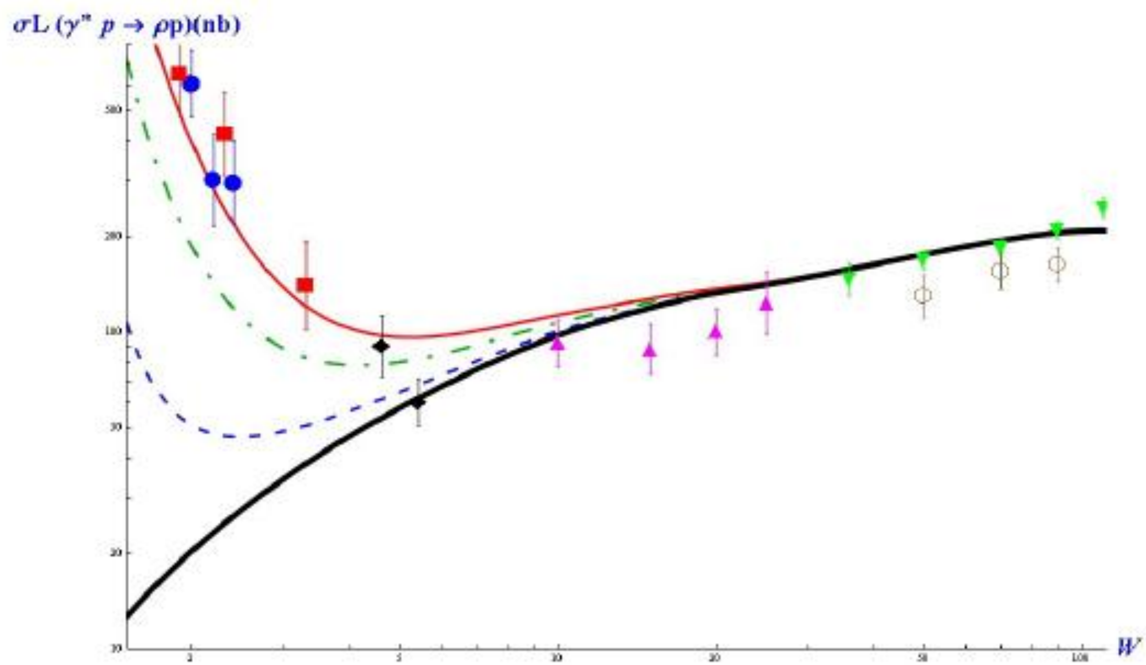


Subtraction in exclusive electroproduction (fixed pole appearance not obvious)

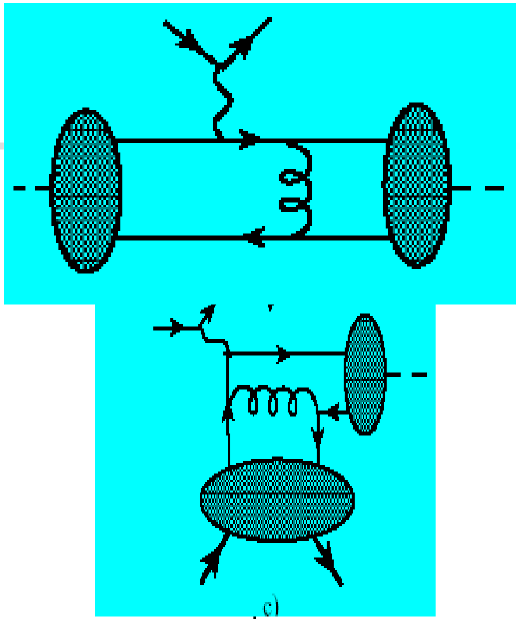
- May qualitatively explain the low energy enhancement (Gabdrakhmanov,OT’12)
- GK model

$$\sigma(W) \approx \sigma_0(W)\left|\frac{A_{Collinear}(W) + a\cdot\Delta}{A_{Collinear}(W)}\right|^2$$

- a=1,3,4.8
- D-term - stable against TM?
- Problems wih flavour/charge dependence



Next step: 2 DAs -> DA+GPD->2 GPD's- Exclusive DY (OT'05)



