Toward an effective field theory approach in energy density functional theory

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ECT* workshop

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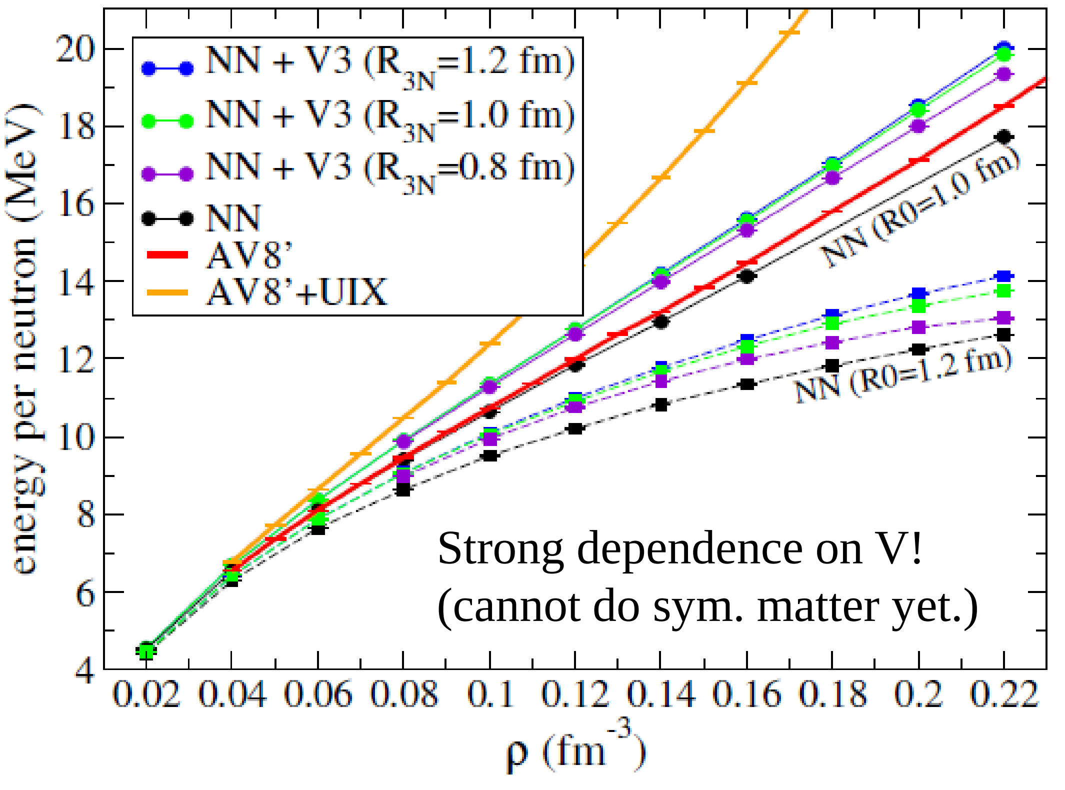
INSTITUT DE PHYSIQUE NUCLÉAIRE ORSAY

Nuclear matter: ab-inito

Equation of state of neutron matter at N^2LO.



S. Gandolfi, talk in ESNT workshop, 2017

Take another expansion

EDF

- Energy density functional (EDF) framework gives reasonable results at mean field, when sufficient amount of parameters (~10) are included.

But,…

- Include more parameters won't necessarily help.

→ Limited predictive power.

⇩

Is there a way to do EFT ? (need to go beyond mean field to perform the test).

Turn off nucleon-nucleon d.o.f.,

Also, no EFT/ERE to guild the power counting

In term of power counting: Just like turn of the light in a cave.

Turn off nucleon-nucleon d.o.f.,

no EFT/ERE to guild the power counting

One hint at $\rho \to 0$

Second hint
from unitarity limit

In term of power counting: Just like turn of the light in a cave.

First hint: a special case where an EFT expansion is known to works

Pure neutron matter at very low density ($k_N a<1$, $\rho<10^{-6}$ fm^{-3}).

Lee & Yang formula (1957) describes the dilute system.

=> Can be re-derived by EFT with matching to ERE

L. Platter, H. Hammer, Ulf. Meissner, Nucl.Phys. A714 (2003), 250-264,

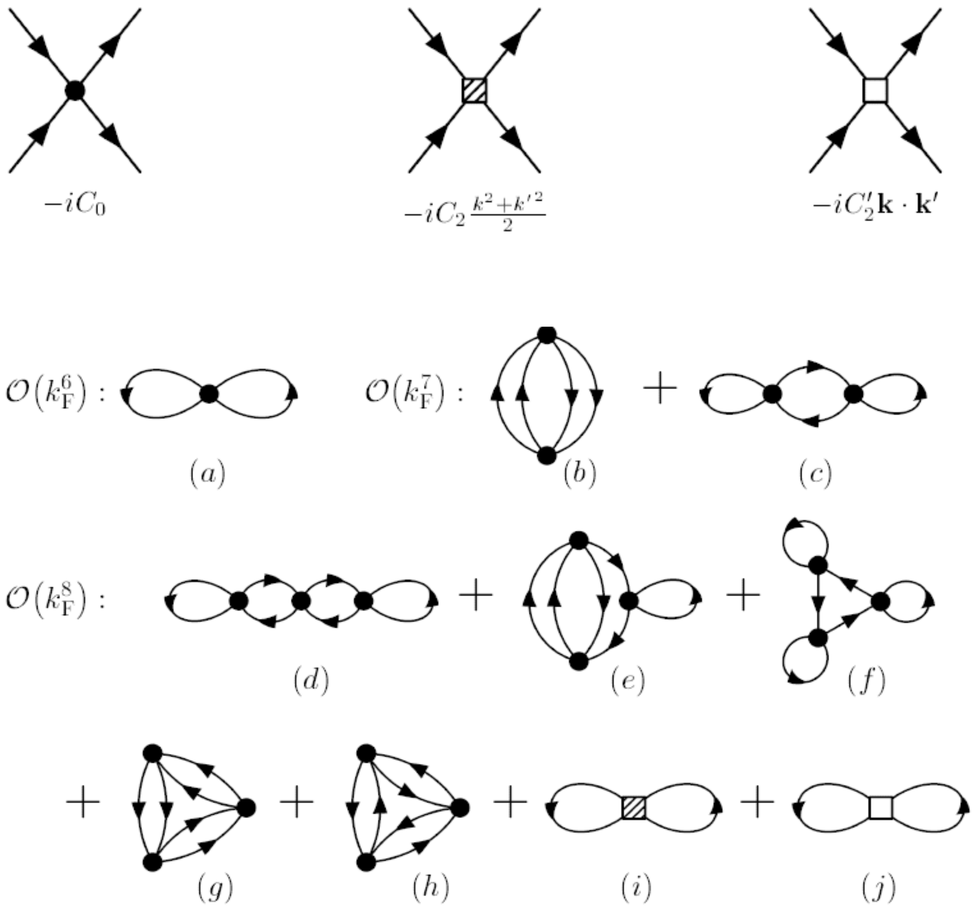
H. Hammer and R.J. Furnstahl, Nucl.Phys. A678 (2000) 277-294.

$$\frac{E_{NM}}{A} = \frac{\hbar^2 k_N^2}{2m}\left[\underbrace{\frac{3}{5}}_{K.E.} + \underbrace{\frac{2}{3\pi}(k_N a)}_{\text{analog to } t_0 \text{ term}} + \underbrace{\frac{4}{35}(11-2\ln 2)(k_N a)^2}_{\textit{automatically} \text{ recover in } 2^{\text{nd}} \text{ of } t_0} + \underbrace{O(k_N^3)}_{\textit{higher} \text{ order}}\right]$$