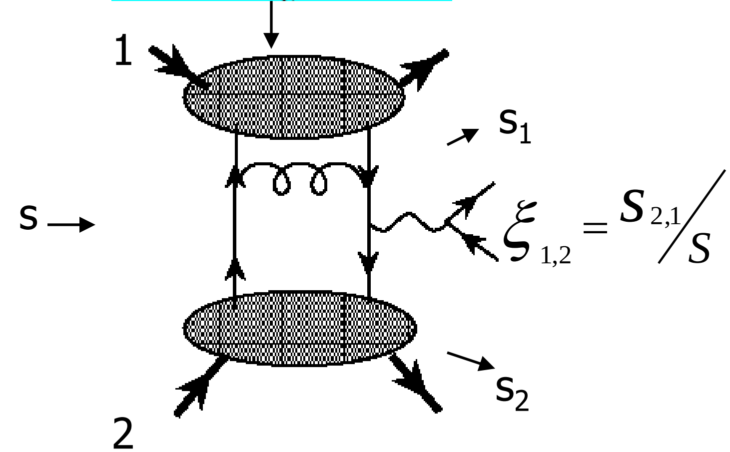
- Exclusive double diffractive DY process
- Analytic continuation:

$$M \sim \int dx \frac{H(x,\xi_1)}{x-\xi_1 \pm i\varepsilon} \int dy \frac{H(y,\xi_2)}{y-\xi_2 \mp i\varepsilon}$$

+ ~D and ~ D^2 subtraction terms

- DIFFERS from direct calculation – NO factorization in physical region

$$M \sim \int\int dxdy \frac{H(x,\xi_1)H(y,\xi_2)}{(x-\xi_1)(y-\xi_2)+i\varepsilon}$$

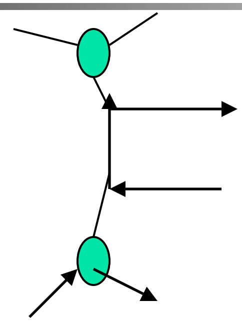
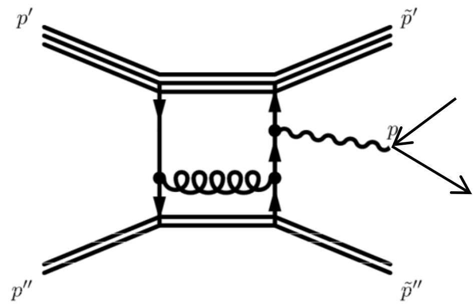


- Real part OK – how to observe?

Interference effects

- Interference (cf talk of C. Weiss)with pure EM (FFxFF) production of (C-even) lepton pair contains only real IR safe part of the amplitude and gives rise to charge asymmetry
- The way to extract GPDxGPD in central region from inclusive DY
- Possible to do at LHC for pp and at COMPASS even without observed pion

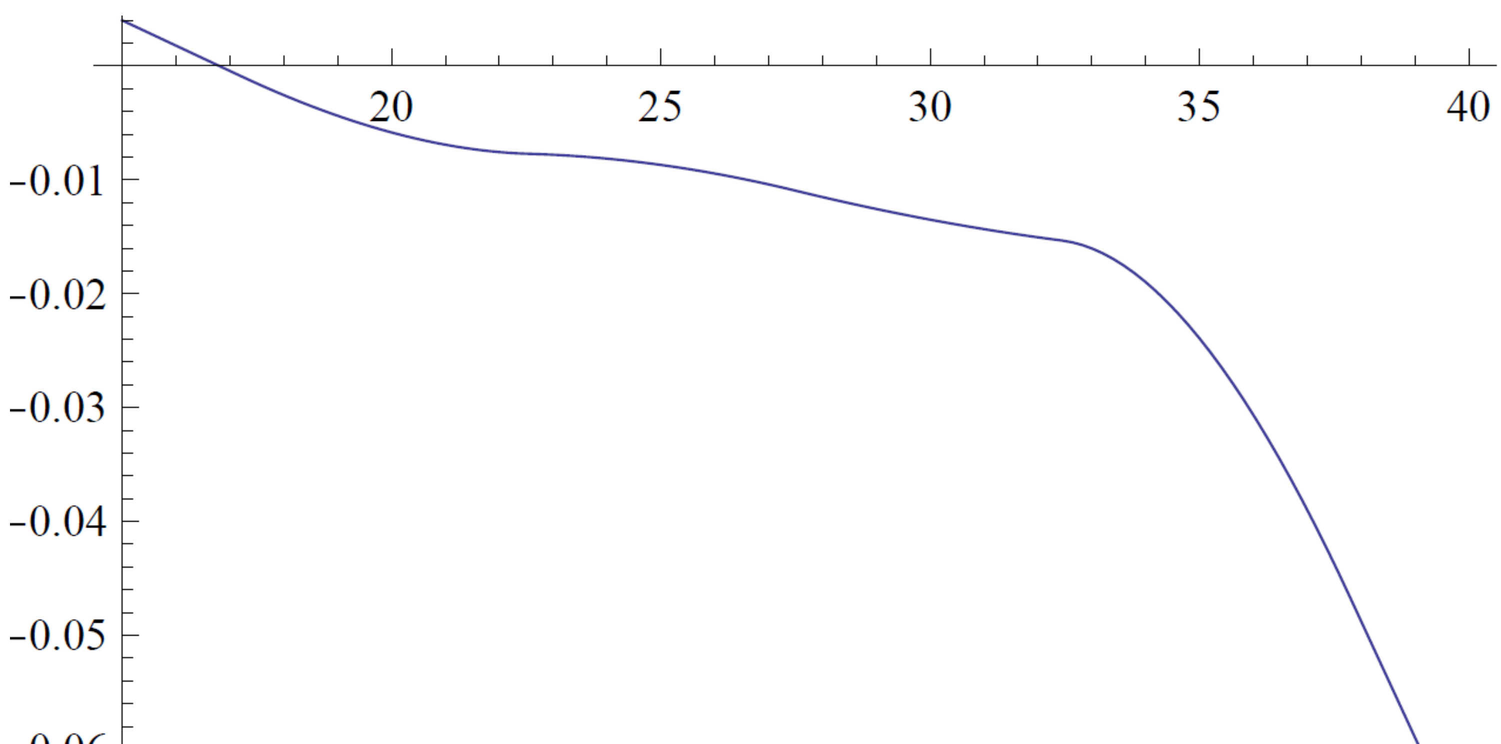
Interference of EM and GPD x GPD mechanisms