Expansion in $\mathbf{k_N a}$

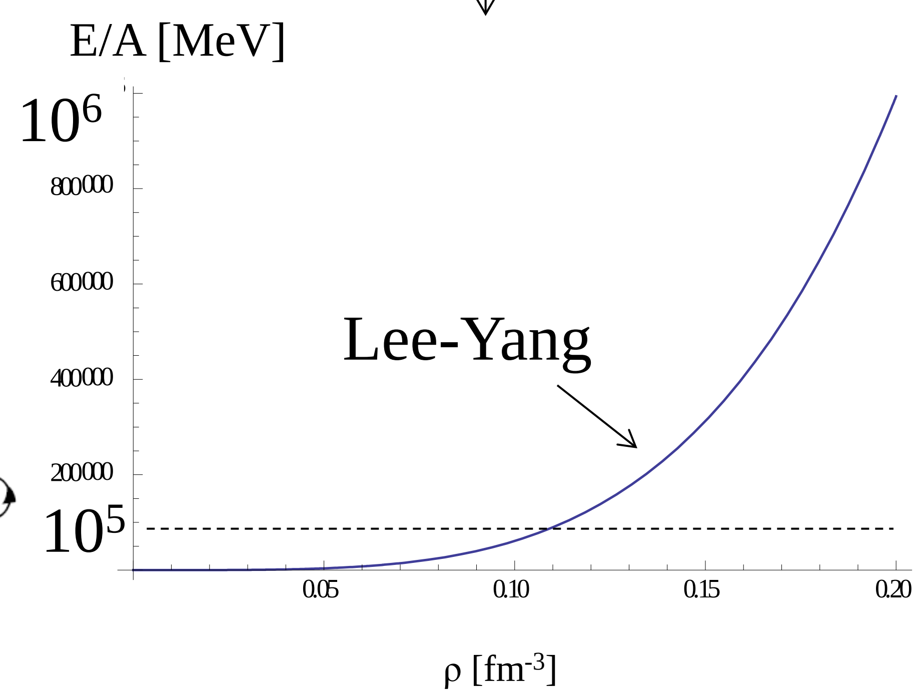
But the valid ρ is way too low!

Diagrams gives V up to $O(k_F^8)$



H. Hammer and R.J. Furnstahl, Nucl.Phys. A678 (2000) 277-294

If take physical value of a=-18.9 fm, then *impossible* to fit pure neutron matter EoS outside region $k_F a << 1$ (adding t1, t2, t3 terms doesn't help).

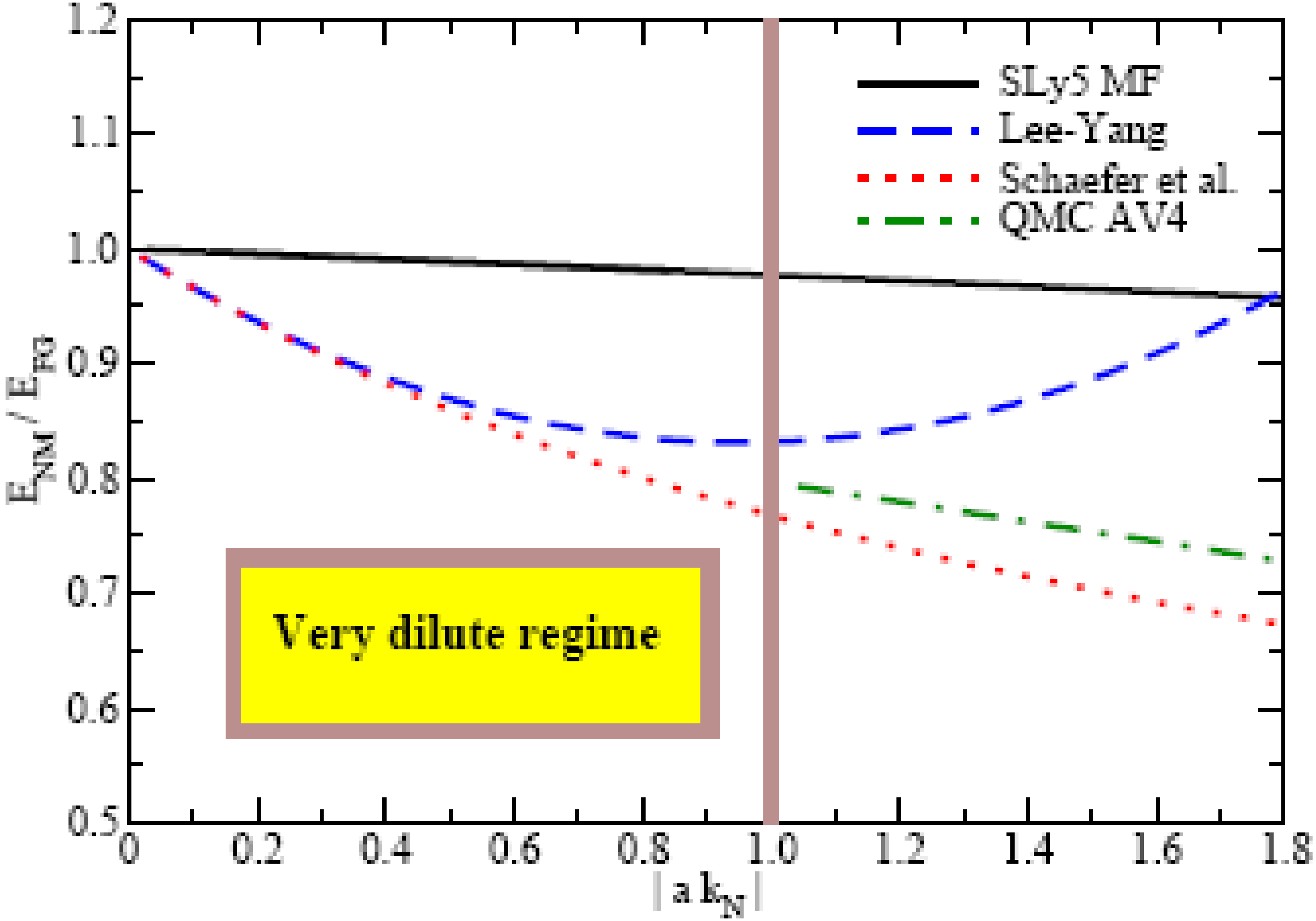


Treatment: Re-sum

To be valid at higher ρ, ($k_N a$) needs to be re-summed. (Steele (2000), Schafer (2005), Kaiser (2011))

$$\frac{E_{NM}}{N} = \frac{\hbar^2 k_N^2}{2m}\left[\frac{3}{5} + \frac{2}{3\pi}\frac{k_N a}{1 - 6k_N a(11 - 2\ln 2)/(35\pi)}\right]$$

Neutron matter only



SLy5 MF
Lee-Yang
Schaefer et al.
QMC AV4
Very dilute regime
E_{NM} / E_{FG}
$|a\, k_N|$
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8

YGLO: Resumed-inspired functional

C.J. Yang, D. Lacroix, M. Grasso, Phys. Rev. C 94 034311(2016)

$$V = \frac{B_\beta}{1 - R_\beta \rho^{1/3} + \underbrace{C_\beta \rho^{2/3}}_{\text{higher order in L\&Y to be resumed*}}} + \underbrace{D_\beta \rho^{2/3}}_{velocity-dep\ \text{term*}} + \underbrace{F_\beta \rho^{\alpha}}_{3^{+}-body}$$

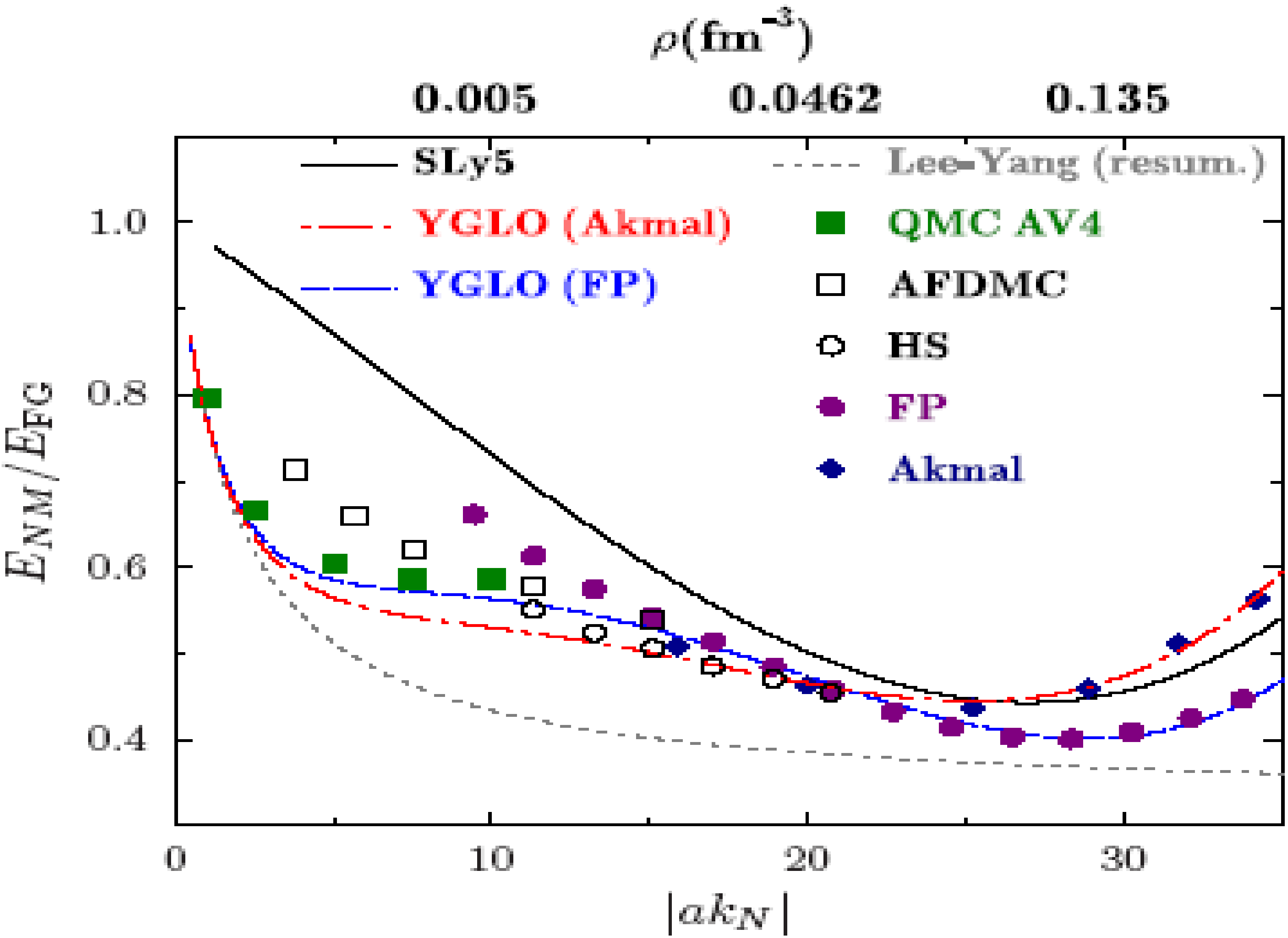
B_β, R_β are fixed to reproduce first two term in Lee & Yang.

$$\Rightarrow B_\beta = 2\pi \frac{\hbar^2}{\mathrm{m}} \frac{\nu - 1}{\nu} a_\beta,\ R_\beta = \frac{6}{35\pi}\left(\frac{6\pi^2}{\nu}\right)^{1/3}(11 - 2\ln 2) a_\beta.$$

(degeneracy: $\nu = 2(4)$ for $\beta = \underbrace{0}_{\text{pure n}} \ (\underbrace{1}_{\text{sym}}))$

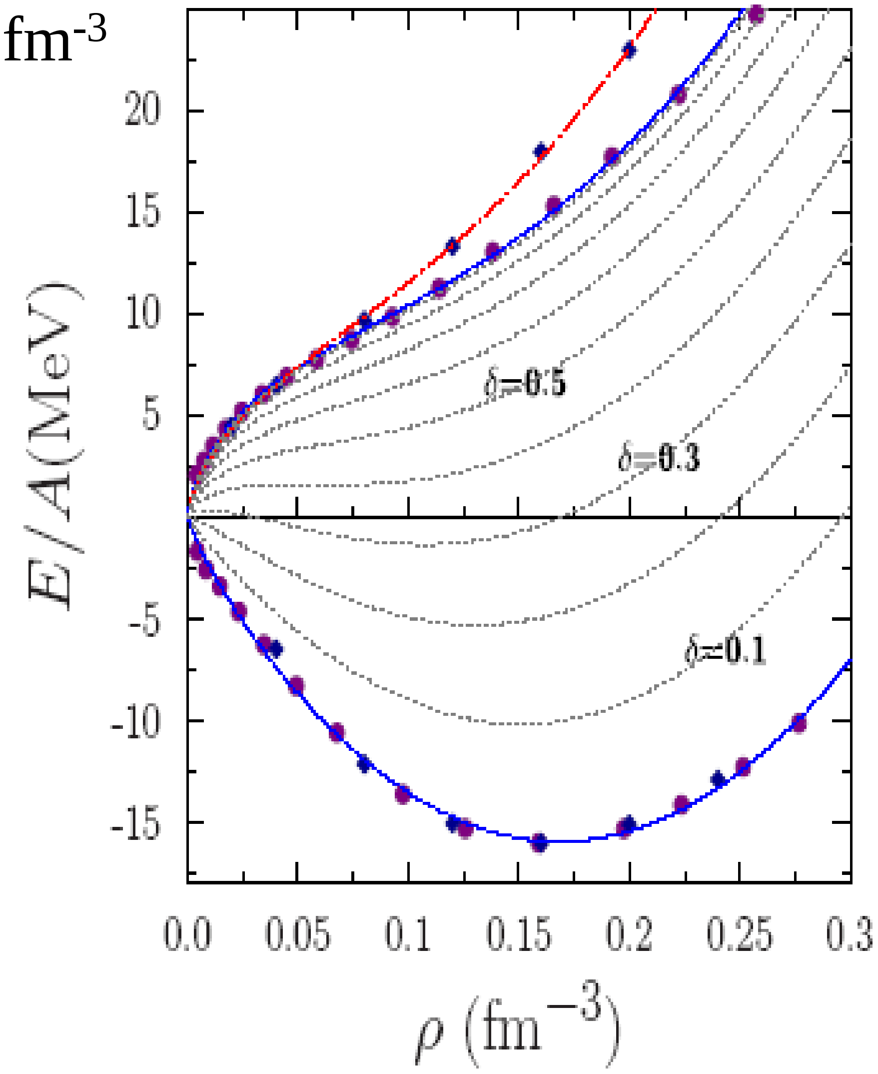
$$a_0 = -18.9\text{fm},\quad \underbrace{a_1 = -20\ \text{fm}}_{avg.\ \text{of}\ a_{\text{nn}},\ a_{\text{pp}},\ a_{\text{np}}\ \text{in}\ {}^1S_0}\ .$$

$$\frac{E}{A} = KE_\beta + \frac{B_\beta \rho}{1 - R_\beta \rho^{1/3} + C_\beta \rho^{2/3}} + D_\beta \rho^{5/3} + F_\beta \rho^{\alpha+1}$$



FP: B. Friedman and V. Pandharipande, Nucl. Phys. A361,502 (1981).
Akmal: A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C 58, 1804 (1998).
HS: K. Hebeler and A. Schwenk Phys. Rev. **C 82**, 014314(2010).

Up to ρ=0.3 fm^{-3}



Able to describe both sym and pure neutron matter EoS up to $2\rho_0$ very well with only 4 free parameters each.