-  (2 diagrams)
- 

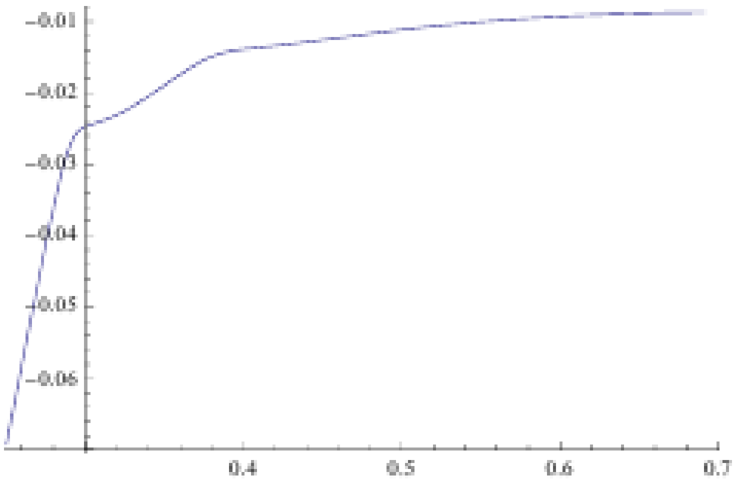
(16 diagrams)

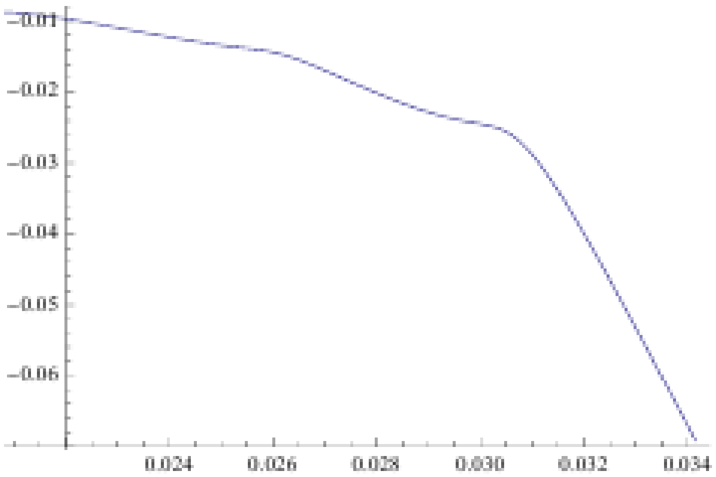
Interference with EM mechanism (preliminary)

- Charge asymmetry (muon-antimuon interchange) vs cm muon angle



(Anti)muon Lab frame asymmetry





CONCLUSIONS/OUTLOOK

- D-term appears a subtraction constant in dispersive holographic equation
- TCS – same equation for DR in Q^2
- DDVCS for 2 spacelike photons – r-dependent subtraction
- D-term in DDVCS via factorization Q^2- (but NOT energy-) dependent imaginary part contributing to beam SSA
- D-term in exclusive DY- charge asymmetry in BH-type interference