Asymmetric case

Parabolic approximation

$$\frac{E_\delta}{A}(\rho) = \frac{E_{sym}}{A}(\rho) + S(\rho)\delta^2,$$

$$(\delta = (\rho_N - \rho_p)/(\rho_N + \rho_p))$$

$$L = 3\rho_0 (dS/d\rho)_{\rho=\rho_0}$$

Before: Lots of models fail

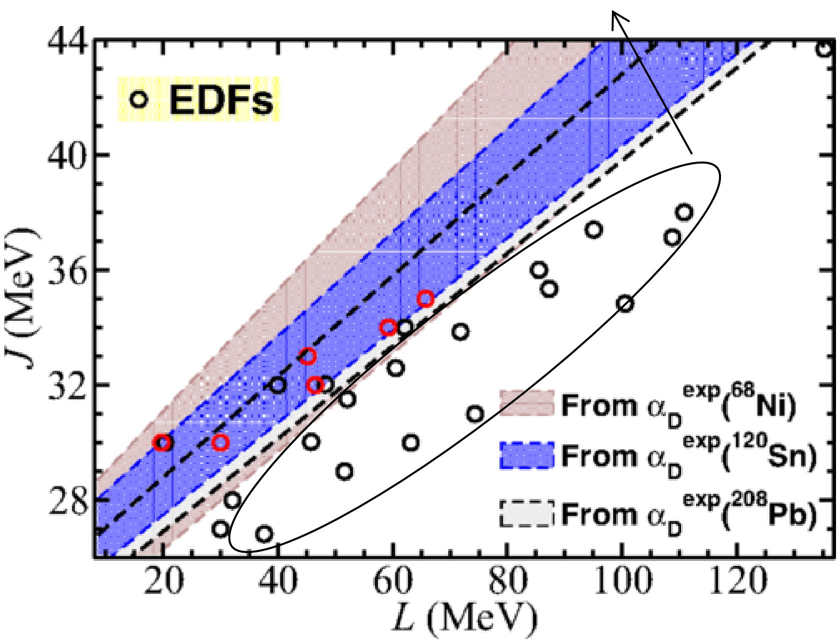


FIG. 4: Symmetry energy at saturation density as a function of its slope L. The black lines delimit the phenomenological area constrained by the experimental determination of the electric dipole polarizability in ^{208}Pb. The blue dotted lines delimit the area constrained by the same measurement in ^{68}Ni, and the red dashed lines refer to the measurement done in ^{120}Sn. The yellow area is the overlap. Inset: density dependence of the Symmetry energy for the two YGLO parametrizations of this work.

X. Roca-Maza, et al., (2015).

Asymmetric case

Our result (prediction)

Satisfies the experimental constraint.

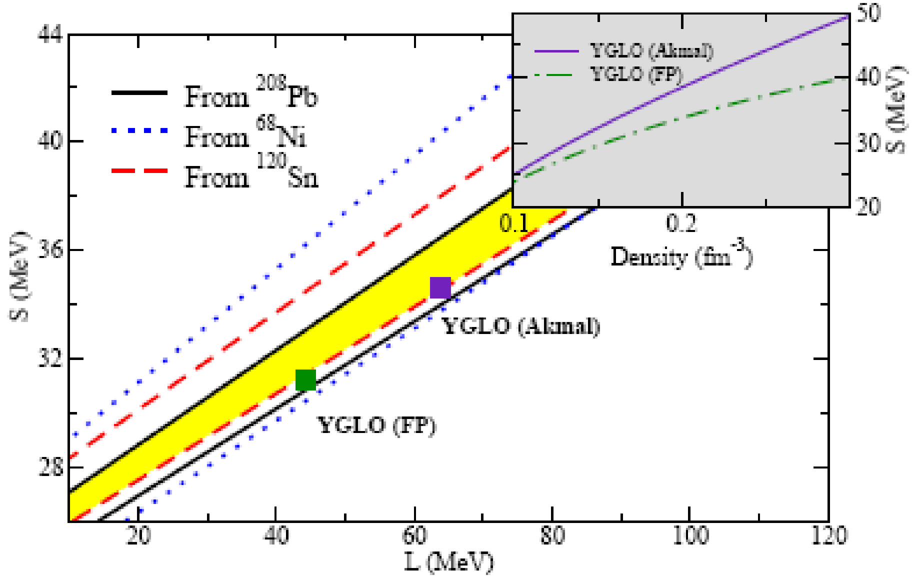


FIG. 4: Symmetry energy at saturation density as a function of its slope L. The black lines delimit the phenomenological area constrained by the experimental determination of the electric dipole polarizability in ^{208}Pb. The blue dotted lines delimit the area constrained by the same measurement in ^{68}Ni, and the red dashed lines refer to the measurement done in ^{120}Sn. The yellow area is the overlap. Inset: density dependence of the Symmetry energy for the two YGLO parametrizations of this work.

Second hint: Unitarity limit

D Lacroix, Phys. Rev. A 94, 043614 (2016).
D Lacroix, A. Boulet, M. Grasso, C. J. Yang, submitted to prc

- Scale invariance tells $\frac{E}{E_{\rm FG}} = \xi$ (Bertch parameter)

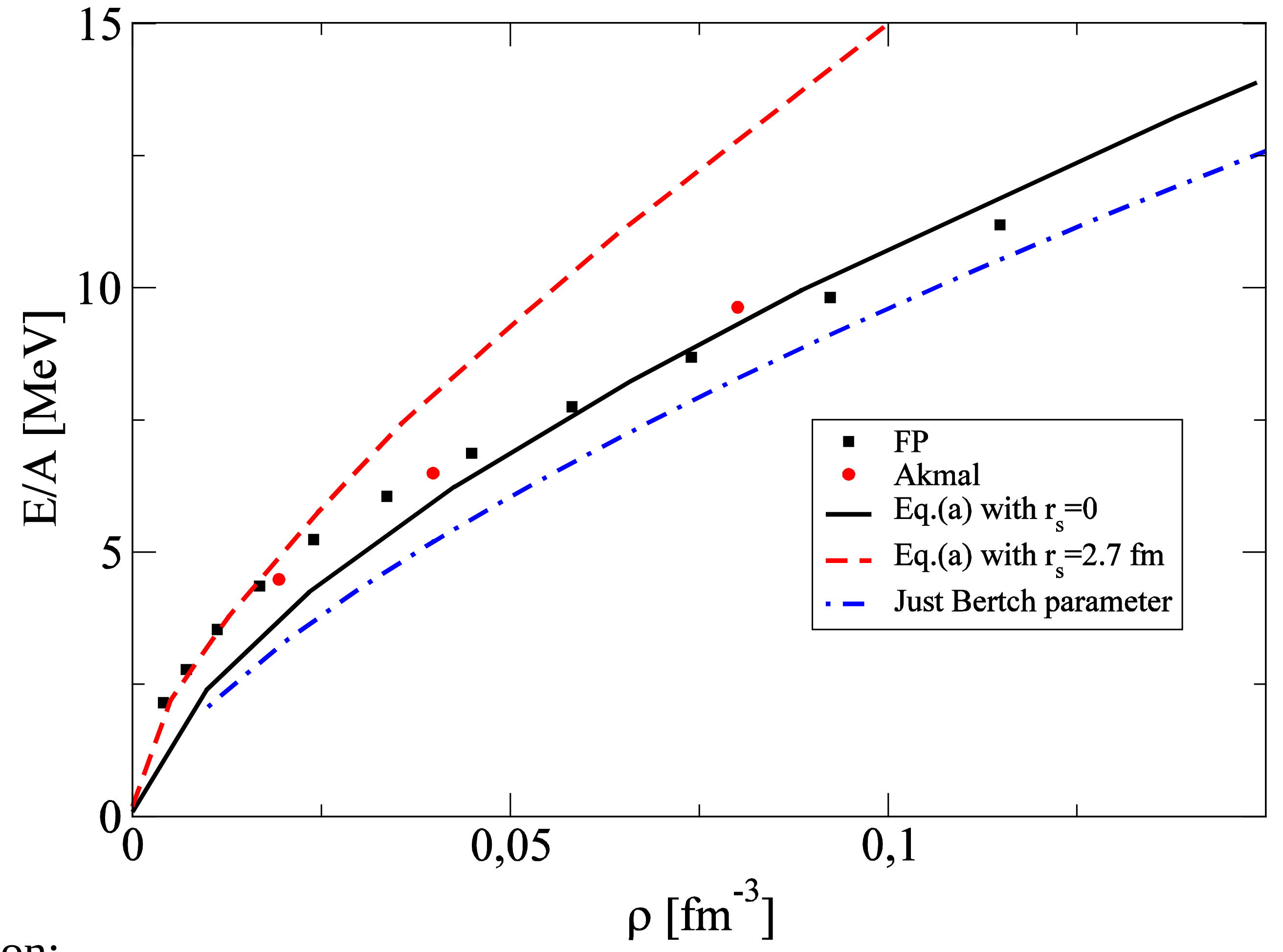
Nuclear system (*a=-18.9 fm*) is close to unitarity.

- $|a|$>>R(range of interaction) $\frac{1}{|a_s|} < k_F < \frac{1}{R}$ => $4*10^{-6} < \rho < 0.002[\text{fm}^{-3}]$
- Functional contains resum of $(a_s k_F)^{-1}$:

$$\frac{E}{E_{\rm FG}} = 1 - \frac{U_0}{1-(a_s k_F)^{-1}U_1} + \frac{R_0(r_e k_F)}{[1-R_1(ak_F)^{-1}]\,[1-R_1(a_s k_F)^{-1}+R_2(r_e k_F)]}$$

No free parameters: U_i, R_i come from QMC data (with $V_{unitarity}$)

Results



Lesson:

- Nuclear (many-body) systems are not too far from the unitarity limit.
- Just a few more parameters might be sufficient to describe data up to $\rho = 0.3\ fm^{-3}$, this explains why Skyrme works!

How to establish an EFT with a Skyrme-like interaction?

What will an EFT-based force look like?

- Leading order (LO): Need to make a guess.
- Based on renormalizability analysis

C.J. Yang, M. Grasso, U. van Kolck, and K. Moghrabi, coming soon!

⇒A good guess would be the t_0-t_3 model (or t_0 model, but it gives a very bad EOS).

Estimation of Breakdown scale

If require $O\left(\left(\frac{k_F}{M_{hi}}\right)^1\right) > O\left(\left(\frac{k_F}{M_{hi}}\right)^2\right)$

to be valid up to ρ=0.3 fm^{-3}.

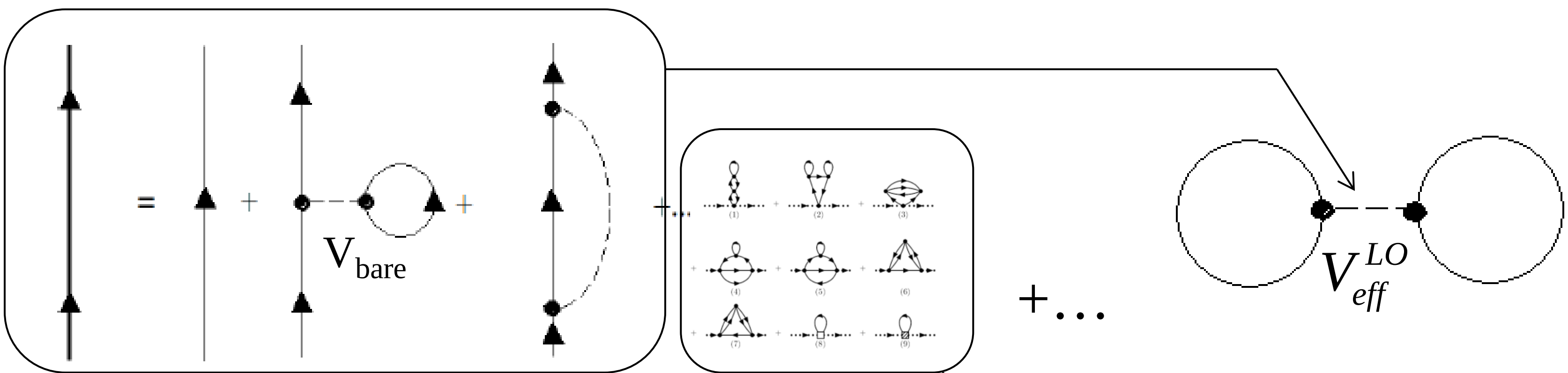
Then M_{hi} need to be at least 400 MeV.

Also, the low bound cannot do better than the unitarity limit.

Then, only applicable for $\rho > 4*10^{-6}$[fm^{-3}].

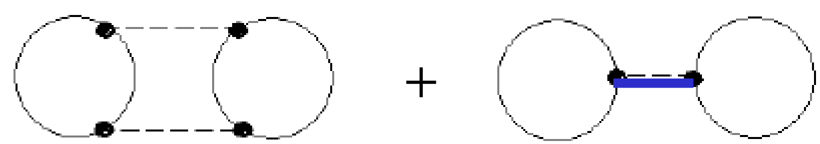
Diagrammatic explanation of How Skyrme works

Dressing of propagator→V_{eff}



Leading order (LO)

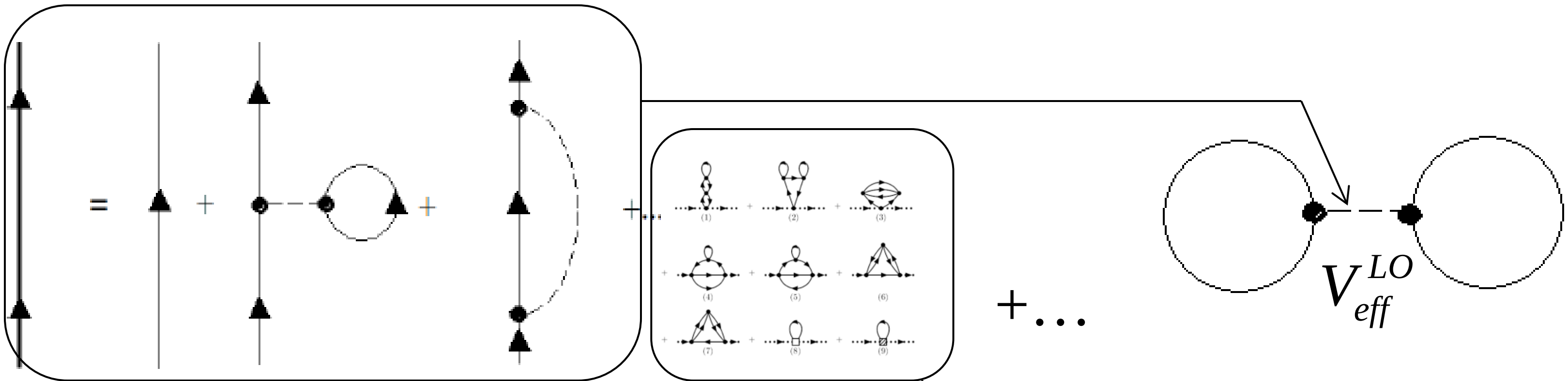
Then, NLO includes:



$$\underbrace{V_{eff}^{LO} G V_{eff}^{LO}}_{\textit{diverge}\text{ at least in }\Lambda k_F^3} + V_{eff}^{NLO}$$

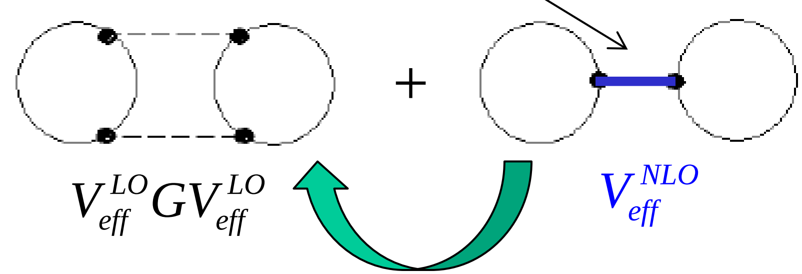
$V_{eff}^{Sly5} G V_{eff}^{Sly5}$ evaluated in: C.J. Yang, M. Grasso, X. Roca-Maza, G. Colo, and K. Moghrabi, PhysRevC.94.034311

Dressing of propagator→V_{eff}



Leading order (LO)

Then, NLO includes:



* V_{eff}^{NLO} contains (at least) contact terms to renormalize $V_{eff}^{LO} G V_{eff}^{LO}$.

Counter term part of the NLO potential

V_{eff}^{NLO} : For t_0-t_3 model, the divergence from $V_{eff}^{LO} G V_{eff}^{LO}$ is:

$$\underbrace{O(k_F^3), O(k_F^{3+3\alpha})}_{k_F^n - dep.\ \text{appears in}\ MF}, \underbrace{O(k_F^{3+6\alpha})}_{new\ k_F^n - dep.}.$$

If want to keep α free, =>Minimun contact term required: $Ck_F^{3+6\alpha}$.

Most general case: $Ak_F^3, Bk_F^{3+3\alpha}, Ck_F^{3+6\alpha}$.

In infinite matter, k_F^{3n} in-distinguishable with $3\pi^2\rho$

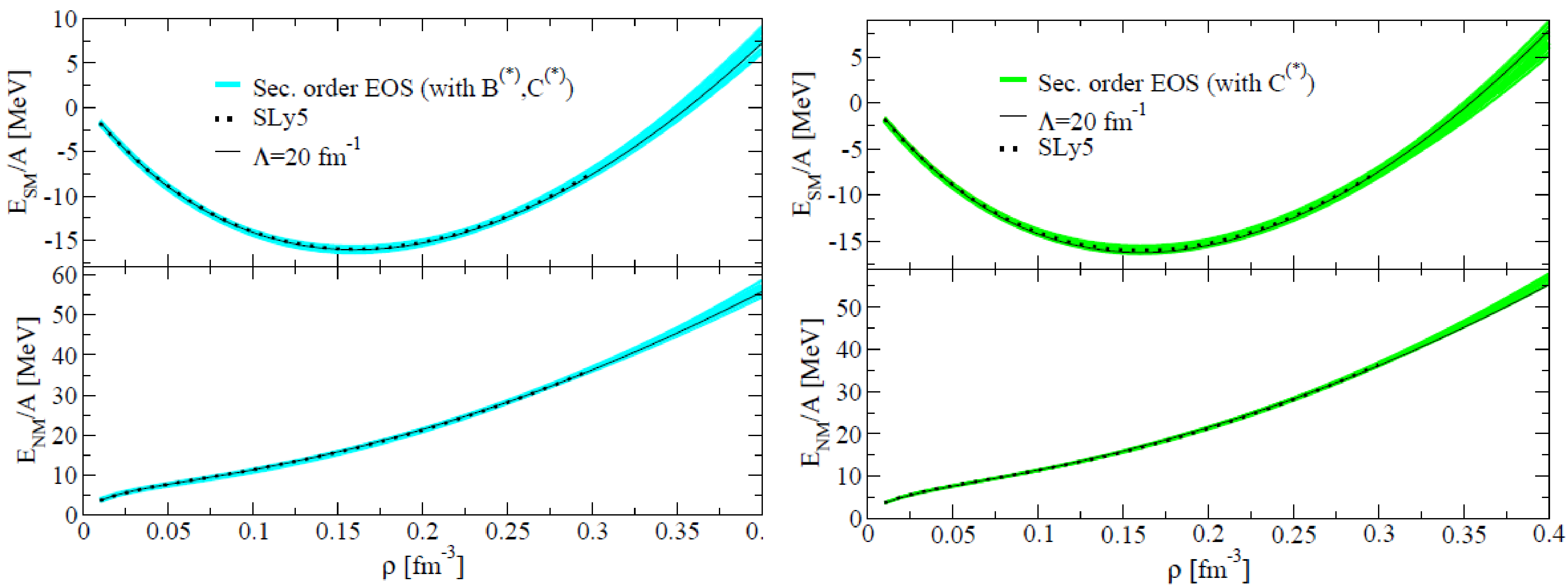
=> k_F^n-term in EOS *could* originated (at interaction level) from $(k-k')^{3n}\rho^v$,

where v is an extra parameter to be decided in the fitting to finite nuclei.

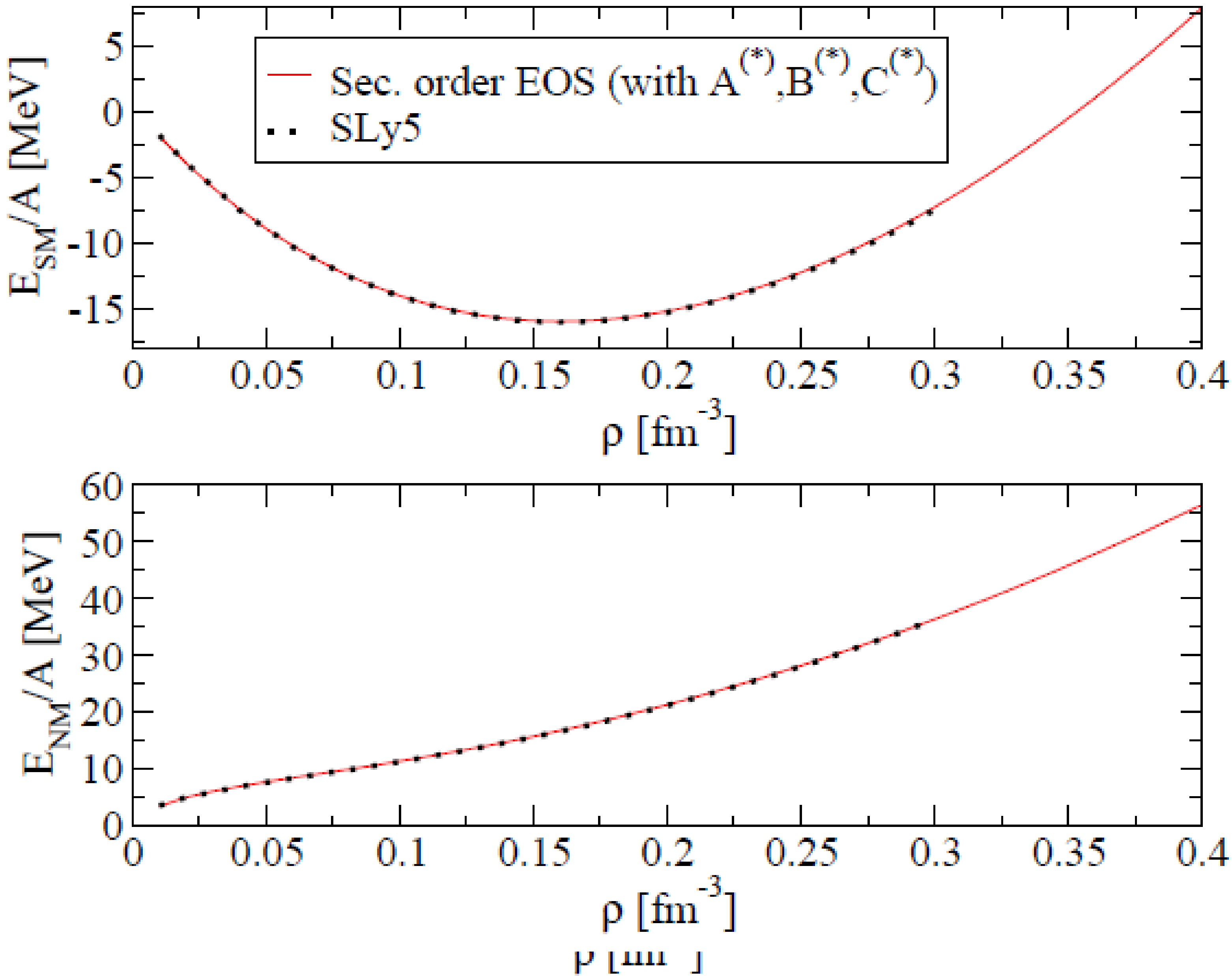
NLO results (based on t_0-t_3 as LO) $\alpha<1/6$ case*

C.J. Yang and M. Grasso, coming soon!

* For $\alpha>1/6$, V_{eff}^{NLO} also includes t_1,t_2 terms.

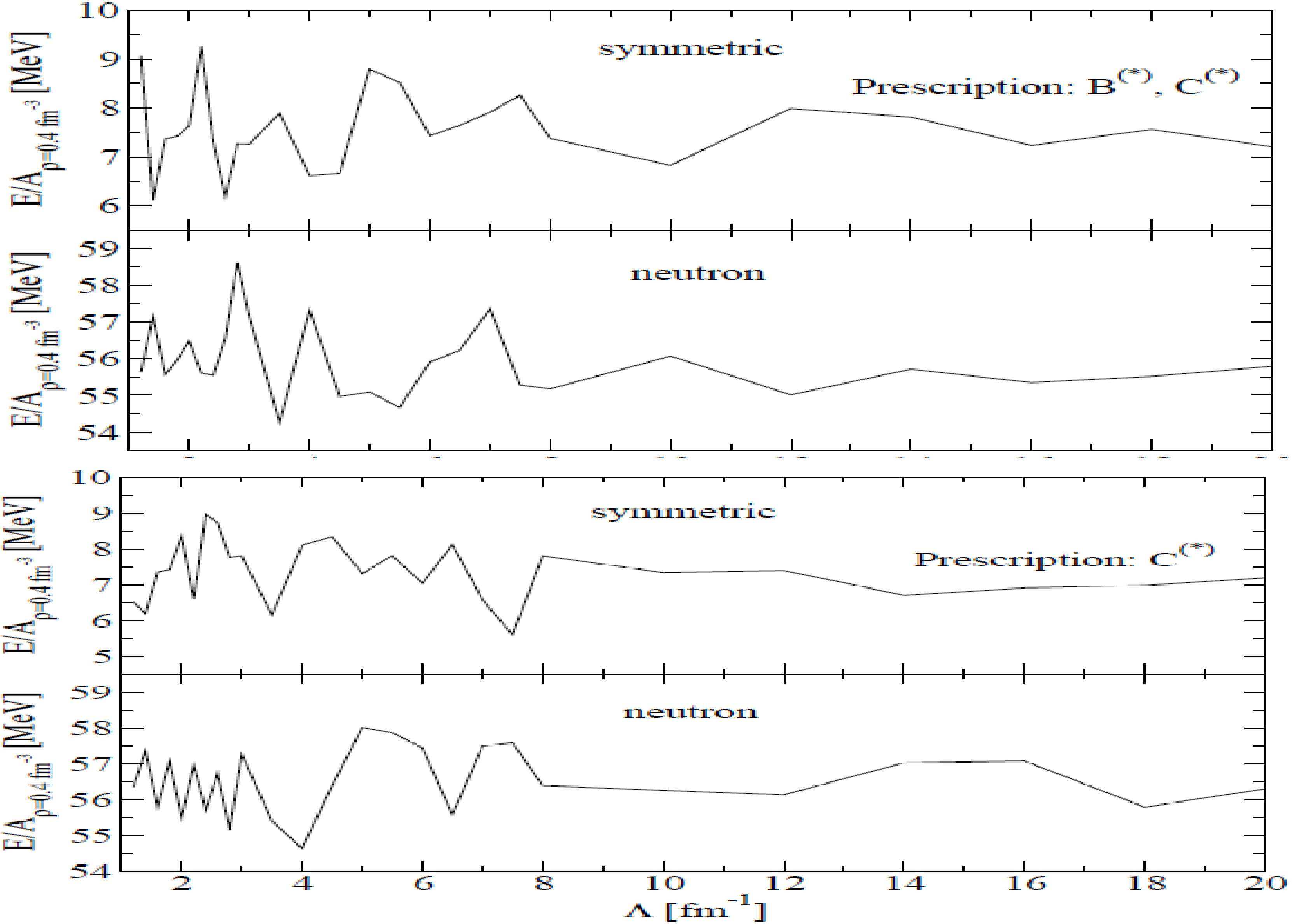


Sec. order EOS (with B(*),C(*))
SLy5
Λ=20 fm-1
E_SM/A [MeV]
E_NM/A [MeV]
ρ [fm-3]
Sec. order EOS (with C(*))
Λ=20 fm-1
SLy5
E_SM/A [MeV]
E_NM/A [MeV]
ρ [fm-3]



Sec. order EOS (with A^(*),B^(*),C^(*))
SLy5
E_SM/A [MeV]
ρ [fm^-3]
E_NM/A [MeV]
ρ [fm^-3]

Renormalization group (RG) check



How to apply to finite nuclei

- One simple version of beyond mean field interaction has been applied via PVC (with the phonon replaced by p-h pair).

  (M. Brenna, G. Colo, X. Roca-Maza, Phys. Rev. C 90, 044316 (2014))
- In principle, a general refitting is needed.

One either perform the fit directly in the chosen beyond mean field scheme, or use subtraction.

- To be fully consistent, **n parameters in the interaction means n subtractions** are needed.

Future prospects

Try to bridge EFT ideas/techniques to mean field (and beyond) within EDF framework.

Mean field with potential models (effective interaction).
(e.g., Skyrme-type)

2nd order corrections

Higher order corrections

Add new effective interactions?

What is the proper form of it?

Is the improvement systematic?

Renormalization-group analysis
+
power counting check

Goal: Systematic treatment of the interactions.

Thank you

2^{nd} order correction (symmetric & neutron matter)

$$\frac{E}{A} = \underbrace{\frac{E^{(0)}}{A}}_{\text{mean field}} + \underbrace{\frac{E^{(2)}}{A}}_{2^{nd}\ \text{order}} + \ldots$$

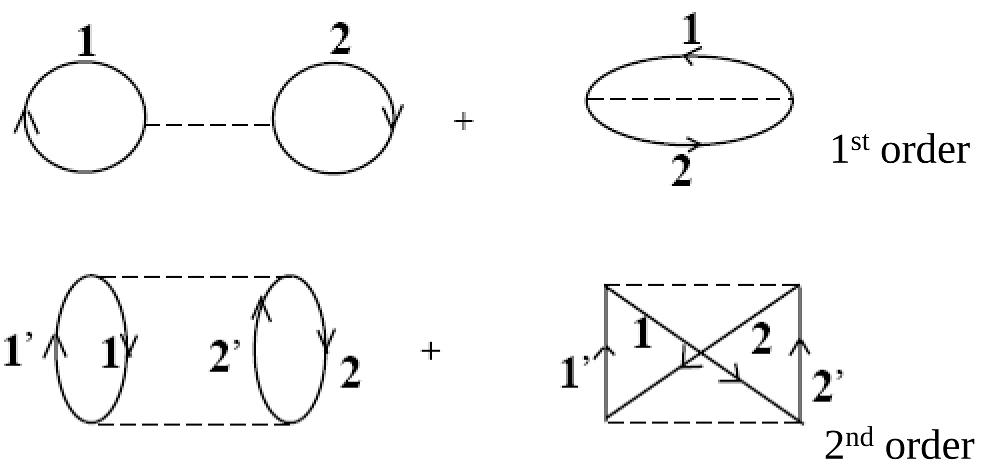
$$\frac{E^{(2)}_{sym}}{A} = \frac{-3m^*}{64\pi k_F^3 (2\pi)^6} \sum_{S,T} (2T+1)(2S+1) \int_{C_I} d^3\mathbf{k}_1 \int d^3\mathbf{k}_2 \int d^3\mathbf{q} [vGv]$$

$$G = \frac{1}{q^2 + \mathbf{q} \bullet (\mathbf{k_1} - \mathbf{k_2})}$$

Contour of integral (C_I):

$$|\mathbf{k}_{1,2}| \in [0, k_{F_{1,2}}]$$

$$|\mathbf{k_1} + \mathbf{q}| > k_{F_1}, |\mathbf{k_2} - \mathbf{q}| > k_{F_2}$$



Results for nuclear matter

In agreement with N.Kaiser, J. Phys. G 42,095111(2015)

$$\frac{\Delta E^{(2)}_{sym(l=0)}}{A} = -\frac{mk_F^4}{110880\hbar^2\pi^4}\left\{\begin{array}{l} \left[\begin{array}{c} -6534+1188ln[2]+3564\lambda-19602\lambda^3-5940\lambda^5 \\ +(1782-20790\lambda^4)ln[\frac{\lambda-1}{\lambda+1}] \\ +(24948\lambda^5-5940\lambda^7)ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right]\widetilde{T}^2_{03} \\ +\left[\begin{array}{c} -\quad 14696+2112ln[2]+5280\lambda-2860\lambda^3 \\ -48840\lambda^5-18480\lambda^7+(2640-55440\lambda^6)ln[\frac{\lambda-1}{\lambda+1}] \\ +(71280\lambda^7-18480\lambda^9)ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right]k_F^2\widetilde{T}_{03}\widetilde{T}_1 \\ +\left[\begin{array}{c} -9886+1128ln[2]+\ 2520\lambda+147\lambda^3-3654\lambda^5 \\ -35280\lambda^7-15120\lambda^9+(1260-41580\lambda^8)ln[\frac{\lambda-1}{\lambda+1}] \\ +(55440\lambda^9-15120\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right]k_F^4\widetilde{T}^2_1 \end{array}\right\}$$

Diverge as Λ^5

$$\frac{\Delta E^{(2)}_{sym(l=1)}}{A} = -\frac{mk_F^8}{73920\hbar^2\pi^4}\left\{\left[\begin{array}{c} -1033+156ln[2]+420\lambda+140\lambda^3-840\lambda^5 \\ -5880\lambda^7-2520\lambda^9+(-210+6930\lambda^8)ln[\frac{\lambda-1}{\lambda+1}] \\ +(9240\lambda^9-2520\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right]\widetilde{T}^2_2\right\},$$

$$\frac{\Delta E^{(2)}_{neutr(l=0)}}{A} = -\frac{mk_{F_N}^4}{166320\hbar^2\pi^4}\left\{\begin{array}{l} \left[\begin{array}{c} -6534+1188ln[2]+3564\lambda-19602\lambda^3-5940\lambda^5 \\ +(1782-20790\lambda^4)ln[\frac{\lambda-1}{\lambda+1}] \\ +(24948\lambda^5-5940\lambda^7)ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right]T^2_{03} \\ +\left[\begin{array}{c} -\quad 14696+2112ln[2]+5280\lambda-2860\lambda^3 \\ -48840\lambda^5-18480\lambda^7+(2640-55440\lambda^6)ln[\frac{\lambda-1}{\lambda+1}] \\ +(71280\lambda^7-18480\lambda^9)ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right]k_{F_N}^2 T_{03}T_1 \\ +\left[\begin{array}{c} -9886+1128ln[2]+\ 2520\lambda+147\lambda^3-3654\lambda^5 \\ -35280\lambda^7-15120\lambda^9+(1260-41580\lambda^8)ln[\frac{\lambda-1}{\lambda+1}] \\ +(55440\lambda^9-15120\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right]k_{F_N}^4 T^2_1 \end{array}\right\}$$

Diverge as Λ^5

$$\frac{\Delta E^{(2)}_{neutr(l=1)}}{A} = -\frac{mk_{F_N}^8}{110880\hbar^2\pi^4}\left\{\left[\begin{array}{c} -1033+156ln[2]+420\lambda+140\lambda^3-840\lambda^5 \\ -5880\lambda^7-2520\lambda^9+(-210+6930\lambda^8)ln[\frac{\lambda-1}{\lambda+1}] \\ +(9240\lambda^9-2520\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right]T^2_2\right\},$$

PART II:
RENORMALIZABILITY

- When $\Lambda \to \infty$, how the 2nd order terms behaves?

$$\frac{\Delta E_f^{(2)}(k_F)}{A} = \frac{3m}{2\pi^4\hbar^2} k_F^4 \left[A_0 + A_1 T_3 k_F^{3\alpha} + A_2 T_3^2 k_F^{6\alpha} + A_3 k_F^2 + A_4 T_3 k_F^{2+3\alpha} + A_5 k_F^4\right],$$

Converge terms

$$\frac{\Delta E_a^{(2)}(k_F,\lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 \left[B_0(\lambda) + B_1(\lambda) T_3 k_F^{3\alpha} + B_2(\lambda) k_F^2\right],$$

Diverge, k_F-dep appears in MF

$$\frac{\Delta E_d^{(2)}(k_F,\lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 \left[C_0 T_3^2 k_F^{6\alpha} + C_1 T_3 k_F^{2+3\alpha} + C_2 k_F^4\right],$$

Diverge, k_F-dep *not* in MF

- Idea: Absorb the Λ-divergence in 2nd order into mean field terms with the same k_F-dependence.

$$\frac{\Delta E_f^{(2)}(k_F)}{A} = \frac{3m}{2\pi^4\hbar^2} k_F^4 \left[A_0 + A_1 T_3 k_F^{3\alpha} + A_2 T_3^2 k_F^{6\alpha} + A_3 k_F^2 + A_4 T_3 k_F^{2+3\alpha} + A_5 k_F^4\right],$$

converge

$$\frac{\Delta E_a^{(2)}(k_F,\lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 \left[B_0(\lambda) + B_1(\lambda) T_3 k_F^{3\alpha} + B_2(\lambda) k_F^2\right],$$

Diverge, k_F-dep appears in MF

$$\frac{\Delta E_d^{(2)}(k_F,\lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 \left[C_0 T_3^2 k_F^{6\alpha} + C_1 T_3 k_F^{2+3\alpha} + C_2 k_F^4\right],$$

Diverge, k_F-dep *not* in MF

eliminate by setting α=1/3 and $t_1=t_2=0$, or seting $t_1=t_2=t_3=0$.

- Idea: Absorb the Λ-divergence in 2^{nd} order into mean field terms with the same k_F-dependence.

$$\frac{\Delta E_f^{(2)}(k_F)}{A} = \frac{3m}{2\pi^4\hbar^2}k_F^4\left[A_0 + A_1 T_3 k_F^{3\alpha} + A_2 T_3^2 k_F^{6\alpha} + A_3 k_F^2 + A_4 T_3 k_F^{2+3\alpha} + A_5 k_F^4\right],$$

converge

$$\frac{\Delta E_a^{(2)}(k_F,\lambda)}{A} = -\frac{m}{8\pi^4\hbar^2}\lambda k_F^3\left[B_0(\lambda) + B_1(\lambda) T_3 k_F^{3\alpha} + B_2(\lambda) k_F^2\right],$$

Diverge, k_F-dep appears in MF

↓

Treatment 1: Absorb divergence into redefinition of parameters.

Treatment 2: Add counter terms correspond to each divergence.